ADD MATH SPM

SPM 2016 Add Math paper 1 question with answer

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1. Table 1 shows the achievement of three classes. 5 Gamma, 5 Omega and 5 Beta in an Additional Mathematics test.

Class	Mean mark	Standard deviation of the marks
5 Gamma	75	4
5 Omega	70	1
5 Beta	75	2

Table 1

Which class shows the most consistent achievement in the test?
Give reason for your answer.

5 Omega because the standard deviation of the mark by 5 Omega is the lowest among three classes.

2. Two fair coins are tossed simultaneously. H denotes the event of obtaining the head and T denotes the event of obtaining tail.

   (a) List the sample space using set notation

   S={(H,H),(H,T),(T,H),(T,T)}

   (b) Given X is a discrete random variable which represent the number of heads obtain, list the possible values of X.

   X = 0, 1, 2

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3. Diagram 3 shows five cards of different letters.

B	0	L	E	H

Diagram 3
Rajah 3

Calculate the number of different ways to arrange all the cards in a row if

(a) there is no restriction.

Number of different ways = 5!
=120

(b) the first card and the last card are consonants.

Number of different ways = 3 x 3! x 2
= 3 x 6 x2
=36

4. Ben and Chandran are qualified to the final of a badminton tournament in their school. The player who first wins any two sets of the match is the winner. The probability Ben wins in any of the sets is $\frac{3}{7}$.

Find the probability that

(a) The winner is determined after two set of the match.

Answer :

$$\text{Probability} = \left(\frac{3}{7} \times \frac{3}{7}\right) + \left(\frac{4}{7} \times \frac{4}{7}\right) = \frac{25}{49}$$

(b) Ben will win the tournament after playing three set of the match.

Answer :

$$\text{Probability} = \left(\frac{4}{7} \times \frac{3}{7} \times \frac{3}{7}\right) + \left(\frac{3}{7} \times \frac{4}{7} \times \frac{3}{7}\right)$$
$$= \frac{72}{343}$$

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5. Given

$$\int_1^h (2x-6)dx = -4$$

find the value of h.

Answer :

$$\int_1^h (2x-6)dx = -4$$

$$\left[\frac{2x^2}{2} - 6x\right]_1^h = -4$$

$$h^2 - 6h - (1-6) + 4 = 0$$

$$h^2 - 6h + 9 = 0$$

$$(h-3)(h-3) = 0$$

$$h - 3 = 0$$

$$h = 3$$

6. The surface area of a cube increases at a constant rate of 15 cm² s^{-1}. Find the rate of change of side length, in cm s^{-1}, when the volume of the cube is 125 cm^3.

Answer :

$V = 125$

Let the length of side be x cm.

$$x^3 = 125 = 5^3$$

$$x = 5$$

$$A = 6x^2$$

$$\frac{dA}{dx} = 12x^2 = 12 \times 5^2 = 300$$

$$\frac{dA}{dt} = \frac{dA}{dx} \cdot \frac{dx}{dt}$$

$$15 = 300 \times \frac{dx}{dt}$$

$$\frac{dx}{dt} = \frac{15}{300} = \frac{1}{20} \text{ cm s}^{-1}$$

7. Diagram 7 shows a part of curve y = $\frac{2x-6}{x=2}$ and a straight line.

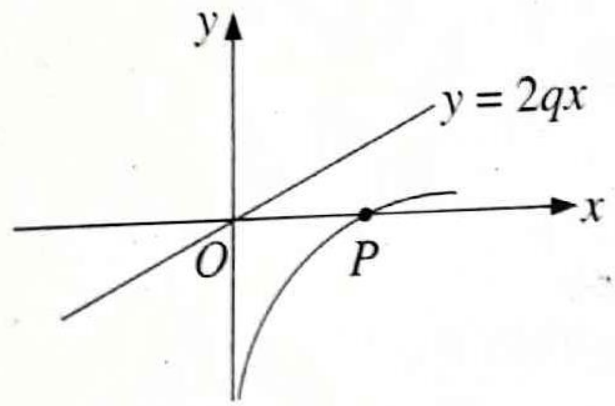

Diagram 7

It is given that the straight line is parallel to the tangent of the curve at point P. Find the value of q.

Answer :

At P, $y = 0$, $\quad \frac{2x-6}{x+2} = 0$

$$2x = 6$$

$$x = 3$$

$$P = (3, 0)$$

$$y = \frac{2x-6}{x+2}$$

$$\frac{dy}{dx} = \frac{(x+2)2 - (2x-6)1}{(x+2)^2}$$

$$= \frac{2x+4-2x+6}{(x+2)^2}$$

$$= \frac{10}{(x+2)^2}$$

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$$\text{At } P,\ \frac{dy}{dx} = \frac{10}{(3+2)^2} = \frac{2}{5}$$

$$\frac{dy}{dx} = 2q$$

$$\frac{2}{5} = 2q$$

$$q = \frac{1}{5}$$

8. The straight line 2y = 3x + I1 + 4 intersects the y-axis at 5k, where h and are constants. Express h in terms of k.

Answer :

$$(0, 5k) : x = 0, y = 5k$$

$$2y = 3x + h + k$$

$$2(5k) = 0 + h + k$$

$$10k - k = h$$

$$h = 9k$$

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9. Diagram 9 shows two straight lines on a Cartesian plane.

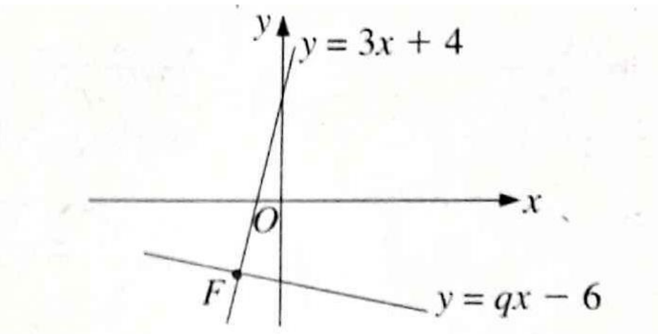

Diagram 9

Both are straight are perpendicular to each other.

(a) State the value of q.

Answer :

$$m_1 m_2 = -1 \qquad 3 \times q = -1$$
$$q = -\frac{1}{3}$$

(b) Find the coordinate of F.

Answer :

(b) $y = 3x + 4$ .... ①

$y = -\frac{1}{3}x - 6$ .... ②

$$3x + 4 = -\frac{1}{3}x - 6$$
$$9x + 12 = -x - 18$$
$$10x = -30$$
$$x = -3$$
$$y = = -9 + 4 = -5$$

$F = (-3, -5)$

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10. Diagram 10 shows a regular hexagon with centre O.

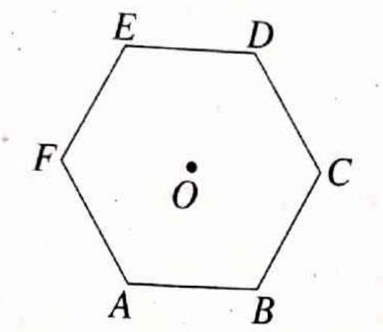


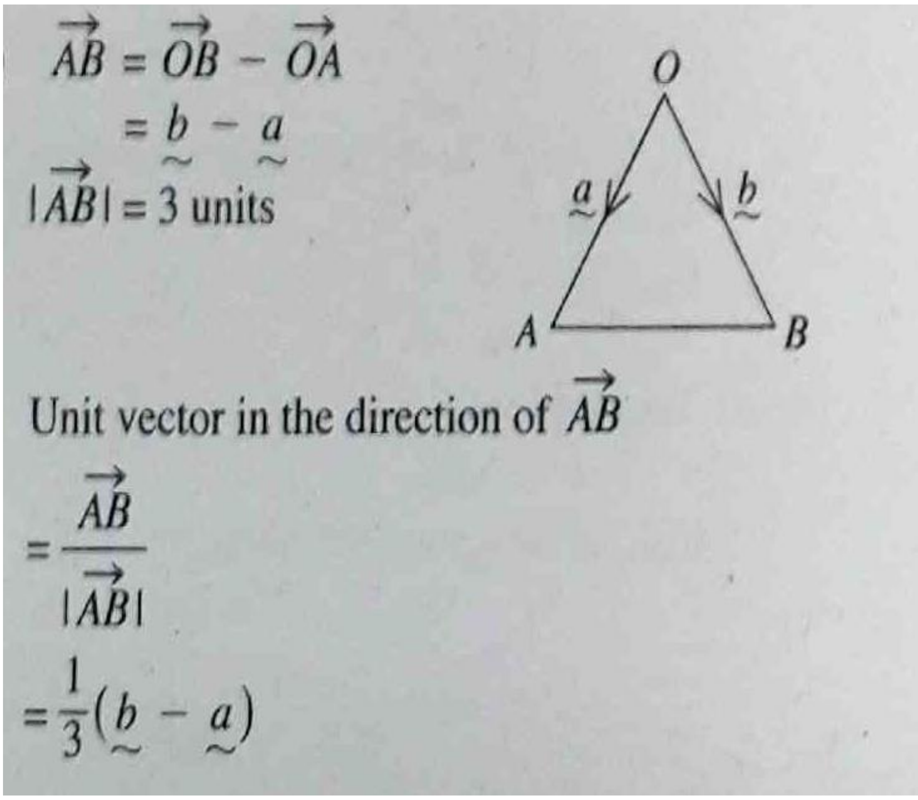
Diagram 10

(a) Express $\overrightarrow{AC} + \overrightarrow{CE} + \overrightarrow{CB}$ as a single vector.

$$\boldsymbol{\overrightarrow{AC} + \overrightarrow{CE} + \overrightarrow{CB} = \overrightarrow{AE} + \overrightarrow{EF} = \overrightarrow{AF}}$$

(b) Given $\overrightarrow{OA} = \underline{a}$,$\overrightarrow{OB} = \underline{b}$ and the length of each side of the hexagon is 3 units find the unit vector in direction of $\overrightarrow{AB}$, in term of $\underline{a}$ and $\underline{b}$.

Answer :



$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$$
$$= \underset{\sim}{b} - \underset{\sim}{a}$$
$$|\overrightarrow{AB}| = 3 \text{ units}$$

Unit vector in the direction of $\overrightarrow{AB}$

$$= \frac{\overrightarrow{AB}}{|\overrightarrow{AB}|}$$
$$= \frac{1}{3}(\underset{\sim}{b} - \underset{\sim}{a})$$

11. Given the function $f : x \rightarrow 3x - 2$, find

(a) The value of $f(x)$ maps onto itself,

Answer :

$$
\begin{aligned}
f(x) &= x \\
3x - 2 &= x \\
3x - x &= 2 \\
2x &= 2 \\
x &= 1
\end{aligned}
$$

(b) The value of h such that $f(2 - h) = 4h$

Answer :

$$
\begin{aligned}
f(x) &= 3x - 2 \\
f(2 - h) &= 3(2 - h) - 2 \\
&= 6 - 3h - 2 \\
&= 8 - 3h \\
8 - 3h &= 4h \\
7h &= 8 \\
h &= \frac{8}{7} = 1\frac{1}{7}
\end{aligned}
$$

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12. Given the function m : $x \rightarrow Px + 1$, h : $x \rightarrow 3x - 5$ and mh$(x) = 3px + q$. Express p in terms of q .

Answer :

$m(x) = px + 1,\ h(x) = 3x - 5$

$$\begin{aligned} mh(x) &= m(3x - 5) \\ &= p(3x - 5) + 1 \\ &= 3px - 5p + 1 \end{aligned}$$

Given $mh(x) = 3px + q$

$3px - 5p + 1 = 3px + q$

Comparing both sides of the equation,

$$\begin{aligned} q &= -5p + 1 \\ 5p &= 1 - q \\ p &= \frac{1 - q}{5} \end{aligned}$$

13. Given the function g : $x \rightarrow 3x + 1$, and $fg: x \rightarrow 9x + 6x - 4$, find

(a) $g^{-1}(x)$.

Answer :

$$\begin{aligned} 3x + 1 &= y \\ 3x &= y - 1 \\ x &= \frac{y - 1}{3} \\ g^{-1}(x) &= \frac{x - 1}{3} \end{aligned}$$

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(b) $f(x)$.

Answer :

$$fg(x) = 9x^2 + 6x - 4$$
$$f(3x + 1) = 9x^2 + 6x - 4$$

Let $3x + 1 = u$, then $x = \dfrac{u-1}{3}$

$$f(u) = 9\left(\frac{u-1}{3}\right)^2 + 6\left(\frac{u-1}{3}\right) - 4$$
$$= \frac{9(u-1)^2}{9} + 2(u-1) - 4$$
$$= u^2 - 2u + 1 + 2u - 2 - 4$$
$$f(u) = u^2 - 5$$
$$f(x) = x^2 - 5$$

14. Given $log_a 7 = r$, express in term of r

(a) $log_a 49$,

Answer :

$$\log_a 49 = \log_a 7^2$$
$$= 2\log_a 7 = 2r$$

(b) $log_7 343a^2$

Answer :

$$\log_7 343a^2 = \log_7 343 + \log_7 a^2$$
$$= \log_7 7^3 + 2\log_7 a$$
$$= 3\log_7 7 + 2\left(\frac{1}{\log_a 7}\right) = 3 + \frac{2}{r}$$

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15. Given $3^p = 5^q = 15^r$, express r in term of p and q .

Answer:

$$3^p = 5^q$$

$$\log 3^p = \log 5^q$$

$$p \log 3 = q \log 5$$

$$\log 3 = \frac{q}{p} \log 5 \qquad \text{..........} \ (1)$$

$$5^q = 15^r$$

$$5^q = (3 \times 5)^r$$

$$5^q = 3^r \times 5^r$$

$$\log 5^q = \log (3^r \times 5^r)$$

$$= \log 3^r + \log 5^r$$

$$q \log 5 = r \log 3 + r \log 5 \qquad \text{.........} \ (2)$$

Substitute (1) into (2) :

$$q \log 5 = r\left(\frac{q \log 5}{p}\right) + r \log 5$$

$$q = \frac{rq}{p} + r$$

$$pq = rq + rp$$

$$pq = r(q + p)$$

$$r = \frac{pq}{p + q}$$

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16. The variable x and y are related by the equation $y = 2x^2 - \frac{q}{x}$, where q is a constant. A straight line is obtained by plotting xy against x^3 , as shown in Diagram 16.

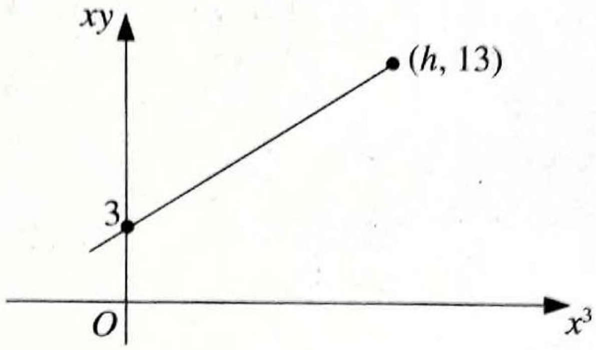

Diagram 16

Find the value of h and of q .

Answer :

$$y = 2x^2 - \frac{q}{x}$$

$$xy = 2x^3 - q$$

$$Y = mX + c$$

$$m = 2, \; c = -q$$

$$m = \frac{13 - 3}{h - 0}$$

$$2 = \frac{10}{h}$$

$$2h = 10$$

$$h = 5$$

$$c = 3$$

$$-q = 3$$

$$q = -3$$

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17. It is given that the quadratic equation $3x^2 + 8x + 7 = 0$ has roots $\alpha\ and\ \beta$.
Form a quadratic equation with roots 3α and 3β .

Answer :

$3x^2 + 8x + 7 = 0$

$a = 3,\ b = 8,\ c = 7$

$$\alpha + \beta = -\frac{b}{a} = -\frac{8}{3}$$

$$\alpha\beta = \frac{c}{a} = \frac{7}{3}$$

Sum of new roofs $= 3\alpha + 3\beta$

$= 3(\alpha + \beta)$

$= 3\left(-\frac{8}{3}\right)$

$= -8$

Product of new roots $= 3\alpha \times 3\beta$

$= 9\alpha\beta$

$= 9\left(\frac{7}{3}\right)$

$= 21$

The equation is $x^2 - (-8)x + 21 = 0$

$x^2 + 8x + 21 = 0$

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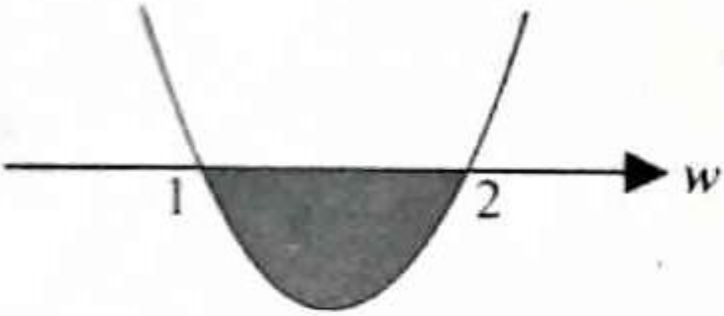
18. Given the quadratic function $f(x) = x^2 + 2wx + 3w - 2$, where w is a constant, is always positive when $p < w < q$.
Find the value of p and of q.

Answer :

$$f(x) = x^2 + 2wx + 3w - 2$$
$$a = 1,\ b = 2w,\ c = 3w - 2$$
$$b^2 - 4ac < 0$$
$$(2w)^2 - 4(1)(3w - 2) < 0$$
$$4w^2 - 12w + 8 < 0$$
$$w^2 - 3w + 2 < 0$$
$$(w - 1)(w - 2) < 0$$



$$1 < w < 2$$
$$p < w < q$$
Thus, $p = 1$, $q = 2$

19. Diagram 19 shows a circle with centre O and radius 8cm.

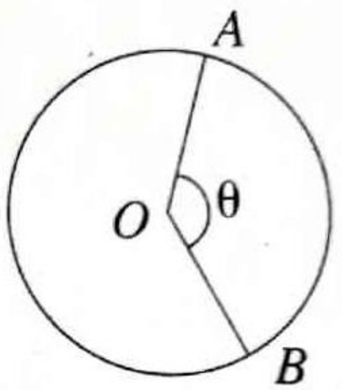


Diagram 19

Given the length of the minor arc AB is 16cm
[use , π = 3.142]

(a) State the value of θ in radians.

Answer :

Length of arc $AB = 16$ cm

$$r \times \theta = 16$$
$$8 \times \theta = 16$$
$$\theta = 2 \text{ radians}$$

(b) Find the area of the major sector OAB, in cm^2 ,correct to four significant figures.

Answer :

$$\theta + x = 2\pi$$
$$x = 2\pi - \theta$$
$$= 2(3.142) - 2 = 4.284 \text{ radians}$$

$$\text{Area of major sector } OAB = \frac{1}{2} r^2 x$$
$$= \frac{1}{2} \times 8^2 \times 4.284$$
$$= 137.088$$
$$= 137.1 \text{ cm}^2$$

20. Solve the equation tan α = 4-3 cot α for $0° \leq \alpha \leq 180°$.

Answer :

tan α = 4 – 3 cot α

tan α = 4 – $\frac{3}{tan\,\alpha}$

tan^2 = 4 tan α 4 tan α – 3

tan^2 α – 4 tan α + 3 = 0

(tan α -1)(tan α - 3) = 0

Tan α = 1

α = 45º

or tan α = 3

α =71º 34’

thus, α = 45º ,71º 34’

21. A stall selling ‘the tarik’ gives choice to the customers of using either condensed milk or evaporated milk in their drinks. On particular day the stalls has 70 cans of condensed milk and 48 can of evaporated milk. The stall used 5 can of condensed milk and 3 can of evaporated milk in a day.
After how many days, the remainder can of both milk are the same?

Answer :

Condensed milk: 70, 65, 60, …

$a = 70,\ d = -5 \qquad T_n = a + (n-1)d$

$= 70 + (n-1)(-5)$

$= 70 - 5n + 5$

$= 75 - 5n$ .... ①

Evaporated milk: 48, 45, 42, …

$a = 48,\ d = -3 \qquad T_n = a + (n-1)d$

$= 48 + (n-1)(-3)$

$= 48 - 3n + 3$

$= 51 - 3n$ .... ②

① = ②: $75 - 5n = 51 - 3n$

$75 - 51 = 5n - 3n$

$2n = 24$

$n = 12$

After 12 days, the remainder numbers of cans are the same.

22. It is given that (x + 1), (2x-7) and $\left(\frac{x+1}{4}\right)$ are three consecutive term of a geometric progression with a common ration of $\frac{1}{2}$.

Find

(a) The value of x

Answer :

$$\frac{2x-7}{x+1} = \frac{1}{2}$$

$$2(2x-7) = x+1$$

$$4x - 14 = x + 1$$

$$4x - x = 14 + 1$$

$$3x = 15$$

$$x = 5$$

(b) The first term if (x+1) is the 12^{th} term of progression.

Answer :

$$T_{12} = x + 1$$

$$ar^{11} = 5 + 1$$

$$a \times \left(\frac{1}{2}\right)^{11} = 6$$

$$a \times \frac{1}{2\,048} = 6$$

$$a = 12\,288$$

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23. Mohan took 4 minutes to complete the first kilometre of a 15 km run . He could not sustain his stamina thus for each subsequent kilometre, he took $\frac{1}{8}$ more time compared to the time he took for the previous kilometre.
The participant who finished the run more than two hours are not qualified for the state level run.
Did Mohan qualified? Show calculation to support your answer.

Answer :

$4, 4.5, 5.0625, \ldots$

$a = 4, r = 1.125, n = 15$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$S_{15} = \frac{4(1.125^{15} - 1)}{1.125 - 1}$$

$$= 155.3 \text{ minutes}$$

$$= 2.588 \text{ hours}$$

Since the time taken is more than 2 hours, Mohan did not qualify.

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24. Customer Association ABC conduct a survey on lifespan of a particular brand light buld. It is found that the probability lifespan of the buld less than six months is p.

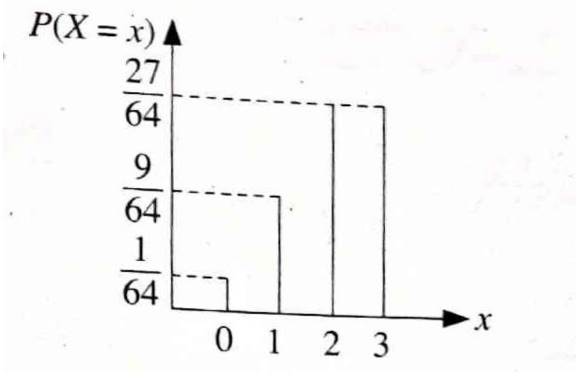

Diagram 24

A sample of 3 light bulbs is selected at random. Diagram 24 shows the result of the survey. Such that X represent the numbe of light bulbs with a lifespan less than six months.

(a) Find the value of p

Answer :

$$n = 3, \quad P(X = r) = {}^3C_r p^r (1 - p)^{3-r}$$

$$P(X = 3) = {}^3C_3 p^3 (1 - p)^0$$

$$\frac{27}{64} = 1 \times p^3 \times 1$$

$$p^3 = \left(\frac{3}{4}\right)^3$$

$$p = \frac{3}{4}$$

(b) Calculate how many light bulbs are still functioning after six months. If 20 light bulbs from the same brand are used.

Answer :

$$P(\text{lifespan more than 6 months}) = 1 - p$$

$$= 1 - \frac{3}{4}$$

$$= \frac{1}{4}$$

$$\text{Number of bulbs} = 20 \times \frac{1}{4} = 5$$

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25. Diagram 25.1 shows the front view of four pieces of wood with the same width. The total front area of the four pieces of wood is 20 cm^2. The four pieces of wood are used to produce a rectangular photo frame as shown in Diagram 25.2

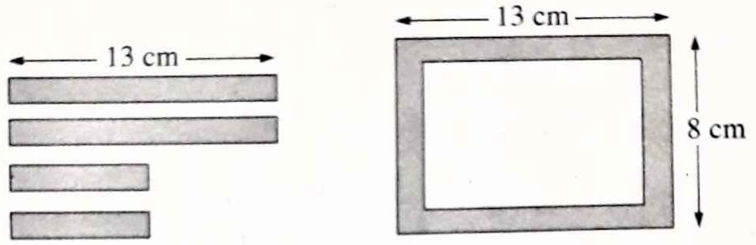
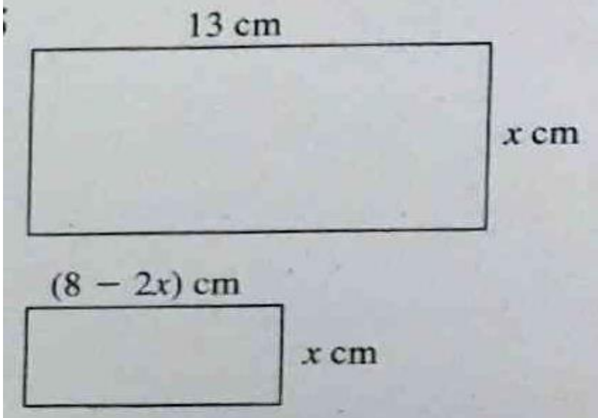


Diagram 25.1 Diagram 25.2

Calculate the width, in cm , of the wood.

Answer :



Total area = 20 cm²

$$2(13 \times x) + 2x(8 - 2x) = 20$$
$$26x + 16x - 4x^2 - 20 = 0$$
$$4x^2 - 42x + 20 = 0$$
$$2x^2 - 21x + 10 = 0$$
$$(2x - 1)(x - 10) = 0$$
$$2x - 1 = 0$$
$$x = \frac{1}{2}$$

or $x - 10 = 0$

$x = 10$ (rejected)

The width of the wood is $\frac{1}{2}$ cm.

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