II Скаларно произведение спрямо ОКС.

$K = O\vec{e_1}\vec{e_2}\vec{e_3} = Oxyz$ е ОКС - ортонормирана координатна система, ако

1) $\vec{e_1} \perp \vec{e_2}, \vec{e_2} \perp \vec{e_3}, \vec{e_1} \perp \vec{e_3}$ (ортогонална)

2) $|\vec{e_1}| = |\vec{e_2}| = |\vec{e_3}| = 1$ (нормирана)

$\vec{e_1}^2 = \vec{e_2}^2 = \vec{e_3}^2 = 1$

$\vec{e_1} \cdot \vec{e_2} = \vec{e_2} \cdot \vec{e_3} = \vec{e_1} \vec{e_3} = 0$

- Нека $\vec{a}(a_1, a_2, a_3)$ спрямо $K \iff \vec{a} = a_1\vec{e_1} + a_2\vec{e_2} + a_3\vec{e_3}$

  $\vec{b}(b_1, b_2, b_3)$ спрямо $K \iff \vec{b} = b_1\vec{e_1} + b_2\vec{e_2} + b_3\vec{e_3}$

$\vec{a} \cdot \vec{b} = a_1 b_1 + a_2 \cdot b_2 + a_3 \cdot b_3$

$\vec{a}^2 = |\vec{a}|^2 = a_1^2 + a_2^2 + a_3^2 \Rightarrow |\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$

- разстояние м/у две точки:

Нека $A_1(x_1, y_1, z_1)$ и $A_2(x_2, y_2, z_2)$

$|\vec{A_1A_2}| = ?$

$\vec{OA_1} = x_1\vec{e_1} + y_1\vec{e_2} + z_1\vec{e_3}$, $\vec{OA_2} = x_2\vec{e_1} + y_2\vec{e_2} + z_2\vec{e_3}$ - наричаме радиус-вектори спрямо ОКС

$O(0,0,0)$ - начало на ОКС

$\vec{A_1A_2} = \vec{OA_2} - \vec{OA_1} \Rightarrow \vec{A_1A_2}(x_2 - x_1, y_2 - y_1, z_2 - z_1)$

$|\vec{A_1A_2}|^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2$

1 зад. ОКС, $K = Oxyz$

$A(-1,-1,1)$ а) $P_{\triangle ABC} = ?$

$B(2,-7,4)$ б) Да се определи вида на $\triangle ABC$ в зависимост от $\sphericalangle$-те

$C(4,-2,6)$

в) т. $H \in AB$, $CH \perp AB$, т.$H(?,?,?)$, т.е. т.H е петата на височината, спусната от C към AB.

реш.: $\to \exists$ ли $\triangle ABC$?

$$\left.\begin{array}{l}\overrightarrow{AB}(3,-6,3)\\ \overrightarrow{AC}(5,-1,5)\end{array}\right\} \text{линейно независими} \left(\frac{3}{5} \neq \frac{-6}{-1} \neq \frac{3}{5}\right)$$

а) $\overrightarrow{AB}(3,-6,3) \Rightarrow |\overrightarrow{AB}| = \sqrt{9+36+9} = \sqrt{6.9} = 3\sqrt{6} = \sqrt{54}$

$\overrightarrow{AC}(5,-1,5) \Rightarrow |\overrightarrow{AC}| = \sqrt{25+1+25} = \sqrt{51}$

$\overrightarrow{BC}(2,5,2) \Rightarrow |\overrightarrow{BC}| = \sqrt{4+25+4} = \sqrt{25+8} = \sqrt{33}$

$P_{\triangle ABC} = 3\sqrt{6} + \sqrt{51} + \sqrt{33}$

б) $|\overrightarrow{AB}| > |\overrightarrow{AC}| > |\overrightarrow{BC}|$

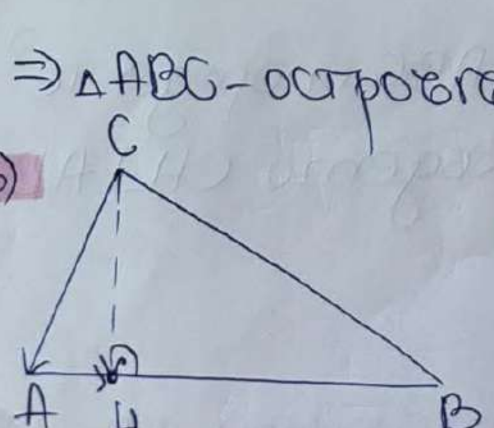
$\Downarrow$

$\sphericalangle ACB > \sphericalangle ABC > \sphericalangle BAC \Rightarrow$ Достатъчно е да разгледаме $\sphericalangle ACB$.

$$\cos\sphericalangle ACB = \frac{\overrightarrow{CA}.\overrightarrow{CB}}{|\overrightarrow{CA}|.|\overrightarrow{CB}|} = \frac{10-5+10}{\sqrt{51}.\sqrt{33}} = \underbrace{\frac{15}{\sqrt{51}\sqrt{33}}}_{>0} \quad \boxed{\begin{array}{l}\overrightarrow{CA}(-5,1,-5)\\ \overrightarrow{CB}(-2,-5,-2)\end{array}}$$

$\Rightarrow \triangle ABC$ - остроъгълен

в)



$\overrightarrow{CH} \perp \overrightarrow{AB} \Leftrightarrow \overrightarrow{CH}.\overrightarrow{AB} = 0$

1н. $\overrightarrow{CH} = \overrightarrow{CA} + \overrightarrow{AH}$

$\overrightarrow{AH} = k.\overrightarrow{AB}$, $\overrightarrow{AH} \uparrow\uparrow \overrightarrow{AB} \Rightarrow k > 0$

$\overrightarrow{CH}.\overrightarrow{AB} = 0 \Rightarrow (\overrightarrow{CA} + \overrightarrow{AH})\overrightarrow{AB} = 0$

$(\overrightarrow{CA} + k\overrightarrow{AB})\overrightarrow{AB} = 0$

$\overrightarrow{CA}.\overrightarrow{AB} + k.\overrightarrow{AB}^2 = 0 \Rightarrow$

$(-5.3 + 1.(-6) + (-5).3) + k54 = 0 \Rightarrow 54k = 36 \Rightarrow k = \frac{2}{3} \Rightarrow \overrightarrow{AH} = \frac{2}{3}\overrightarrow{AB} \Rightarrow \overrightarrow{AH}(2,-4,2)$

$\overrightarrow{AH} = \overrightarrow{OH} - \overrightarrow{OA} \Rightarrow \overrightarrow{OH} = \overrightarrow{AH} + \overrightarrow{OA} \Rightarrow \overrightarrow{OH}(1,-5,3) \Rightarrow$ т. $H(1,-5,3)$

2н (с осн. зад.) $\overrightarrow{CH} = \alpha\overrightarrow{CA} + \beta\overrightarrow{CB}$ $(A, H, B \in 1$ права$)$

$\alpha + \beta = 1 \Rightarrow \beta = 1 - \alpha$ $(\exists!(\alpha,\beta))$

Нека $\overrightarrow{CH} = \alpha.\overrightarrow{CA} + (1-\alpha)\overrightarrow{CB}$

$\overrightarrow{CH} \perp \overrightarrow{AB} \Leftrightarrow \overrightarrow{CH}.\overrightarrow{AB} = 0$

$(\alpha\overrightarrow{CA} + (1-\alpha)\overrightarrow{CB}).\overrightarrow{AB} = 0$ $\qquad \overrightarrow{CB}(-2,-5,-2)$

$\alpha.\overrightarrow{CA}.\overrightarrow{AB} + (1-\alpha)\overrightarrow{CB}.\overrightarrow{AB} = 0$

$\alpha.(-15-6-15) + (1-\alpha)(-6+30-6) = 0$

$-36\alpha + (1-\alpha).18 = 0$

$-54\alpha = -18$

$\alpha = \frac{\cancel{18}^{\,1}}{\cancel{54}_{\,3}} = \frac{1}{3}$

$\overrightarrow{CH} = \frac{1}{3}\overrightarrow{CA} + \frac{2}{3}\overrightarrow{CB}$

$\overrightarrow{OH} - \overrightarrow{OC} = \frac{1}{3}(\overrightarrow{OA} - \overrightarrow{OC}) + \frac{2}{3}(\overrightarrow{OB} - \overrightarrow{OC})$

$\overrightarrow{OH} = \frac{1}{3}\overrightarrow{OA} + \frac{2}{3}\overrightarrow{OB}$

$\left(-\frac{1}{3}, -\frac{1}{3}, \frac{1}{3}\right) + \left(\frac{4}{3}, -\frac{14}{3}, \frac{8}{3}\right) = (1,-5,3)$

$\Rightarrow \overrightarrow{OH}(1,-5,3) \Rightarrow$ т. $H(1,-5,3)$

2зад. Спрямо ОКС, $K = Oxyz$ (за упр.)

$A(0,2,4)$

$B(3,-4,-2)$

$C(5,-2,6)$

а) $P_{\triangle \cancel{ABC}}$

б) вида

в) $H(?,?$

(като 1-ва задача)

3зад. ОКС, $K = Oxyz$

$A(0,0,-2)$

$B(4,0,-4)$

$C(2,0,0)$

$D(5,3,-3)$

а) т. $M(?,?,?)$ и т. $N(?,$

$M \in AB$, $N \in CD$, MN

б) т. $H(?,?,?)$, къде

височината

тетраедъра AB

A, B, C, D - компланарни ли са?

$$\begin{vmatrix} 0 & 0 & -2 & 1 \\ 4 & 0 & -4 & 1 \\ 2 & 0 & 0 & 1 \\ 5 & 3 & -3 & 1 \end{vmatrix} = 3\begin{vmatrix} 0 & -2 & 1 \\ 4 & -4 & 1 \\ 2 & 0 & 1 \end{vmatrix}$$

$= 3(-4+8+8) = 3.12 = 36 \neq 0$

$\overrightarrow{AH}=\overrightarrow{OH}-\overrightarrow{OA} \Rightarrow \overrightarrow{OH}=\overrightarrow{AH}+\overrightarrow{OA} \Rightarrow \overrightarrow{OH}(1,-5,3) \Rightarrow$ т. $H(1,-5,3)$

(2н.) (осн. зад.) $\left| \begin{array}{l} \overrightarrow{CH}=\alpha\overrightarrow{CA}+\beta\overrightarrow{CB} \quad (A, H, B \in 1 \text{ права}) \\ \alpha+\beta=1 \Rightarrow \beta=1-\alpha \quad (\exists!(\alpha,\beta)) \end{array} \right.$

Нека $\overrightarrow{CH}=\alpha.\overrightarrow{CA}+(1-\alpha)\overrightarrow{CB}$

$\overrightarrow{CH}\perp\overrightarrow{AB} \Leftrightarrow \overrightarrow{CH}.\overrightarrow{AB}=0$

$(\alpha\overrightarrow{CA}+(1-\alpha)\overrightarrow{CB}).\overrightarrow{AB}=0$ $\qquad \overrightarrow{CB}(-2,-5,-2)$

$\alpha.\overrightarrow{CA}.\overrightarrow{AB}+(1-\alpha)\overrightarrow{CB}.\overrightarrow{AB}=0$

$\alpha.(-15-6-15)+(1-\alpha)(-6+30-6)=0$

$-\cancel{36}^{1}\alpha+(1-\alpha)\cancel{18}=0$

$-54\alpha=-18$

$\alpha=\dfrac{\cancel{18}^{1}}{\cancel{54}_{3}}=\dfrac{1}{3}$

$\overrightarrow{CH}=\frac{1}{3}\overrightarrow{CA}+\frac{2}{3}\overrightarrow{CB}$

$\overrightarrow{OH}-\overrightarrow{OC}=\frac{1}{3}(\overrightarrow{OA}-\overrightarrow{OC})+\frac{2}{3}(\overrightarrow{OB}-\overrightarrow{OC})$

$\overrightarrow{OH}=\frac{1}{3}\overrightarrow{OA}+\frac{2}{3}\overrightarrow{OB}$

$\left(-\frac{1}{3},-\frac{1}{3},\frac{1}{3}\right)+\left(\frac{4}{3},-\frac{14}{3},\frac{8}{3}\right)=(1,-5,3)$

$\Rightarrow \overrightarrow{OH}(1,-5,3) \Rightarrow$ т. $H(1,-5,3)$

2зад. (за упр.) Спрямо ОКС, $K=Oxyz$

$A(0,2,4)$ $\quad$ а) $P_{\triangle ABC}=?$

$B(3,-4,-2)$ $\quad$ б) вида на $\triangle ABC$ според ∢-те?

$C(5,-2,6)$ $\quad$ в) $H(?,?,?)$, където $CH\perp AB$

(като 1-ва задача)

3зад. ОКС, $K=Oxyz$

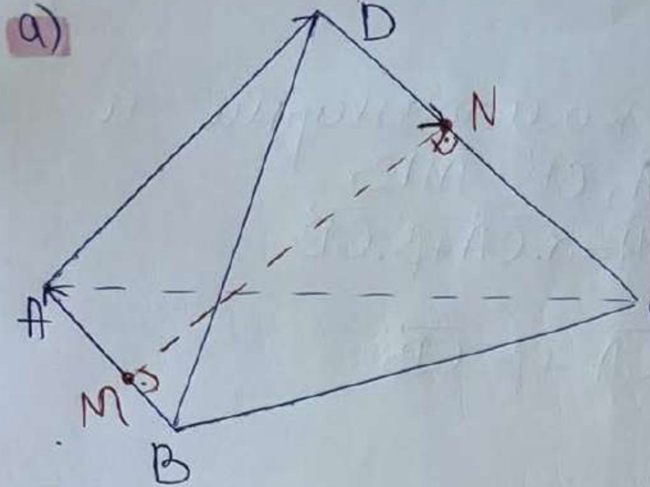
$A(0,0,-2)$ $\quad$ а) т. $M(?,?,?)$ и т. $N(?,?,?)$, такива че $M\in AB$, $N\in CD$, $MN\perp AB$, $MN\perp CD$

$B(4,0,-4)$

$C(2,0,0)$ $\quad$ б) т. $H(?,?,?)$, където H е петата на височината през върха D на тетраедъра $ABCD$.

$D(5,3,-3)$

a)



$\overrightarrow{MN}=\overrightarrow{MA}+\overrightarrow{AD}+\overrightarrow{DN}$

$\overrightarrow{MA}\parallel\overrightarrow{BA}\Leftrightarrow\overrightarrow{MA}=k.\overrightarrow{BA}$

$\overrightarrow{DN}\parallel\overrightarrow{DC}\Leftrightarrow\overrightarrow{DN}=l.\overrightarrow{DC}$

$\overrightarrow{BA}(-4,0,2)\Rightarrow|\overrightarrow{BA}|^2=20$

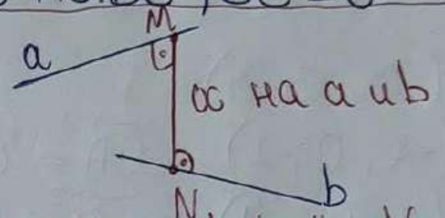
$\overrightarrow{AD}(5,3,-1)$

$\overrightarrow{DC}(-3,-3,3)\Rightarrow|\overrightarrow{DC}|^2=9.3=27$

$\overrightarrow{MN}=k\overrightarrow{BA}+\overrightarrow{AD}+l.\overrightarrow{DC}$

$$\left|\begin{array}{l}\overrightarrow{MN}\perp\overrightarrow{BA}\Leftrightarrow\overrightarrow{MN}.\overrightarrow{BA}=0\\ \overrightarrow{MN}\perp\overrightarrow{DC}\Leftrightarrow\overrightarrow{MN}.\overrightarrow{DC}=0\end{array}\right.\Rightarrow\left|\begin{array}{l}(k.\overrightarrow{BA}+\overrightarrow{AD}+l.\overrightarrow{DC})\overrightarrow{BA}=0\\ (k\overrightarrow{BA}+\overrightarrow{AD}+l.\overrightarrow{DC})\overrightarrow{DC}=0\end{array}\right.$$

$$\left|\begin{array}{l}k.\overrightarrow{BA}^2+\overrightarrow{AD}.\overrightarrow{BA}+l.\overrightarrow{DC}.\overrightarrow{BA}=0\\ k\overrightarrow{BA}.\overrightarrow{DC}+\overrightarrow{AD}.\overrightarrow{DC}+l.\overrightarrow{DC}^2=0\end{array}\right.$$



MN-ос отс. на a и b, $|MN|=d(a,b)$

$$\left|\begin{array}{l}k.20+(-20-2)+l.(12+6)=0\\ k(12+6)+(-15-9-3)+l.27=0\end{array}\right.\Rightarrow\left|\begin{array}{l}20k-22+18l=0\ /:2\\ 18k-27+27l=0\ /:(-3)\end{array}\right.$$

$$+\left|\begin{array}{l}10k-11+9l=0\\ -6k+9-9l=0\end{array}\right.\Rightarrow\left|\begin{array}{l}4k-2=0\Rightarrow k=\frac{1}{2}\\ -6k+9-9l=0\Rightarrow-\frac{6}{2}+9-9l=0\end{array}\right.$$

$6-9l=0$

$l=\frac{2}{3}$

$\Rightarrow\overrightarrow{MA}=\frac{1}{2}\overrightarrow{BA}$ и $\overrightarrow{DN}=\frac{2}{3}\overrightarrow{DC}$

$\Rightarrow M(2,0,-3)$ //

(средното аритметично на коорд. на т. A и т. B, защото M е среда на AB)

1н.

$N\left(\frac{2.2+5}{3},\frac{2.0+3}{3},\frac{2.0-3}{3}\right)$

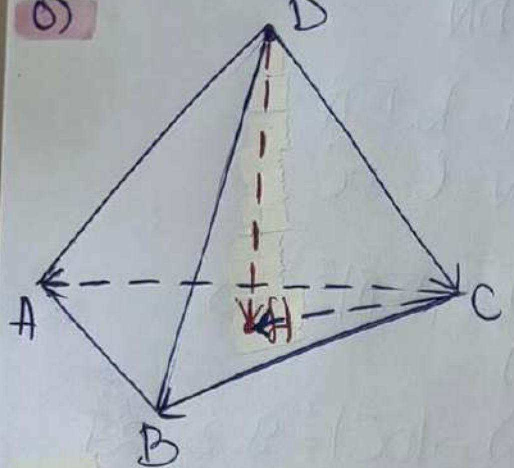
$N(3,1,-1)$ //

2н. $\overrightarrow{ON}-\overrightarrow{OD}=\frac{2}{3}\overrightarrow{DC}$

$\overrightarrow{ON}=\overrightarrow{OD}+\frac{2}{3}\overrightarrow{DC}$

$$\left|\begin{array}{l}x_N=5+\frac{2}{3}.(-3)=3\\ y_N=3+\frac{2}{3}.(-3)=1\\ z_n=-3+\frac{2}{3}.(3)=-1\end{array}\right.\Rightarrow N(3,1,-1)$$

б)



$\overrightarrow{DH} = \overrightarrow{DC} + \overrightarrow{CH}$

$\overrightarrow{CH}, \overrightarrow{CA}$ и $\overrightarrow{CB}$ - компланарни и $\overrightarrow{CA}, \overrightarrow{CB}$ - ЛНЗ

$\Rightarrow \exists! (\alpha, \beta): \overrightarrow{CH} = \alpha.\overrightarrow{CA} + \beta.\overrightarrow{CB}$

$\Rightarrow \overrightarrow{DH} = \overrightarrow{DC} + \alpha.\overrightarrow{CA} + \beta.\overrightarrow{CB}$

$DH \perp (ABC) \Rightarrow \left| \begin{array}{l} \overrightarrow{DH} \perp \overrightarrow{CA} \Leftrightarrow \\ \overrightarrow{DH} \perp \overrightarrow{CB} \Leftrightarrow \end{array} \right| \begin{array}{l} \overrightarrow{DH}.\overrightarrow{CA} = 0 \\ \overrightarrow{DH}.\overrightarrow{CB} = 0 \end{array} \Rightarrow$

$\left| \begin{array}{l} (\overrightarrow{DC} + \alpha.\overrightarrow{CA} + \beta\overrightarrow{CB})\overrightarrow{CA} = 0 \\ (\overrightarrow{DC} + \alpha.\overrightarrow{CA} + \beta.\overrightarrow{CB}).\overrightarrow{CB} = 0 \end{array} \right. \Rightarrow \left| \begin{array}{l} \overrightarrow{DC}.\overrightarrow{CA} + \alpha.\overrightarrow{CA}^2 + \beta\overrightarrow{CB}.\overrightarrow{CA} = 0 \\ \overrightarrow{DC}.\overrightarrow{CB} + \alpha.\overrightarrow{CA}.\overrightarrow{CB} + \beta\overrightarrow{CB}^2 = 0 \end{array} \right.$

$\overrightarrow{DC}(-3, -3, 3)$

$\overrightarrow{CA}(-2, 0, -2) \Rightarrow \overrightarrow{CA}^2 = 8$

$\overrightarrow{CB}(2, 0, -4) \Rightarrow \overrightarrow{CB}^2 = 20$

$\Rightarrow \left| \begin{array}{l} (\cancel{6} + 0 - \cancel{6}) + \alpha.8 + \beta.(-4 + 0 + 8) = 0 \\ (-6 + 0 - 12) + \alpha.(-4 + 8) + \beta.20 = 0 \end{array} \right.$

$\left| \begin{array}{l} 8\alpha + 4\beta = 0 \Rightarrow 2\alpha + \beta = 0 \\ -18 + 4\alpha + 20\beta = 0 \Rightarrow -9 + 2\alpha + 10\beta = 0 \end{array} \right.$

$9\beta = 9 \Rightarrow \beta = 1, \alpha = -\frac{1}{2}$

$\overrightarrow{CH} = -\frac{1}{2}\overrightarrow{CA} + \overrightarrow{CB}$

$\overrightarrow{OH} - \cancel{\overrightarrow{OC}} = -\frac{1}{2}(\overrightarrow{OA} - \overrightarrow{OC}) + \overrightarrow{OB} - \cancel{\overrightarrow{OC}}$

$\overrightarrow{OH} = -\frac{1}{2}\overrightarrow{OA} + \frac{1}{2}\overrightarrow{OC} + \overrightarrow{OB}$

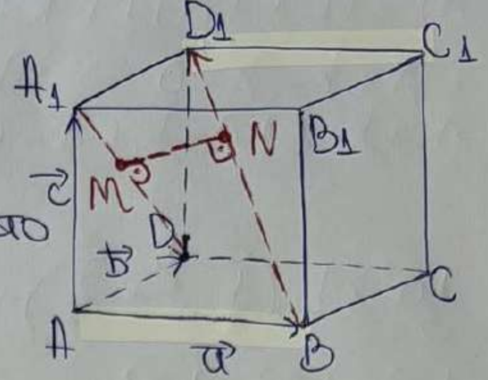
$\left| \begin{array}{l} x_H = -\frac{1}{2}.0 + \frac{1}{2}.2 + 4 = 5 \\ y_H = -\frac{1}{2}.0 + \frac{1}{2}.0 + 0 = 0 \\ z_H = -\frac{1}{2}.(-2) + \frac{1}{2}.0 + (-4) = -3 \end{array} \right\} \Rightarrow т.H(5, 0, -3)$

$\underbrace{}_{A} \quad \underbrace{}_{C} \quad \underbrace{}_{B}$

Зад. Даден е куб $ABCDA_1B_1C_1D_1$ с ръб с дължина 1 $(AB=1)$

а) Да се намери ъгълът м/у A_1D и BD_1

б) Да се намери разстоянието м/у A_1D и BD_1.



реш: Въвеждаме ОКС:

$\overrightarrow{AB}=\vec{a}$, $\overrightarrow{AD}=\vec{b}$, $\overrightarrow{AA_1}=\vec{c}$

$|\vec{a}|=|\vec{b}|=|\vec{c}|=1$, $\sphericalangle(\vec{a},\vec{b})_e=\sphericalangle(\vec{a},\vec{c})_e=\sphericalangle(\vec{b},\vec{c})_e=\frac{\pi}{2}$

а) $\overrightarrow{A_1D}=\overrightarrow{A_1A}+\overrightarrow{AD}=-\vec{c}+\vec{b}$ $(\overrightarrow{A_1D}=\overrightarrow{AD}-\overrightarrow{AA_1}=\vec{b}-\vec{c})$

$\overrightarrow{BD_1}=\overrightarrow{BA}+\overrightarrow{AD}+\overrightarrow{DD_1}=-\vec{a}+\vec{b}+\vec{c}$

$$\cos\sphericalangle(A_1D,BD_1)_e=\frac{\overrightarrow{A_1D}.\overrightarrow{BD_1}}{|\overrightarrow{A_1D}|.|\overrightarrow{BD_1}|}=\frac{0}{\sqrt{6}}=0$$

$\overrightarrow{A_1D}.\overrightarrow{BD_1}=(\vec{b}-\vec{c})(-\vec{a}+\vec{b}+\vec{c})=\vec{b}^2-\vec{c}^2=1-1=0$

$\left(|\overrightarrow{A_1D}|=\sqrt{\overrightarrow{A_1D}^2}=\sqrt{\vec{b}^2+\vec{c}^2}=\sqrt{2},\ |\overrightarrow{BD_1}|=\sqrt{(-\vec{a}+\vec{b}+\vec{c})^2}=\sqrt{3}\right)$

$\Rightarrow \overrightarrow{A_1D}\perp\overrightarrow{BD_1}$

б) Нека т.$M\in A_1D$, т.$N\in BD_1$: $MN\perp A_1D$ и $MN\perp BD_1$

тъй като т.$M\in A_1D\Rightarrow \exists!\ k\in\mathbb{R}$: $\overrightarrow{MD}=k.\overrightarrow{A_1D}=k(\vec{b}-\vec{c})$

т.$N\in BD_1\Rightarrow\exists!\ \lambda\in\mathbb{R}$: $\overrightarrow{BN}=\lambda.\overrightarrow{BD_1}=\lambda(-\vec{a}+\vec{b}+\vec{c})$

$\Rightarrow\overrightarrow{MN}=\overrightarrow{MD}+\overrightarrow{DA}+\overrightarrow{AB}+\overrightarrow{BN}=k(\vec{b}-\vec{c})+(-\vec{b})+\vec{a}+\lambda(-\vec{a}+\vec{b}+\vec{c})=$

$=\vec{a}(1-\lambda)+\vec{b}(k-1+\lambda)+\vec{c}(-k+\lambda)$

$\overrightarrow{MN}\perp\overrightarrow{A_1D}\iff\overrightarrow{MN}.\overrightarrow{A_1D}=0$

$\overrightarrow{MN}\perp\overrightarrow{BD_1}\iff\overrightarrow{MN}.\overrightarrow{BD_1}=0$

$$\begin{cases}[\vec{a}(1-\lambda)+\vec{b}(k-1+\lambda)+\vec{c}(-k+\lambda)].(\vec{b}-\vec{c})=0\\ [\vec{a}(1-\lambda)+\vec{b}(k-1+\lambda)+\vec{c}(-k+\lambda)].(-\vec{a}+\vec{b}+\vec{c})=0\end{cases}$$

$$\Rightarrow\begin{cases}\vec{b}^2(k-1+\lambda)-\vec{c}^2(-k+\lambda)=0\\ -\vec{a}^2(1-\lambda)+\vec{b}^2(k-1+\lambda)+\vec{c}^2(-k+\lambda)=0\end{cases}\Rightarrow$$

$$\begin{cases} k-1+\lambda-(-k+\lambda)=0 \\ -(1-\lambda)+(k-1+\lambda)+(-k+\lambda)=0 \end{cases}$$

$$\begin{cases} k-1+\cancel{\lambda}+k-\cancel{\lambda}=0 \Rightarrow 2k=1 \Rightarrow k=\frac{1}{2} \\ -1+\lambda+\cancel{k}-1+\lambda-\cancel{k}+\lambda=0 \Rightarrow 3\lambda=2 \end{cases}$$

$$\lambda=\frac{2}{3}$$

$$\vec{MN}=\frac{1}{3}\vec{a}+\vec{b}\left(\frac{1}{2}^{(3)}-\frac{1}{1}^{(6)}+\frac{2}{3}^{(2)}\right)+\vec{c}\left(-\frac{1}{2}^{(3)}+\frac{2}{3}^{(2)}\right)=$$

$$=\frac{1}{3}\vec{a}+\frac{1}{6}\vec{b}+\frac{1}{6}\vec{c}$$

$$|\vec{MN}|=\sqrt{\vec{MN}^2}=\sqrt{\left(\frac{1}{3}\vec{a}+\frac{1}{6}\vec{b}+\frac{1}{6}\vec{c}\right)^2}=\sqrt{\frac{1}{9}\vec{a}^2+\frac{1}{36}\vec{b}^2+\frac{1}{36}\vec{c}^2}=$$

$$=\sqrt{\frac{1}{9}^{(4)}+\frac{1}{36}+\frac{1}{36}}=\sqrt{\frac{6}{36}}=\frac{\sqrt{6}}{6}$$