Part4- Voting

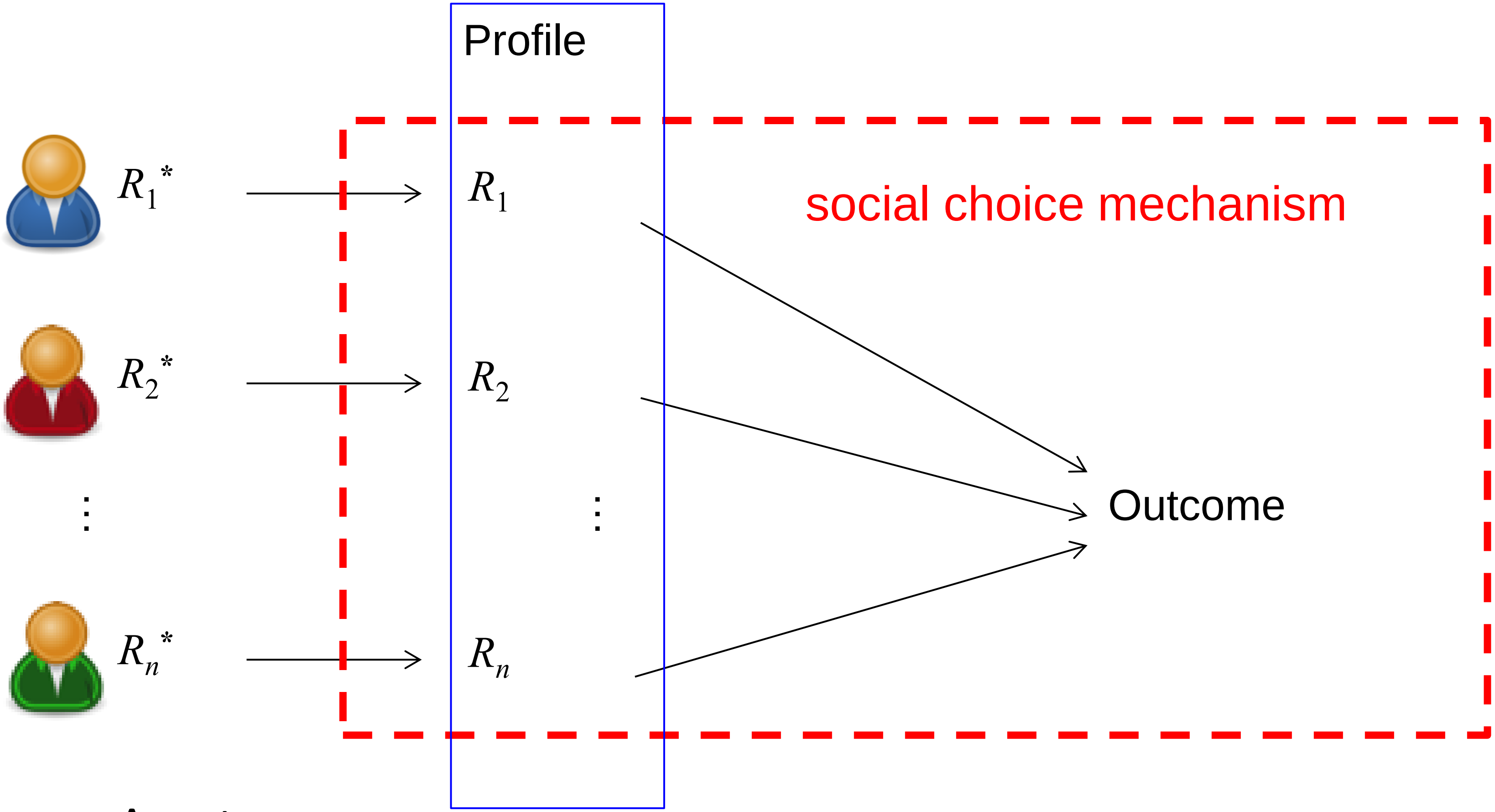
Based on slides by Lirong Xia, Ester David and Avinatan Hassidim

Social choice

"social choice is a theoretical framework for analysis of combining individual opinions, preferences, interests, or welfares to reach a collective decision or social welfare in some sense."

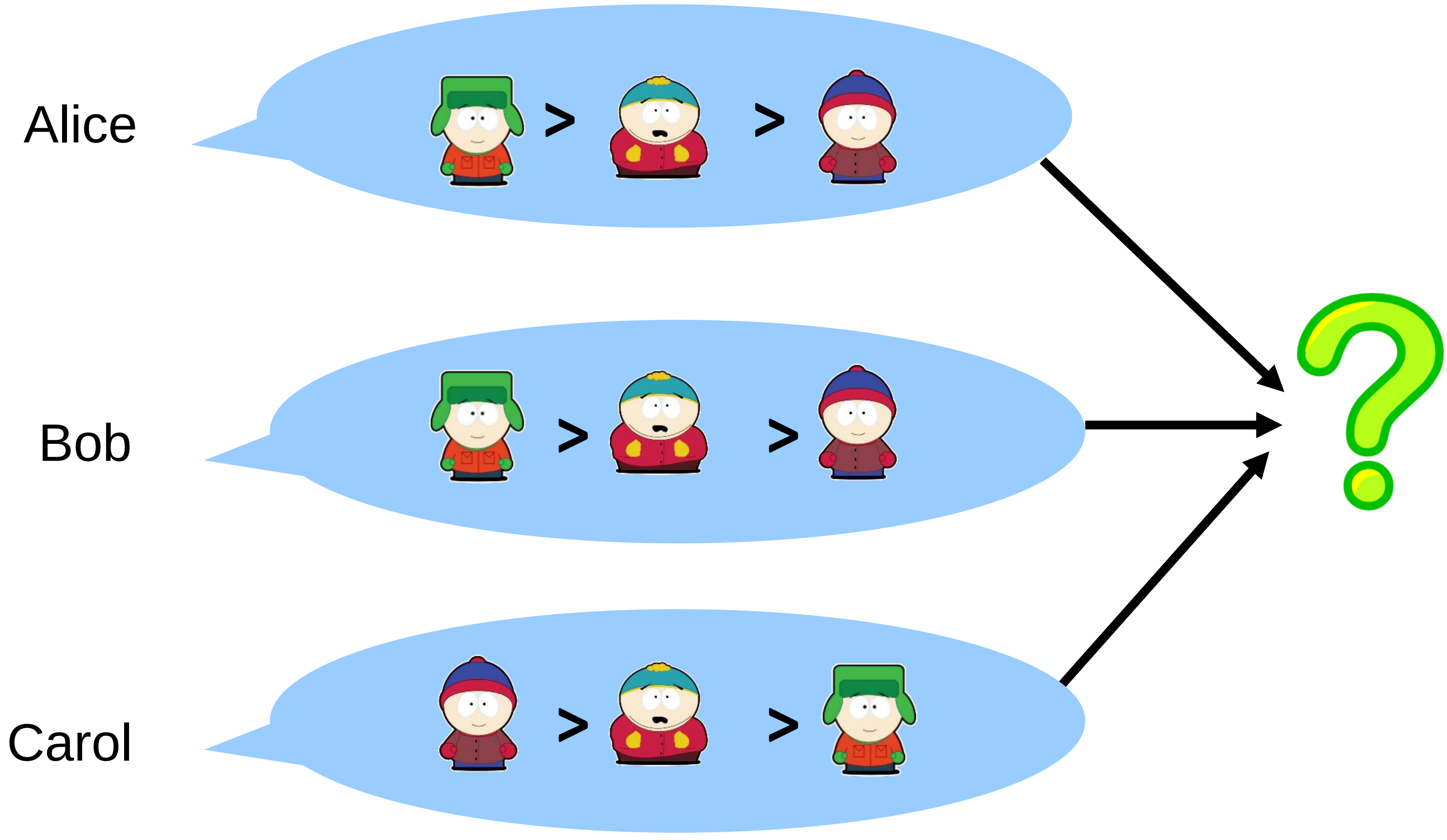
---Wikipedia Dec 28, 2021

Social choice problems



- Agents
- Alternatives
- Outcomes
- Preferences (true and reported)
- Social choice mechanism

Example: Political elections



Why is this social choice?

- Agents: {Alice, Bob, Carol}
- Alternatives: { , ,  }
- Outcomes: winners (alternatives)
- Preferences (vote): rankings over alternatives
- Mechanisms: voting rules

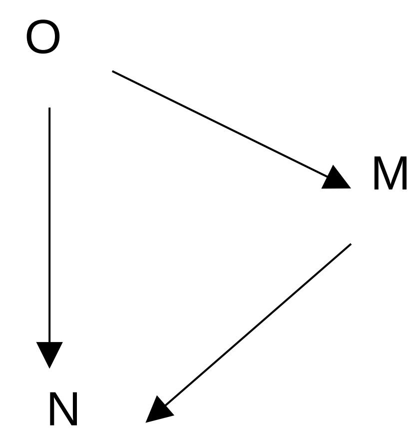
- Can vote over just about anything
  - political representatives, award nominees, where to go for dinner tonight, joint plans, allocations of tasks/resources, …
  - Also can consider other applications: e.g., aggregating search engines' rankings into a single ranking

More formally

- Agents: n voters, N={1,…,n}
- Alternatives: m candidates,
  - A={a_1,…,a_m} or {a, b, c, d,…}
- Preferences: R_j* and R_j are full rankings over A
  - Extensions include indifference and incompleteness
- Voting rule: a function that maps each profile to outcome(s)
- Outcomes:
  - winners (alternatives): O ⊆ A. Social choice function
    - If always |O|=1, the SCF is resolute
  - rankings over alternatives: O ∈ Rankings(A). Social welfare function
    - If the ranking is always strict, the SWF is resolute

Recall: binary relation

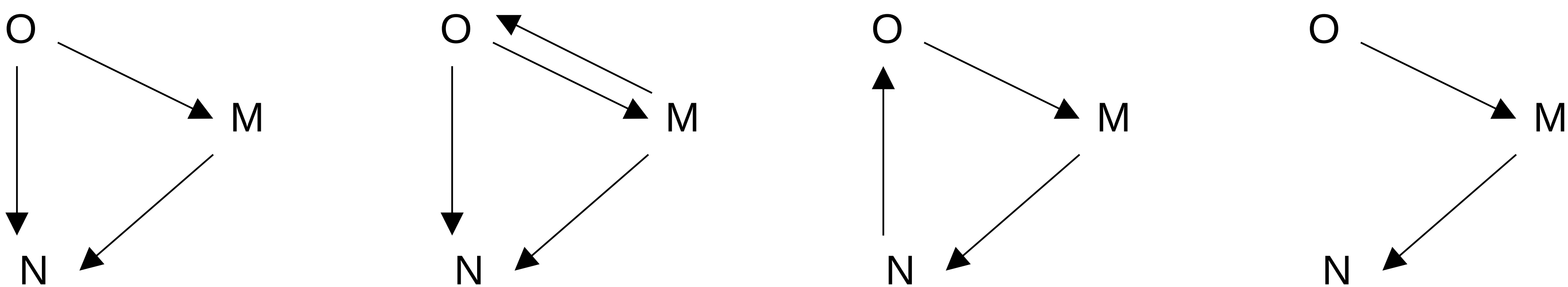
- Given a set of alternatives A
- A binary relation R is a subset of A×A
  - (a,b) ∈R means "a is preferred to b"
  - Also write $a>_R b$
- Example
  - A = {O, M, N}
  - R = {(O,M), (O,N), (M,N)}
- Graphical representation
  - Vertices are A
  - There is an edge a→b if and only if (a,b) ∈R



Linear orders - full rankings

Linear orders (rankings without ties): binary relations that satisfies

- Antisymmetry (no ties): $a >_R b$ and $b >_R a$ implies $a = b$
- Transitivity: $a >_R b$ and $b >_R c$ implies $a >_R c$
- Totality: for all a,b, one of $a >_R b$ or $b >_R a$ must hold



Yes

no,
Antisymmetry

no,
transitivity

no,
totality

Voting rules

- Majority rule: if a candidate is ranked first by most votes, that candidate should win
  – But what if there is no such candidate?

- Plurality: candidate with most votes wins
  – Otherwise known as "first past the post"

- Some (informal) criticisms
  – Ignores preferences other than favorite
  – Encourages voters to vote tactically
    - "*My candidate cannot win so I'll vote for my second favorite*"

Is the winner indeed a “good” one?

$$P = \left\{ \boxed{a > b > c} \times 4,\ \boxed{b > c > a} \times 3,\ \boxed{b > a > c} \times 2,\ \boxed{c > a > b} \times 2 \right\}$$

Plurality(P)=b

But the majority of the voters prefer a to b (6 out of 11).
They also prefer a to c.

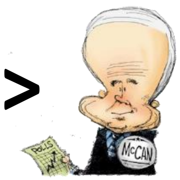
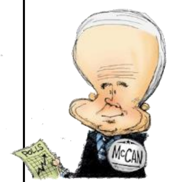
One possible generalization of Majority

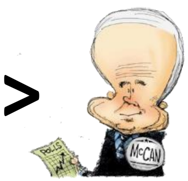
Jean Charles de Borda, 1733-1799

Borda: given m candidates

- ith ranked candidate score m-i
- Candidate with greatest sum of scores wins

Borda - example

$P=\{$  >  >  ×4,  >  >  ×3

 >  >  ×2,  >  >  ×2 $\}$

Borda scores  : 4×2+4×1=12  : 2×2+7×1=11  : 5×2=10

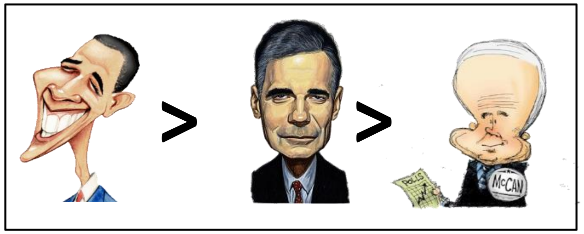
Borda(P)= 

Positional scoring rules

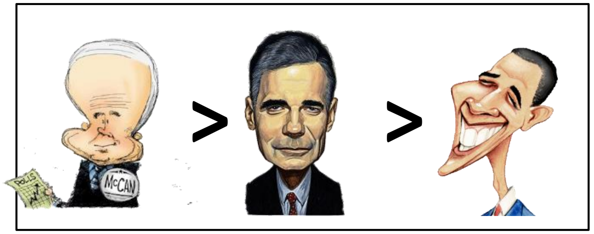
- Characterized by a score vector $s_1,\ldots,s_m$ in non-increasing order
- For each vote R, the alternative ranked in the i-th position gets s_i points
- The alternative with the most total points is the winner
- Special cases
  - Borda: score vector $(m\text{-}1, m\text{-}2, \ldots, 0)$
  - k-approval: score vector $(\underbrace{1\ldots1}_{k}, 0\ldots0)$
  - Plurality: score vector $(1, 0\ldots0)$
  - Veto: score vector $(1\ldots1, 0)$

Example

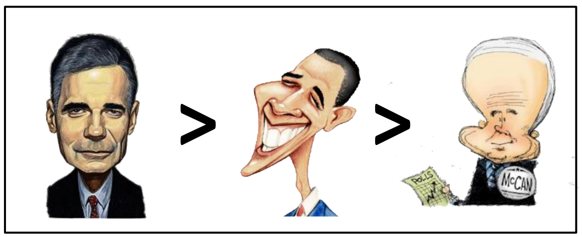
$P=\{$



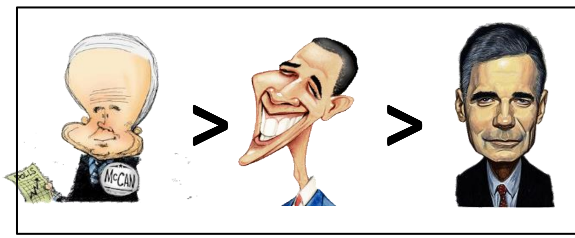
$\times 4$,



$\times 3$



$\times 2$,



$\times 2$ $\}$

Borda

Plurality
(1- approval)

Veto
(2-approval)

Is the winner indeed a “good” one?

$$P = \left\{ \boxed{\textbf{a} > \textbf{b} > \textbf{c}} \times 4,\ \boxed{\textbf{b} > \textbf{c} > \textbf{a}} \times 3,\ \boxed{\textbf{b} > \textbf{a} > \textbf{c}} \times 2,\ \boxed{\textbf{c} > \textbf{a} > \textbf{b}} \times 2 \right\}$$

Borda(P)=b , Veto(P)=b

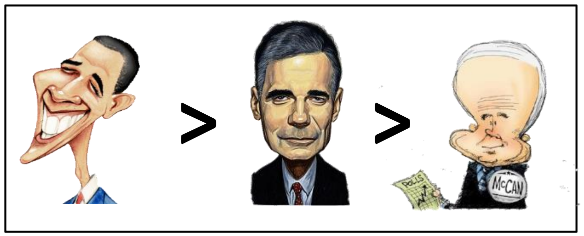
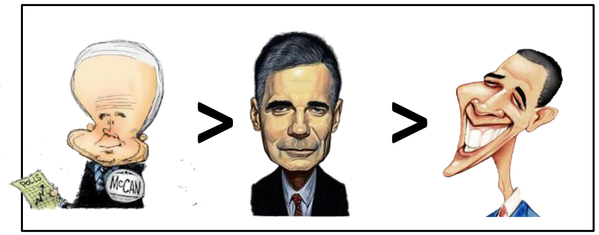
But the majority of the voters prefer a to b (6 out of 11).

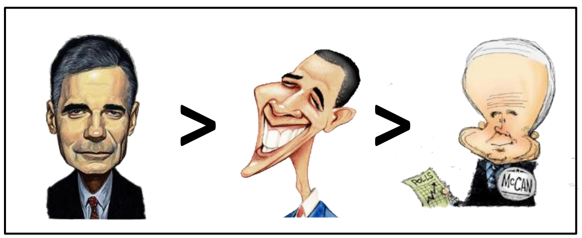
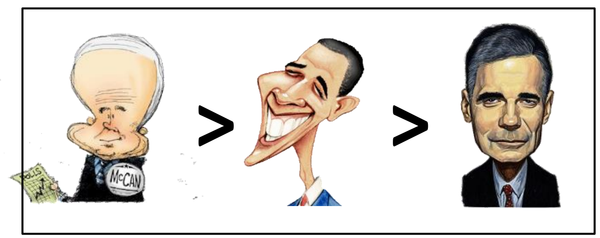
They also prefer a to c.

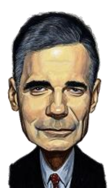
Another possible generalization of Majority

- Plurality with runoff: the election has two rounds
  - First round, all alternatives except the two with the highest plurality scores drop out
  - Second round, the alternative preferred by more voters wins
- [used in France, Iran, North Carolina State]

Example: Plurality with runoff

$P=\{$  $\times 4$,  $\times 3$

 $\times 2$,  $\times 2$ $\}$

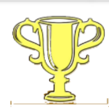
- First round:  drops out
- Second round:  defeats 

Different from Plurality!

Single transferable vote (STV)

- Also called instant run-off voting or alternative vote
- The election has m-1 rounds, in each round,
  - The alternative with the lowest plurality score drops out, and is removed from all votes
  - The last-remaining alternative is the winner
- [used in Australia and Ireland]

$a > b > c > d$	$d > a > b > c$	$c > d > a > b$	$b > c > d > a$
10	7	6	3

 a

Should we consider pairwise elections directly?

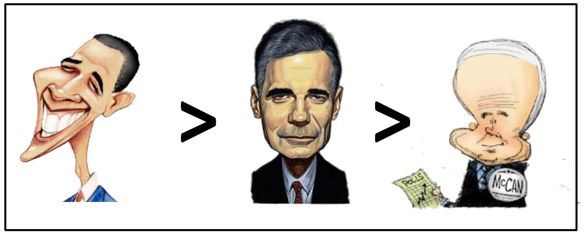
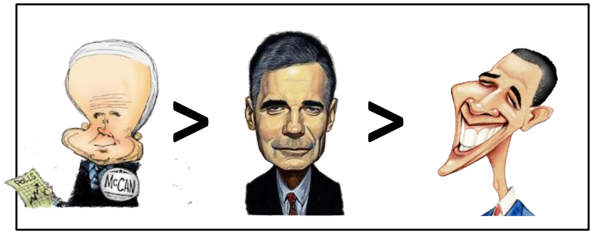
Marie Jean Antoine Nicolas de Caritat, marquis de Condorcet (1743 – 1794)

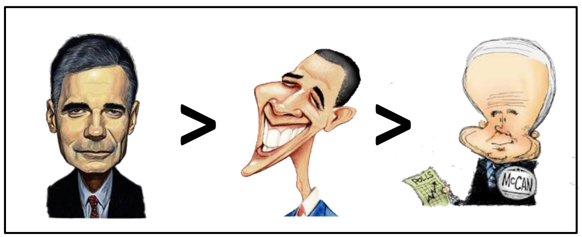
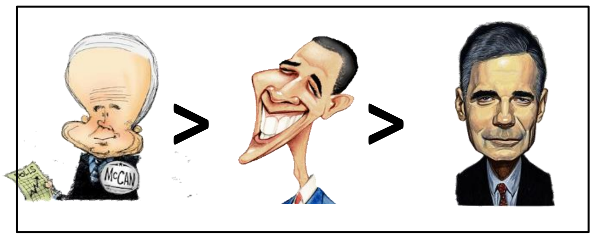
- We saw several voting rules that are trying to generalize the concept of "majority" in different ways.
- Condorcet proposed another way, which relies on **pairwise elections.**
- a beats b in pairwise elections, if more voters prefer a over b.

Should we consider pairwise elections directly?

- We can define several voting rules based on the idea of pairwise elections. For example, the Copeland protocol.
- The Copeland score of an alternative is its total “pairwise wins” minus its “pairwise loses”.
- The winner is the alternative with the highest Copeland score.

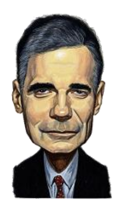
Example: Copeland

P={  ×4,  ×3

 ×2,  ×2 }

Copeland score:

 : 2

 : 0

 : -2

Which is best?

- So many voting rules to choose from ..
- How do we choose a rule from all of these rules?
- How do we know that there does not exist another, “perfect” rule?
- Let us look at some criteria that we would like our voting rule to satisfy
- The axiomatic approach (again…)

Fairness axioms

- Anonymity: names of the voters do not matter
  - Fairness for the voters
- Non-dictatorship: there is no dictator, whose top-ranked alternative is always the winner, no matter what the other votes are
  - Fairness for the voters
- Neutrality: names of the alternatives do not matter
  - Fairness for the alternatives
- If we use a tie-breaking rule for converting a SCF to a resolute SCF, the resulting function might not be neutral (or anonymous)!

Monotonicity criteria

- Informally, monotonicity means that “ranking a candidate higher should help that candidate,” but there are multiple nonequivalent definitions

- A weak monotonicity requirement: if
  - candidate w wins for the current votes,
  - we then improve the position of w in some of the votes and leave everything else the same,

  then w should still win.

Weak monotonicity

- Does STV satisfy the weak monotonicity criterion?
  - 7 votes b > c > a
  - 7 votes a > b > c
  - 6 votes c > a > b
- c drops out first, its votes transfer to a, a wins
- But if 2 votes b > c > a change to a > b > c, b drops out first, its 5 votes transfer to c, and c wins.
- What about plurality with runoff?
- What about Copeland?

Strong monotonicity

- A strong monotonicity requirement: if
  - candidate w wins for the current votes,
  - we then change the votes in such a way that for each vote, if a candidate c was ranked below w originally, c is still ranked below w in the new vote
  - then w should still win.
- Note the other candidates can jump around in the vote, as long as they don’t jump ahead of w

May's theorem (1952)

- Thm: With 2 candidates, a voting rule is anonymous, neutral and monotonic iff it is the plurality rule

  – Since these properties are uncontroversial, this about decides what to do with 2 candidates!

  – Proof: Plurality rule is clearly anonymous, neutral and monotonic
  – Other direction is more interesting
  – For simplicity, assume an odd number of voters

May's theorem (1952)

- Thm: With 2 candidates, a voting rule is anonymous, neutral and monotonic iff it is the plurality rule
  - Proof: Anonymous and neutral implies only number of votes matters
  - Two cases:
    - N(A>B) = N(B>A)+1 and A wins.
      - By monotonicity, A wins whenever N(A>B) > N(B>A)
    - N(A>B) = N(B>A)+1 and B wins
      - Swap one vote A>B to B>A. By monotonicity, B still wins. But now N(B>A) = N(A>B)+1. By neutrality, A wins. This is a contradiction.

Weak Pareto efficiency criterion

- If all agents prefer *a* to *b*, the voting rule will never choose *b* to be the winner.
- Note: the voting rule does not have to choose *a*.
- However, if all votes rank *a* first, then *a* should win.
- This criterion is also called unanimity.
- Does Plurality satisfy weak Pareto efficiency?
- Does Copeland satisfy weak Pareto efficiency?

Condorcet criterion

- A candidate is a Condorcet winner if a beats any other candidate in a pairwise election.
- A voting rule is Condorcet consistent if the Condorcet winner is always selected.
- We already saw that Plurality and Borda are not Condorcet consistent

Condorcet paradox

$P = \{ \boxed{a > b > c} \times 4, \quad \boxed{b > c > a} \times 3$

$\boxed{b > a > c} \times 2, \quad \boxed{c > a > b} \times 2 \}$

Plurality(P), Borda(P)=b

- Candidate a is the Condorcet winner.

Condorcet paradox

$$P=\left\{ \boxed{a > b > c} \times 4,\ \boxed{b > c > a} \times 3,\ \boxed{b > a > c} \times 2,\ \boxed{c > a > b} \times 4 \right\}$$

Plurality(P), Borda(P)=b

- Candidate a is the Condorcet winner.
- What if we add two voters that prefer c > a > b?
- Majority prefer a to b, and prefer b to c, and prefer c to a!

Condorcet criterion

- A voting rule is Condorcet consistent if the Condorcet winner is always selected, **when there is one**.
- If there is a Condorcet winner, then it is unique.

Condorcet criterion

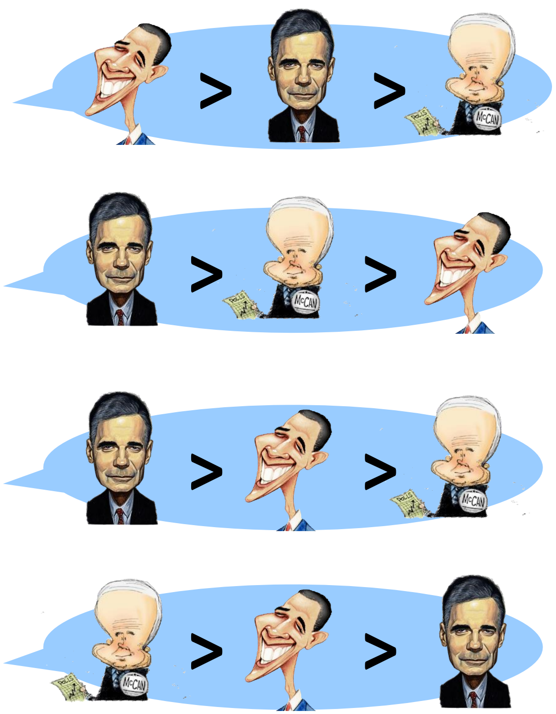
- Theorem (Fishburn-1974). No positional scoring rule with strict ordering of weights satisfies Condorcet criterion:
  - suppose $s_1 > s_2 > s_3$

3 Voters

2 Voters

1 Voter

1 Voter



 is the Condorcet winner

CONTRADICTION

 $: 3s_1 + 2s_2 + 2s_3$

 $: 3s_1 + 3s_2 + 1s_3$

Majority criterion

- If a candidate is ranked first by most votes, that candidate should win.
  - Relationship to Condorcet criterion?

- Some rules do not even satisfy this
- E.g. Borda:
  - a > b > c > d > e
  - a > b > c > d > e
  - c > b > d > e > a
- a is the majority winner, but it does not win under Borda

Muller-Satterthwaite impossibility theorem [1977]

- Is Copeland the best voting rule?
- Theorem: Suppose there are at least 3 candidates. Then there exists no resolute rule that simultaneously:
  - satisfies weak Pareto efficiency,
  - is non-dictatorial, and
  - is monotone (in the strong sense).

Social welfare function

- Let’s look on our voting rules as social welfare functions.
- How to generalize our previous criteria:
  - Anonymity and neutrality: the same.
  - Non-dictatorship: there does not exist a voter such that the rule simply always copies that voter’s ranking.
  - Weak Pareto efficiency → Pareto efficiency: if all votes rank *a* above *b*, then the rule should rank *a* above *b*.

Independence of irrelevant alternatives

- Result between *a* and *b* only depends on the agents' preferences between *a* and *b*.

- Formally, for two profiles $D_1 = (R_1, \ldots, R_n)$ and $D_2 = (R_1', \ldots, R_n')$ and any pair of alternatives a and b,
  - if for all voter j, the pairwise comparison between a and b in R_j is the same as that in R_j'
  - then the pairwise comparison between a and b are the same in $f(D_1)$ as in $f(D_2)$
  - even if voters' preferences between other pairs like *a* and *x*, *b* and *y*, or *x* and *y* change

Arrow's impossibility theorem [1951]

- Theorem: Suppose there are at least 3 candidates. Then there exists no rule that simultaneously:
  - satisfies Pareto efficiency,
  - is non-dictatorial, and
  - satisfies independent of irrelevant alternatives.

- The idea: we have to break Condorcet cycles, but how we do this, inevitably leads to trouble
- A genius observation
  - Led to the Nobel prize in economics

Strategic behavior (of the agents)

- Manipulation: an agent (manipulator) casts a vote that does not represent her true preferences, to make herself better off

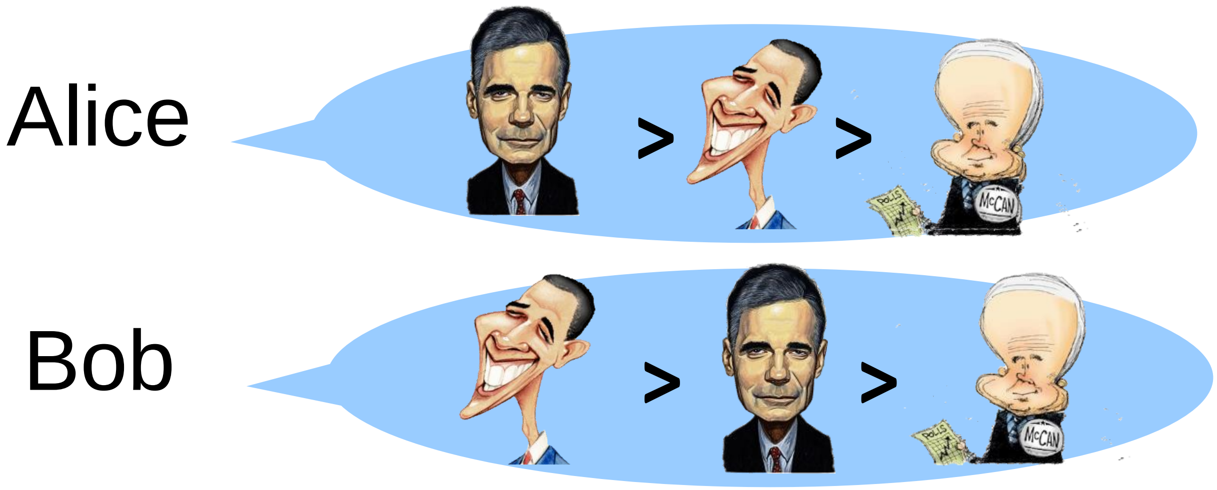
- A voting rule is strategy-proof if there is never a (beneficial) manipulation under this rule

- Do you think Plurality is strategy-proof?

Manipulation under Plurality

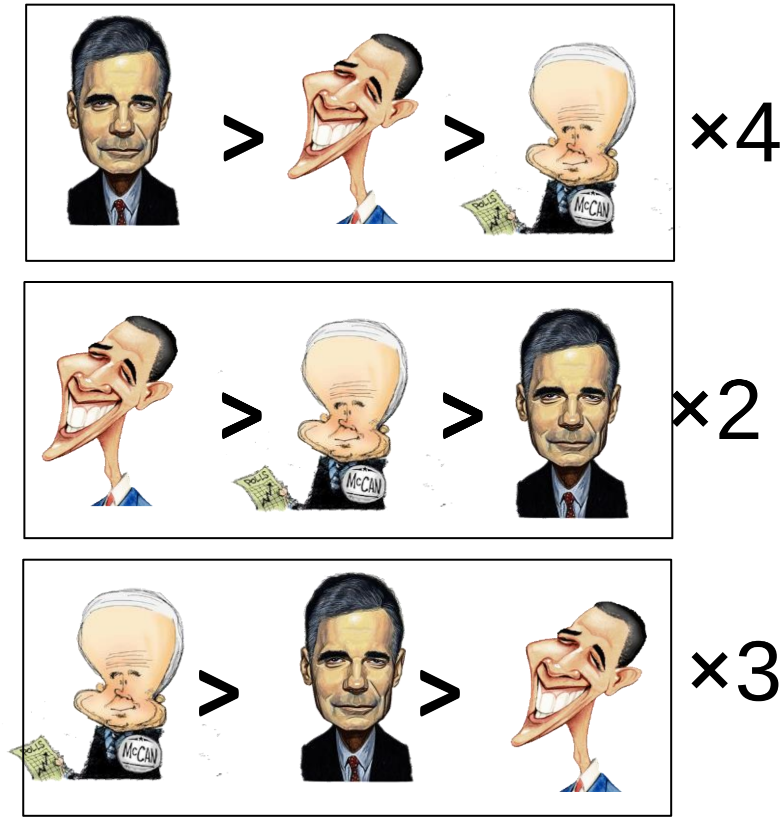
- Suppose a voter prefers a > b > c
- Also suppose she knows that the other votes are
  - 2 times b > c > a
  - 2 times c > a > b
- Voting truthfully will lead to a tie between b and c
- She would be better off voting e.g. b > a > c, guaranteeing b wins

Manipulation under Borda



What if we change the tie-breaking mechanism?

Manipulation under STV



N>O>M → O>N>M

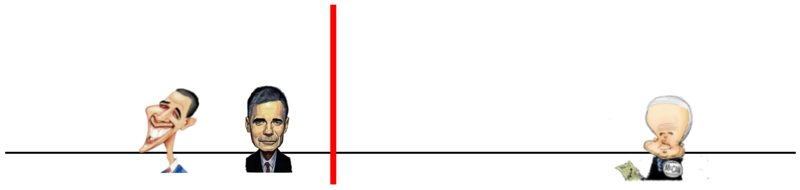
Gibbard-Satterthwaite impossibility theorem (1973, 1975)

- Suppose there are at least 3 candidates
- There exists no resolute rule that is simultaneously:
  - onto (for every candidate, there are some votes that would make that candidate win),
  - nondictatorial, and
  - nonmanipulable

- This is a powerful negative result

A few ways out

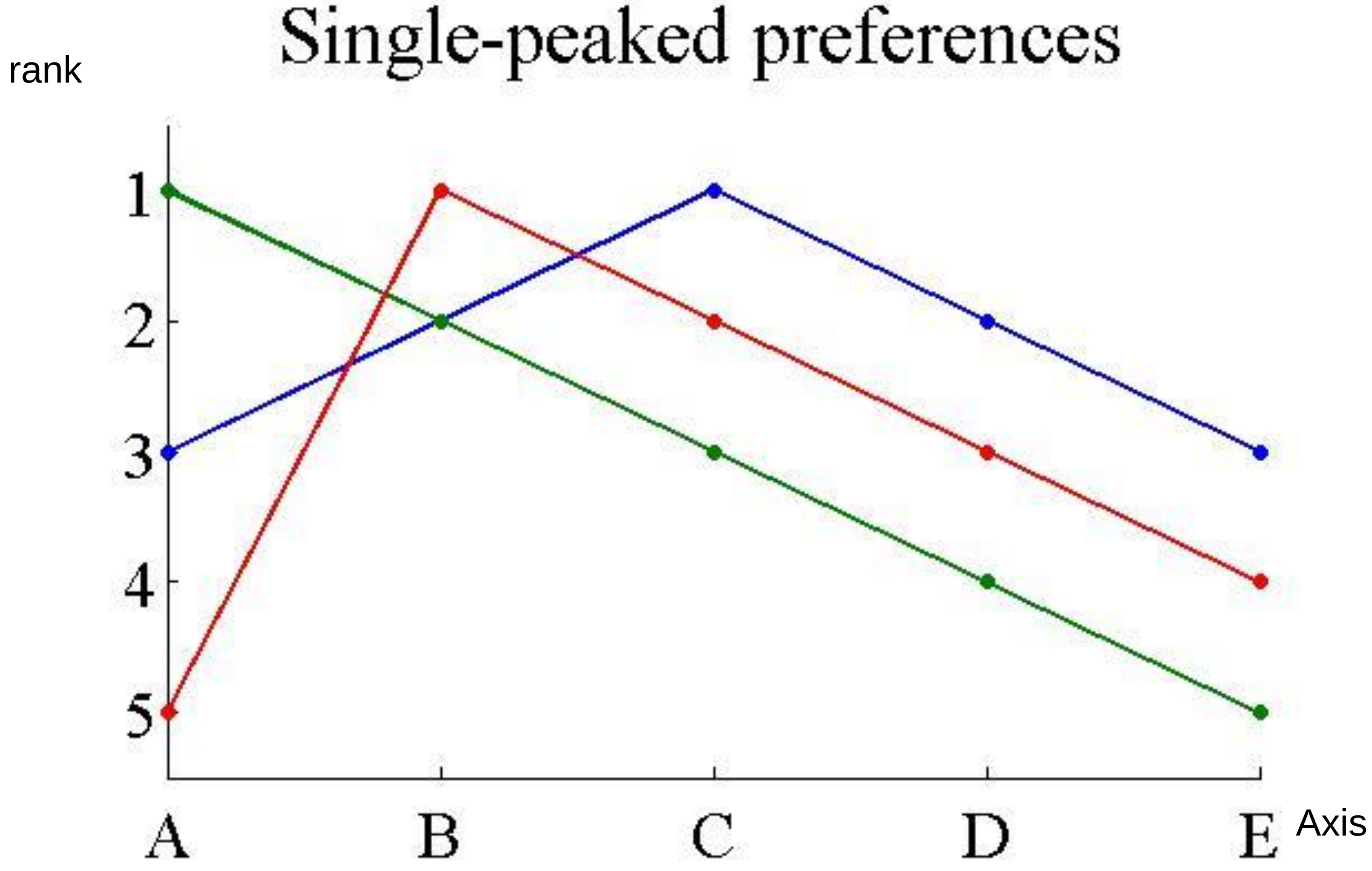
- Relax non-dictatorship: use a dictatorship
- Restrict the number of alternatives to 2
- Relax unrestricted domain: mainly pursued by economists
- For example, single-peaked preferences:



Single-peaked preferences

- There exists a social axis S
  - linear order over the alternatives
- Each voter's preferences R are compatible with the social axis S
  - there exists a "peak" a such that
    - [b$\prec$c$\prec$a in S] implies [c$\succ$b in R]
    - [a$\succ$c$\succ$b in S] implies [c$\succ$b in R]
    - alternatives closer to the peak are more preferred
  - different voters may have different peaks

Examples



A Strategy-proof rule for single-peaked preferences

- The median rule
  - given a profile of “peaks”
  - choose the median in the social axis
- For example, suppose that S = a > e > c > d > b
  - v1: a > e > c > d > b
  - v2: e > a > c > d > b
  - v3: d > b > c > e > a
  - v4: b > d > c > e > a
  - v5: d > c > e > b > a
- Who is the winner?

- Theorem: The Median rule is group strategy-proof (and thus also strategy-proof)
- Proof:
  - Let c* be the true winner
  - Assume that a subset V' of voters can manipulate, and the new winner is c'
  - Clearly, the voters whose peaks are c* are not in V'
  - Let V'r and V'l be the sets of voters whose peaks are to the right and to the left of c*, respectively
  - W.L.O.G. assume that c' is to the left of c*
  - Since no report from V'l can move the median left → V'r cannot be empty. Let *i* be a voter in V'r with a peak *p*.
  - However, since *p* is to the right of c*, c' < c* < *p* in S → c' < c* according to the preferences of *i*. That is, *i* loses from this manipulation. Thus, V'r is empty. Contradiction.

Another way out?

- A randomized rule that maps every profile of votes to a probability distribution over the alternatives
- We first need to define strategyproofness in the context of randomized rules, which is not trivial.
- E.g., a voter prefers a > b > c
  - b is preferred over a 50-50 lottery over a and c?
  - b is preferred over a uniform lottery over all candidates?
- A conservative definition: a randomized rule is strategy-proof if and only if for every utility function over the alternatives that is consistent with the voter's preferences over the (pure) alternatives, the voter maximizes her utility by reporting these true preferences

Another impossibility…

Theorem (Gibbard 1977)

Any strategy-proof randomized rule is a randomization over a collection of the following types of rules:

- unilateral rules, under which at most one voter's vote affects the outcome;
- duple rules, under which there are at most two alternatives that have a possibility of winning (i.e., that win under some profile).

Randomized rules

- Randomization is not the answer
  - It results in the discarding of all but one of the votes, or in the discarding of all but two of the alternatives
- However, this is not entirely unreasonable
  - We can randomly choose a dictator
  - We can randomly choose two alternatives and have a majority election between them

Computational perspective

- We first need to verify that the voting rules are not too complicated so that nobody can easily compute the winner.
- The winner determination problem:
  - Given: a voting rule f
  - Input: a preference profile P and an alternative c
  - input size: $nm\log m$
  - Output: is c the winner of f under P?

- We want a voting rule where the winner determination is in P.

Computational perspective

- The winner determination problem for all of the voting rules that we saw is in P. 

- There are some interesting and important voting rules where the winner determination problem is NP-hard. 
  - Dodgson rule
  - Kemeny rule

Dodgson voting rule

- We saw that there is not always a Condorcet winner.
- A Dodgson winner is a candidate who is "closest" to being a unique Condorcet winner.
- That is, the Dodgson score of a candidate, *a*, is the smallest number of sequential exchanges of adjacent candidates in preference orders such that after those exchanges *a* is a Condorcet winner.

Example: Dodgson voting rule

- 2 voters that vote for: a > b > c

- The Dodgson scores of a,b,c are 0,2,4 respectively.

- Now suppose we have 2 voters with
  - a > b > c
  - b > a > c

- The Dodgson scores of a,b,c are 1,1,4 respectively.

Kemeny voting rule

- The idea: create an overall ranking of the candidates that has as few *disagreements* as possible.
- Kendall tau distance
  - $K(R1, R2)$= # {different pairwise disagreements}
  - Also called bubble-sort distance, since it is also the number of swaps that the bubble sort algorithm would make to place one list in the same order as the other list

$$K(\, b \succ c \succ a \,,\, a \succ b \succ c \,) = \, ?$$

Kemeny voting rule

- Kemeny(P)=argmin$_W$ $\mathrm{K}(P,W)$=argmin$_W$ $\Sigma_{R\in \mathrm{P}}\mathrm{K}(R,W)$
- For single winner, choose the top-ranked alternative in Kemeny(P)
- E.G.,

$a \succ b \succ c \succ d$	$b \succ a \succ c \succ d$	$d \succ a \succ b \succ c$	$c \succ d \succ b \succ a$
1	1	2	2

- $\mathrm{K}(P, a \succ b \succ c \succ d)$=0+1+2*3+2*5=17

Computational perspective

- Gibbard-Satterthwaite only tells us that for some instances, successful manipulations exist.
- It does not say that these manipulations are always easy to find.
- If it is computationally too hard for a manipulator to compute a manipulation, she is best off voting truthfully
  - Similar as in cryptography

- For which common voting rules manipulation is computationally hard?

A formal computational problem

- The simplest version of the manipulation problem:
- Constructive Unweighted Manipulation (C-UM):
  - We are given a voting rule r, a manipulator v* $\in V$, the (unweighted) votes of the other voters, and an alternative p.
  - We are asked if we can cast a vote for v* that will make p win.
- E.g., for the Borda rule:
  - Voter 1 votes A > B > C
  - Voter 2 votes B > A > C
  - Voter 3 votes C > A > B
  - Voter 4 is the manipulator
- Current Borda scores are: A: 4, B: 3, C: 2
- Can we make B win?
- Answer: YES. Vote B > C > A (Borda scores: A: 4, B: 5, C: 3)

Few observations

- The problem is a decision problem. We would like to know if there exists such a vote for v*.
- The problem is defined more strongly than the simple definition of manipulation. The question is not whether v* can guarantee a victory of a more preferred candidate, but whether he can guarantee the victory of a specific candidate.
- The Gibbard-Satterthwaite theorem specifies that one of the voters can manipulate. We don’t know if this is true under the given circumstances, where v*, p, and the voting profile were given.
- The goal is to make p win. There are actually two version of this objective:
  – Make p among the set of winners - the nonunique winner model
  – Make p the single winner - the unique winner model

Basic Result

- C-UM can be solved in efficiently for many voting rules, due to Bartholdi et al. 1989

- Definition: We say that a voting rule satisfies the BTT conditions if
  - It can be run in polynomial time
  - For every profile *P* and every alternative a, the rule assigns a score S(*P*,a) to a
  - For every profile *P*, the alternative with the maximum score wins
  - The following monotonicity condition holds: for any *P*,*P'* and for any alternative a, if for each voter i we have that $\{b: a \succ_i b\} \subseteq \{b: a \succ_i' b\}$ then S(*P*,a) ≤ S(*P'*,a)

    That is, if we modify a vote in a way that does not rank anyone ahead of a that was previously ranked behind a, then a's score cannot have decreased.

Manipulation Algorithm

- Theorem (BTT-1989)

The C-UM problem can be solved in polynomial time for any rule satisfying the BTT conditions

- Algorithm

Initialization: Place p at the top of the preference order

Iterative Step: find some remaining alternative b that can be ranked in the next lower position without preventing p from winning (To check this, complete the rest of the vote arbitrarily).

- If no such alternative can be found, declare failure;
- if the vote is completed, declares success;
- Otherwise, repeat for the next position

Why the algorithm is correct? (or, where do we use the BTT conditions?)

STV

- Does STV satisfies the BTT condition?

- No! C-UM is NP-C even with a single manipulator [Bartholdi and Orlin, SCW-91]

- However, in practice this is not such a hard problem
  - Conitzer et al. (2007) give an $O(n1.62^m)$ time recursive algorithm to compute the set of alternatives that can win an STV election
  - Walsh (2010) showed that this algorithm could often quickly compute manipulations of the STV rule even with hundreds of alternatives

Destructive manipulation

- Destructive Unweighted Manipulation (D-UM):
  - We are given a voting rule r, a manipulator v* $\in V$, the (unweighted) votes of the other voters, and an alternative d.
  - We are asked if we can cast a vote for v* that will make sure that d is not the winner

- Given an algorithm for C-UM in the nonunique winner model we can efficiently solve the D-UM problem in the unique winner model

Coalitions of Manipulators

- We can also extend the single manipulation problem to a coalitional manipulation problem:
- Constructive Unweighted Coalitional Manipulation (C-UCM):
  - We are given a voting rule r, a number of manipulator k, the (unweighted) votes of the other voters, and an alternative p.
  - We are asked if we can cast k votes for the manipulators that will make p win.

- Can be extended to a destructive version, D-UCM
- Both UM and UCM can be extended to weighted versions

Results for C-UCM and D-UCM

- C-UCM with Copeland and Borda: NP-C if there at least 2 manipulators [FHS AAMAS-08,10], [DKN+ AAAI-11] [BNW IJCAI-11].

- However, D-UCM can be solved in polynomial time for any rule satisfying the BTT conditions

- How? With the following algorithm:
  - For any candidate c (except for d)
    - All the manipulators place c at the top position, and d at the bottom
    - The other candidates are placed arbitrarily
    - If d is not winning, a manipulation is found
  - No manipulation exists

Manipulating C-UCM with Borda

- Let’s examine again the assumption that computational complexity is a barrier against manipulation
  - Recall that NP-hardness is a worst-case concept
- Suppose you want to add k manipulators for Borda who would promote p
- Clearly, p should be ranked first in all of them
- If we could rank no one else, we would be happy
  - But we must give points to other candidates
- The goal is to give points to bad candidates who won’t win

Greedy algorithm for UCM for Borda - Reverse

- Reverse is a simple greedy algorithm, which constructs the vote of each manipulator in turn.

- For each manipulator:
  - Place p on top
  - Place the remaining candidates in reverse order of their current Borda scores

Example

- Suppose that P = {(c>a>b>d),(b>c>a>d)}
- Borda scores:

	Current
a	3
b	4
c	5
d	0

- We would like that d will win. The tie-breaking rule is a fixed order: d>c>b>a
- Is it possible with two manipulators?

Example

- Suppose that P = {(c>a>b>d),(b>c>a>d)}
- Borda scores:

	Current			
a	3			
b	4			
c	5			
d	0			

- The votes of the manipulators:

V1				
V2				
V3				

Example

- Suppose that P = {(c>a>b>d),(b>c>a>d)}
- Borda scores:

	Current			
a	3	5		
b	4	5		
c	5	5		
d	0	3		

- The votes of the manipulators:

V1	d	a	b	c
V2				
V3				

Example

- Suppose that P = {(c>a>b>d),(b>c>a>d)}
- Borda scores:

	Current			
a	3	5	5	
b	4	5	6	
c	5	5	7	
d	0	3	6	

- The votes of the manipulators:

V1	d	a	b	c
V2	d	c	b	a
V3				

Example

- Suppose that P = {(c>a>b>d),(b>c>a>d)}
- Borda scores:

	Current			
a	3	5	5	7
b	4	5	6	7
c	5	5	7	7
d	0	3	6	9

- The votes of the manipulators:

V1	d	a	b	c
V2	d	c	b	a
V3	d	a	b	c

The Performance of Reverse

- We can consider the number of manipulators as our objective, which we would like to minimize.
- Theorem [ZPR AIJ-09]: If exists a manipulation with k-1 manipulators reverse will succeed with k manipulators
- We can also interpret this result as an indication of the performance of Reverse:
  - Reverse will succeed on any given instance such that the same instance but with one less manipulator is manipulable
  - That is, there is a small window of instances on which Reveres may fail; the algorithm is proven to succeed on all other instances

Can we do better?

- A successful manipulation can be described by a manipulation matrix.
- A manipulation matrix is an k by m matrix A where $A(i,j) = s$ if and only if the i-th manipulator adds a score of s to candidate j.
- Thus,
  - Each of the n rows is a permutation of 0 to m − 1
  - The sum of the j-th column is less than or equal to the maximum score candidate j can receive without defeating p
- How does Reverse build the manipulation matrix?

Relaxed Manipulation Matrix

- A relaxed manipulation matrix is an *k* by *m* matrix A_r that contains *k* copies of 0 to m−1, and the sum of the *j*-th column is less than or equal to the maximum score candidate *j* can receive without defeating *p*
- That is, a row in A_r can repeat a number, provided other rows compensate by not having the number at all.
- Does a row in A_r represent a legitimate vote?
- Theorem: given an *k* by *m* relaxed manipulation matrix A_r, it is possible to efficiently construct an *k* by *m* manipulation matrix A with the same column sums.

Largest Fit (Davies et al. 2011)

- Idea: assigns the largest unallocated score to the largest gap. That is,
  - Assign k scores of m-1 to column p
  - Assign the remaining k(m−1) scores in reverse order to the columns corresponding to the candidate with the current smallest score who has not yet received k votes
  - Convert the relaxed manipulation matrix into a manipulation matrix

Our Previous Example

- Suppose that P = {(c>a>b>d),(b>c>a>d)}
- Borda scores:

	Current	
a	3	
b	4	
c	5	
d	0	

- Relaxed manipulation matrix:

	a	b	c	d

Our Previous Example

- Suppose that P = {(c>a>b>d),(b>c>a>d)}
- Borda scores:

	Current	
a	3	
b	4	
c	5	
d	0	6

- Relaxed manipulation matrix:

	a	b	c	d
				3
				3

Our Previous Example

- Suppose that P = {(c>a>b>d),(b>c>a>d)}
- Borda scores:

	Current	
a	3	5
b	4	
c	5	
d	0	6

- Relaxed manipulation matrix:

	a	b	c	d
	2			3
				3

Our Previous Example

- Suppose that P = {(c>a>b>d),(b>c>a>d)}
- Borda scores:

	Current	
a	3	5
b	4	6
c	5	
d	0	6

- Relaxed manipulation matrix:

	a	b	c	d
	2	2		3
				3

Our Previous Example

- Suppose that P = {(c>a>b>d),(b>c>a>d)}
- Borda scores:

	Current	
a	3	5
b	4	6
c	5	6
d	0	6

- Relaxed manipulation matrix:

	a	b	c	d
	2	2	1	3
				3

Our Previous Example

- Suppose that P = {(c>a>b>d),(b>c>a>d)}
- Borda scores:

	Current	
a	3	6
b	4	6
c	5	6
d	0	6

- Relaxed manipulation matrix:

	a	b	c	d
	2	2	1	3
	1			3

Our Previous Example

- Suppose that P = {(c>a>b>d),(b>c>a>d)}
- Borda scores:

	Current	
a	3	6
b	4	6
c	5	6
d	0	6

- Relaxed manipulation matrix:

	a	b	c	d
	2	2	1	3
	1	0		3

Our Previous Example

- Suppose that P = {(c>a>b>d),(b>c>a>d)}
- Borda scores:

	Current	
a	3	6
b	4	6
c	5	6
d	0	6

- Relaxed manipulation matrix:

	a	b	c	d
	2	2	1	3
	1	0	0	3

Our Previous Example

- Suppose that P = {(c>a>b>d),(b>c>a>d)}
- Borda scores:

	Current	
a	3	6
b	4	6
c	5	6
d	0	6

- We thus get the following manipulation:

V1	d	b	a	c
V2	d	a	c	b

Average Fit (Davies et al. 2011)

- Idea: consider both the size of the gap and the number of scores still to be added to each column.
- That is, if two columns have the same gap, we want to choose the column that contains the fewest scores.
- We thus look on the average score required to fill each gap. That is, the size of the gap divided by the number of scores still to be added to the column. Let g_c be this number.

Average Fit (Davies et al. 2011)

- Assign k scores of m-1 to column p
- Compute g_c for each column
- While there are unassigned scores:
  - Choose the column with the largest g_c (tie-break on the column containing fewest scores)
  - Assign the largest unassigned score that will fit into the gap
  - Update g_c of this column
- Convert the relaxed manipulation matrix into a manipulation matrix

Results (Davies et al. 2011)

m	# Inst.	Reverse	Largest Fit	Average Fit	Largest Fit beat Average Fit
4	2771	2611	2573	2771	0
8	5893	5040	5171	5852	2
16	5966	4579	4889	5883	3
32	5968	4243	4817	5879	1
64	5962	3980	4772	5864	3
128	5942	3897	4747	5821	2
Total	32502	24350	26969	32070	11
%		75	83	99	<1

Figure 1: Number of uniform elections for which each method found an optimal manipulation.

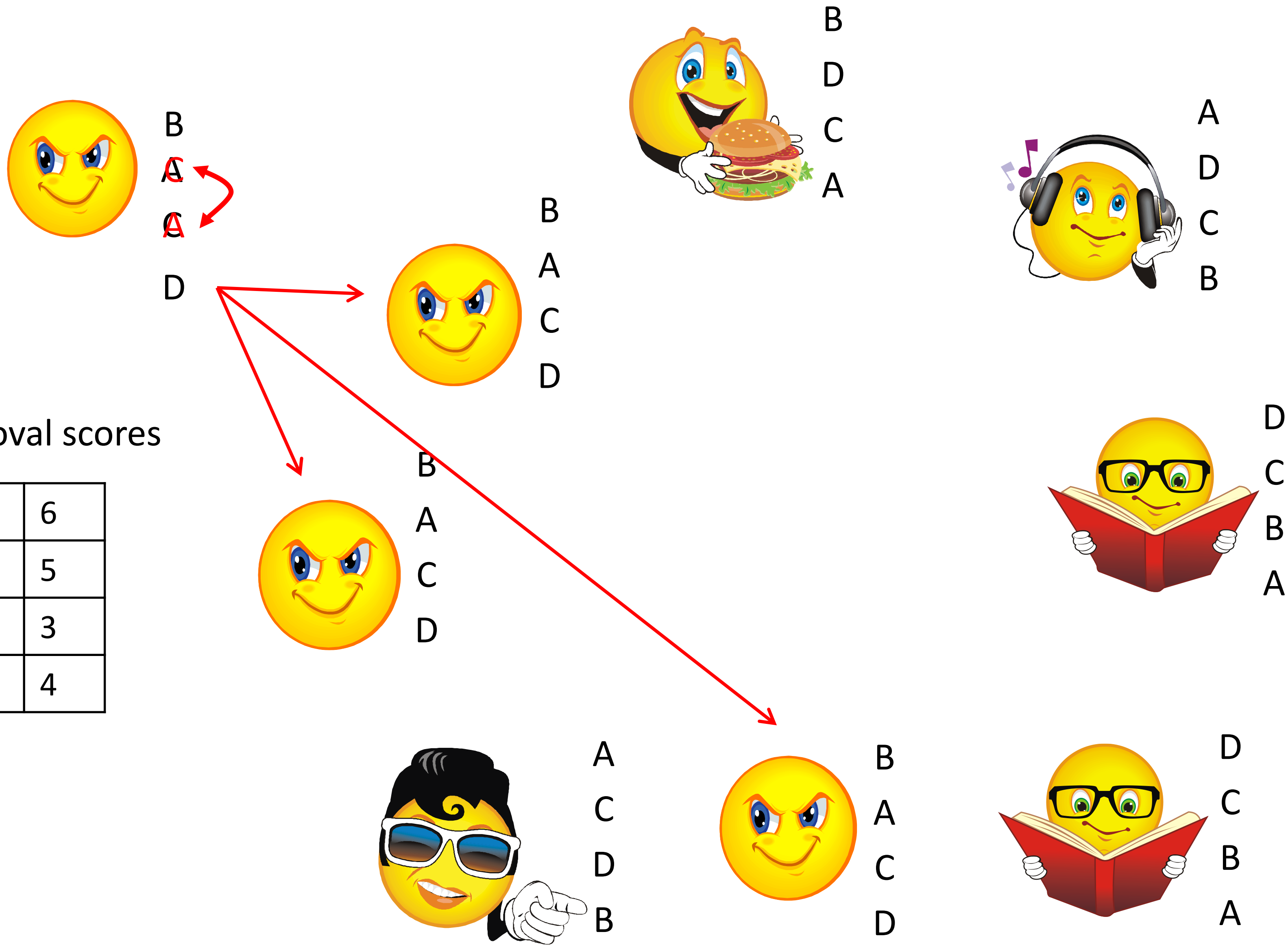
m	# Inst.	Reverse	Largest Fit	Average Fit	Largest Fit beat Average Fit
4	3929	3666	2604	3929	0
8	5501	4709	2755	5496	0
16	5502	4357	2264	5477	1
32	5532	4004	2008	5504	0
64	5494	3712	1815	5475	0
128	5571	3593	1704	5565	0
Total	31529	24041	13150	31446	1
%		76	42	99.7	<1

Figure 2: Number of urn elections for which each method found an optimal manipulation.

Is Coalitional Manipulation a Real Model?

- Manipulators don’t have any preferences
- The manipulating coalition is given
  - How does it form? Who is the coordination center?
  - How to reach an agreement?
  - How to make sure every manipulator will obey?

A New Framework



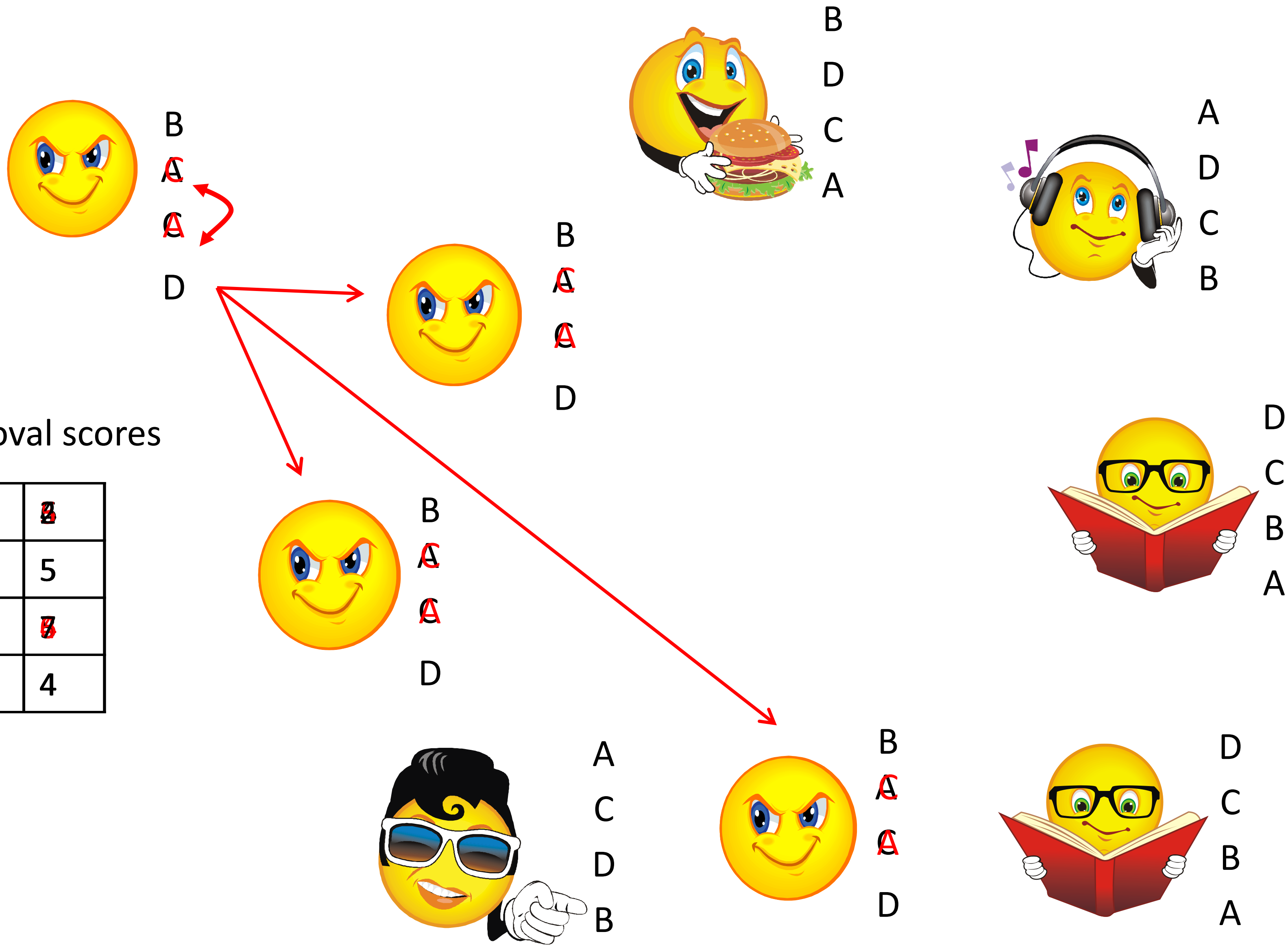
2-approval scores

A	6
B	5
C	3
D	4

Safe Strategic Voting

- Incentive to manipulate: promote a candidate
- Disincentive: you can’t force voters to manipulate
- Successful manipulation – a Safe strategic vote

Unsuccessful Manipulation



2-approval scores

A	~~6~~ 2
B	5
C	~~3~~ 7
D	4

Safe Strategic Voting

- Definition

A safe manipulation in profile L, is a vote L_i' of voter *i*, such that:

1. For some subset I of voters of type L_i , $\mathrm{f}(\mathrm{L}_{-\mathrm{I}}, L_i') \succ_i f(L)$
2. For any subset I of voters of type L_i , $\mathrm{f}(\mathrm{L}_{-\mathrm{I}}, L_i') \not\succcurlyeq_i f(L)$

   where $L_i' = (L_i', \ldots, L_i')$

- That is, it is possible to gain if the “right” number of followers cast L_i', but no number of followers may cause harm.

Another Impossibility…

- Theorem (Slinko and White 2008)

A deterministic and onto voting rule for at least three candidates has no safe manipulations, if and only if it is dictatorial.

- This is an extension of the Gibbard-Satterthwaite theorem to safe manipulations

- However, maybe it is hard to find safe manipulations?

Algorithmic Questions

- How hard is to find a safe vote to manipulate? (ExistSafe)
- How hard is to decide if a given vote is safe? (IsSafe)

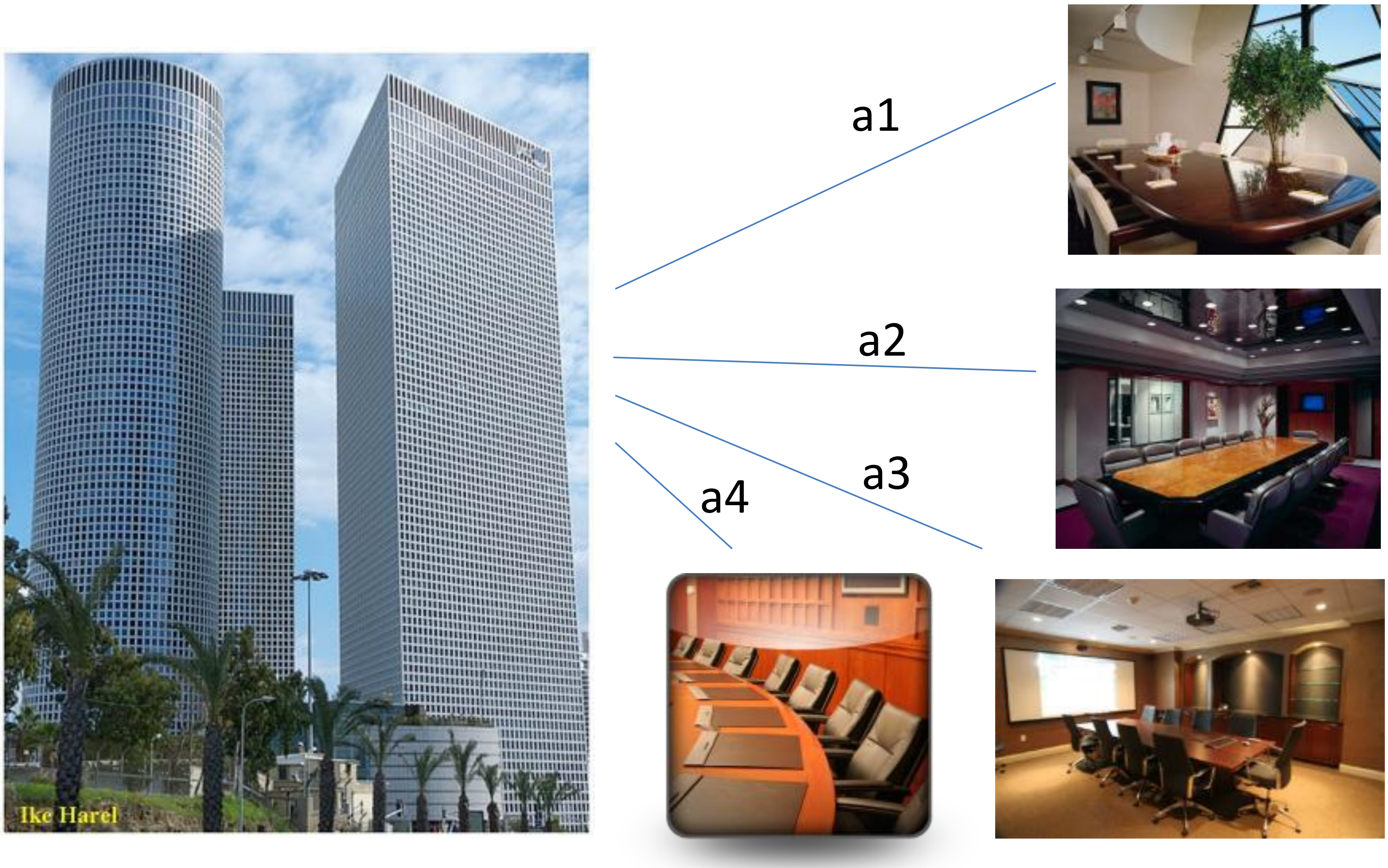
- Different settings
  - Unweighted or weighted voters
  - Number of candidates
  - Tie-breaking rule
    - Order-based: $\exists \succ, c_{i_1} \succ \ldots \succ c_{i_m}$ s.t. $\forall S \subseteq C$ T(S) = $\max_{\succ}(S)$
    - Arbitrary: T({2,3}) = 2 but T({2,3,4}) = 3
  - Voting rule

Results [Hazon and Elkind SAGT-10]

Voters	Number of Candidates	Rule	Problem			
			IsSafe		ExistSafe	
			order-based	any	order-based	any
weighted	constant or any	Plurality	P	CoNP-Hard	P	CoNP-Hard
		Veto	P			
		K-approval	P	CoNP-Hard	P	CoNP-Hard
		Bucklin		CoNP-Hard	P	CoNP-Hard
		Borda	CoNP-Hard			
unweighted	constant	Plurality	P			
		Veto				
		K-approval				
		Bucklin				
		Borda				
	any	Plurality	P		P	
		Veto			P	
		K-approval			P	NP-Hard
		Bucklin			P	
		Borda				

Application: Reducing Energy Consumption

How to Choose a Meeting Room?



The Voting Process

a3 > a4 > a1 > a2

a4 > a2 > a1 > a3

a2 > a3 > a1 > a4

a1 > a2 > a3 > a4

The Voting Process

a3 > a4 > a1 > a2

a4 > a2 > a1 > a3

a2 > a3 > a1 > a4

a1 > a2 > a3 > a4

a1 > a2

The Voting Process

a3 > a4 > a1 > a2

a4 > a2 > a1 > a3

a2 > a3 > a1 > a4

a1 > a2 > a3 > a4

a3 > a1 > a4 > a2

?

a1 > a2

The Persuasion Problems

- Given:
  - A set of alternatives
  - A set of voters with their preferences
  - A preferences list of the sender
- Is there a “good” set of suggestions?
- K-Persuasion: send at most k suggestions

Add Safety Requirement

- What if not all the voters accept the suggestions?

- Safe-Persuasion
  - Is there a “good” and safe set of suggestions?

- K-Safe-persuasion: send at most k suggestions

Persuasion ≠ Manipulation

- In coalitional manipulation
  - The manipulators always obey their suggestions
  - There is no requirement that they will benefit from it
  - How the manipulators attain full knowledge?
- In persuasion
  - Voters can accept or decline the sender's suggestions
  - Send suggestion only to voters that will benefit from it, and we add safety requirement
  - The sender is the election organizer

Complexity Results [HLK IJCAI-13]

	Persuasion	K-Persuasion	Safe-Persuasion	K-Safe-Persuasion
Plurality	P	P	P	P
Veto	P	P	P	P
K-Approval	P	NP-complete	NP-hard	NP-hard
Bucklin	P	NP-complete	NP-hard	NP-hard
Borda	NP-complete	NP-complete	NP-hard	NP-hard