

# 2-D Wave Equation

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The wave equation can be stated in the following form,

$$\frac{\partial^2 p}{\partial t^2} = c^2 \nabla^2 p, \quad (1)$$

where  $p$  is a physical measure such as a pressure, or the height of the water in a tub. This equation can be discretized over a 2d grid using a finite differencing scheme. Assume the pressures are located across a rectangular 2-D grid in  $x$  and  $y$  dimensions. We call the value of the pressure  $p_{x,y}^t$  for time  $t$  at location  $(x, y)$  in the grid. One solution to the differential equation (1) is the following:

$$p_{x,y}^{t+1} + p_{x,y}^{t-1} - 2p_{x,y}^t = c^2 (p_{x+1,y}^t + p_{x-1,y}^t + p_{x,y+1}^t + p_{x,y-1}^t - 4p_{x,y}^t). \quad (2)$$

The left hand side term  $\frac{\partial^2 p}{\partial t^2}$  is represented in terms of the values of  $p$  at three consecutive timesteps at grid location  $(x, y)$ . The right hand side terms are discretized using the  $p$  values for the 4 locations adjacent to  $(x, y)$  in the grid. One can simply solve for  $p_{x,y}^{t+1}$  in equation (2) to produce an update rule,

$$p_{x,y}^{t+1} = c^2 (p_{x+1,y}^t + p_{x-1,y}^t + p_{x,y+1}^t + p_{x,y-1}^t - 4p_{x,y}^t) - p_{x,y}^{t-1} + 2p_{x,y}^t. \quad (3)$$

This update rule specifies how the  $p$  value for each location in the grid should be computed, using  $p$  values from the two previous time steps. This update rule is easy to code in an application. The application simply maintains two timesteps' worth of  $p$  grids, and using these, the next timestep's  $p$  grid can be computed.