

# STAT 201 Exam 3

**You can do it!**

## **Question break down (40 Questions total)**

- 13 CI questions (Ch 13 & 14) (1 of them interpretation)
- 12 Chapter 15&16 Qs (1 of them interpretation)
- 6 Chapter 17 Qs (1 of them interpretation)
- 7 Chapter 19 Qs (1 of them interpretation)
- 2 Choose the right tool Qs

# Confidence intervals

“We are 95% confident the true population proportion is in the range (0.4, 0.7).”

$$\hat{p} \pm z * SE(\hat{p})$$

$$\bar{y} \pm t_{df} * SE(\bar{y})$$

- Sample statistic
  - the actual observed value of the proportion ( $\hat{p}$ ) or the mean ( $\bar{y}$ )
  - We use this as the best guess of the population parameter and add some give or take (aka Margin of Error)
- Critical value
  - A value looked up based on how confident we want to be
    - $z$  is looked up based on normal distribution;  $t_{df}$  is looked up based on t-distribution and  $df$
    - $df$  is “degrees of freedom”, here  $df = n - 1$
  - Bigger confidence -> bigger critical value -> bigger margin of error -> wider CI
- Standard error
  - More evidence (aka smaller  $n$ ) -> lower SE -> smaller margin of error
  - More variation -> higher SE -> larger margin of error

# Confidence intervals

“We are 95% confident the true population mean is in the range (4, 10).”

$$\hat{p} \pm \underbrace{z * SE(\hat{p})}_{\text{Margin of Error}}$$

$$\bar{y} \pm \underbrace{t_{df} * SE(\bar{y})}_{\text{Margin of Error}}$$

“Margin of Error”

- Affected by level of confidence (bigger confidence, bigger interval, 🧑🍳🍕)
- Affected by sample size (more evidence -> smaller ME -> smaller interval)
- CIs are the **sample stat** plus or minus **ME**, this means that the width of the CI is  $2 * \text{ME}$  (start at middle and take 1 **ME** sized step in both directions)
  - Given CI of (5, 15) we can find ME and sample stat with:
    - $2 * \text{ME} = \text{CI}_{\text{hi}} - \text{CI}_{\text{low}} \rightarrow 2 * \text{ME} = 15 - 5 \rightarrow 2 * \text{ME} = 10 \rightarrow \text{ME} = 5$
    - $\text{sample stat} = \text{CI}_{\text{hi}} - \text{ME} \rightarrow \text{sample stat} = 15 - 5 = 10$

CI for population proportion

$$\hat{p} \pm \underbrace{z * SE(\hat{p})}_{\text{Margin of Error}}$$

### Assumptions

- Success/failure
  - at least 10 successes ( $\hat{p}$ ) and 10 failures ( $\hat{q}$  -> aka  $1 - \hat{p}$ )

CI for population mean

$$\bar{y} \pm \underbrace{t_{df} * SE(\bar{y})}_{\text{Margin of Error}}$$


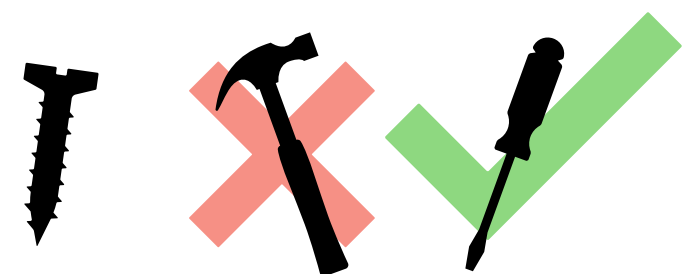
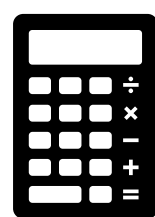
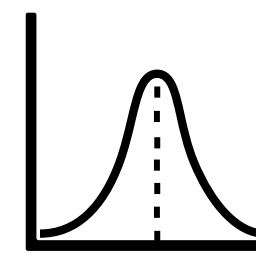
### Assumptions

- Nearly normal
  - Distribution should resemble the normal distribution. When sample size is larger (40ish) we can have larger skew without worry.

### Shared Assumptions

- Randomization
  - Sample is a random sample (we want to be representative, not biased)
- 10%
  - sample size ( $n$ ) must be less than or equal to 10% of population

# Hypothesis testing - steps

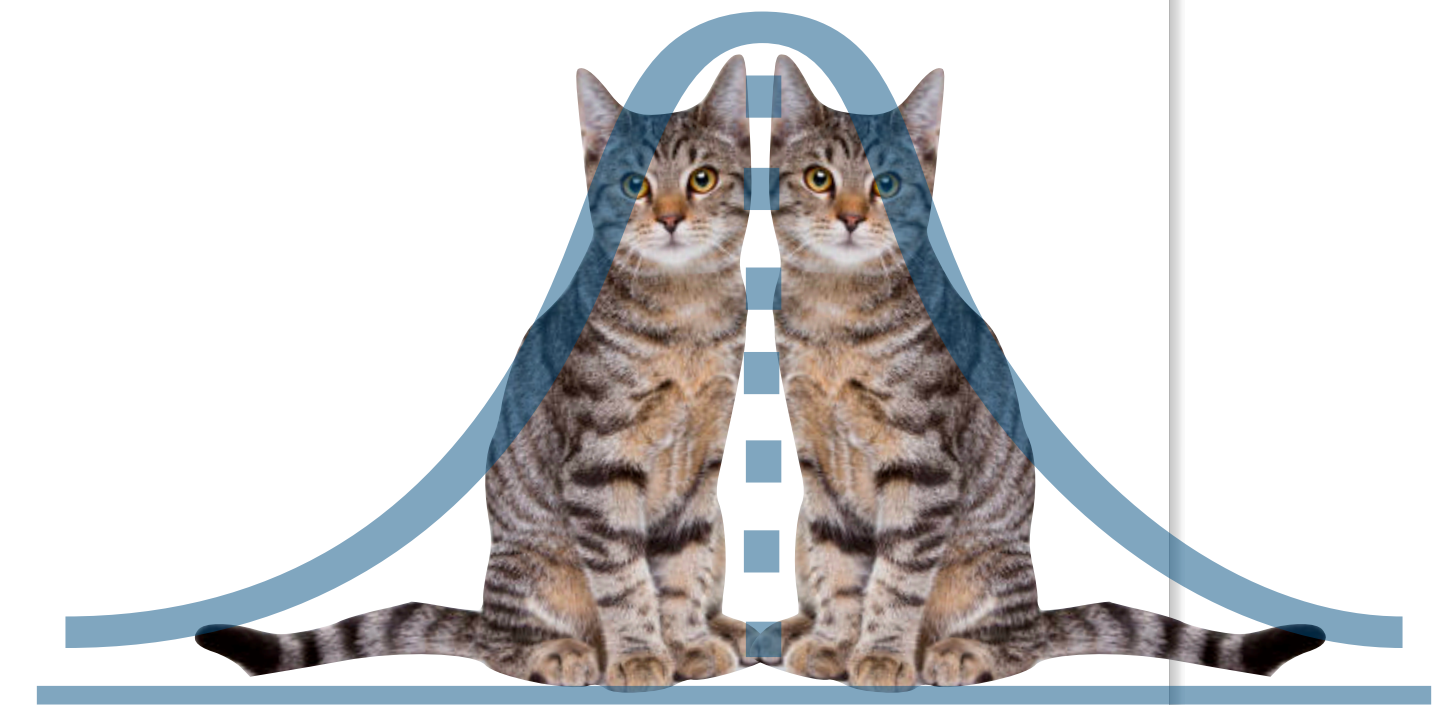
- **Hypotheses** 
  - State the hypotheses (aka the 2 possible outcomes of the test)
- **Model** 
  - Check your assumptions (use the right tool for the job)
- **Mechanics** 
  - Formula time! (or JMP time)
- **Conclusions** 
  - Read the  $p$ -value - “If  $p$  is low,  $H_0$  must go”
    - If  $p < \alpha$  (aka “level of significance”) reject null hypothesis; there is evidence supporting  $H_a$
    - If  $p > \alpha$  (aka “level of significance”) fail to reject null hypothesis; there is not enough evidence supporting  $H_a$

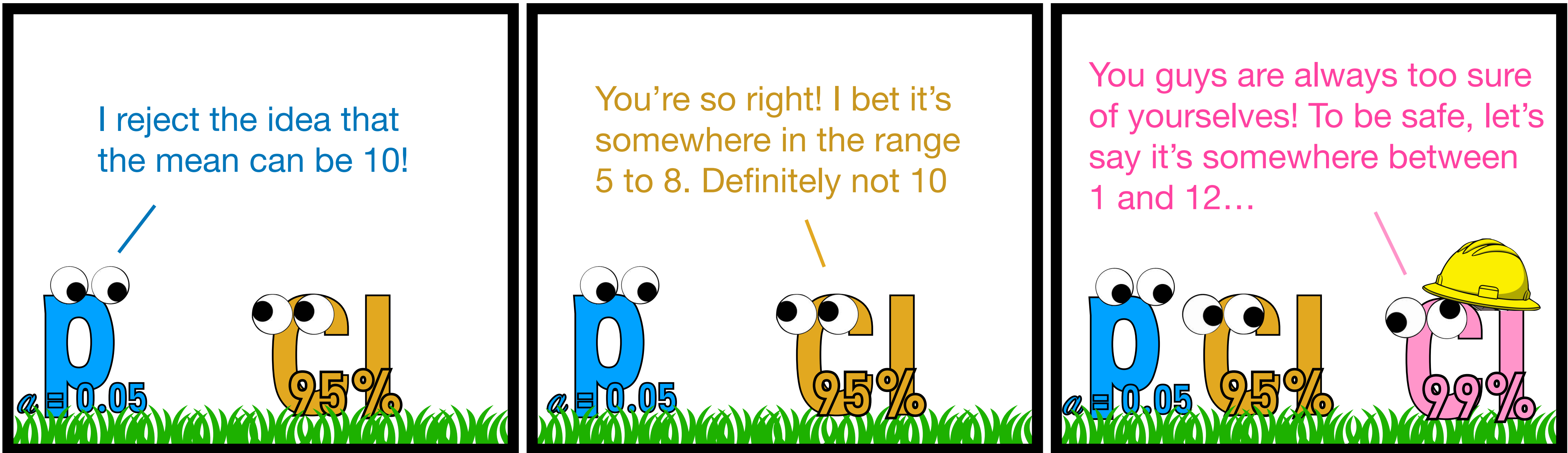


# Hypothesis testing - hypotheses

**“We reject the hypothesis that the population mean is 0.”**

- Example “null” hypotheses ( $H_0$ ):
  - $\mu = 10$  “the true population mean is 10”
  - $p = 0.4$  “the true population proportion is 0.4”
  - $\mu_a - \mu_b = 0$  “the true difference in population means is 0”
- Example “alternative” hypotheses ( $H_a$ ):
  - “Two-tailed” tests - use both “tails” of distribution to measure probability
    - $\mu \neq 10$ ;  $p \neq 0.4$ ;  $\mu_a - \mu_b \neq 0$
  - “One-tailed” tests - use only 1 “tail” of distribution to measure probability
    - $\mu > 10$ ;  $p > 0.4$ ;  $\mu_a - \mu_b > 0$
    - $\mu < 10$ ;  $p < 0.4$ ;  $\mu_a - \mu_b < 0$





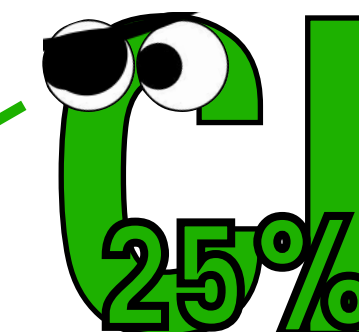
## • Tests and Confidence Intervals will agree\* 🤝

- \*requires alpha and confidence to match (eg  $\alpha = 0.05$  & 95% CI or  $\alpha = 0.01$  and 99% CI)

## • If the test rejects a possible value -> CI won't have that value in its range

- If we reject null that mean is 10 then the CI's range will not include 10, maybe (5, 8)
- If we fail to reject that the mean is 10 then the CI's range will include 10, maybe (1, 12)

Psst! I might not have the best betting history, but I really think it's somewhere between 6 and 7.



That guy knows what he's talking about. Trust me.



# Hypothesis testing - errors

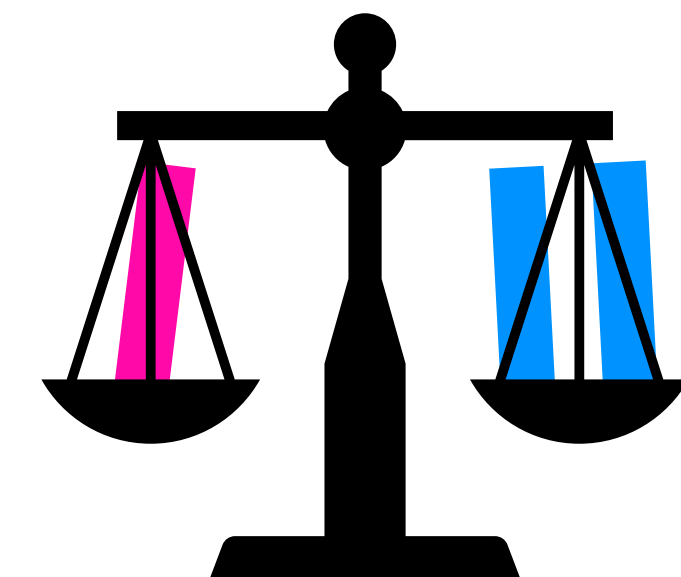
Type I error

Rejecting the null when it is true

Type II error

Failing to reject the null when it is false

- The probability of making a Type I is denoted as  $\alpha$  (aka alpha or “level of significance”)
  - When  $p < \alpha$  we’re saying “the probability of this happening is very low if the null hypothesis is indeed true”, but rare events can just happen 🙄 and this can lead to type I errors
- The probability of making Type II is denoted as  $\beta$  (aka beta)
- Managing the errors is a balancing act
  - If you lower  $P(\text{Type I})$  you raise  $P(\text{Type II})$
  - If you raise  $P(\text{Type I})$  you lower  $P(\text{Type II})$







# Hypothesis test utility belt



If  $p$  is low  $H_0$  must go!

1. Hypotheses
2. Model
3. Mechanics
4. Conclusions

Type I error

Rejecting the null when it is true

Type II error

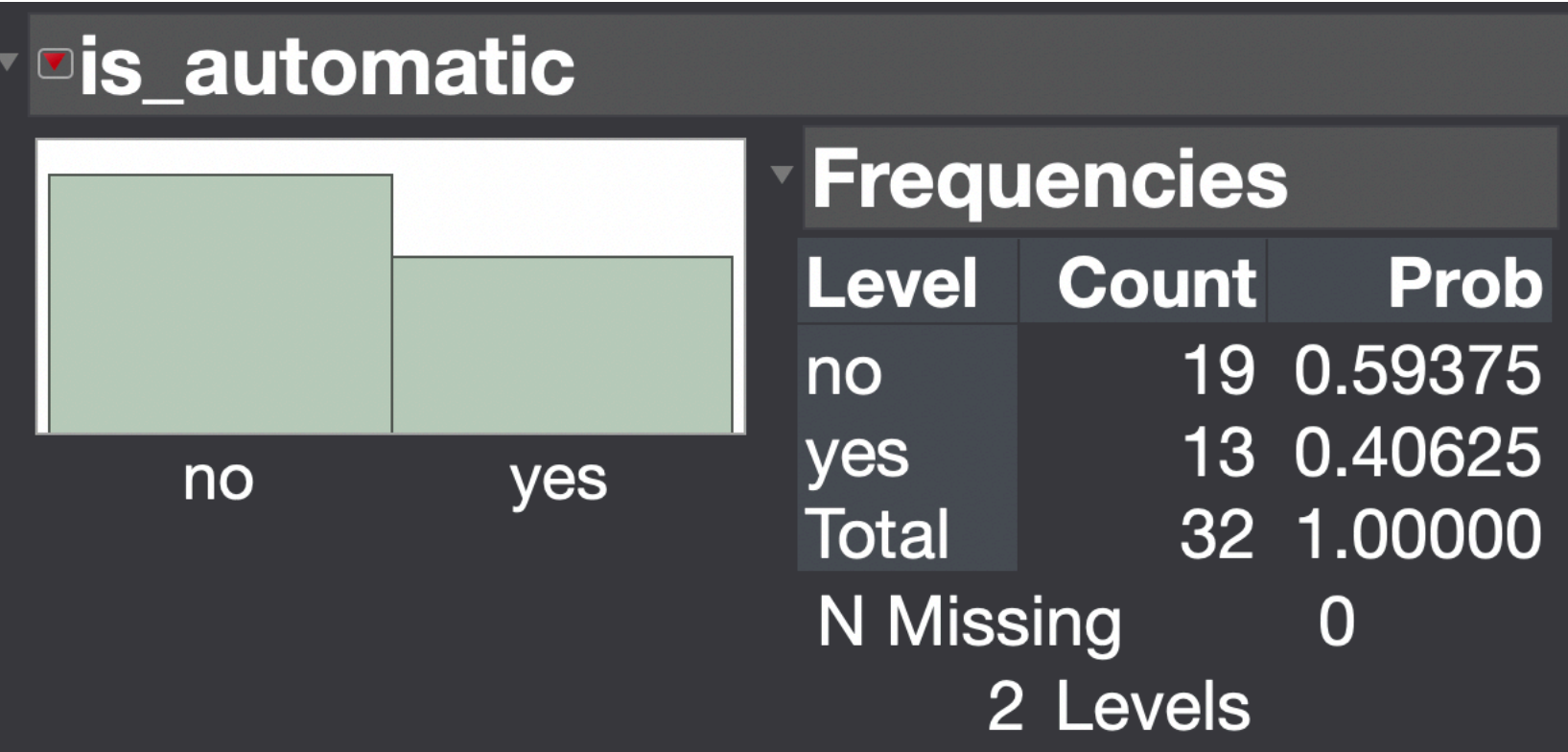
Failing to reject the null when it is false

test	data	example null (fail to reject null if $p\text{-value} > \alpha$ )	example alts (reject null if $p\text{-value} < \alpha$ )	p-value comes from
Proportion z-test	1 categorical var w/2 categories	$p = 0.5$	$p \neq 0.5$ $p > 0.5$ $p < 0.5$	normal dist (z)
1-sample t-test	1 numeric var	$\mu = 10$	$\mu \neq 10$ $\mu > 10$ $\mu < 10$	student's t-dist ( $t_{df}$ )
2-sample t-test	1 numeric var & 1 categorical var w/2 categories	$\mu_{\text{group1}} - \mu_{\text{group2}} = 0$	$\mu_{\text{group1}} - \mu_{\text{group2}} \neq 0$ $\mu_{\text{group1}} - \mu_{\text{group2}} > 0$ $\mu_{\text{group1}} - \mu_{\text{group2}} < 0$	student's t-dist ( $t_{df}$ )
Chi-square test	2 categorical vars	Counts are independent	Counts are not independent	Chi-Square dist ( $\chi^2_{df}$ )

Make sure to review the assumptions of each!

- For all tests: Random & 10%
- For prop z test: Success/failure
- For t-tests: nearly normal (both groups for 2 sample) & independence (for 2 sample)
- For chi-square: expected cell frequency





“We are 95% confident the true population proportion of manual cars is captured in the range (0.42, 0.74).”

Confidence Intervals					
Level	Count	Prob	Lower CI	Upper CI	1-Alpha
no	19	0.59375	0.4226	0.744804	0.950
yes	13	0.40625	0.255196	0.5774	0.950
Total	32				

Note: Computed using score confidence intervals.

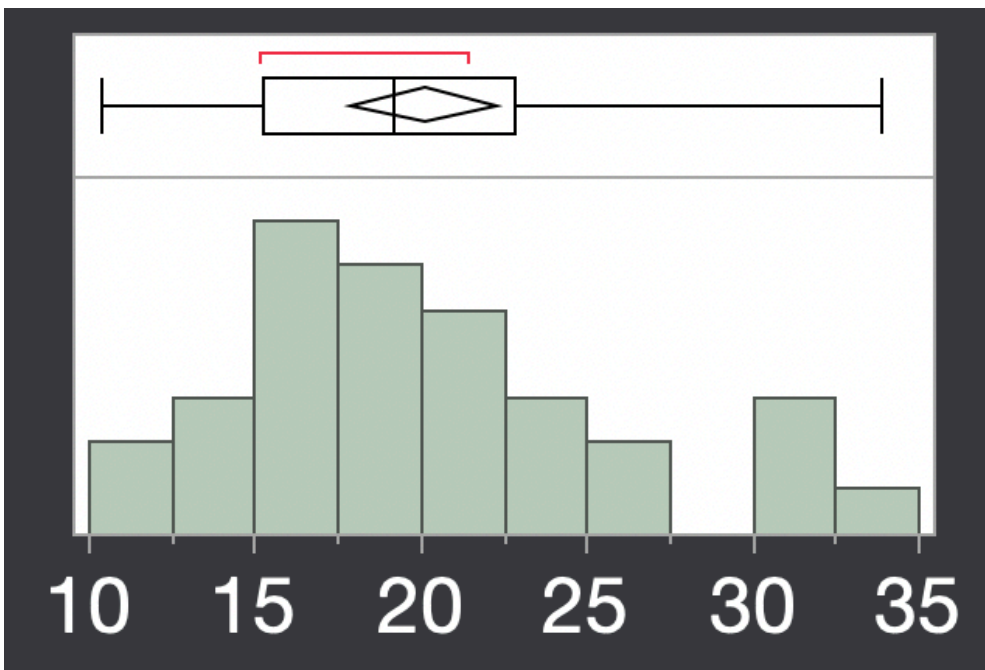
CI interpretation ✓-list

- Level of confidence (95%)
- Mention what the interval is for (population proportion)
- Context of data (manual cars)
- The range of the interval ((0.42, 0.74))

You don't have to say “captured in the range,” but it's a pretty safe move

Interpret this one yourself for practice

95% confidence intervals.  
They will agree with a hypothesis test using alpha = 0.05



“We are 95% confident the true population mean of mpg is captured in the range (17.9, 22.3).”

Quantiles			Summary Statistics	
100.0%	maximum	33.9	Mean	20.090625
99.5%		33.9	Std Dev	6.0269481
97.5%		33.9	Std Err Mean	1.065424
90.0%		30.4	Upper 95% Mean	22.263571
75.0%	quartile	22.8	Lower 95% Mean	17.917679
50.0%	median	19.2	N	32
25.0%	quartile	15.275		
10.0%		13.6		
2.5%		10.4		
0.5%		10.4		
0.0%	minimum	10.4		

CI interpretation ✓-list

- Level of confidence (95%)
- Mention what the interval is for (population mean)
- Context of data (mpg)
- The range of the interval ((17.9, 22.3))

You don't have to say “captured in the range,” but it's a pretty safe move

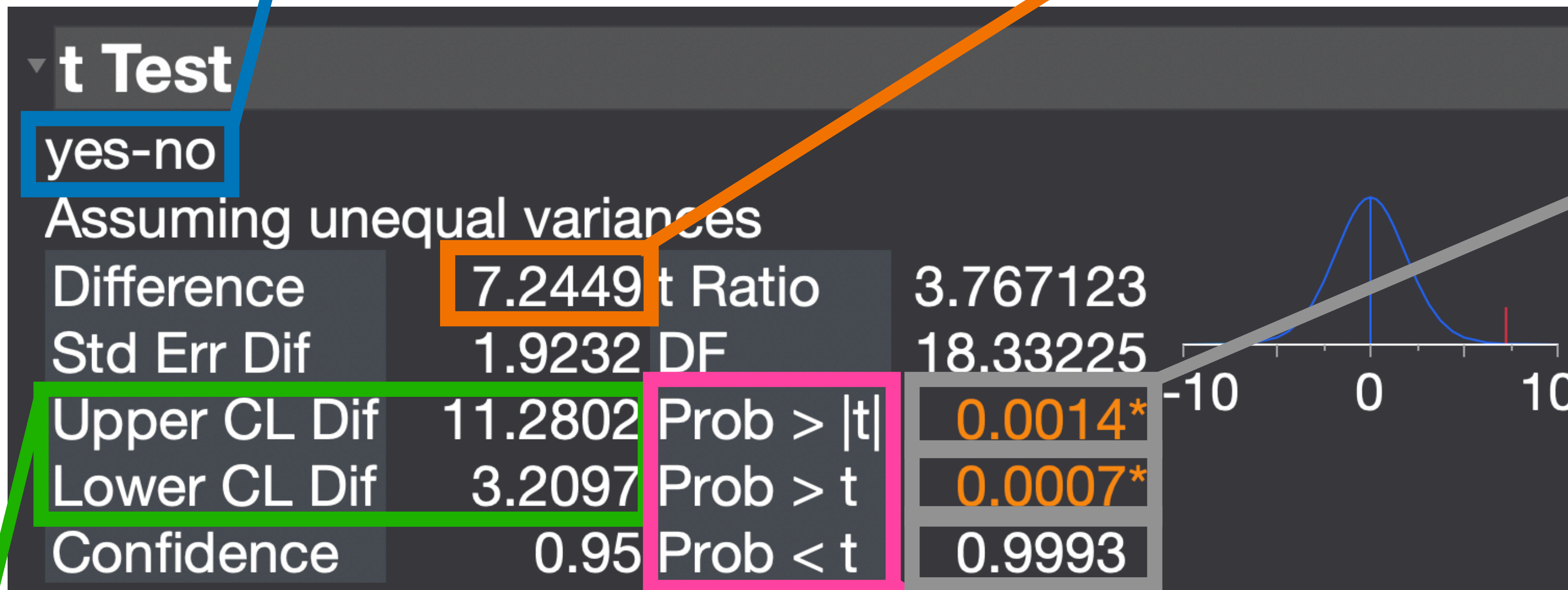
95% confidence intervals.  
They will agree with a hypothesis  
test using  $\alpha = 0.05$



🚨 Verry important to look here! 🚨  
Since we have yes-no:

- Positive numbers in CI indicate yes>no on average
- Dictates which p-value to use

The observed difference in mean.  
Based on the cars we had in the sample:  
 $\text{AVG}(\text{mpg}_{\text{yes}}) - \text{AVG}(\text{mpg}_{\text{no}}) = 7.2$   
We'll use the CI and the hypothesis test to see what we can expect from the whole population



Use when:  $H_a : \mu_{\text{yes}} - \mu_{\text{no}} \neq 0$   
Use when:  $H_a : \mu_{\text{yes}} - \mu_{\text{no}} > 0$   
Use when:  $H_a : \mu_{\text{yes}} - \mu_{\text{no}} < 0$   
Match the order of subtraction with **yes-no**

“We are 95% confident the true difference of means between manuals and automatics is captured in the range (3.2, 11.3). It appears automatics have higher mpg on average”

**p-values!** (aka probability values) If our null hypotheses was  $H_o : \mu_{\text{yes}} - \mu_{\text{no}} = 0$  (aka “the means are the same”) read these as “the probability we’d see a difference this big in our sample assuming there is no difference (aka assuming  $H_o$ )”

“We reject the null hypothesis that there is no difference of means, there is evidence that the average for yes is greater than the average for no at the 0.05 level of significance ( $p < \alpha$ )”