Capitol Technology University
Department of Electrical Engineering
EL 204 Digital Electronics (Spring 2020)
HW # 3 (Due 03/09/2020)

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1) Using Boolean algebra, to prove the following Boolean expressions:
   a. $AB + (\bar{A} + \bar{B})C = AB + C.$
   b. $ABCD + AB(\overline{CD}) + (\overline{AB})CD = AB + CD.$
   c. $A + B(AC + (B + \bar{C})D) = A + BD.$

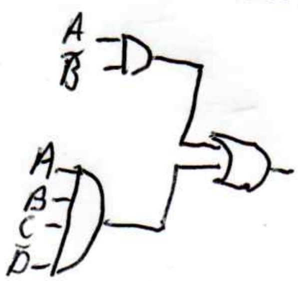
2) Given the following truth table, find out the binary expression for output **F** in **standard SOP** form. Simplify the expression and reduce it to **SOP** form using Boolean Algebra. Create the AND/OR two-level implementation for the minimized SOP expression.

$F = A\bar{B}\bar{C}\bar{D} + A\bar{B}\bar{C}D + A\bar{B}C\bar{D} + A\bar{B}CD + ABC\bar{D}$

$A\bar{B}\bar{C}(\bar{D}+D) + A\bar{B}C(\bar{D}+D) + ABC\bar{D}$

$A\bar{B}(C+\bar{C}) + ABC\bar{D}$

$A\bar{B} + ABC\bar{D}$



A	B	C	D	F
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1	0	1	1	1
1	1	0	0	0
1	1	0	1	0
1	1	1	0	1
1	1	1	1	0

3) What is the domain of the Boolean expression $(A + \bar{B})(B + C)$? Convert this expression into **standard POS** form, and write down the corresponding truth table.

domain = ABC

$F(A,B,C) = (A+\bar{B}+C\bar{C})(A\bar{A}+B+C)$

$= (A+\bar{B}+C)(A+\bar{B}+\bar{C})(A+B+C)(\bar{A}+B+C)$

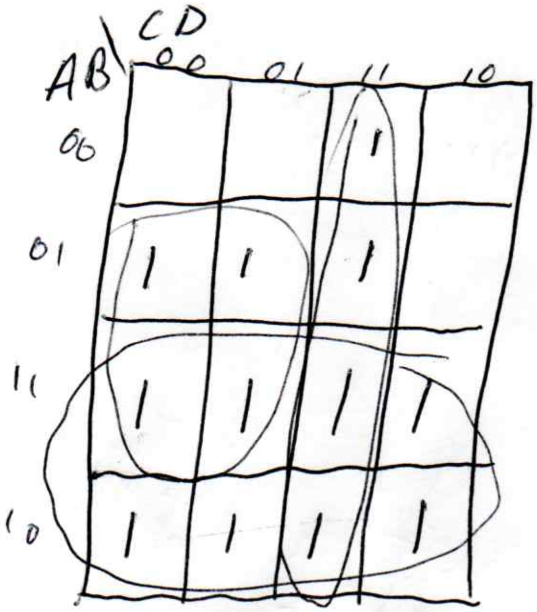
4) Use a Karnaugh map for the following:
   a. Find the minimum SOP form of Boolean expression

   $\bar{A}B(\bar{C}\bar{D} + \bar{C}D) + AB(\bar{C}\bar{D} + \bar{C}D) + A\bar{B}\bar{C}D$

   b. Find the minimum POS form of Boolean expression

   $(A + \bar{B})(\bar{A} + C)(A + \bar{B} + \bar{C})(\bar{A} + \bar{B} + C)$

5) Reduce the function specified in the given truth table to its minimum SOP by using a Karnaugh map.



A	B	C	D	F
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	1
0	1	0	0	1
0	1	0	1	1
0	1	1	0	0
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

$\boxed{CD + B\bar{C} + A}$

① a) $AB+(\bar{A}+\bar{B})C = \underline{AB+C}$

$AB+\bar{A}C+\bar{B}C$

$AB+\bar{A}BC+\bar{A}\bar{B}C+\bar{B}C$

$AB+\bar{A}BC+\bar{B}C(\bar{A}+1)$

$AB+\bar{A}BC+\bar{B}C$

$B(A+\bar{A}C)+\bar{B}C$

$B(A+C)+\bar{B}C$

$AB+BC+\bar{B}C$

$AB+C(B+\bar{B})$

$\underline{AB+C}$

b) $ABCD+AB(\overline{CD})+(\overline{AB})CD = \underline{AB+CD}$

$ABCD+AB(\bar{C}+\bar{D})+(\bar{A}+\bar{B})CD =$

$ABCD+AB\bar{C}+AB\bar{D}+\bar{A}CD+\bar{B}CD$

$ABCD+AB\bar{C}D+AB\bar{C}\bar{D}+AB\bar{D}+\bar{A}CD+\bar{B}CD$

$ABD(C+\bar{C})+AB\bar{C}\bar{D}+AB\bar{D}+\bar{A}CD+\bar{B}CD$

$AB(\cancel{D}+\bar{D})+AB\bar{C}\bar{D}+\bar{A}CD+\bar{B}CD$

$AB+AB\bar{C}\bar{D}+\bar{A}CD+\bar{B}CD$

$AB\cancel{(1+\bar{C}\bar{D})}+\bar{A}CD+\bar{B}CD$

$AB+\bar{A}CD+\bar{B}CD$

$AB+\bar{A}BCD+\cancel{\bar{A}\bar{B}CD}+\cancel{\bar{A}\bar{B}CD}+A\bar{B}CD$

$AB+CD(\cancel{\bar{A}B}+\cancel{A\bar{B}})$

$\underline{AB+CD}$

c) $A+B(AC+(B+\bar{C})D) = \underline{A+BD}$

$A+B(AC+BD+\bar{C}D)$

$A+BAC+BD+B\bar{C}D$

$A+ABC+BD\cancel{(\bar{C}+1)}$

$A(1+BC)+BD$

$A\cancel{(1+B)(1+C)}+BD$

$\underline{A+BD}$

(4) a) $\bar{A}B\bar{C}\bar{D} + \bar{A}B\bar{C}D + AB\bar{C}\bar{D} + AB\bar{C}D + A\bar{B}\bar{C}D$

AB \ CD	00	01	11	10
00				
01	1	1		
11	1	1		
10		1		

$\boxed{B\bar{C} + A\bar{C}D}$

b) $(A+\bar{B}+C)(A+\bar{B}+\bar{C})(\bar{A}+B+C)(\bar{A}+\bar{B}+C) \rightarrow$ Standard POS

AB \ C	0	1
00		1
01		1
11		
10	1	1

$(\bar{a}+c)(a+\bar{b})$