Finding the Lovasz Number

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The Lovasz Number of a graph $\mathbf{G}$, denoted $\vartheta(\mathbf{G})$, is the upper bound on the Shannon capacity of the graph ([1]). For an adjacency matrix $\mathbf{B} = [B_{ij}]$ the problem of finding the Lovasz number is given by the following primal SQLP problem

$$\begin{array}{ll} \underset{\mathbf{X}}{\text{minimize}} & tr(\mathbf{CX}) \\ \text{subject to} & \\ & \begin{array}{rcl} tr(\mathbf{X}) & = & 1 \\ X_{ij} & = & 0 \quad \text{if } B_{ij} = 1 \\ \mathbf{X} & \in & \mathcal{S}^n \end{array} \end{array}$$

The function `lovasz` takes as input an adjacency matrix `B`, and returns the input variables necessary for the Lovasz number to be found using `sqlp`.

```
R> out <- lovasz(B)
R> blk <- out$blk
R> At <- out$At
R> C <- out$C
R> b <- out$b

R> sqlp(blk,At,C,b)
```

Numerical Example

To compute the Lovasz number using `sqlp`, we need only the (weighted) adjacency matrix representing a graph object.

```
R> data(Glovasz)
```

```
      V1 V2 V3 V4 V5 V6 V7 V8 V9 V10
 [1,]  0  0  0  1  0  0  1  1  0   0
 [2,]  0  0  0  1  0  0  1  0  1   1
 [3,]  0  0  0  0  0  0  0  1  0   0
 [4,]  1  1  0  0  0  0  0  1  0   1
 [5,]  0  0  0  0  0  0  1  1  1   1
 [6,]  0  0  0  0  0  0  0  0  1   0
 [7,]  1  1  0  0  1  0  0  1  1   1
 [8,]  1  0  1  1  1  0  1  0  0   0
 [9,]  0  1  0  0  1  1  1  0  0   1
[10,]  0  1  0  1  1  0  1  0  1   0
```

The Lovasz number for the associated graph is the value of the primal objective function. Again, since the objective function was negated to make the primal problem a minimization, we negate the value of the objective function.

```
R> out <- lovasz(Glovasz)
R> blk <- out$blk
R> At <- out$At
R> C <- out$C
R> b <- out$b

R> out <- sqlp(blk,At,C,b)
R> -out$pobj

[1] 5
```

References

[1] László Lovász. On the shannon capacity of a graph. *IEEE Transactions on Information theory*, 25(1):1–7, 1979.