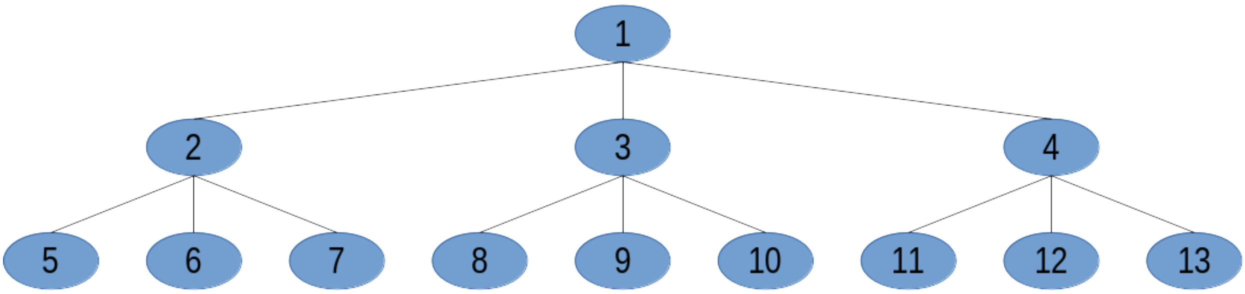
Solutions Sheet

Exercise 2.2

(a) The state space consists of all states and all possible transitions between those states. In this special case, the state space is a tree as evidenced by the fact that there is no loop and states have “successors” (children).



(b)

BFS: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10

DFS with limit two (as the tree only has levels 0 to 2, the depth limit does not have any effect here): 1, 2, 5, 6, 7, 3, 8, 9, 10

IDS: 1, 1, 2, 3, 4, 1, 2, 5, 6, 7, 3, 8, 9, 10

Exercise 5. 1

Using a truth table, determine whether the following formulas are tautologous, contingent, or inconsistent:

(a) $(p \vee q) \wedge (\neg p \vee \neg q)$

p	q	$p \vee q$	$\neg p \vee \neg q$	(a)
F	F	F	T	F
F	T	T	T	T
T	F	T	T	T
T	T	T	F	F

As we have some rows with “F” and some with “T”, we find that the formula is contingent.

(b) $(p \rightarrow q) \rightarrow (p \wedge \neg q)$

P	Q	$p \rightarrow q$	$p \wedge \neg q$	(b)
F	F	T	F	F
F	T	T	F	F
T	F	F	T	T
T	T	T	F	F

Again, as the formula becomes true for some assignments of variables and false for some other assignments, it is contingent.

Exercise 5.2

Which of the following are correct?

(a) False $\vDash$ True

X ⊨ Y means “X entails Y” (see definition on lecture slides). For practical purposes, we can handle it like the implies operator “→” although it is on a higher / meta-level (making statements about logical statements). “False ⊨ True” is correct because “True” is true in all models where “False” is true.

(b) True ⊨ False

This statement is incorrect because “False” is not true in all models where “True” is true (in fact, it is not true in any model).

(c) (A ∧ B) ⊨ (A ↔ B)

A	B	A ∧ B	A ↔ B
F	F	F	T
F	T	F	F
T	F	F	F
T	T	T	T

This statement is correct because “A ∧ B” is correct in all models (corresponding to rows of the truth table here) where “A ↔ B” is true.

(d) A ↔ B ⊨ A ∨ B

This statement is incorrect, as can be shown using a truth table.

(e) A ↔ B ⊨ ¬ A ∨ B

This statement is correct, as can be shown using a truth table.

(f) (A ∨ B) ∧ (¬ C ∨ ¬ D ∨ E) ⊨ (A ∨ B ∨ C) ∧ (B ∧ C ∧ D ⇒ E)

If “A ∨ B” is true, then “A ∨ B ∨ C” must also be true (why?). Likewise, if “ ¬ C ∨ ¬ D ∨ E” is true, then “¬B ∨ ¬ C ∨ ¬ D ∨ E” must also be true, and this is equivalent to “B ∧ C ∧ D ⇒ E” considering the definition of the implies-operator. Therefore the statement is correct.

(g) (A ∨ B) ∧ (¬ C ∨ ¬ D ∨ E) ⊨ (A ∨ B) ∧ (¬ D ∨ E)

This statement is incorrect, which can be quickly spotted as the “¬ C” is missing in the second bracket of the right hand side (RHS) statement. Imagine a model where A is true and C is false. This would make the left hand side statement true, but not necessarily the RHS statement.

(h) (A ∨ B) ∧ ¬ (A ⇒ B) is satisfiable.

The formula is satisfiable for A = T, B = F, so this statement is correct.

Exercise 5.3

Convert into conjunctive normal form:

(a) C ∧ (A ↔ (B ∨ D))

C ∧ (¬A ∨ B ∨ D) ∧ (A ∨ ¬B) ∧ (A ∨ ¬D)

(b) C ∨ (¬A → (B ∨ D))

A ∨ B ∨ C ∨ D

Exercise 6.1

Convert the following set of sentences to clausal form. Then use resolution to prove ¬ A ∧ ¬ B from them:

S1: A ↔ (B ∨ E)
S2: E ⇒ D
S3: C ∧ F ⇒ ¬ B
S4: E ⇒ B
S5: B ⇒ F
S6: B ⇒ C

Solution:

T1:
(a) $\neg A \lor B \lor E$
(b) $A \lor \neg B$
(c) $A \lor \neg E$
T2:
$D \lor \neg E$
T3:
$\neg B \lor \neg C \lor \neg F$
T4:
$B \lor \neg E$
T5:
$\neg B \lor F$
T6:
$\neg B \lor C$
The question really asks to prove two statements, ¬ A and ¬ B. We start with the latter.
Resolution algorithm: add the negated statement ¬(¬ B) = B as T7 to the knowledge base and try to derive a contradiction:
(1) T5 and T7: F (T9)
(2) T6 and T7: C (T10)
(3) T3 and T9: $\neg B \lor \neg C$ (T11)
(4) T10 and T11: ¬B (T12)
(5) T7 and T12: empty clause. We have proved ¬ B.
Now we restart the whole process for ¬(¬ A) = A as T7. We can also add ¬ B as T8 because that is what we proved in the previous step.
(1) T1a and T7: $B \lor E$ (T9)
(2) T8 and T9: E (T10)
(3) T4 and T10: B (T11)
(4) T8 and T11: empty clause. We have proved ¬ A.

Exercise 6.2

Represent the following sentences in first-order logic using a consistent vocabulary.

Note: there are many possible ways to answer these questions. No matter which one you choose, you should always aim to fully use the elements of first order logic to express these statements in a rather fine-grained way.

(a) Every student who takes French passes it.
$\forall x\ Student(x) \land Takes(x, French) \rightarrow Passes(x, French)$
(b) The best score in Greek is always higher than the best score in French.
$\exists y\ Takes(y, Greek) \land \forall x\ Takes(x, French) \rightarrow Score(y) > Score(x)$
(c) No person buys an expensive policy.
$\forall x\ \forall y\ Person(x) \land Policy(y) \land Expensive(y) \rightarrow \neg\ (Buys(x, y)$
(d) There is an agent who sells policies only to people who are not ensured.
$\exists x\ Agent(x) \land \forall y\ \forall z\ \forall t\ Person(y) \land Policy(z) \land Time(t) \land Sells(x, y, z, t) \rightarrow \neg Ensured(y, t)$
(e) Politicians can fool some of the people all of the time, and they can fool all of the people some of the time, but they can't fool all of the people all of the time.
$\forall x\ \forall t\ \exists y\ Politician(x) \land Person(y) \land Time(t) \rightarrow Fools(x, y, t)$
$\land\ \forall x\ \forall y\ \exists t\ Politician(x) \land Person(y) \land Time(t) \rightarrow Fools(x, y, t)$
$\land\ \neg(\forall x\ \forall y\ \forall t\ Politician(x) \land Person(y) \land Time(t) \rightarrow Fools(x, y, t))$
(Note: The statement “$\forall x\ \forall t\ \exists y\ Politician(x) \land Person(y) \land Time(t) \rightarrow Fools(x, y, t)$” should be more correctly written as “$\forall x\ \forall t\ \exists y\ Person(y) \land (Politician(x) \land Time(t) \rightarrow Fools(x, y, t))$” because in the first form, it fails to tell as anything if we don’t have another statement telling us that

there exists a person (“$\exists$ w Person(w)”). Likewise for the second statement and time t. However, we might get away here by assuming that this is part of a knowledge base where objects that are persons and objects that are times indeed exist).