

# MA3505 Multivariate Statistics Project 1

April 28, 2016

**1 Introduction and exploratory data analysis for the variables.**

**2 Analysis to answer each research question**

**2.1 Question 1**

## 2.2 Question 2

```
model2 = lm(cbind(chol, thaldur, thaltime, met, thalach, thalrest, tpeakbps,
                  tpeakbpd, trestbpd, oldpeak, rldv5, rldv5e) ~ proto + restecg + dig
            + prop + nitr + pro + diuretic, data=datall)
```

Using the above code I created the necessary multivariate regression model. I was able to use this model to get the following table of coefficients:

	chol	thaldur	thaltime	met	thalach
(Intercept)	2.182e+02	2.964e+00	1.941e+00	4.167e+00	1.216e+02
proto	5.933e-01	7.621e-02	7.395e-02	1.284e-02	1.241e-01
restecg	-1.965e+01	1.506e-01	4.325e-01	1.305e-01	-2.202e-01
dig	6.033e+00	3.130e+00	2.889e+00	1.519e+00	-8.249e+00
prop	1.800e+01	3.734e-01	6.787e-01	4.875e-02	-6.759e+00
nitr	-1.390e+01	-3.582e-01	-3.903e-01	3.416e-02	-5.949e+00
pro	-6.872e+01	1.142e+00	9.582e-01	4.637e-01	1.990e-02
diuretic	-4.914e+01	1.516e+00	6.732e-01	3.744e-01	1.610e+01
	thalrest	tpeakbps	tpeakbpd	trestbpd	oldpeak
(Intercept)	7.475e+01	1.607e+02	9.326e+01	8.488e+01	1.937e+00
proto	4.602e-02	2.114e-01	3.306e-02	1.262e-02	-3.094e-03
restecg	1.481e+00	3.762e+00	-1.245e+00	1.159e+00	-2.081e-01
dig	2.175e+00	-7.984e+00	-1.854e+01	-4.949e+00	4.202e-01
prop	-2.692e-01	5.788e-02	-1.988e+00	5.123e-01	-1.674e-02
nitr	-8.676e+00	-9.099e+00	-3.690e+00	-3.270e+00	2.621e-01
pro	2.958e+00	4.851e+00	7.011e+00	2.962e-01	-8.122e-01
diuretic	-8.346e-01	6.602e+00	2.153e+00	1.116e+00	-2.439e-02
	rldv5	rldv5e			
(Intercept)	1.487e+01	1.497e+01			
proto	-6.571e-04	-6.529e-03			
restecg	1.703e-01	2.203e-01			
dig	-2.153e+00	-2.219e+00			
prop	1.272e+00	1.175e+00			
nitr	6.343e-01	-6.043e-01			
pro	-1.583e+00	7.800e-01			
diuretic	-1.303e-01	3.239e+00			

However this is not very useful, so I used the **summary()** function to enable me to achieve a more detailed view of my analysis. Below I have tried my best to explain the detailed view for each response variable.

Response chol :

Call:

lm(formula = chol ~ proto + restecg + dig + prop + nitr + pro +  
diuretic , data = datall)

Residuals:

Min	1Q	Median	3Q	Max
-221.153	-37.934	-0.852	55.190	310.650

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	218.1866	16.6717	13.087	< 2e-16 ***
proto	0.5933	0.1884	3.149	0.00207 **
restecg	-19.6467	15.5463	-1.264	0.20877
dig	6.0327	42.3211	0.143	0.88689
prop	17.9968	25.2562	0.713	0.47750
nitr	-13.8953	24.8506	-0.559	0.57710
pro	-68.7201	27.9126	-2.462	0.01524 *
diuretic	-49.1356	33.0596	-1.486	0.13983

Signif. codes:	0	***	0.001	**	0.01	*	0.05	.	0.1	1
----------------	---	-----	-------	----	------	---	------	---	-----	---

Residual standard error: 85.12 on 120 degrees of freedom  
(771 observations deleted due to missingness)

Multiple R-squared: 0.2402, Adjusted R-squared: 0.1958

F-statistic: 5.418 on 7 and 120 DF, p-value: 2.026e-05

From the table above we can see that the predictor that had the most affect in the value of the **chol** response was **proto**. As *chol* refers to the amount of cholesterol in a person's system and *proto* refers to the type of exercise that they do, it is not a major surprise that this is the most important as in theory the higher the intensity of the your exercise program the lower your cholesterol will be. The second most important variable is **pro**; this is an indicator variable that tells us if someone uses *calcium channel blocker used during exercise* (it is used in cholesteryl ester hydrolysis which helps reduce cholesterol) during their exercise routine.

```
Response thaldur :
Call:
lm(formula = thaldur ~ proto + restecg + dig + prop + nitr +
    pro + diuretic, data = datall)
```

Residuals:

Min	1Q	Median	3Q	Max
-4.312	-1.681	-0.310	1.422	6.440

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )	
(Intercept)	2.964440	0.443361	6.686	7.65e-10	***
proto	0.076214	0.005011	15.209	< 2e-16	***
restecg	0.150581	0.413433	0.364	0.7163	
dig	3.129630	1.125473	2.781	0.0063	**
prop	0.373380	0.671654	0.556	0.5793	
nitr	-0.358181	0.660868	-0.542	0.5888	
pro	1.141536	0.742300	1.538	0.1267	
diuretic	1.516199	0.879177	1.725	0.0872	.

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 '1'

Residual standard error: 2.264 on 120 degrees of freedom

(771 observations deleted due to missingness)

Multiple R-squared: 0.6909, Adjusted R-squared: 0.6728

F-statistic: 38.31 on 7 and 120 DF, p-value: < 2.2e-16

The predictor variable in this instance is **thaldur** which represents the length of time a person spends on an exercise test, it is therefore no surprise that **proto** is the most important predictor as the harder the exercise test the less time you will be able to do it for. The second most significant predictor **dig** refers to whether or not the person is taking a drug called *digitails* during exercise. Studies have shown that the use of this drug during exercise increases blood flow which could allow someone to exercise for longer (*experts are not sure if it is a performance enhancing drug as trial results vary*).

Response thaltime :

Call:

```
lm(formula = thaltime ~ proto + restecg + dig + prop + nitr +  
    pro + diuretic, data = datall)
```

Residuals:

Min	1Q	Median	3Q	Max
-4.469	-1.639	-0.139	1.053	7.352

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )	
(Intercept)	1.941466	0.449229	4.322	3.21e-05	***
proto	0.073951	0.005077	14.565	< 2e-16	***
restecg	0.432490	0.418906	1.032	0.3039	
dig	2.888715	1.140370	2.533	0.0126	*
prop	0.678710	0.680544	0.997	0.3206	
nitr	-0.390289	0.669615	-0.583	0.5611	
pro	0.958162	0.752125	1.274	0.2051	
diuretic	0.673187	0.890814	0.756	0.4513	

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 2.294 on 120 degrees of freedom  
(771 observations deleted due to missingness)

Multiple R-squared: 0.6704, Adjusted R-squared: 0.6511

F-statistic: 34.86 on 7 and 120 DF, p-value: < 2.2e-16

**thaltime** refers to the time at which a person's ST depression was measured. It is therefore no surprise that **proto** has the highest effect as different exercises will take different amount of times to complete meaning that if *thaltime* is always measured at the end of the exercise test people who do different tests will have different times but those who take the same test should have very similar times. **dig** is the next significant variable which sort of makes sense as you most likely have to wait for the drug to leave your system before your ST depression can be measured.

```

Response met :
Call:
lm(formula = met ~ proto + restecg + dig + prop + nitr + pro +
    diuretic, data = datall)

Residuals:
    Min       1Q   Median       3Q      Max
-3.7325 -1.0919 -0.1298  0.8792  5.8206

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  4.166578   0.336945  12.366  <2e-16 ***
proto        0.012843   0.003808   3.372   0.0010 **
restecg      0.130481   0.314201   0.415   0.6787
dig          1.518904   0.855338   1.776   0.0783 .
prop         0.048745   0.510444   0.095   0.9241
nitr         0.034160   0.502247   0.068   0.9459
pro          0.463725   0.564133   0.822   0.4127
diuretic     0.374352   0.668158   0.560   0.5763
---
Signif. codes:  0      ***      0.001      **      0.01      *      0.05      .      0.1      1

Residual standard error: 1.72 on 120 degrees of freedom
(771 observations deleted due to missingness)
Multiple R-squared:  0.1108,    Adjusted R-squared:  0.05892
F-statistic: 2.136 on 7 and 120 DF,  p-value: 0.04484

```

The predictor **met** refers to the *metabolic equivalent of resting oxygen consumption while sitting* and therefore it is not much of a surprise that the response **proto** is the most significant. It is also not that surprising that it is as significant as before, as the trial that produced these results most likely used people of varying athletic abilities for each test in order to make the results more accurate.

Response thalach :

Call:

```
lm(formula = thalach ~ proto + restecg + dig + prop + nitr +  
    pro + diuretic, data = datall)
```

Residuals:

Min	1Q	Median	3Q	Max
-42.497	-10.060	-0.925	13.735	53.075

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	121.6141	3.6895	32.962	< 2e-16 ***
proto	0.1241	0.0417	2.977	0.00352 **
restecg	-0.2202	3.4405	-0.064	0.94908
dig	-8.2489	9.3659	-0.881	0.38022
prop	-6.7587	5.5893	-1.209	0.22896
nitr	-5.9491	5.4996	-1.082	0.28154
pro	0.0199	6.1772	0.003	0.99743
diuretic	16.0994	7.3163	2.200	0.02969 *

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 18.84 on 120 degrees of freedom  
(771 observations deleted due to missingness)

Multiple R-squared: 0.1867, Adjusted R-squared: 0.1392

F-statistic: 3.934 on 7 and 120 DF, p-value: 0.0006723

The predictor **thalach** refers to the maximum heart rate that a person achieves during their exercise test and as such it is no surprise that the response variable that is the most significant when calculating it is **proto**. This is because the more intense the exercise test is the more oxygen your body is going to need thus you will have a higher heart rate. Again it is not surprising that *proto* is only a 2\* rather than a 3\* significance level as your maximum heart rate will depend on how athletic you are, the more athletic the lower your max heart rate will be. **diuretic** is the other significant response variable and it refers to whether or not the subject uses diuretic used during exercise. Diuretic is considered to be a performance enhancing drug so it is therefore no surprise that it only has a 1\* significance level due to the fact that the analysis up to now has shown that there is a high probability that athletes are involved in this trial and would be band by WADA if they were caught using it.

Response thalrest :

Call:

```
lm(formula = thalrest ~ proto + restecg + dig + prop + nitr +  
    pro + diuretic, data = datall)
```

Residuals:

Min	1Q	Median	3Q	Max
-28.204	-8.542	-1.909	8.172	55.796

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	74.75201	2.60040	28.746	<2e-16 ***
proto	0.04602	0.02939	1.566	0.120
restecg	1.48140	2.42487	0.611	0.542
dig	2.17533	6.60112	0.330	0.742
prop	-0.26923	3.93939	-0.068	0.946
nitr	-8.67576	3.87612	-2.238	0.027 *
pro	2.95844	4.35374	0.680	0.498
diuretic	-0.83465	5.15655	-0.162	0.872

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 13.28 on 120 degrees of freedom  
(771 observations deleted due to missingness)

Multiple R-squared: 0.09876, Adjusted R-squared: 0.04619

F-statistic: 1.879 on 7 and 120 DF, p-value: 0.07885

The **thalrest** variable refers to the subjects resting heart rate and the only variable that has any significant effect on the outcome of this result is **nitr** which tells us whether or not the subject uses nitrates used during their exercise. I am not quite sure what the use of nitrates has to do with the resting heart rates but I do know that they are added to ‘unhealthy foods’ such as *bacon, sandwich meats and salami* which could indicate that they are not very athletic but a high resting heart does not mean that someone is less athletic.



In this trial the subjects the measuring of their peak blood pressure was split into two different variables: **tpeakbps** and **tpeakbpd**, google wasn't able to explain why this is the case.

```

Response tpeakbps :

Call:
lm(formula = tpeakbps ~ proto + restecg + dig + prop + nitr +
    pro + diuretic, data = datall)

Residuals:
    Min       1Q   Median       3Q      Max
-46.56  -15.18   -2.97   13.36   58.73

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept) 160.70295    4.33227   37.094 < 2e-16 ***
proto         0.21138    0.04897    4.317 3.27e-05 ***
restecg       3.76224    4.03984    0.931  0.354
dig          -7.98425   10.99749   -0.726  0.469
prop          0.05788    6.56303    0.009  0.993
nitr         -9.09857    6.45763   -1.409  0.161
pro           4.85054    7.25334    0.669  0.505
diuretic      6.60213    8.59083    0.769  0.444
---
Signif. codes:  0      ***      0.001      **      0.01      *      0.05      .      0.1      1

Residual standard error: 22.12 on 120 degrees of freedom
(771 observations deleted due to missingness)
Multiple R-squared:  0.1992,    Adjusted R-squared:  0.1525
F-statistic: 4.266 on 7 and 120 DF,  p-value: 0.0003059

```

For the variable that had the most significant affect on **tpeakbps** was (as normal it seems in this trial) **proto**. This is most likely because of the fact that exercise can lower your blood pressure and therefore the subjects that are able to take the more intensive exercise tests were likely to have a lower peak blood pressure.

Response tpeakbpd :

Call :

```
lm(formula = tpeakbpd ~ proto + restecg + dig + prop + nitr +  
    pro + diuretic , data = datall)
```

Residuals:

Min	1Q	Median	3Q	Max
-60.687	-7.517	-0.023	8.638	36.329

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	93.26322	2.68207	34.773	< 2e-16 ***
proto	0.03306	0.03031	1.091	0.27768
restecg	-1.24519	2.50103	-0.498	0.61948
dig	-18.54131	6.80844	-2.723	0.00743 **
prop	-1.98759	4.06311	-0.489	0.62561
nitr	-3.69032	3.99786	-0.923	0.35782
pro	7.01102	4.49047	1.561	0.12108
diuretic	2.15344	5.31850	0.405	0.68627

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 '1'

Residual standard error: 13.69 on 120 degrees of freedom

(771 observations deleted due to missingness)

Multiple R-squared: 0.1321, Adjusted R-squared: 0.0815

F-statistic: 2.61 on 7 and 120 DF, p-value: 0.01525

The response variable that was most significant when working out the predictor **tpeakbpd** was **dig**. This makes sense as studies have shown that the use of the drug digitalis during exercise lowers a person's blood pressure.

```

Response trestbpd :
Call:
lm(formula = trestbpd ~ proto + restecg + dig + prop + nitr +
    pro + diuretic, data = datall)

Residuals:
    Min       1Q   Median       3Q      Max
-35.510  -6.141  -1.298   5.543  24.175

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  84.87854    1.88379   45.057  <2e-16 ***
proto         0.01262    0.02129    0.593   0.554
restecg       1.15852    1.75663    0.660   0.511
dig          -4.94866    4.78200   -1.035   0.303
prop          0.51233    2.85378    0.180   0.858
nitr         -3.27042    2.80795   -1.165   0.246
pro           0.29623    3.15394    0.094   0.925
diuretic      1.11644    3.73552    0.299   0.766
---
Signif. codes:  0   ***    0.001   **    0.01   *    0.05   .    0.1    1

Residual standard error: 9.618 on 120 degrees of freedom
(771 observations deleted due to missingness)
Multiple R-squared:  0.0366,    Adjusted R-squared:  -0.01959
F-statistic: 0.6514 on 7 and 120 DF,  p-value: 0.7126

```

The predictor variable **trestbpd** refers to the subjects resting blood pressure. As this must be taken before any exercise is started it makes sense that none of the responses are significant in determining what this value shall be due to them being mainly related to the exercise test the subject takes.

Response oldpeak :

Call:

```
lm(formula = oldpeak ~ proto + restecg + dig + prop + nitr +  
    pro + diuretic, data = datall)
```

Residuals:

Min	1Q	Median	3Q	Max
-1.9343	-0.6280	-0.0506	0.3642	3.5801

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )	
(Intercept)	1.937386	0.169250	11.447	< 2e-16	***
proto	-0.003094	0.001913	-1.617	0.10841	
restecg	-0.208133	0.157825	-1.319	0.18976	
dig	0.420187	0.429642	0.978	0.33004	
prop	-0.016737	0.256399	-0.065	0.94806	
nitr	0.262057	0.252282	1.039	0.30101	
pro	-0.812187	0.283368	-2.866	0.00491	**
diuretic	-0.024390	0.335620	-0.073	0.94219	

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.8642 on 120 degrees of freedom  
(771 observations deleted due to missingness)

Multiple R-squared: 0.1081, Adjusted R-squared: 0.05603

F-statistic: 2.077 on 7 and 120 DF, p-value: 0.0511

The predictor variable **oldpeak** refers to *ST depression induced by exercise relative to rest* (which I understand from google to be a fancy way of saying that the subject gets a small heart attack during exercise). It makes sense then that the most significant variable in deciding what the value of which if it is high can cause heart attacks. *oldpeak* is going to be is **pro** as helps to lower cholesterol

The next two predictors, **rldv5** and **rldv5e**, refer to *height at rest* and *height at peak exercise*. I don't know what *height* they are referring to (I am assuming it is not just how tall they are as that would be dull to measure at rest and during peak exercise as it would not change) and luckily none of the response variables are significant in working out what the values of the variables will be.

Response rldv5 :

Call:

```
lm(formula = rldv5 ~ proto + restecg + dig + prop + nitr + pro +
    diuretic, data = datall)
```

Residuals:

Min	1Q	Median	3Q	Max
-10.7927	-3.3161	-0.7927	3.0914	16.1580

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	14.8748145	1.0543301	14.108	<2e-16 ***
proto	-0.0006571	0.0119165	-0.055	0.956
restecg	0.1703404	0.9831619	0.173	0.863
dig	-2.1530704	2.6764216	-0.804	0.423
prop	1.2718279	1.5972216	0.796	0.427
nitr	0.6342939	1.5715709	0.404	0.687
pro	-1.5831511	1.7652196	-0.897	0.372
diuretic	-0.1302669	2.0907202	-0.062	0.950

Signif. codes: 0 \*\*\* 0.001 \*\* 0.01 \* 0.05 . 0.1 1

Residual standard error: 5.383 on 120 degrees of freedom

(771 observations deleted due to missingness)

Multiple R-squared: 0.016, Adjusted R-squared: -0.0414

F-statistic: 0.2787 on 7 and 120 DF, p-value: 0.9612

Response rldv5e :

Call:

```
lm(formula = rldv5e ~ proto + restecg + dig + prop + nitr + pro +
    diuretic, data = datall)
```

Residuals:

Min	1Q	Median	3Q	Max
-11.1533	-3.5409	-0.4798	2.7664	14.0371

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	14.969428	1.048021	14.284	<2e-16 ***
proto	-0.006529	0.011845	-0.551	0.583
restecg	0.220336	0.977279	0.225	0.822
dig	-2.219224	2.660406	-0.834	0.406
prop	1.174742	1.587664	0.740	0.461
nitr	-0.604272	1.562167	-0.387	0.700
pro	0.779970	1.754657	0.445	0.657
diuretic	3.238914	2.078210	1.559	0.122

Signif. codes: 0 \*\*\* 0.001 \*\* 0.01 \* 0.05 . 0.1 1

Residual standard error: 5.351 on 120 degrees of freedom

(771 observations deleted due to missingness)

Multiple R-squared: 0.03959, Adjusted R-squared: -0.01643

F-statistic: 0.7067 on 7 and 120 DF, p-value: 0.6663

## 2.3 Question 3

Due to that each dataset is missing different variables from the data, we have decided that in order to maximise the amount of variables we have, we are going to be using each dataset independent of the others.

For each dataset we removed the dummy variables and variables that were missing at least a percentage of data. This percent was different for each data set and we were aiming for approximate at least double the number of observations to the number of variables.

### 2.3.1 Cleveland

After removing dummy variables and variables with at least 90% NA data, we are left with 45 variables and 201 observations.

Cleveland variance inflation factor

age	sex	cp	trestbps	htn	chol	cigs	years
2.070591	2.379469	1.683710	2.935706	1.734144	1.326342	2.346224	2.315459
fbs	famhist	restecg	ekgmo	ekgday	ekgyr	dig	prop
1.281244	1.291443	1.338021	14.903816	3.357399	78.992867	1.296383	1.679766
nitr	pro	diuretic	thaldur	thaltme	met	thalach	thalrest
1.546570	1.415979	1.480903	9.549788	1.422540	10.328475	2.868773	1.713892
tpeakbps	tpeakbpd	trestbpd	exang	xhypo	oldpeak	slope	rldv5e
2.829387	2.173463	2.785971	1.734917	1.870852	2.831028	2.291928	1.557587
ca	thal	cmo	cday	cyr	lmt	ladprox	laddist
1.841289	2.051953	15.389866	3.413846	80.511913	1.401270	1.496650	1.526869
cxmain	oml	rcaprox	rcadist				
1.543251	1.789705	1.764053	1.835745				

From the variance inflation factor we see the variables **ekgmo**, **ekgyr**, **cmo** and **cyr** are highly collinear with other variables in the model.

Scree plot – Cleveland

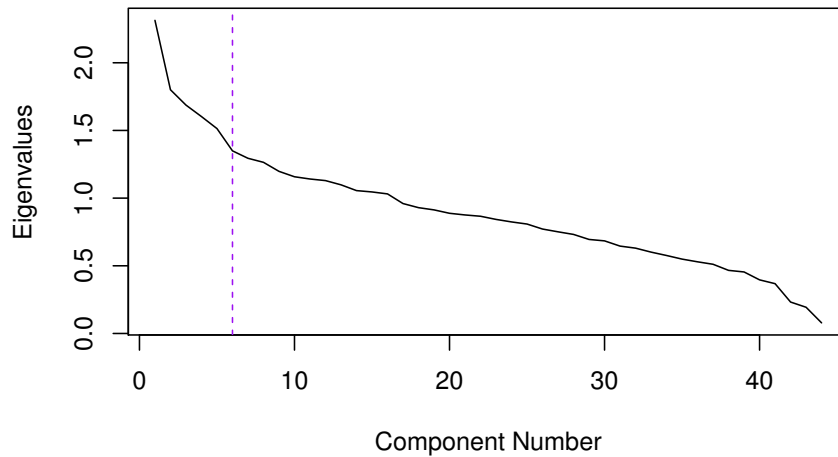


Figure 1: Scree plot for PCA of Cleveland

From the scree plot in Figure 1 we see that we keep 6 components.

We have the loadings of each components as follows.

Cleveland PCA loadings

Loadings :

Comp.1	Comp.2	Comp.3	Comp.4	Comp.5	Comp.6	Comp.7	Comp.8	Comp.9	Comp.10
--------	--------	--------	--------	--------	--------	--------	--------	--------	---------

age	0.192		-0.196			0.167		-0.122	0.163	0.372
sex		-0.195	0.306	0.193					-0.303	
cp	0.208							0.384		-0.116
trestbps	0.133	-0.144	-0.297	0.222	0.107	0.119		-0.149		
htn			0.222		-0.189	0.390		0.117		
chol			-0.184					0.184	0.213	0.222
cigs		-0.200	0.181	0.231	-0.292		-0.128		-0.214	
years		-0.189	0.145	0.223	-0.330	0.138			-0.156	
fbs			-0.128	0.143		-0.214	0.132			0.129
famhist					0.123	0.140		0.133	0.162	-0.136
restecg		-0.103	-0.132				-0.128	0.238		
ekgmo		-0.244		-0.433	-0.109		0.220	-0.161	-0.144	
ekgday					0.384	0.326	0.255	0.298	-0.109	
ekgyr		0.414		0.193			0.268		-0.129	-0.212
dig		0.105				0.195	-0.112	-0.150	-0.230	
prop	0.102	0.107			0.162	-0.263	-0.247	0.105		0.173
nitr	0.142	0.107		-0.128			-0.141		-0.180	0.131
pro		0.236				-0.115		0.154	-0.222	
diuretic					0.128	-0.417			-0.136	0.199
thaldur	-0.301	-0.109	0.237	0.125	0.184	-0.149				
thaltme			0.154		0.153			-0.191	0.359	-0.189
met	-0.295	-0.137	0.228	0.135	0.181	-0.167				
thalach	-0.298	-0.172					0.130	0.101	0.106	-0.141
thalrest			-0.229		-0.254		0.221	0.145	0.194	-0.113
tpeakbps		-0.211	-0.236	0.297				-0.211		
tpeakbpd		-0.167	-0.330	0.142			-0.128	0.161	-0.148	-0.159
trestbpd		-0.222	-0.314	0.112	0.130		-0.106		-0.167	-0.194
exang	0.224			-0.100					-0.207	-0.157
xhypo	0.104	0.153		-0.229		-0.102			-0.113	
oldpeak	0.280				0.185			-0.219	0.150	-0.159
slope	0.232				0.230		-0.120	-0.217		-0.263
rldv5e				0.126	0.127	0.166		-0.295		
ca	0.213			0.113	-0.124		0.283		0.211	0.251
thal	0.231	-0.163	0.167	0.102					-0.143	
cmo		-0.243		-0.433	-0.116		0.209	-0.162	-0.122	
cday					0.391	0.294	0.280	0.243	-0.135	
cyr		0.415		0.195			0.261		-0.130	-0.218
lmt	0.130	-0.106					0.132			-0.126
ladprox	0.183		0.147				-0.234	0.150	0.132	
laddist	0.206		0.107			-0.114	0.254			
cxmain	0.189		0.150	0.104					0.111	0.201
oml	0.249					-0.108	0.202			
rcaprox	0.191					-0.237	0.162	0.151	0.181	-0.291
rcadist	0.196		0.103				0.183	-0.130		0.250

We see that the first principle component is mostly formed of **thaldur**, **thalach**, **met** and **oldpeak** variables.

The second principle component is mostly formed of **cyr** and **ekgyr** variables.

The third principle component is mostly formed of **tpeakbpd**, **trestbpd**, **sex** and **trestbps** variables.

The fourth principle component is mostly formed of **ekgmo** and **cmo** variables.

The fifth principle component is mostly formed of **cday**, **ekgday**, **years** and **cigs** variables.

The sixth principle component is mostly formed of **diuretic**, **htn**, **ekgday**

### 2.3.2 Hungary

After removing dummy variables and variables with at least 79% NA data, we are left with 36 variables and 88 observations.

Hungary variance inflation factor

age	sex	painloc	painexer	relrest	cp	trestbps
2.434590	2.080449	2.654746	10.046548	6.565678	19.690793	4.275217
htn	chol	fbs	restecg	ekgmo	ekgday	ekgyr
1.861599	2.510843	2.038199	1.494730	26.201141	3.202566	179.069024
prop	nitr	pro	diuretic	proto	thaldur	thaltime
4.687494	8.190981	9.216954	3.508473	40.100317	160.858151	159.083733
met	thalach	thalrest	tpeakbps	tpeakbpd	trestbpd	exang
8.256667	3.592979	1.974234	3.419070	3.548859	3.556353	3.194635
oldpeak	slope	rldv5	rldv5e	cmo	cday	cyr
2.059364	2.928694	9.159900	8.276601	26.609872	2.619470	173.894964

From the variance inflation factor we see that the variables **painexer**, **cp**, **ekgmo**, **ekgyr**, **proto**, **thaldur**, **thaltime**, **cmo** and **cyr** are highly collinear with other variables in the model.

Scree plot – Hungary

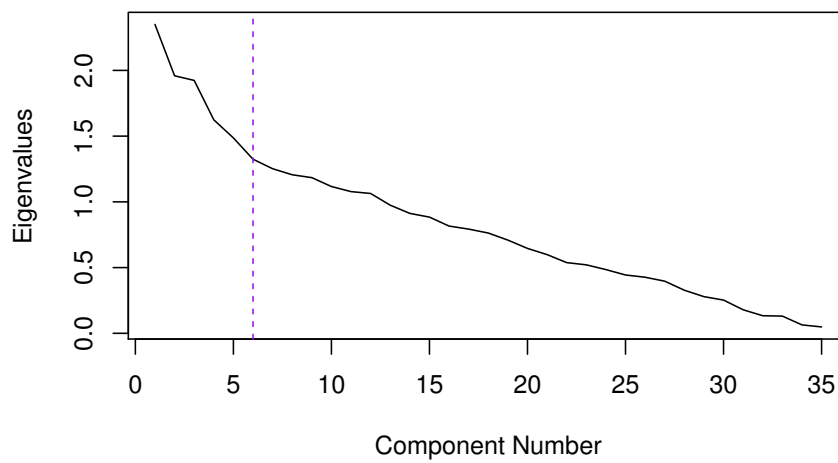


Figure 2: Scree plot for PCA of Hungary

From the scree plot in Figure 2 we see that we keep 6 components.

We have the loadings of each components as follows.

Hungary PCA loadings

Loadings :										
	Comp.1	Comp.2	Comp.3	Comp.4	Comp.5	Comp.6	Comp.7	Comp.8	Comp.9	Comp.10
age	0.162		0.184		-0.253		-0.105	-0.185	-0.114	0.137
sex		-0.144	-0.164	-0.236					0.232	
painloc	0.143		-0.261	-0.112	0.116					0.214
painexer	0.212		-0.335			-0.146				
relrest	0.228		-0.332					-0.159		-0.108
cp	0.229		-0.357			-0.163		-0.115		
trestbps	0.213		0.179	-0.290	0.113			-0.214		0.198
htn		-0.160	0.101			0.247		0.125	0.230	0.214
chol				-0.139	-0.207	-0.105		0.186	-0.529	
fbs		-0.170		-0.127	-0.193	0.168		0.265	-0.304	-0.195
restecg									-0.135	0.304
ekgmo	-0.191		-0.177		-0.276		0.344	-0.380		
ekgday			-0.108		-0.254	0.479	-0.110		0.255	
ekgyr	0.126	-0.312	0.189			-0.181	0.237		0.101	-0.326
prop	0.132	-0.253		0.301	0.117		0.112			0.194
nitr		-0.286		0.402						0.163



pro		-0.309		0.355				-0.105	0.119
diuretic			0.129	-0.101	0.149		0.275	0.123	0.377
proto	-0.312	-0.277	-0.136	-0.121					
thaldur	-0.305	-0.277	-0.135	-0.130					
thaltime	-0.303	-0.270	-0.138	-0.128			-0.111		0.113
met	-0.306	-0.227					-0.192		
thalach	-0.259			-0.135	0.126		0.376	0.177	-0.142
thalrest				-0.126			0.539	0.233	-0.141
tpeakbps		-0.225		-0.292		0.170		-0.264	
tpeakbpd		-0.191	0.271	-0.231		0.134		-0.199	
trestbpd	0.157		0.155	-0.301	0.137			-0.207	-0.150
exang	0.237		-0.216	-0.140					-0.175
oldpeak		0.113		-0.168	0.217	0.146			0.259
slope	0.156		-0.245			0.280		0.287	
rldv5	-0.147	0.126			0.439	0.283		-0.130	-0.232
rldv5e	-0.128			0.116	0.444	0.289		-0.179	-0.172
cmo	-0.175		-0.194		-0.284	0.103	0.297	-0.395	
cday		-0.123			-0.229	0.387	0.133	0.142	-0.211
cyr	0.130	-0.316	0.179			-0.181	0.222		-0.331

We see that the first principle component is mostly formed of **proto**, **met**, **thaldur** and **thaltime** variables.

The second principle component is mostly formed of **cyr**, **ekgyr** and **pro** variables.

The third principle component is mostly formed of **cp**, **painexer** and **relrest** variables.

The fourth principle component is mostly formed of **nitr** and **pro** variables.

The fifth principle component is mostly formed of **rldv5e** and **rldv5** variables.

The sixth principle component is mostly formed of **ekgday** and **cday** variables.

### 2.3.3 Longbeach

After removing dummy variables and variables with at least 50% NA data, we are left with 50 variables and 94 observations.

Longbeach variance inflation factor

age	sex	painloc	painexer	relrest	cp	trestbps	htn
3.228090	1.931427	2.577184	7.718893	6.621863	14.044802	3.617851	2.921354
chol	smoke	cigs	years	lbs	famhist	restecg	ekgmo
2.045184	4.098390	3.361309	4.794619	3.248168	2.369061	2.192354	3.658305
ekgday	ekgyr	dig	prop	nitr	pro	diuretic	proto
2.449554	39.950248	2.509731	2.528256	1.901292	1.732049	2.476101	4.533802
thaldur	met	thalach	thalrest	tpeakbps	tpeakbpd	trestbpd	exang
18.058001	16.434757	3.931213	2.659439	3.784661	2.735284	3.161172	2.222555
xhypo	oldpeak	rldv5	rldv5e	cmo	cday	cyr	lmt
2.096734	4.015243	7.661567	6.930116	4.934749	2.272059	43.093455	1.678378
ladprox	laddist	diag	cxmain	ramus	oml	om2	rcaprox
2.126046	1.837113	1.766507	1.847673	2.269851	2.527439	2.913807	2.246981
rcadist							
2.142703							

From the variance inflation factor we see that the variables **cp**, **ekgyr**, **thaldur**, **met** and **cyr** are highly collinear with other variables in the model.

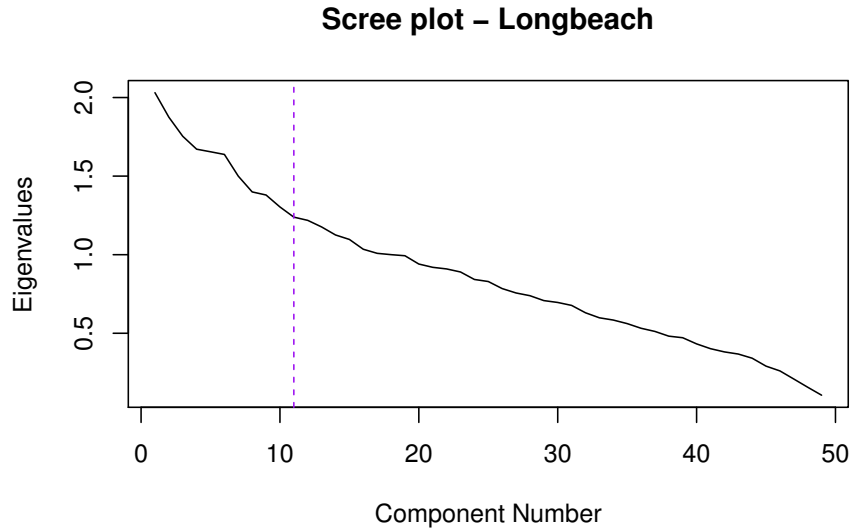


Figure 3: Scree plot for PCA of Longbeach

From the scree plot in Figure 3 we see that we keep 11 components.

We have the loadings of each components as follows.

Longbeach PCA loadings

Loadings :	Comp.1	Comp.2	Comp.3	Comp.4	Comp.5	Comp.6	Comp.7	Comp.8	Comp.9	Comp.10
age	-0.204		0.197			-0.182		-0.205		
sex			-0.106	-0.215			0.147			0.146
painloc		-0.133	0.209			0.255		0.136	-0.105	
painexer	-0.192	-0.223			0.288	0.188	-0.162			
relrest	-0.181	-0.203			0.114	0.314		0.197		
cp	-0.181	-0.276			0.216	0.288	-0.128	0.184		
trestbps	-0.196		0.310	-0.133		-0.202				
htn			0.313	-0.134	-0.219					0.106
chol			0.118	0.166	0.175	-0.156	-0.232			-0.194
smoke	0.160		-0.191	-0.334						-0.186
cigs			-0.250	-0.320						0.240
years	0.155	-0.117	-0.133	-0.344					0.103	
fbs		0.111	0.196			-0.168	-0.154			0.315
famhist				-0.124	-0.235			0.159		-0.316
restecg	0.125		0.132					-0.128	0.257	0.229
ekgmo		-0.189	-0.134				0.178		-0.289	
ekgday			-0.149				0.357		0.264	0.166
ekgyr	-0.357	0.130	-0.161	-0.127			-0.195			
dig	0.166				-0.111	-0.241		0.111	0.227	-0.183
prop			0.137	-0.106		0.115	0.179	-0.179	0.135	-0.116
nitr			0.123		-0.215	0.181		-0.255	-0.139	
pro				-0.177	-0.253			0.170	-0.112	
diuretic		-0.140	0.162	-0.221			0.106		0.187	-0.109
proto	-0.288	0.135	-0.240		-0.102			-0.171		
thaldur		0.402					0.102	0.153	-0.190	-0.113
met		0.353			0.126		0.131	0.237	-0.172	
thalach		0.151		-0.129	0.323		0.279	0.172		
thalrest					0.349		0.169		0.119	
tpeakbps		0.264	0.229	-0.167			0.154			0.158
tpeakbpd	0.139		0.223		0.124	0.141	0.235	0.144		
trestbpd			0.200	-0.181		-0.154		0.135		

exang	-0.105	-0.260				-0.141	0.135		-0.110	-0.174
xhypo	-0.128				0.153	-0.177			-0.148	
oldpeak	-0.250				0.204	-0.208	0.116			-0.274
rldv5	-0.238			0.274	-0.118		0.280		0.249	-0.137
rldv5e	-0.238			0.262	-0.191		0.255		0.224	-0.153
cmo		-0.244	-0.131	0.117		-0.124	0.210		-0.291	
cday					0.135	0.129	0.160	-0.310		0.299
cyr	-0.352	0.156	-0.160	-0.134			-0.185			
lmt	-0.104						-0.149	0.166		
ladprox		-0.103		-0.127	0.229	-0.220				0.140
laddist						0.123	0.102		-0.233	
diag	-0.105				-0.109			0.384		0.174
cxmain		-0.126				-0.233			-0.118	
ramus		0.103	0.113	-0.139	0.140		-0.114	-0.232		-0.116
om1		-0.105	0.100	-0.144					-0.347	-0.161
om2			0.249	-0.217	0.100	0.133		-0.310		-0.142
rcaprox		-0.139		-0.118		-0.347		-0.113		
rcadist	-0.196				-0.192		0.131		-0.177	0.172
	Comp.11	Comp.12	Comp.13	Comp.14	Comp.15	Comp.16	Comp.17	Comp.18	Comp.19	
age	-0.163		-0.136	0.110		0.110		0.185		-0.230
sex		-0.264		0.125			-0.206	-0.150		-0.363
painloc				0.210	0.397			-0.138		-0.199
painexer						-0.246				
relrest	-0.147						0.229	0.114		
cp						-0.138				
trestbps	0.112		-0.129	-0.101						
htn	-0.123		0.241	-0.196	0.100	0.102				
chol		0.180	-0.233				0.113	-0.105		-0.178
smoke	-0.107						0.203			
cigs			-0.160		0.131			-0.118		0.198
years			-0.264				0.156	0.211		
fbs	-0.182	0.233		0.139	0.176			0.135		0.250
famhist		0.269		0.199	0.119		-0.113			0.209
restecg	-0.169					-0.216	-0.139			-0.238
ekgmo	-0.384	0.110	-0.184	-0.205	-0.258					
ekgday	0.168	0.147		0.172	0.161					
ekgyr		0.133				0.168	-0.111			
dig			0.121	-0.146		-0.255	-0.247			-0.129
prop		0.242	-0.265	-0.255	-0.141	-0.265		-0.185		-0.188
nitr				0.322	-0.116			-0.135		
pro		0.168	0.127	0.208	-0.186	-0.188	-0.190			
diuretic	-0.194		0.170	-0.138	0.170	0.322	0.248	0.107		-0.126
proto							-0.156			
thaldur	-0.152				0.114	-0.138	0.111	-0.183		-0.152
met	-0.139				0.171	-0.230	0.215	-0.143		-0.166
thalach				0.135		0.102		0.290		0.111
thalrest		0.183			-0.220	0.151	-0.344			-0.102
tpeakbps	-0.110	-0.145		0.166	-0.242			0.162		-0.112
tpeakbpd	0.102	-0.105		0.146	-0.169	0.353	-0.157			
trestbpd	0.371	0.130	-0.147	-0.210	-0.110		-0.134	-0.124		0.114
exang	0.206	-0.116	-0.215	0.208	0.156	-0.106				
xhypo		0.349		-0.156	0.253	0.155		0.123		
oldpeak		-0.159	0.101		0.100			0.143		0.175
rldv5		-0.105								0.119
rldv5e					-0.108		0.104			
cmo	-0.337		-0.142				-0.116			
cday	0.209	0.127			0.122		0.258	-0.119		
cyr		0.146				0.164				
lmt		-0.226	-0.455	-0.245		0.126				
ladprox		-0.137	0.224				0.117	-0.310		0.236

laddist	0.331	-0.207		-0.191	0.182	-0.167	-0.168	0.349	-0.174
diag		-0.262						-0.327	0.139
cxmain	0.104	0.130		0.137	-0.356	-0.184	0.350	0.191	-0.105
ramus	-0.196	-0.260		0.165	0.104	-0.133	-0.169	0.115	0.279
om1			0.313	-0.280		0.122		-0.129	
om2						-0.257	-0.104		0.209
rcaprox				0.184	0.156			-0.297	
rcadist		0.124	0.214				-0.106		

We see that the first principle component is mostly formed of **ekgyr**, **cyr** and **proto** variables.

The second principle component is mostly formed of **thaldur**, **met** variables.

The third principle component is mostly formed of **htn**, **trestbps** and **cigs** variables.

The fourth principle component is mostly formed of **years**, **smoke** and **cigs** variables.

The fifth principle component is mostly formed of **thalrest**, **thalach** and **painexer** variables.

The sixth principle component is mostly formed of **rcaprox**, **relrest** and **cp** variables.

The seventh principle component is mostly formed of **ekgday**, **rldv5** and **thalach** variables.

The eighth principle component is mostly formed of **diag**, **cday** and **om2** variables.

The ninth principle component is mostly formed of **om1**, **cmo** and **ekgmo** variables.

The tenth principle component is mostly formed of **famhist**, **fbs** and **cday** variables.

The eleventh principle component is mostly formed of **ekgmo**, **trestbpd**, **cmo** and **laddist** variables.

### 2.3.4 Switzerland

After removing dummy variables and variables with at least 13% NA data, we are left with 39 variables and 101 observations.

Switzerland variance inflation factor

age	sex	painloc	painexer	relrest	cp	trestbps	restecg
2.369562	1.738028	3.014841	5.301607	5.348634	5.703978	3.512376	2.391685
ekgmo	ekgday	ekgyr	dig	prop	nitr	pro	diuretic
15.412883	4.698930	11.069307	1.660195	2.140460	2.363016	2.250616	1.810968
thaldur	thalach	thalrest	tpeakbps	tpeakbpd	trestbpd	exang	xhypo
4.680438	4.923162	3.031050	4.382267	2.124042	2.928830	1.982196	2.170784
oldpeak	cmo	cday	cyr	lmt	ladprox	laddist	diag
2.282318	17.422769	4.254917	6.008334	1.696866	2.296964	1.815151	1.777471
cxmain	ramus	om1	om2	rcaprox	rcadist		
2.269139	2.183169	2.849526	1.660434	1.868517	1.905493		

From the variance inflation factor we see that the variables **ekgmo**, **ekgyr** and **cmo** are highly collinear with other variables in the model.

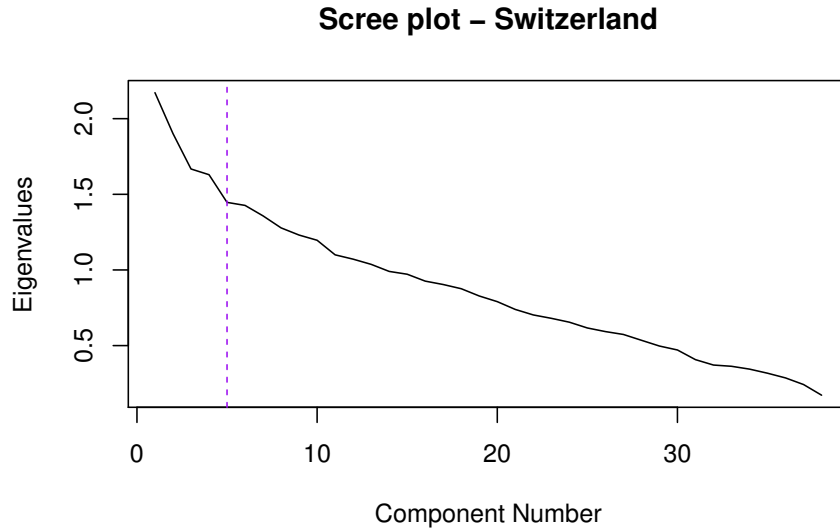


Figure 4: Scree plot for PCA of Switzerland

From the scree plot in Figure 4 we see that we keep 5 components.

We have the loadings of each components as follows.

Switzerland PCA loadings

Loadings :										
	Comp.1	Comp.2	Comp.3	Comp.4	Comp.5	Comp.6	Comp.7	Comp.8	Comp.9	Comp.10
age			0.366		0.115		-0.106	0.158	0.220	-0.138
sex		-0.107			-0.238			-0.326	0.118	
painloc	-0.209	-0.265	-0.145	-0.197						0.160
painexer	-0.245	-0.238	-0.183	-0.211			0.160			
relrest	-0.215	-0.193	-0.187	-0.264			0.147	0.144		
cp	-0.214	-0.292	-0.203	-0.250						
trestbps	-0.154	0.126	0.369	-0.192		0.114		0.169	0.116	
restecg		0.110	0.170		-0.182			0.101	0.202	-0.451
ekgmo	-0.337	0.199			-0.120	-0.168				
ekgday		-0.143	-0.219		-0.172	0.420	-0.178	0.222		
ekgyr	0.316	-0.219	0.142	-0.179	0.137		0.171			
dig				0.140		0.167	0.281	0.187		0.342
prop	0.111	-0.227		0.129		-0.201	-0.174		0.204	0.143
nitr	0.137	-0.214				-0.307		0.181	0.332	
pro		-0.111	0.106		-0.198	-0.331	-0.201	0.325		0.104
diuretic	-0.105	-0.119	0.133	0.113	-0.114	-0.201	-0.162	0.173	-0.162	
thaldur	0.231			-0.107	-0.355	-0.161	0.192	-0.166		0.214
thalach	0.184	0.172	-0.191	-0.187	-0.242		0.247	0.232	-0.236	
thalrest		0.269	-0.156				0.222	0.411	-0.151	-0.160
tpeakbps		0.215	0.117	-0.372	-0.176			0.197		
tpeakbpd	-0.116	0.125	0.148	-0.352		0.100			0.189	
trestbpd	-0.206	0.106	0.199	-0.179		0.214	-0.123		0.163	0.152
exang	-0.188					0.153	-0.315		-0.251	-0.117
xhypo			0.114	0.170	0.226	0.229	0.290	0.269		0.182
oldpeak		-0.110		-0.298			0.151		-0.205	-0.187
cmo	-0.345	0.185			-0.128	-0.128		-0.109		
cday		-0.129	-0.189		-0.188	0.381	-0.231	0.228		
cyr	0.275	-0.194	0.136	-0.154	0.202		0.144	-0.138		
lmt		-0.103	-0.142	0.123	0.119	-0.116	0.135		0.166	-0.333
ladprox	-0.150	-0.113		0.186	0.132			0.122		-0.195
laddist		-0.150	0.117		-0.348					0.198

diag		-0.187	0.196		-0.179		-0.323	
cxmain	-0.102	-0.128			-0.190	0.219	-0.208	-0.366
ramus	-0.139	-0.107	0.237	0.129			0.108	-0.311
om1		-0.186	0.154	0.153	-0.247		0.315	0.124
om2		-0.161	0.212					-0.338
rcaprox	-0.189		0.119		0.159		0.133	0.116
rcadist	-0.106			0.137	-0.255		0.277	0.220

We see that the first principle component is mostly formed of **cmo**, **ekgmo**, **ekgyr** and **cyr** variables.

The second principle component is mostly formed of **cp**, **thalrest**, **painloc**, and **painexer** variables.

The third principle component is mostly formed of **trestbps** and **age** variables.

The fourth principle component is mostly formed of **tpeakbps**, **tpeakbpd** and **oldpeak** variables.

The fifth principle component is mostly formed of **thaldur** and **laddist** variables.

## 2.4 Question 4

In order to distinguish patients based on their disease through discriminant analysis, I had to create four models, one for each of the datasets.

### 2.4.1 Cleveland

```
Call:
lda(num ~ ., data = clevdata)

Prior probabilities of groups:
      0      1      2      3      4
0.50248756 0.18407960 0.12935323 0.12935323 0.05472637

Group means:
      age      sex      cp      trestbps      htn      chol      cigs      years
0 53.52475 0.5247525 2.841584 128.7921 0.5544554 249.3465 14.52475 13.77228
1 55.86486 0.8918919 3.459459 132.7838 0.7027027 252.4324 21.67568 17.27027
2 58.00000 0.8076923 3.692308 133.0385 0.7307692 274.0000 15.30769 15.15385
3 56.03846 0.8076923 3.807692 136.7308 0.4230769 252.1923 15.88462 12.73077
4 59.54545 0.9090909 3.636364 140.0000 0.6363636 233.5455 26.27273 22.63636
      fbs      famhist      restecg      ekgmo      ekgday      ekgyr      dig
0 0.14851485 0.5445545 1.0000000 5.900990 14.89109 82.30693 0.06930693
1 0.08108108 0.6756757 1.1351351 6.756757 13.29730 82.02703 0.00000000
2 0.19230769 0.8076923 0.9230769 5.192308 12.34615 82.38462 0.00000000
3 0.23076923 0.5384615 1.1538462 5.692308 16.80769 82.03846 0.00000000
4 0.09090909 0.5454545 1.6363636 7.636364 14.18182 82.09091 0.00000000
      prop      nitr      pro      diuretic      thaldur      thaltime      met      thalach
0 0.2772277 0.2079208 0.11881188 0.1386139 8.836634 4.271287 10.227723 157.2376
1 0.3783784 0.3513514 0.10810811 0.1081081 8.294595 6.027027 9.837838 145.3784
2 0.5769231 0.3076923 0.15384615 0.1153846 7.873077 6.215385 8.807692 132.3846
3 0.3846154 0.4230769 0.03846154 0.1923077 6.707692 4.642308 8.076923 132.9231
4 0.3636364 0.2727273 0.00000000 0.0000000 6.027273 4.863636 7.363636 139.4545
      thalrest      tpeakbps      tpeakbpd      trestbpd      exang      xhypo      oldpeak      slope
0 77.28713 170.0891 78.96040 84.23762 0.1782178 0.00990099 0.7435644 1.504950
1 72.37838 172.3243 77.59459 85.05405 0.5405405 0.02702703 1.2864865 1.702703
2 69.76923 158.4615 81.69231 85.26923 0.5769231 0.07692308 1.7653846 1.961538
3 75.11538 160.2692 79.26923 87.65385 0.6538462 0.07692308 2.2884615 2.038462
4 74.63636 165.4545 84.09091 86.63636 0.5454545 0.00000000 2.0272727 2.000000
      rldv5e      ca      thal      cmo      cday      cyr      ladprox      laddist
0 122.9010 0.3267327 3.792079 5.811881 15.22772 82.30693 1.000000 1.000000
1 131.3243 0.8648649 5.432432 6.756757 14.67568 82.02703 1.270270 1.324324
2 127.0000 1.3076923 6.115385 5.153846 13.61538 82.42308 1.423077 1.384615
3 138.9231 1.4230769 6.538462 5.923077 16.50000 82.07692 1.461538 1.730769
4 107.8182 1.4545455 6.181818 7.636364 15.36364 82.00000 1.272727 1.545455
      cxmain      oml      rcaprox      readist
0 1.000000 1.000000 1.000000 1.000000
1 1.135135 1.081081 1.216216 1.108108
2 1.384615 1.346154 1.576923 1.230769
3 1.500000 1.730769 1.461538 1.576923
4 1.545455 1.545455 1.454545 1.272727

Coefficients of linear discriminants:
      LD1      LD2      LD3      LD4
age      -0.0064021152 -0.0425071101 0.017806402 -1.388222e-02
sex      0.2188951081 -0.8385787717 0.966289676 -1.080167e-01
cp      0.0836865201 -0.0684255751 0.213747047 -2.619771e-01
trestbps 0.0086722361 -0.0110214859 0.009667569 7.267368e-03
htn      0.1339883778 -0.5852997536 -0.309526408 3.116354e-01
chol     -0.0001853615 -0.0026943630 -0.007365030 1.378357e-03
```

cigs	0.0014297509	-0.0040736758	0.023693179	-5.710203e-03
years	-0.0058677471	0.0064698922	-0.010415602	2.424284e-02
fbs	-0.0677560105	0.4462514441	-0.809205331	1.160757e-01
famhist	-0.1536120593	-0.6331094392	-0.046988498	-7.834152e-03
restecg	0.0020724640	0.0306173919	0.343424560	1.242487e-01
ekgmo	-0.0333931677	-0.1894708072	0.003215433	1.790893e-01
ekgday	0.0307768259	0.0202459475	0.004269374	-1.967420e-02
ekgyr	0.2849909378	-0.1910807911	3.244719860	3.043414e+00
dig	-0.5880796669	1.2356661900	-0.935588867	4.891422e-01
prop	-0.2044468618	-0.4228151708	0.314419800	2.952549e-01
nitr	-0.0500314492	-0.1132297678	-0.008236496	-6.704832e-01
pro	-0.0422771375	-0.3051736743	-0.212490269	-7.934840e-02
diuretic	0.0903449725	-0.1880557316	-0.424110167	-3.147873e-01
thaldur	0.0123720933	-0.0940568662	-0.296638460	2.257168e-01
thaltime	0.0352787900	-0.1062383655	0.033833393	2.174663e-03
met	-0.0227104963	0.0333032838	0.052992717	-2.510038e-01
thalach	0.0004083536	0.0016079361	0.014067133	-2.553563e-05
thalrest	-0.0143560702	0.0142803750	0.001611765	-7.030841e-03
tpeakbps	-0.0071065254	-0.0069409621	0.008450720	-1.586376e-02
tpeakbpd	0.0126391705	0.0036640956	-0.017755491	4.596684e-02
trestbpd	-0.0092676913	0.0131619284	-0.014903990	-3.778290e-02
exang	0.3742483643	-0.6846868868	-0.162847360	-3.792109e-01
xhypo	-0.3070163922	-0.9550240951	-0.837879117	-9.084565e-01
oldpeak	-0.0507070406	0.3552810935	-0.016595052	7.428472e-02
slope	0.2158763397	-0.3990518860	0.152776941	1.517427e-01
rldv5e	-0.0016341882	-0.0009474799	-0.004657443	-6.008307e-03
ca	0.0604185429	-0.1322843859	0.044212892	3.194839e-02
thal	0.1790094224	-0.0163313489	-0.036193049	-1.153858e-01
cmo	0.0652759054	0.1570432167	0.099289729	-1.540472e-01
cday	-0.0202700434	0.0109527280	0.006883157	1.056202e-02
cyr	-0.2318889441	0.0552735975	-3.287530952	-2.820889e+00
ladprox	2.1449671081	-0.7520705684	-0.456454141	-3.504782e-01
laddist	2.2474742138	0.3244757867	0.480884470	-6.849638e-01
cxmain	2.4577588717	0.6761926661	0.110008133	9.377253e-01
oml	2.3487229073	1.7069044343	-0.588196080	6.720648e-01
rcaprox	2.5240275251	-0.9443404888	-0.769549935	8.529602e-01
rcadist	1.8594484941	0.6017481881	-0.073459411	-4.100481e-01

Proportion of trace:

LD1	LD2	LD3	LD4
0.9090	0.0477	0.0268	0.0164



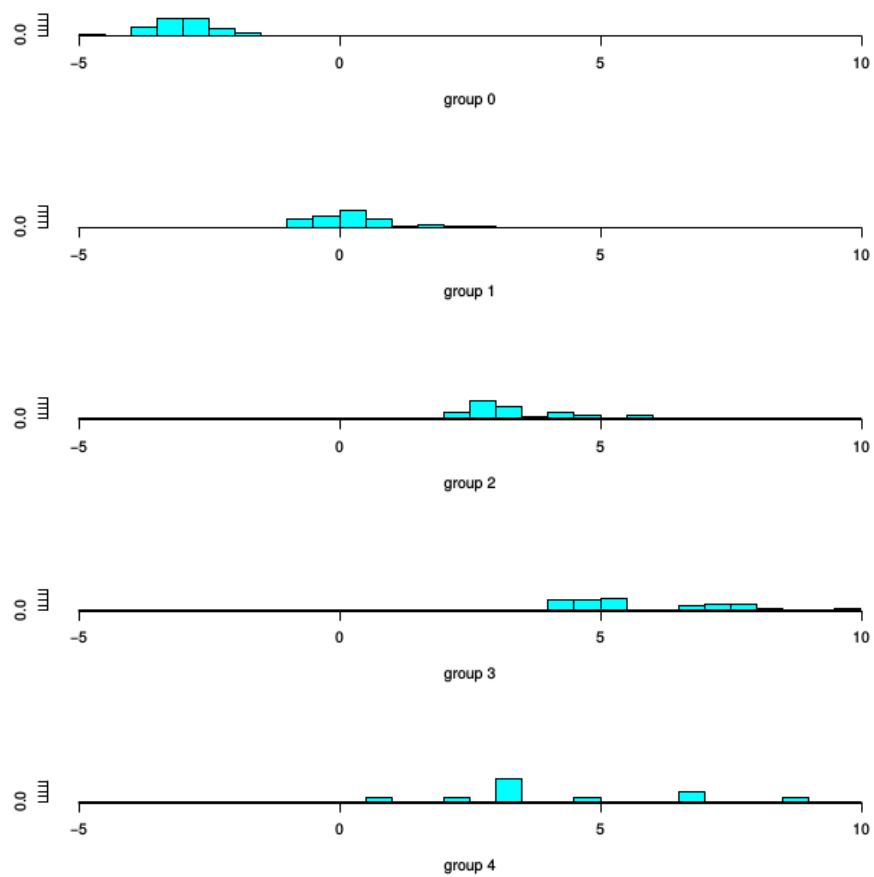


Figure 5: Histergram for discriminant analysis of Cleveland

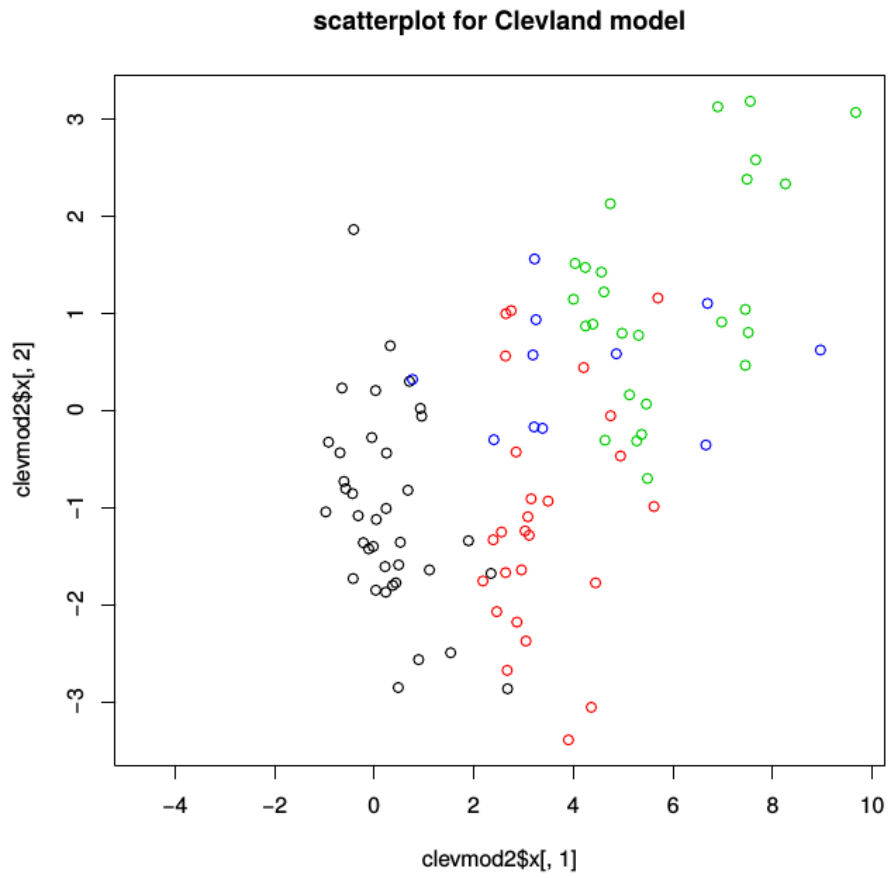


Figure 6: Scatter plot for discriminant analysis of Cleveland

## 2.4.2 Hungary

Call:

```
lda(num ~ ., data = ex3dathun)
```

Prior probabilities of groups:

	0	1	2	3	4
	0.2954545	0.2045455	0.1590909	0.2272727	0.1136364

Group means:

	age	sex	painloc	painexer	relrest	cp	trestbps	htn
0	51.53846	0.4615385	0.8461538	0.4230769	0.4230769	2.884615	131.6923	0.3076923
1	47.72222	0.8333333	1.0000000	0.8333333	0.8888889	3.722222	130.9444	0.5000000
2	51.07143	0.9285714	1.0000000	1.0000000	0.9285714	4.000000	145.8571	0.7142857
3	51.15000	0.9000000	0.9500000	0.7000000	0.7000000	3.400000	140.8000	0.4000000
4	50.40000	0.8000000	1.0000000	0.9000000	1.0000000	3.900000	138.5000	0.5000000

	chol	fbs	restecg	ekgmo	ekgday	ekgyr	prop	nitr
0	246.9615	0.0000000	0.2307692	6.192308	14.96154	85.03846	0.07692308	0.11538462
1	246.0000	0.1111111	0.1111111	5.333333	16.61111	85.55556	0.11111111	0.22222222
2	288.4286	0.2142857	0.1428571	4.642857	16.07143	85.57143	0.14285714	0.07142857
3	250.4500	0.1500000	0.3000000	5.750000	17.60000	85.35000	0.05000000	0.25000000
4	275.0000	0.1000000	0.4000000	7.200000	16.80000	84.90000	0.20000000	0.20000000

	pro	diuretic	proto	thaldur	thaltme	met	thalach	thalrest
0	0.03846154	0.00000000	83.65385	9.038462	8.038462	5.269231	136.1923	81.26923
1	0.22222222	0.00000000	81.94444	9.055556	7.972222	5.111111	129.8333	76.77778
2	0.07142857	0.07142857	75.00000	8.428571	7.285714	4.235714	119.0714	82.57143
3	0.20000000	0.05000000	76.25000	7.975000	6.800000	4.820000	121.5000	71.95000
4	0.20000000	0.00000000	60.00000	5.900000	4.950000	3.800000	127.0000	82.30000

	tpeakbps	tpeakbpd	trestbpd	exang	oldpeak	slope	rldv5	rldv5e
0	176.1538	95.15385	84.96154	0.5000000	1.242308	1.653846	15.53846	15.46154
1	171.4444	96.94444	83.55556	0.8333333	1.611111	2.000000	15.00000	14.94444
2	181.1429	100.35714	90.71429	0.9285714	1.785714	2.071429	14.00000	13.57143
3	182.0000	98.95000	89.05000	0.7500000	1.750000	2.000000	12.90000	13.55000
4	171.0000	94.20000	89.40000	0.9000000	2.300000	2.000000	17.80000	16.50000

	cmo	cday	cyr
0	6.307692	15.30769	85.03846
1	5.722222	16.11111	85.55556
2	4.642857	20.28571	85.57143
3	6.300000	18.10000	85.30000
4	7.200000	21.60000	85.00000

Coefficients of linear discriminants:

	LD1	LD2	LD3	LD4
age	-0.0319039718	0.0949747891	0.013319391	0.025531882
sex	1.0915165611	-1.0511591741	-0.128089043	-0.217640165
painloc	0.1592508146	-0.5982393519	0.751516162	-1.356165886
painexer	1.0443921480	-1.8274651017	0.423915039	-0.129822932
relrest	-0.5353907890	0.3080857927	1.366855699	-1.292874896
cp	0.2051091438	1.0860954298	-0.347952792	0.582142933
trestbps	-0.0291801216	-0.0210943388	-0.005915740	0.010631108
htn	0.7977232482	0.7125540372	0.687763392	0.090163649
chol	0.0059740613	0.0003533943	-0.008706924	0.001824229
fbs	0.3020817737	-1.2745529471	0.340669290	-0.124606147
restecg	-0.0034817198	0.0287925187	-0.860566466	-0.477512401
ekgmo	0.1766896834	0.2064361753	-0.235624384	-0.163174965
ekgday	-0.0008617825	-0.0371982064	-0.025145255	-0.019070970
ekgyr	-2.1415512648	-2.4433469890	1.742048061	0.200436115
prop	-1.7298024975	2.4134058221	0.644332191	0.279975382
nitr	-1.3114932699	-1.8866584974	-1.084697545	0.526330742
pro	2.2930978213	-0.2622377510	-0.638507586	-1.800873718

diuretic	4.0807841243	-1.4825164759	-3.921468835	0.528639304
proto	-0.0268455617	0.0109964005	-0.052095246	-0.002381793
thaldur	-0.1915704888	-0.2807526053	0.116598448	0.766573761
thaltme	0.2397031414	0.3449238438	0.509998503	-0.660418467
met	0.1538161967	-0.1558001805	-0.188012336	0.045169327
thalach	-0.0113911821	-0.0075009574	0.012289493	-0.030730660
thalrest	-0.0148107903	0.0404184418	0.014743907	0.032907539
tpeakbps	0.0055246183	-0.0024652818	-0.014276027	0.015577249
tpeakbpd	0.0147301260	0.0121621412	0.034171915	-0.027803617
trestbpd	0.0447580846	-0.0017554695	-0.059802256	0.041413630
exang	0.3026614220	-0.0396448017	0.081711393	-0.223888568
oldpeak	0.5557096564	0.6919891057	-0.374908659	-0.319812279
slope	1.5871575130	-0.1881557988	0.947157542	0.617202485
rldv5	0.0870335285	0.2322222300	0.056690032	-0.001700644
rldv5e	-0.0568400417	-0.1650279852	-0.053695184	-0.020264457
cmo	-0.1012481889	-0.1914088336	0.070107909	0.059942757
cday	0.0266986647	0.0502210848	-0.025879692	0.029755031
cyr	2.4408858944	2.4327046963	-1.459350228	-0.307030741

Proportion of trace:

LD1	LD2	LD3	LD4
0.5086	0.2406	0.1662	0.0846

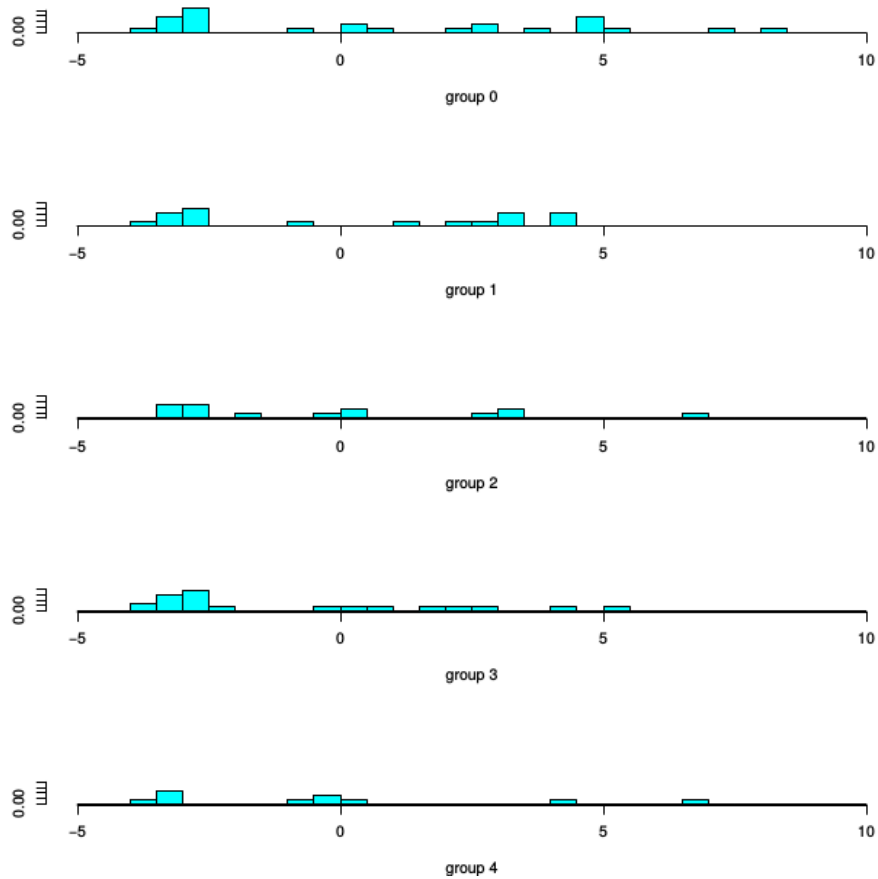


Figure 7: Histergram for discriminant analysis of Hungary

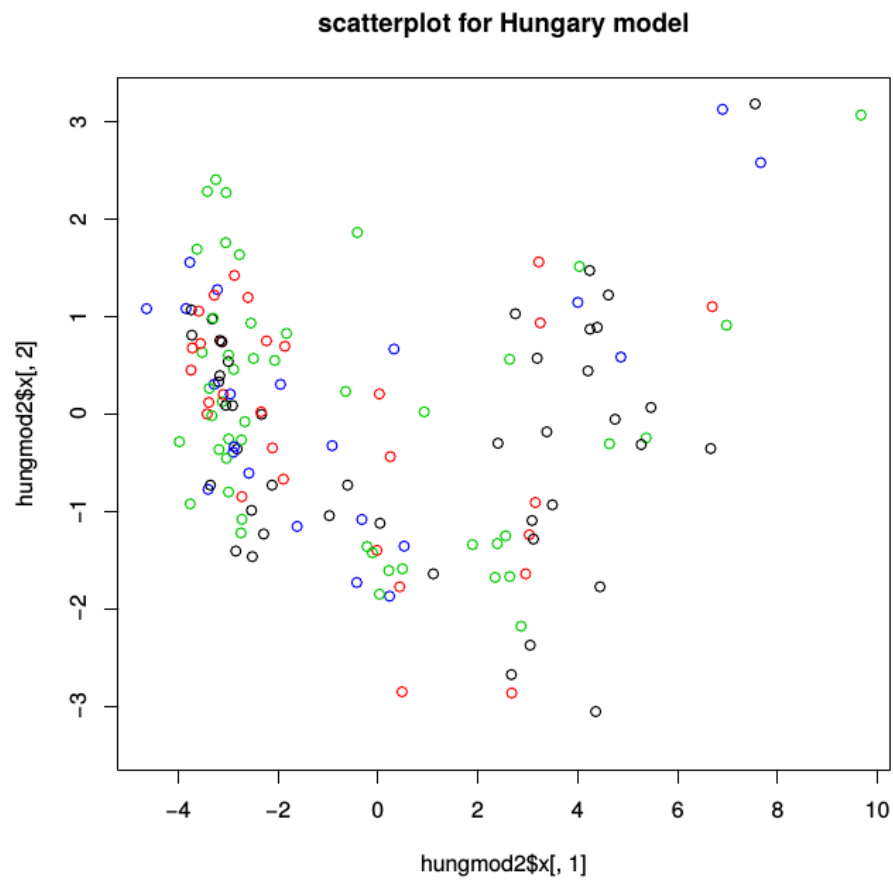


Figure 8: Scatter plot for discriminant analysis of Hungary

### 2.4.3 Longbeach

Call:

```
lda(num ~ ., data = ex3datlon)
```

Prior probabilities of groups:

0	1	2	3	4
0.1808511	0.3191489	0.2553191	0.2234043	0.0212766

Group means:

	age	sex	painloc	painexer	relrest	cp	trestbps	htn	
0	58.88235	0.8823529	1.0000000	0.5882353	0.8235294	3.470588	124.5882	0.5294118	
1	56.63333	0.9333333	0.9666667	0.8666667	0.9333333	3.833333	131.4667	0.5333333	
2	60.58333	1.0000000	0.9583333	0.7916667	0.8333333	3.625000	136.6250	0.6666667	
3	62.95238	1.0000000	0.9047619	0.8571429	0.9047619	3.714286	140.9048	0.7142857	
4	63.50000	1.0000000	1.0000000	1.0000000	1.0000000	4.000000	148.0000	0.5000000	
	chol	smoke	cigs	years	fbs	famhist	restecg	ekgmo	
0	198.9412	0.5294118	24.70588	27.23529	0.4117647	0.7058824	0.7647059	5.647059	
1	194.0000	0.4333333	19.90000	17.00000	0.3666667	0.4666667	0.7000000	6.700000	
2	174.0417	0.5833333	18.33333	24.41667	0.2500000	0.4166667	0.7083333	6.208333	
3	217.4286	0.5714286	23.80952	27.76190	0.3333333	0.3809524	0.6666667	7.000000	
4	251.0000	1.0000000	30.00000	38.50000	0.5000000	1.0000000	0.5000000	6.000000	
	ekgday	ekgyr	dig	prop	nitr	pro	diuretic	proto	
0	15.05882	84.58824	0.11764706	0.4117647	0.7058824	0.1764706	0.3529412	3.588235	
1	16.13333	84.40000	0.06666667	1.0666667	0.5666667	0.2000000	0.1666667	3.566667	
2	13.33333	84.83333	0.12500000	0.4583333	0.5416667	0.2916667	0.3750000	4.000000	
3	15.28571	84.42857	0.09523810	0.4285714	0.6190476	0.2380952	0.3809524	3.904762	
4	26.50000	86.50000	0.50000000	0.0000000	1.0000000	0.5000000	0.5000000	5.000000	
	thaldur	met	thalach	thalrest	tpeakbps	tpeakbpd	trestbpd	exang	
0	5.858824	5.876471	115.3529	63.94118	159.7059	88.35294	78.23529	0.4705882	
1	6.758333	6.740000	121.9667	69.70000	162.9667	91.66667	80.63333	0.6000000	
2	6.504167	6.375000	122.0000	69.62500	172.0417	90.62500	83.29167	0.7083333	
3	5.730952	5.619048	116.8571	67.04762	159.4286	83.66667	83.47619	0.9047619	
4	4.250000	3.500000	124.0000	76.50000	170.0000	84.00000	79.00000	1.0000000	
	xyhypo	oldpeak	rldv5	rldv5e	cmo	cday	cyr	lmt	
0	0.00000000	0.7529412	15.88235	17.52941	6.000000	10.52941	84.64706	10.47059	
1	0.00000000	0.9166667	16.70000	16.06667	6.333333	16.13333	84.53333	1.00000	
2	0.04166667	1.3791667	14.75000	14.83333	6.791667	14.58333	84.91667	1.00000	
3	0.04761905	1.7142857	15.42857	15.47619	7.952381	14.09524	84.52381	1.00000	
4	0.00000000	3.500000	24.00000	19.50000	7.000000	6.50000	86.50000	2.00000	
	ladprox	laddist	diag	cxmain	ramus	oml	om2	rcaprox	rcadist
0	1.000000	1.000000	1.117647	1.000000	1.000	1.000000	1.000000	1.000000	1.000000
1	1.166667	1.133333	1.133333	1.333333	1.000	1.100000	1.033333	1.233333	1.100000
2	1.458333	1.166667	1.125000	1.458333	1.125	1.291667	1.083333	1.708333	1.083333
3	1.761905	1.238095	1.238095	1.857143	1.000	1.285714	1.095238	1.857143	1.238095
4	1.500000	1.000000	1.000000	2.000000	1.000	1.000000	1.000000	2.000000	1.000000

Coefficients of linear discriminants:

	LD1	LD2	LD3	LD4
age	0.0990101557	0.035457144	-0.029404453	-0.037368451
sex	-0.7001377631	-1.223212780	2.472604131	-0.385002333
painloc	1.3527943877	2.689379586	-1.442324304	0.925270543
painexer	1.6779571229	1.095644137	1.132485802	0.132422919
relrest	1.0061607957	2.683724569	1.044784632	-0.315270284
cp	-1.3452114777	-2.179502234	-0.074000882	0.276431196
trestbps	0.0152057624	0.043147540	0.019492777	0.004725401
htn	0.2007845282	-0.213314404	-0.049905568	0.516879641
chol	0.0022668552	0.003074371	0.000609473	-0.004283149
smoke	1.3848919525	0.579322247	0.717647892	-0.203604359

cigs	-0.0021092781	0.040230996	-0.017470605	-0.046310142
years	-0.0189346337	-0.009935330	-0.025142432	0.020697154
fbs	-0.5602845218	-0.600564294	0.917260811	-0.583766088
famhist	0.2368404852	0.633387492	0.062625332	-0.435654580
restecg	0.3148426505	0.054613753	0.230598184	-0.620653987
ekgmo	0.1066755233	0.146072767	0.174751517	0.051019305
ekgday	0.0302154060	0.042708813	0.006610623	0.007431638
ekgyr	-0.6058539073	1.011772666	-0.482944430	1.028674481
dig	0.1934068922	1.829523966	0.420059417	1.532752361
prop	-0.0258580607	-0.128530397	-0.013717278	-0.004001685
nitr	-0.2979245919	0.890522089	0.221162357	-0.115880411
pro	0.2277745757	0.495507245	-0.293334921	0.886596619
diuretic	-0.5265872971	-1.108015873	-0.622279383	-0.614113919
proto	0.0389888030	-0.304394784	-0.045447788	0.044131177
thaldur	0.0974633776	0.347965964	0.471833171	-0.506561997
met	0.0520777936	-0.547177132	-0.234459599	0.387387794
thalach	0.0012019440	-0.009656180	0.001428743	0.009332465
thalrest	0.0349744126	0.037022030	0.013999657	0.005532030
tpeakbps	-0.0005518933	0.012532493	-0.016658042	0.026948500
tpeakbpd	0.0110131580	0.013873221	0.019296564	0.018578019
trestbpd	-0.0101443057	-0.111258996	0.003785584	0.004558456
exang	-0.2885342130	-0.491316817	0.711632300	-0.290403026
xhypo	-2.0890734776	-1.546304470	-2.533162072	2.585953825
oldpeak	0.0562423983	0.729631495	-0.150881175	-0.454416331
rldv5	0.1212482697	0.141480974	0.244757211	0.074758659
rldv5e	-0.1098458488	-0.119370257	-0.225942614	0.008272690
cmo	-0.0044982969	-0.215097791	-0.184654932	0.011105799
cday	0.0154098050	-0.018575403	0.045237609	0.014481709
cyr	0.7728103647	-0.622781492	0.438416369	-0.517516020
lmt	-0.0076644410	0.016199837	-0.011799002	-0.006409811
ladprox	5.1429956452	-0.236848734	-0.756165644	-0.403482082
laddist	5.4700303913	-0.538647061	-0.111674357	-0.040224894
diag	0.1674878034	-0.867356837	-0.233108173	-0.424424671
cxmain	4.9690488611	0.288788245	0.491785610	-0.752048882
ramus	2.2794956046	-2.518189778	-1.110123699	4.143654026
om1	2.7419747798	-0.569683759	0.382398821	0.098133650
om2	1.3014833954	0.573435941	-1.109721994	-1.917714264
rcaprox	4.3481392232	0.005190937	-0.578608990	1.523789635
rcadist	1.9383005198	0.312683826	0.793005347	-2.408552088

Proportion of trace:

LD1	LD2	LD3	LD4
0.9072	0.0418	0.0262	0.0248

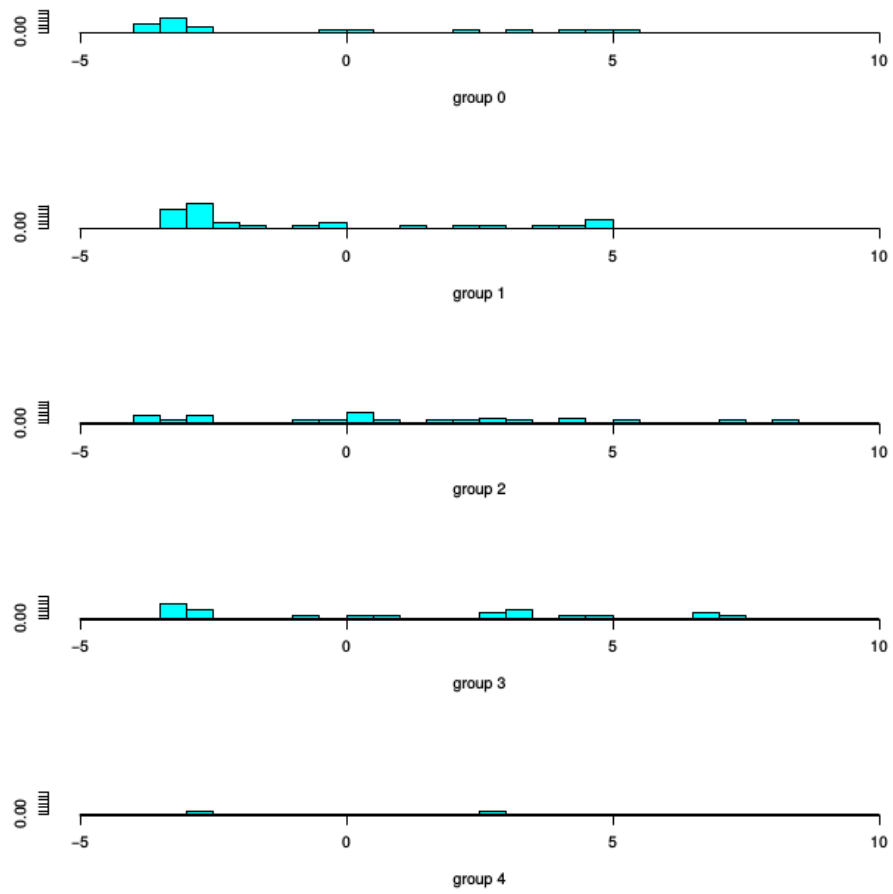


Figure 9: Histergram for discriminant analysis of Longbeach



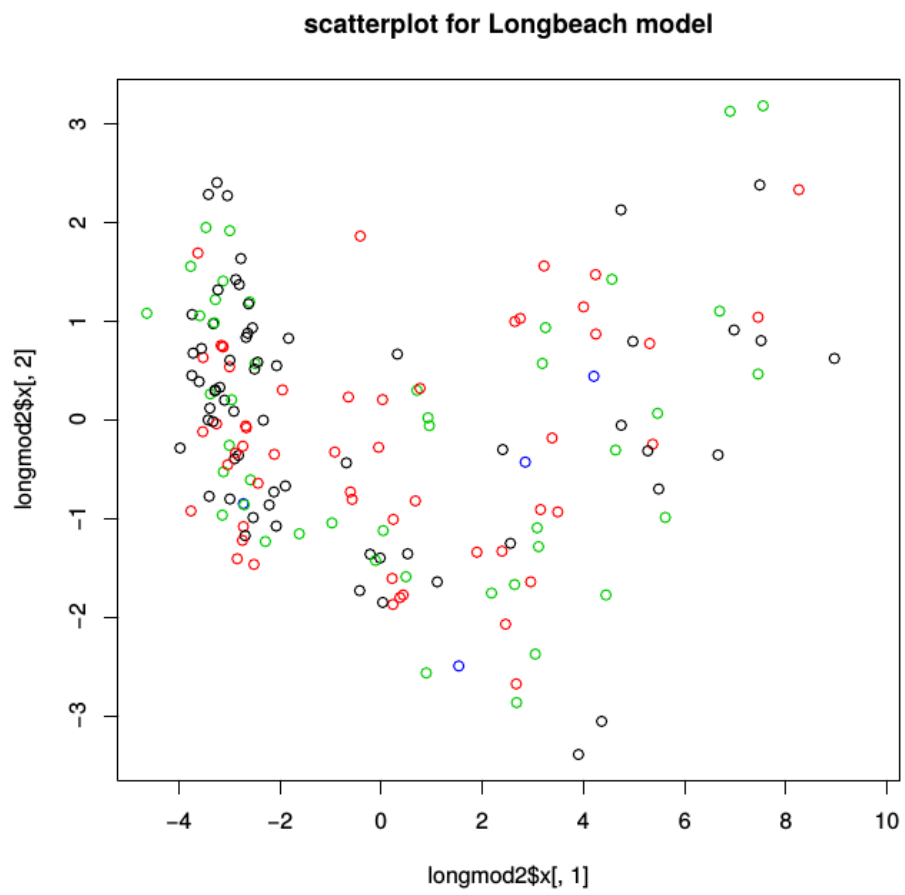


Figure 10: Scatter plot for discriminant analysis of Longbeach

## 2.4.4 Switzerland

Call:

```
lda(num ~ ., data = swidata)
```

Prior probabilities of groups:

	0	1	2	3	4
	0.05940594	0.39603960	0.26732673	0.22772277	0.04950495

Group means:

	age	sex	painloc	painexer	relrest	cp	trestbps	restecg
0	52.00000	1.0000000	0.6666667	0.3333333	0.3333333	3.166667	117.5000	0.1666667
1	56.80000	0.8750000	0.9500000	0.8500000	0.8000000	3.675000	129.3750	0.3250000
2	53.59259	0.9259259	0.8888889	0.8888889	0.8888889	3.740741	132.5926	0.4074074
3	58.65217	0.9565217	1.0000000	0.9130435	0.7826087	3.782609	140.6522	0.3478261
4	52.00000	1.0000000	1.0000000	1.0000000	1.0000000	4.000000	120.0000	0.4000000

	ekgmo	ekgday	ekgyr	dig	prop	nitro	pro	diuretic
0	2.500000	21.33333	84.83333	0.0000000	0.3333333	0.3333333	0.5000000	0.0000000
1	4.100000	16.27500	84.65000	0.0000000	0.4250000	0.4000000	0.4000000	0.1000000
2	5.814815	13.66667	84.48148	0.03703704	0.3703704	0.2962963	0.4814815	0.2222222
3	5.695652	13.95652	84.52174	0.0000000	0.3913043	0.2608696	0.4347826	0.4347826
4	5.800000	18.60000	84.60000	0.0000000	0.6000000	1.0000000	0.4000000	0.2000000

	thaldur	thalach	thalrest	tpeakbps	tpeakbpd	trestbpd	exang	xhypo
0	10.00000	143.1667	68.00000	175.8333	82.50000	75.00000	0.1666667	0.0000000
1	7.557500	123.7500	69.52500	171.7500	86.37500	81.62500	0.4750000	0.0000000
2	7.518519	125.3333	72.11111	169.8148	88.14815	83.51852	0.4074074	0.03703704
3	6.947826	104.2174	64.52174	160.0000	86.08696	87.17391	0.6086957	0.13043478
4	6.900000	109.8000	74.20000	138.0000	78.00000	76.00000	0.4000000	0.2000000

	oldpeak	cmo	cday	cyr	ladprox	laddist	diag	cxmain
0	0.3333333	2.333333	21.33333	84.83333	1.000000	1.000000	1.166667	1.000000
1	0.3675000	4.075000	17.12500	84.70000	1.300000	1.175000	1.075000	1.100000
2	0.7555556	5.814815	13.77778	84.37037	1.592593	1.481481	1.259259	1.333333
3	0.8608696	5.739130	14.21739	84.52174	1.782609	1.521739	1.478261	1.521739
4	0.9600000	6.000000	14.20000	84.60000	1.800000	1.200000	1.000000	1.400000

	ramus	om1	om2	rcaprox	readist
0	1.000000	1.000000	1.000000	1.000000	1.000000
1	1.025000	1.000000	1.000000	1.275000	1.125000
2	1.185185	1.000000	1.148148	1.518519	1.222222
3	1.521739	1.347826	1.130435	1.739130	1.521739
4	1.200000	1.200000	1.000000	1.400000	1.200000

Coefficients of linear discriminants:

	LD1	LD2	LD3	LD4
age	-0.0007858366	0.0628048943	-0.071526268	-0.0417811598
sex	0.8186870033	0.2412555376	0.363652833	0.3287857513
painloc	-0.3837830485	2.2727914494	-1.184109094	-3.4202311632
painexer	0.9118113566	-1.4374997954	0.056616312	-2.5683124594
relrest	-1.2647469151	-1.2871470954	0.420021842	-0.3980500354
cp	0.3521125467	0.6077931365	0.237965504	1.4684042267
trestbps	0.0052216770	-0.0003397642	0.022936687	-0.0002354636
restecg	-0.2637513280	-0.3449011590	0.449630173	-0.2538746008
ekgmo	0.0417657735	-0.0734676135	0.099042462	0.2295703277
ekgday	-0.0056874619	0.0230409744	0.050349943	0.0416943072
ekgyr	0.5069361444	0.5686400550	0.296652266	0.3918472529
dig	-2.5341631270	-2.9792700230	-1.502320680	0.4888530492
prop	-0.2807770793	-0.0863581677	0.267336756	-0.1333181334
nitro	0.4315496561	0.2855985294	1.759176079	-0.4000743686
pro	-0.0005018530	-0.7611546405	-0.194037649	0.5060712017
diuretic	0.7636514352	0.7979700757	-0.147302771	0.0096484846
thaldur	-0.0033692288	-0.0929494992	0.081758599	0.0702982571

thalach	-0.0087387451	-0.0094572112	-0.009259432	-0.0050115022
thalrest	0.0504091039	0.0242042236	0.022939061	-0.0254912923
tpeakbps	-0.0081181662	-0.0013325295	-0.020142259	0.0032913902
tpeakbpd	-0.0111369375	-0.0140911285	-0.002362359	0.0210386963
trestbpd	0.0361591182	-0.0007605452	-0.024619507	-0.0320079275
exang	-0.1520069720	0.6009664626	-0.429942733	-0.4604614290
xhypo	-0.3341779201	1.5596277090	1.378414692	1.2645700698
oldpeak	0.1908935156	-0.0967478433	0.388405128	0.2164803514
cmo	0.0277780256	0.2412607985	0.073061630	-0.1960612646
cday	-0.0151970362	-0.0148062415	-0.026759876	-0.0152737194
cyr	0.2318645756	0.6265312268	0.095369608	-0.6515047003
ladprox	1.7277441803	-1.2217191986	0.314767813	-0.1033884127
laddist	1.1672585854	-0.8125308678	-0.278617954	-0.6830410921
diag	0.2504224578	-0.3244353820	-0.673472037	0.3895221999
cxmain	1.7362037233	-0.3507318707	-0.017925974	0.4765159079
ramus	1.1291841912	0.2856883362	0.286684835	1.0561701368
om1	-0.1893535303	3.8891176666	-0.704398542	0.5598622966
om2	0.4071110870	-2.5992153764	-0.676202349	-0.4854881569
rcaprox	1.4316751371	-0.7666014034	-0.317212280	-0.2971793102
rcadist	1.0688251201	-0.2259274588	-0.629453212	0.1276147609

Proportion of trace:

LD1	LD2	LD3	LD4
0.6579	0.1599	0.1086	0.0736

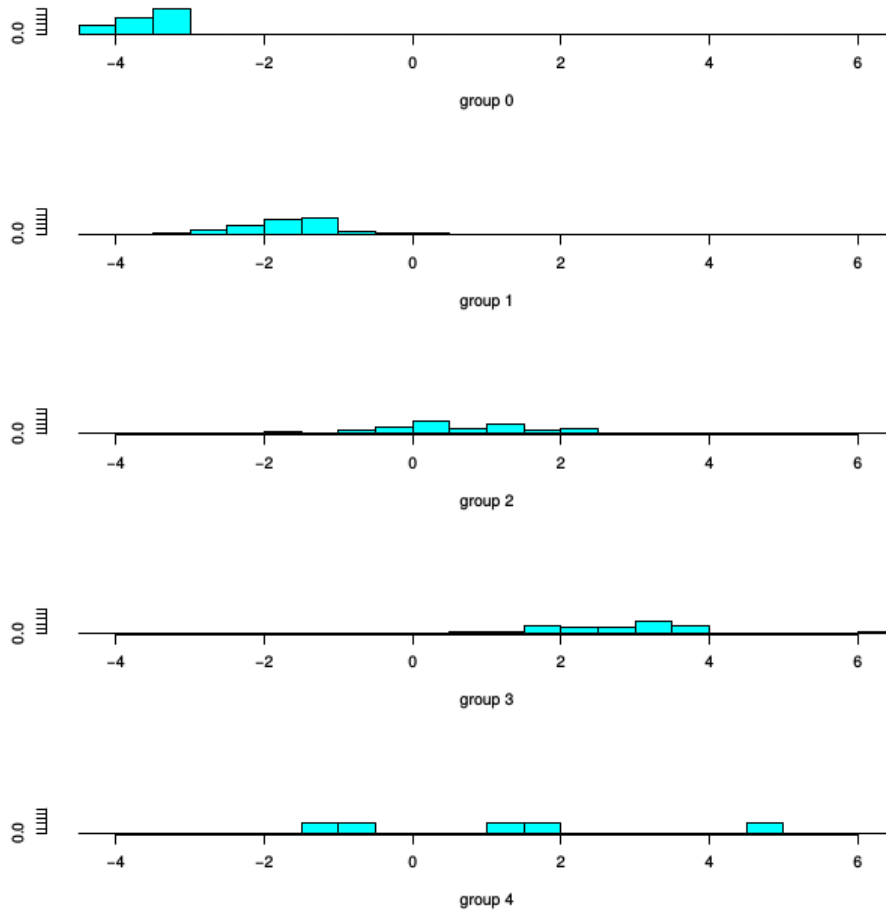


Figure 11: Histergram for discriminant analysis of Switzerland

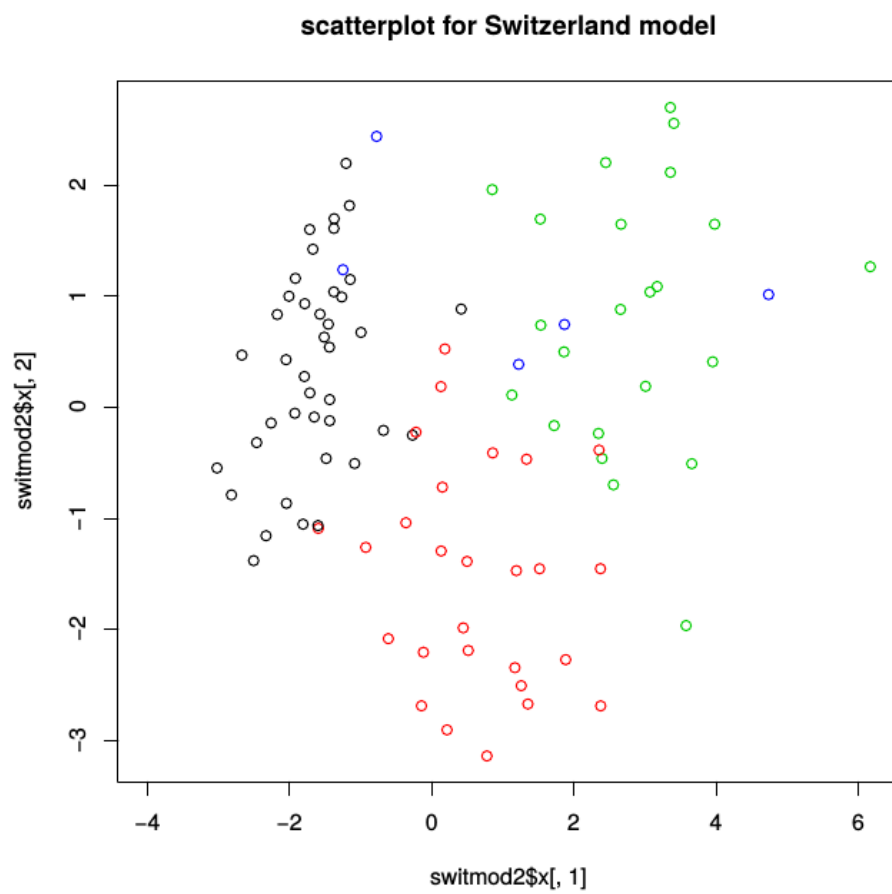


Figure 12: Scatter plot for discriminant analysis of Switzerland

### 3 Summary