5. Минимални Крайни Автомати

Def Релация на Myhill-Nerode

Нека $A = (\Sigma, Q, I, \delta, F)$ е дет. краен автомат

Тогава деф. $\equiv_A \subseteq Q \times Q$ така:

$$(\forall p, q \in Q)\Big[\, p \equiv_A q \iff (\forall w \in \Sigma^*)\big((\exists f \in F)(p \overset{w}{\mapsto} f) \iff (\exists f \in F)(q \overset{w}{\mapsto} f) \big) \Big]$$

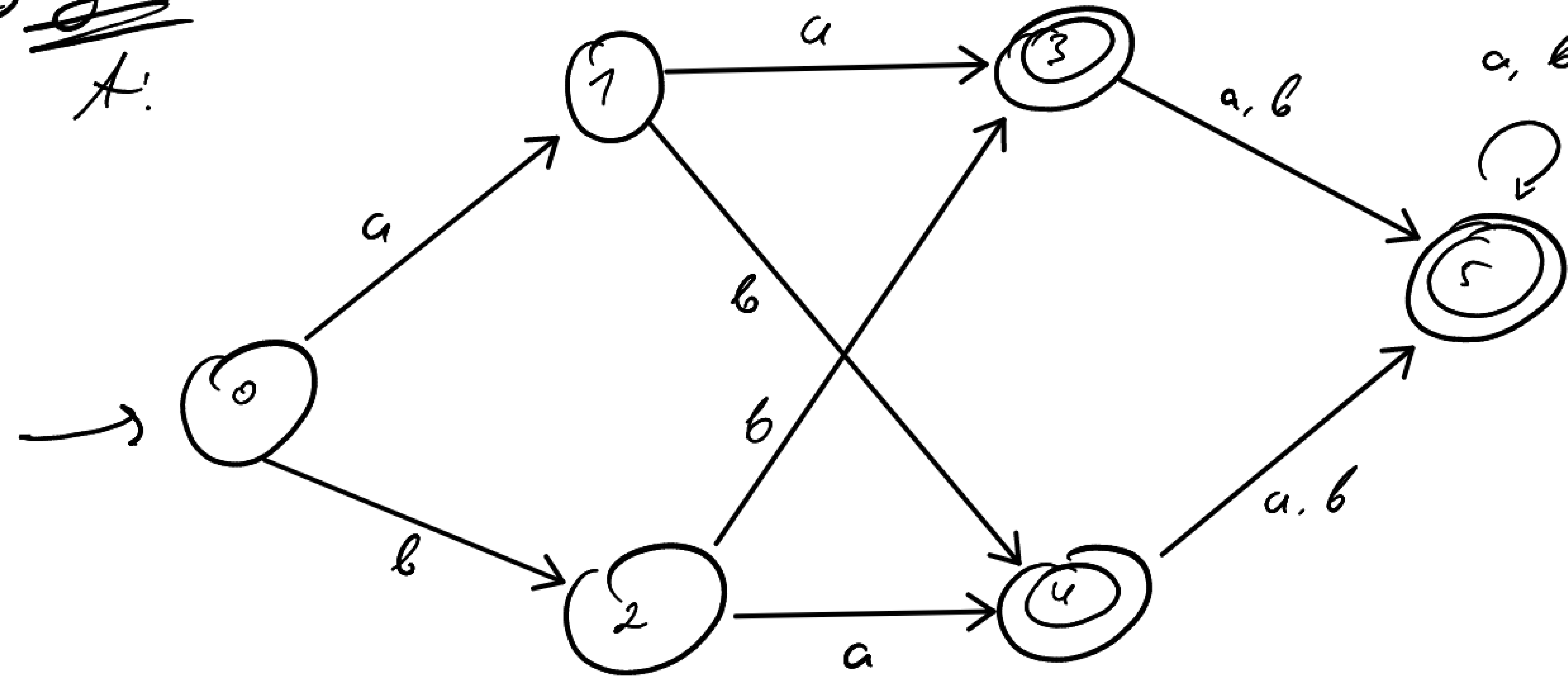
Тогава автоматът

$$A_m = (\Sigma,\ Q/_{\equiv_A},\ [q_0],\ \Delta,\ F/_{\equiv_A})$$

е мин. тот. дет. кр. авт. с $L(A_m) = L(A)$

Задача:

A:



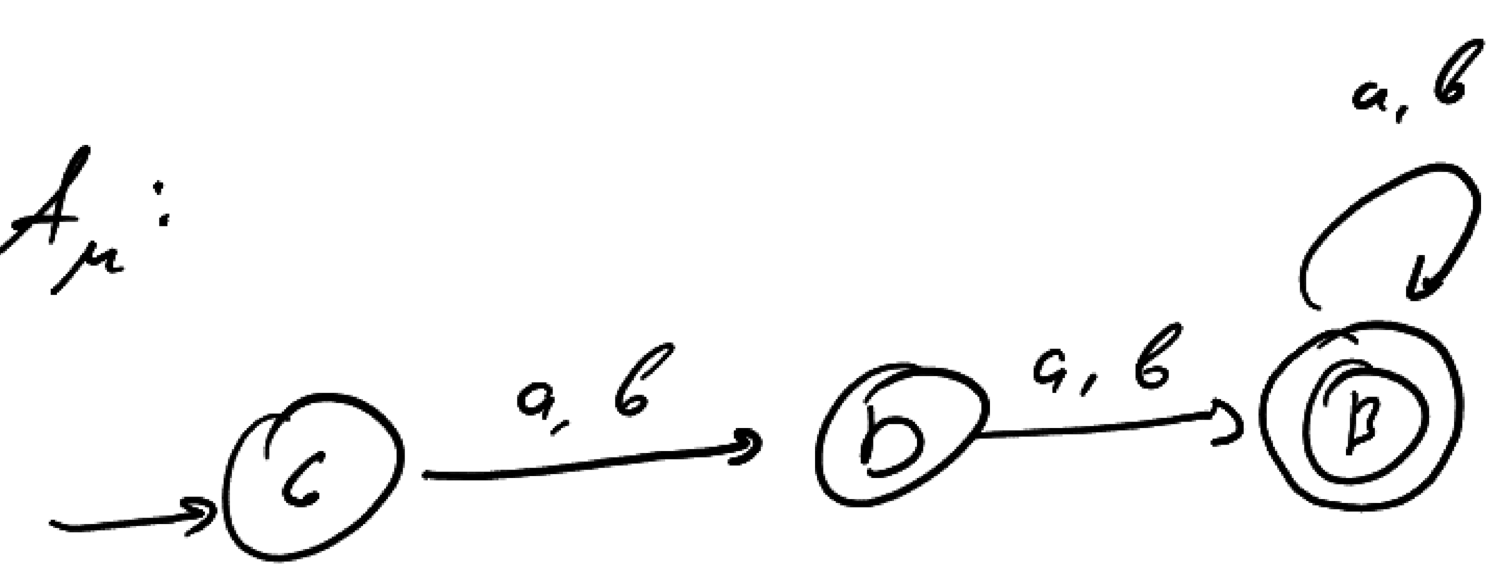
A	a	b
0	A	A
1	B	B
2	B	B

B	a	b
3	B	B
4	B	B
5	B	B

C	a	b
0	D	D

D	a	b
1	B	B
2	B	B

A_m:



Задача Да се построи МДКА с език L:

а) $\Sigma = \{a, b, c\}$

$L = \{ w \in \Sigma^* \mid w$ не завършва на abc и започва с $aaa \}$

б) $\Sigma = \{a, b, c, \ldots, z\}$

$L = \Sigma^* . \{hers, his, he\}$

в) $\Sigma = \{a, b, c\}$

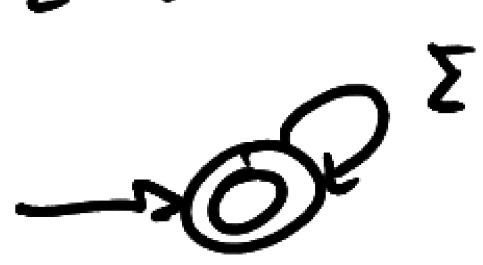
$L = \Sigma^* \setminus (\{ab\}^* \cup (\{a, c\}\Sigma^* \cap \Sigma^*\{b\}))$

Решение

а) $L = \{w \in \Sigma^* \mid w$ не зав. на $'abc'\} \cap \{w \in \Sigma^* \mid w$ започва с $'aaa'\}$

$= \Sigma^* \setminus (\Sigma^* . \{abc\}) \cap \{aaa\} . \Sigma^*$

A_{Σ^*}: $L(A_{\Sigma^*}) = \Sigma^*$

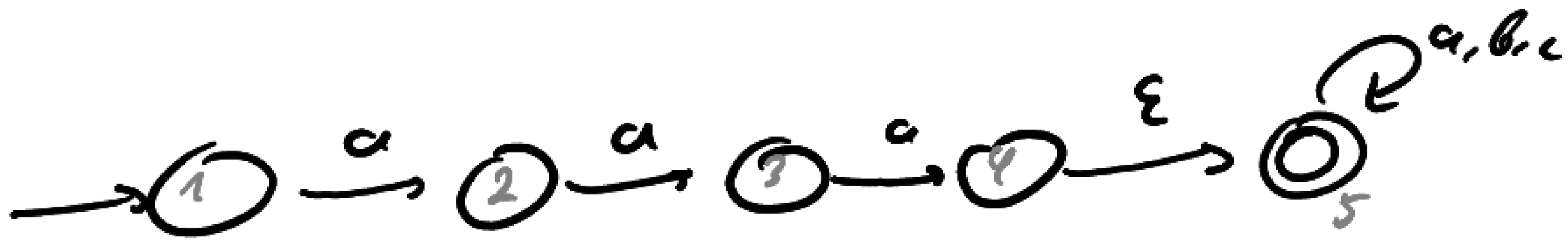


A_{abc}: $L(A_{abc}) = \{abc\}$

A_{aaa}: $L(A_{aaa}) = \{aaa\}$

$A_1: \quad L(A_1) = \Sigma^* \cdot \{abc\} = L(A_{\Sigma^*}) \circ L(A_{abc})$

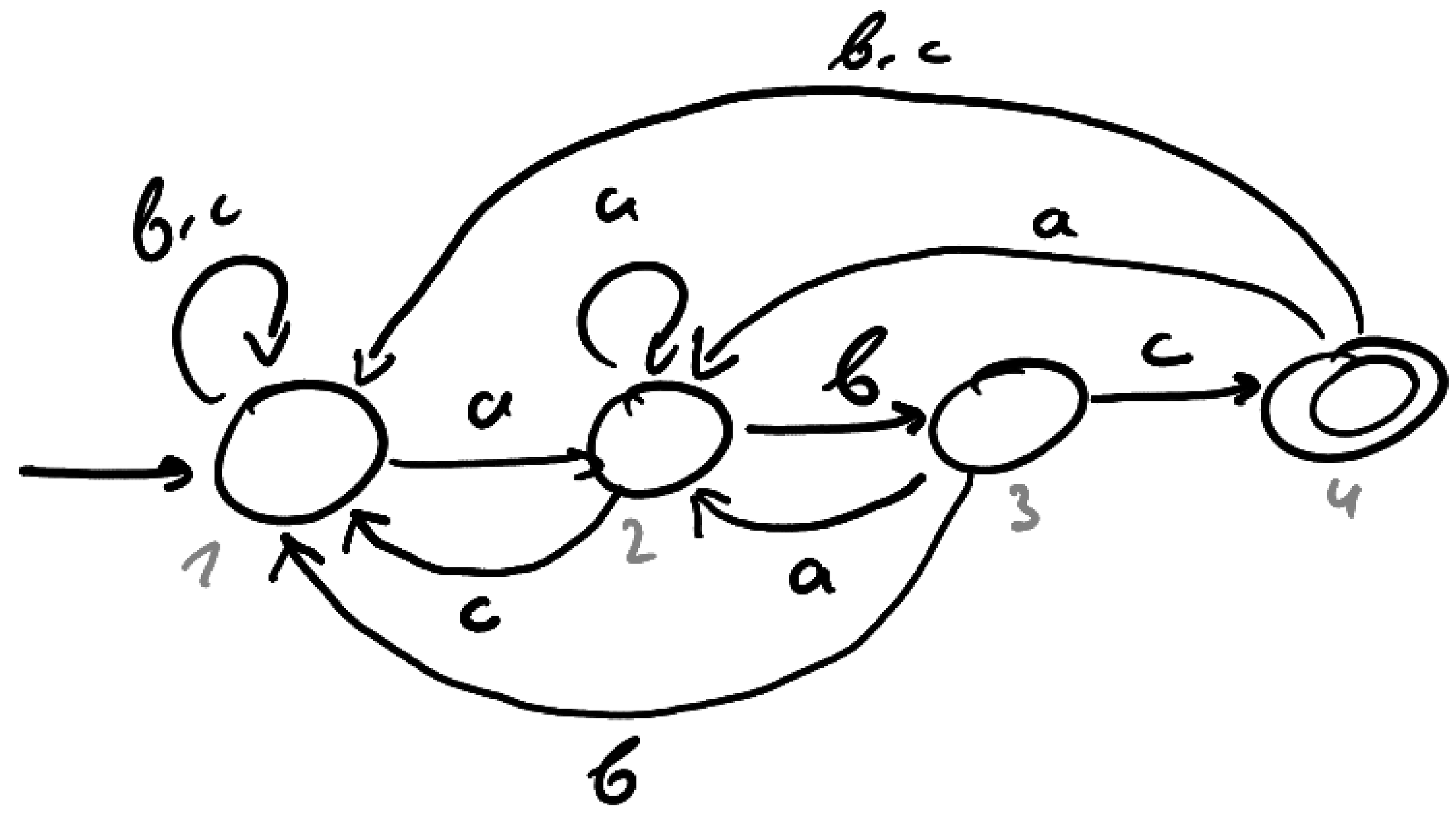
$A_2: \quad L(A_2) = \{aaa\} \cdot \Sigma^* = L(A_{aaa}) \circ L(A_{\Sigma^*})$



Тъй като конструкцията за сечение изисква детерминираност, ще детерминирам A_1 и A_2

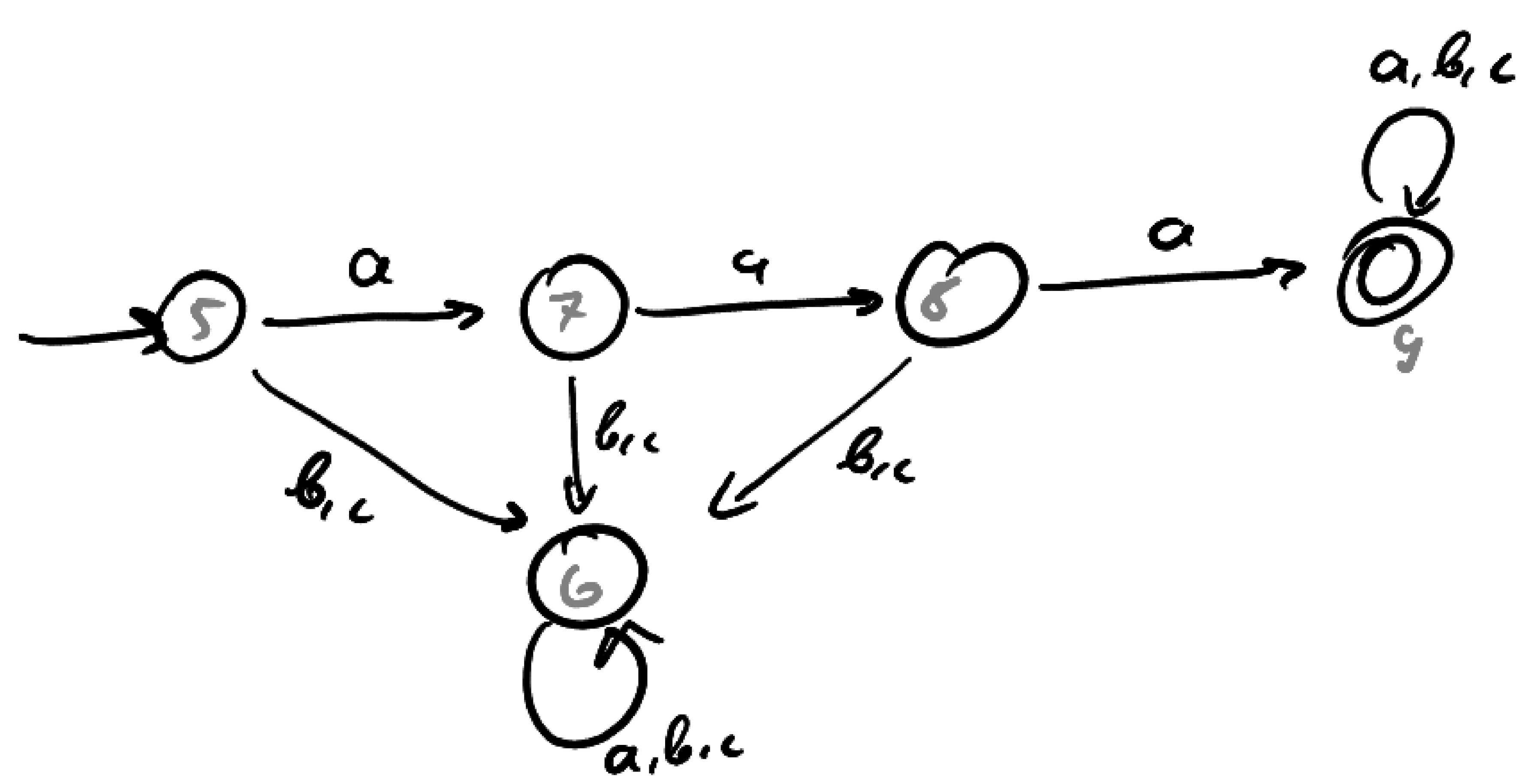
A_1^D:

		a	b	c
→ 1:	{1, 2}	1, 3, 2 (2)	1, 2 (1)	1, 2 (1)
2:	{1, 2, 3}	1, 3, 2 (2)	1, 4, 2 (3)	1, 2 (1)
3:	{1, 2, 4}	1, 3, 2 (2)	1, 2 (1)	1, 5, 2 (4)
4:	{1, 2, 5}	1, 3, 2 (2)	1, 2 (1)	1, 2 (1)

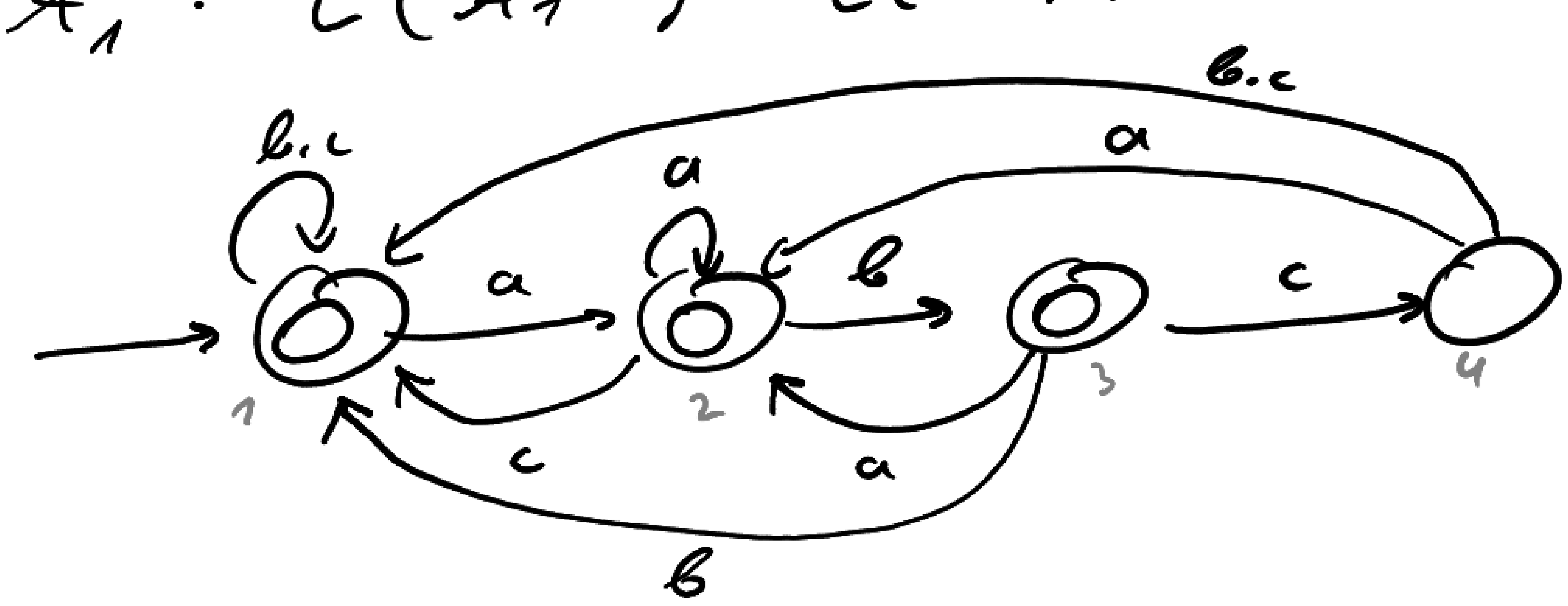


A_2^D:

		a	b	c
→ 5:	{1}	2 (7)	∅ (6)	∅ (6)
6:	∅	∅ (6)	∅ (6)	∅ (6)
7:	{2}	3 (8)	∅ (6)	∅ (6)
8:	{3}	4, 5 (9)	∅ (6)	∅ (6)
9:	{4, 5}	4, 5 (9)	4, 5 (9)	4, 5 (9)



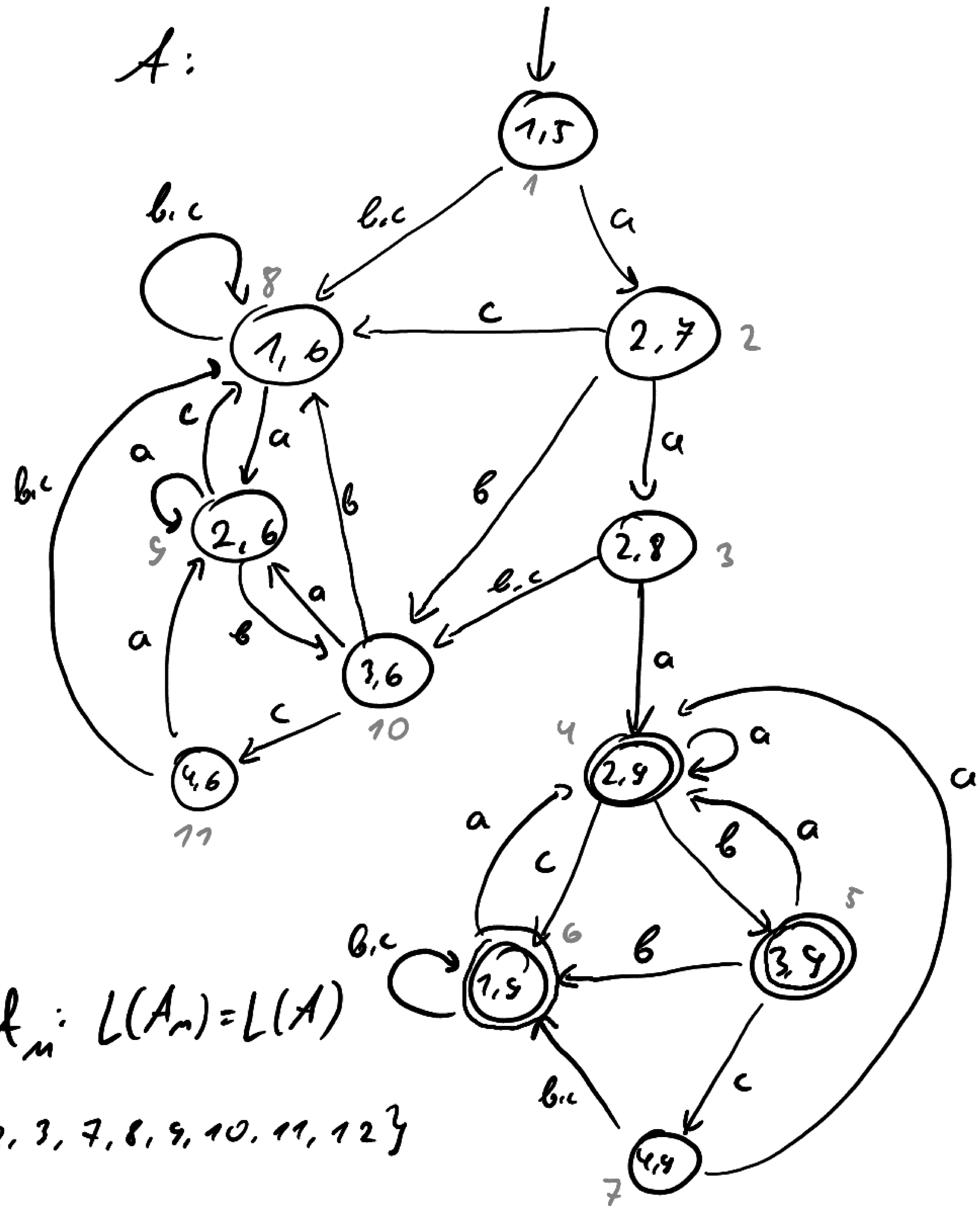
$\overline{A_1^D}: \quad L(\overline{A_1^D}) = \overline{L(A_1^D)} = \Sigma^* \setminus L(A_1^D)$



Сега остава да построим A: $L(A) = L(\overline{A_1^D}) \cap L(A_2^D)$

	a	b	c
→ (1,5)	(2,7)	(1,6)	(1,6)
(1,6)	(2,6)	(1,6)	(1,6)
(2,7)	(2,8)	(3,6)	(1,6)
(2,8)	(2,9)	(3,6)	(1,6)
(3,6)	(2,6)	(1,6)	(4,6)
◎ (2,9)	(2,9)	(3,9)	(1,9)
(2,6)	(2,6)	(3,6)	(1,6)
(4,6)	(2,6)	(1,6)	(1,6)
◎ (3,9)	(2,9)	(1,9)	(4,9)
◎ (1,9)	(2,9)	(1,9)	(1,9)
(4,9)	(2,9)	(1,9)	(1,9)

A:



И сега минимизация : A_m: $L(A_m) = L(A)$

$\underset{\sim}{A} = \{4, 5, 6\}$ $\underset{\sim}{B} = \{1, 2, 3, 7, 8, 9, 10, 11, 12\}$

A	a	b	c
4	A	A	A
5	A	A	B
6	A	A	A

B	1	2	3	7	8	9	10	11
a	B	B	A	A	B	B	B	B
b	B	B	B	A	B	B	B	B
c	B	B	B	A	B	B	B	B

$\underset{\sim}{C} = \{4, 6\}$, $D = \{5\}$, $\underset{\sim}{E} = \{1, 2, 8, 9, 10, 11\}$, $F = \{3\}$ $G = \{7\}$

C	a	b	c
4	C	D	C
6	C	C	C

E	a	b	c
1	E	E	E
2	F	E	E
8	E	E	E
9	E	E	E
10	E	E	E
11	E	E	E

$\underset{\sim}{H} = \{1, 8, 9, 10, 11\}$, $I = \{2\}$, $J = \{4\}$, $K = \{6\}$

H	a	b	c
1	I	H	H
8	H	H	H
9	H	H	H
10	H	H	H
11	H	H	H

$L = \{1\}$

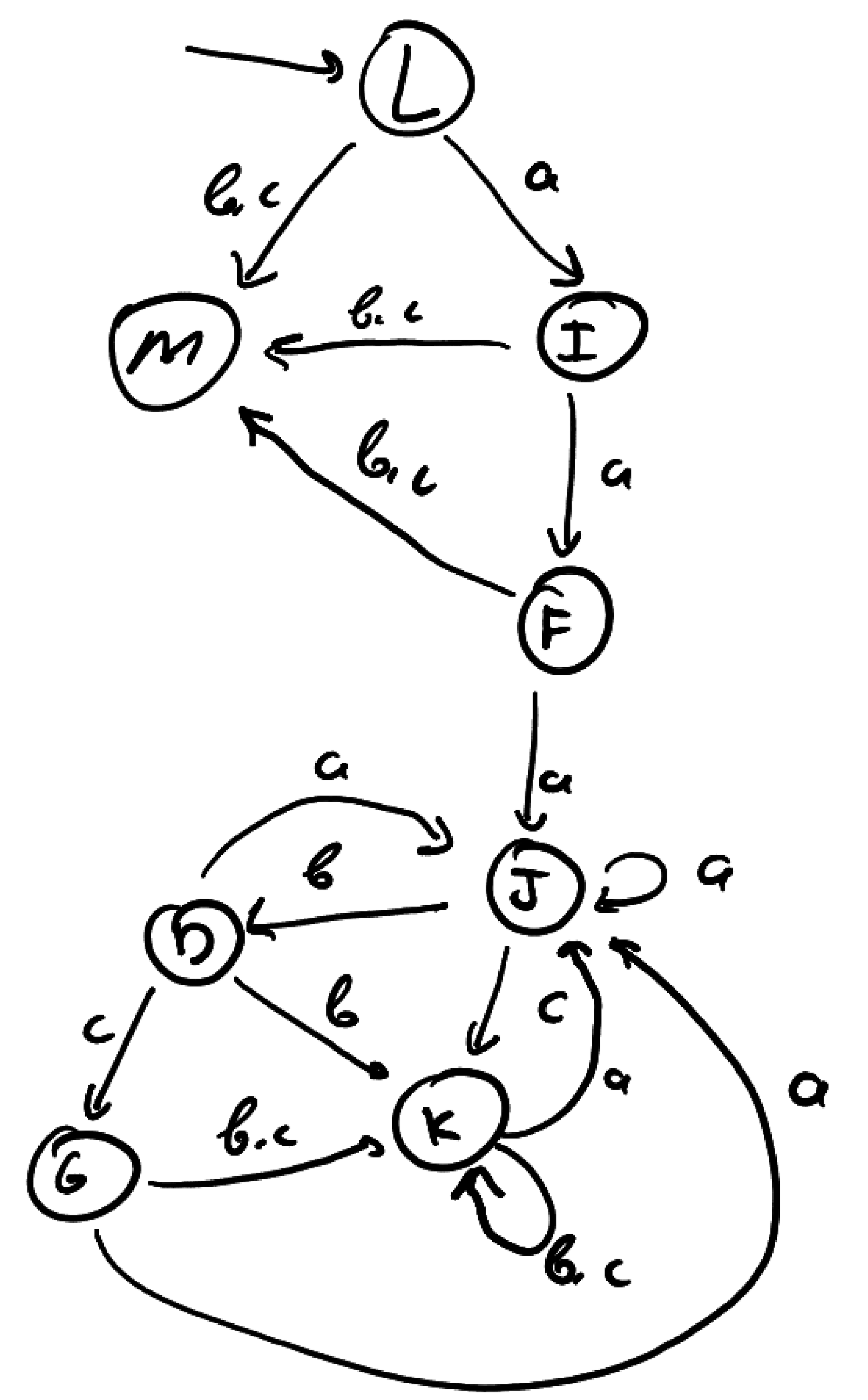
$M = \{8, 9, 10, 11\}$

M	a	b	c
8	M	M	M
9	M	M	M
10	M	M	M
11	M	M	M

край

	a	b	c
◎ D	J	K	G
F	J	M	M
G	J	K	K
I	F	M	M
◎ J	J	D	K
◎ K	J	K	K
→ L	I	M	M
M	M	M	M

A_M:



A_M е минимален тотален детерминиран краен автомат с език L.

Втори начин

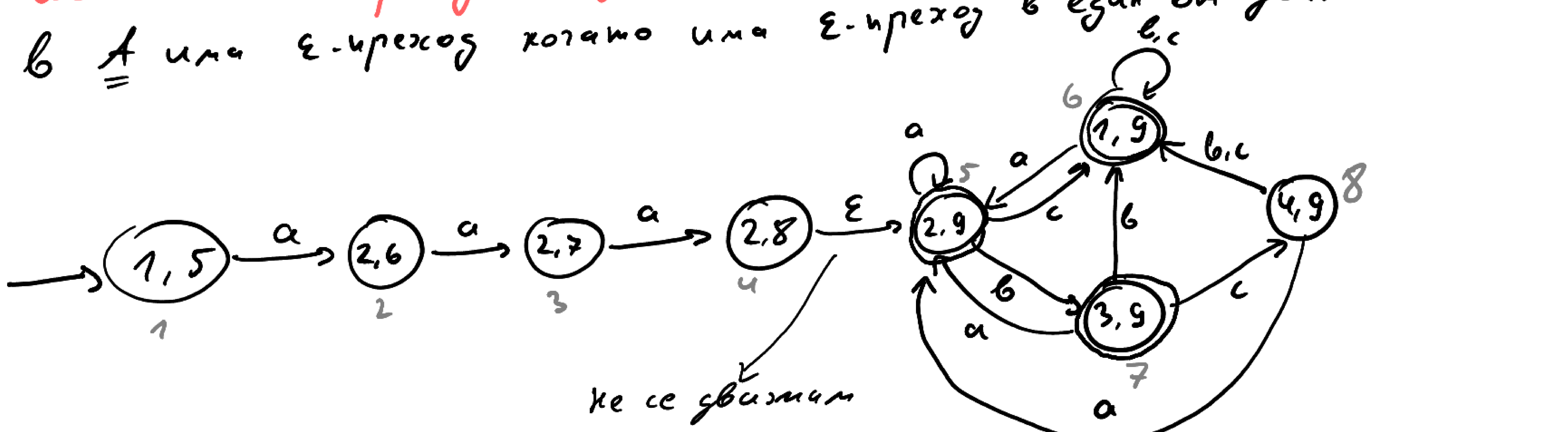
Можете да не детерминираме A_2 преди сечението (както го направихме и на дъската на упражнение)

$A: \; L(A) = L(\overline{A_1^D}) \cap L(A_2)$

Сечението на недетерминирани автомати е гадно

! в $\underline{A}$ има преход с буква $\underline{a}$ т.с.т.к. и в двата автомата има преход с тази буква.

в $\underline{A}$ има ε-преход когато има ε-преход в един от двата автомата



не се движим в A_1^D, правим ε-преход в A_2

Детерминизираме:

$A^D : L(A^D) = L(A)$

		a	b	c
1	$\to \{1\}$	2	∅	∅
2	$\{2\}$	3	∅	∅
3	∅	∅	∅	∅
4	$\{3\}$	4,5	∅	∅
5	$\{4,5\}$	5	7	6
6	$\{5\}$	5	7	6
7	$\{6\}$	5	6	6
8	$\{7\}$	5	6	8
9	$\{8\}$	5	6	6

преименуваме $\longrightarrow$

	a	b	c
→ 1	2	3	3
2	4	3	3
3	3	3	3
4	5	3	3
5	6	8	7
6	6	8	7
7	6	7	7
8	6	7	9
9	6	7	7

и Минимизираме

$A = \{1, 2, 3, 4, 9\}$ $B = \{5, 6, 7, 8\}$

A	a	b	c
1	A	A	A
2	A	A	A
3	A	A	A
4	B	A	A
9	B	B	B

B	a	b	c
5	B	B	B
6	B	B	B
7	B	B	B
8	B	B	A

$C = \{1, 2, 3\}$ $D = \{4\}$ $E = \{9\}$ $F = \{5, 6, 7\}$, $G = \{8\}$

C	a	b	c
1	C	C	C
2	D	C	C
3	C	C	C

F	a	b	c
5	F	G	F
6	F	G	F
7	F	F	F

$H = \{1, 3\}$, $I = \{2\}$, $J = \{5, 6\}$ $K = \{7\}$

H	a	b	c
1	I	H	H
3	D	H	H

J	a	b	c
5	J	G	H
6	J	G	H

К се пада

$L = \{1\}$

$M = \{3\}$

J	a	b	c
5	J	G	K
6	J	G	K

Получаваме следния автомат

$A_M^{\text{д}}$	a	b	c
D	J	M	M
E	J	K	K
G	J	K	E
I	D	M	M
J	J	G	K
K	J	K	K
$\rightarrow$ L	I	M	M
M	M	M	M

A_M: