

# Logarithmic Chebyshev Approximation

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For a  $p \times n$  ( $p > n$ ) matrix  $\mathbf{B}$  and  $p \times 1$  vector  $\mathbf{f}$ , the Logarithmic Chebyshev Approximation problem is stated as the following optimization problem ([1])

$$\begin{aligned} & \underset{\mathbf{x}, t}{\text{minimize}} && t \\ & \text{subject to} && 1/t \leq (\mathbf{x}^\top \mathbf{B}_{i\cdot})/\mathbf{f}_i \leq t, \quad i = 1, \dots, p \end{aligned}$$

where  $\mathbf{B}_{i\cdot}$  denotes the  $i^{\text{th}}$  row of the matrix  $\mathbf{B}$ . Note that we require each element of  $\mathbf{B}_{\cdot j}/\mathbf{f}$  to be greater than or equal to 0 for all  $j$ .

The function `logcheby` takes as input a matrix  $\mathbf{B}$  and vector  $\mathbf{f}$ , and returns the input variables necessary to solve the Logarithmic Chebyshev Approximation problem using `sqlp`.

```
R> out <- logcheby(B,f)
R> blk <- out$blk
R> At <- out$At
R> C <- out$C
R> b <- out$b
```

```
R> sqlp(blk,At,C,b)
```

## Numerical Example

As a numerical example, consider the following

```
R> data(Blogcheby)
```

	V1	V2	V3	V4	V5
[1,]	9.148	9.040	3.796	6.756	5.816
[2,]	9.371	1.387	4.358	9.828	1.579
[3,]	2.861	9.889	0.374	7.595	3.590
[4,]	8.304	9.467	9.735	5.665	6.456
[5,]	6.417	0.824	4.318	8.497	7.758
[6,]	5.191	5.142	9.576	1.895	5.636
[7,]	7.366	3.902	8.878	2.713	2.337
[8,]	1.347	9.057	6.400	8.282	0.900
[9,]	6.570	4.470	9.710	6.932	0.856
[10,]	7.051	8.360	6.188	2.405	3.052
[11,]	4.577	7.376	3.334	0.430	6.674
[12,]	7.191	8.111	3.467	1.405	0.002

```
[13,] 9.347 3.881 3.985 2.164 2.086
[14,] 2.554 6.852 7.847 4.794 9.330
[15,] 4.623 0.039 0.389 1.974 9.256
[16,] 9.400 8.329 7.488 7.194 7.341
[17,] 9.782 0.073 6.773 0.079 3.331
[18,] 1.175 2.077 1.713 3.755 5.151
[19,] 4.750 9.066 2.611 5.144 7.440
[20,] 5.603 6.118 5.144 0.016 6.192
```

```
R> data(flogcheby)
```

```
      V1
[1,] 0.626
[2,] 0.217
[3,] 0.217
[4,] 0.389
[5,] 0.942
[6,] 0.963
[7,] 0.740
[8,] 0.733
[9,] 0.536
[10,] 0.002
[11,] 0.609
[12,] 0.837
[13,] 0.752
[14,] 0.453
[15,] 0.536
[16,] 0.537
[17,] 0.001
[18,] 0.356
[19,] 0.612
[20,] 0.829
```

Note that it must be the case that each element of  $\mathbf{B}_{\cdot j}/\mathbf{f}$  must be greater than or equal to 0 for every column of  $\mathbf{B}$ .

```
R> out <- logcheby(Blogcheby, flogcheby)
R> blk <- out$blk
R> At <- out$At
R> C <- out$C
R> b <- out$b
```

```
R> out <- sqlp(blk,At,C,b)
```

Here, the outputs of interest are the optimal value of the objective function (which again we need to negate due to the negation of the objective function), and the vector  $\mathbf{X}$ , which is stored in the output variable `y`.

```
R> -out$pobj
```

```
[1] 23.08812
```

```
R> m <- ncol(Blogcheby)
```

```
R> x <- out$y[1:m]

      [,1]
[1,] 0.001106650
[2,] 0.002661286
[3,] 0.001050662
[4,] 0.002180275
[5,] 0.001435069
```

## References

- [1] Lieven Vandenbergh, Stephen Boyd, and Shao-Po Wu. Determinant maximization with linear matrix inequality constraints. *SIAM journal on matrix analysis and applications*, 19(2):499–533, 1998.