STAT 201 Exam 3

You can do it!

Question break down (40 Questions total)

- 13 CI questions (Ch 13 & 14) (1 of them interpretation)
- 12 Chapter 15&16 Qs (1 of them interpretation)
- 6 Chapter 17 Qs (1 of them interpretation)
- 7 Chapter 19 Qs (1 of them interpretation)
- 2 Choose the right tool Qs

Confidence intervals

"We are 95% confident the true population proportion is in the range (0.4, 0.7)."

$$\hat{p} \pm z^* SE(\hat{p}) \qquad \bar{y} \pm t_{df}^* SE(\bar{y})$$

- Sample statistic
  - the actual observed value of the proportion ($\hat{p}$) or the mean ($\bar{y}$)
  - We use this as the best guess of the population parameter and add some give or take (aka Margin of Error)
- Critical value
  - A value looked up based on how confident we want to be
    - z is looked up based on normal distribution; t_{df} is looked up based on t-distribution and df
    - df is "degrees of freedom", here df = n - 1
  - Bigger confidence -> bigger critical value -> bigger margin of error -> wider CI
- Standard error
  - More evidence (aka smaller n) -> lower SE -> smaller margin of error
  - More variation -> higher SE -> larger margin of error

Confidence intervals

"We are 95% confident the true population mean is in the range (4, 10)."

$$\hat{p} \pm \underbrace{z^* SE(\hat{p})}_{\text{"Margin of Error"}} \qquad \bar{y} \pm \underbrace{t_{df}^* SE(\bar{y})}_{\text{"Margin of Error"}}$$

- Affected by level of confidence (bigger confidence, bigger interval, 👨‍🍳🍕)
- Affected by sample size (more evidence -> smaller ME -> smaller interval)

- CIs are the sample stat plus or minus ME, this means that the width of the CI is 2*ME (start at middle and take 1 ME sized step in both directions)
  - Given CI of (5, 15) we can find ME and sample stat with:
    - **2*ME = $\mathbf{CI_{hi}}$ - $\mathbf{CI_{low}}$** -> 2*ME = 15 - 5 -> 2*ME = 10 -> ME = 5
    - **sample stat = $\mathbf{CI_{hi}}$ - ME** -> sample stat = 15 - 5 = 10

CI for population proportion

$$\hat{p} \pm \underbrace{z^{*} SE(\hat{p})}_{\text{Margin of Error}}$$

CI for population mean

$$\bar{y} \pm \underbrace{t_{df}^{*} SE(\bar{y})}_{\text{Margin of Error}}$$

Assumptions

- Success/failure
  - at least 10 successes ($\hat{p}$) and 10 failures ($\hat{q}$ -> aka 1 - $\hat{p}$)

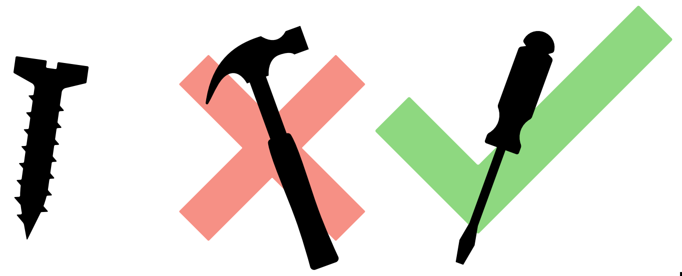
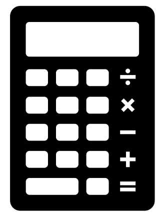
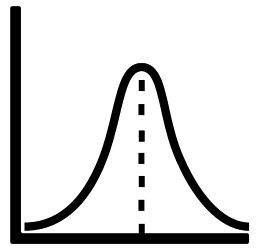
Assumptions

- Nearly normal
  - Distribution should resemble the normal distribution. When sample size is larger (40ish) we can have larger skew without worry.

Shared Assumptions

- Randomization
  - Sample is a random sample (we want to be representative, not biased)
- 10%
  - sample size (n) must be less than or equal to 10% of population

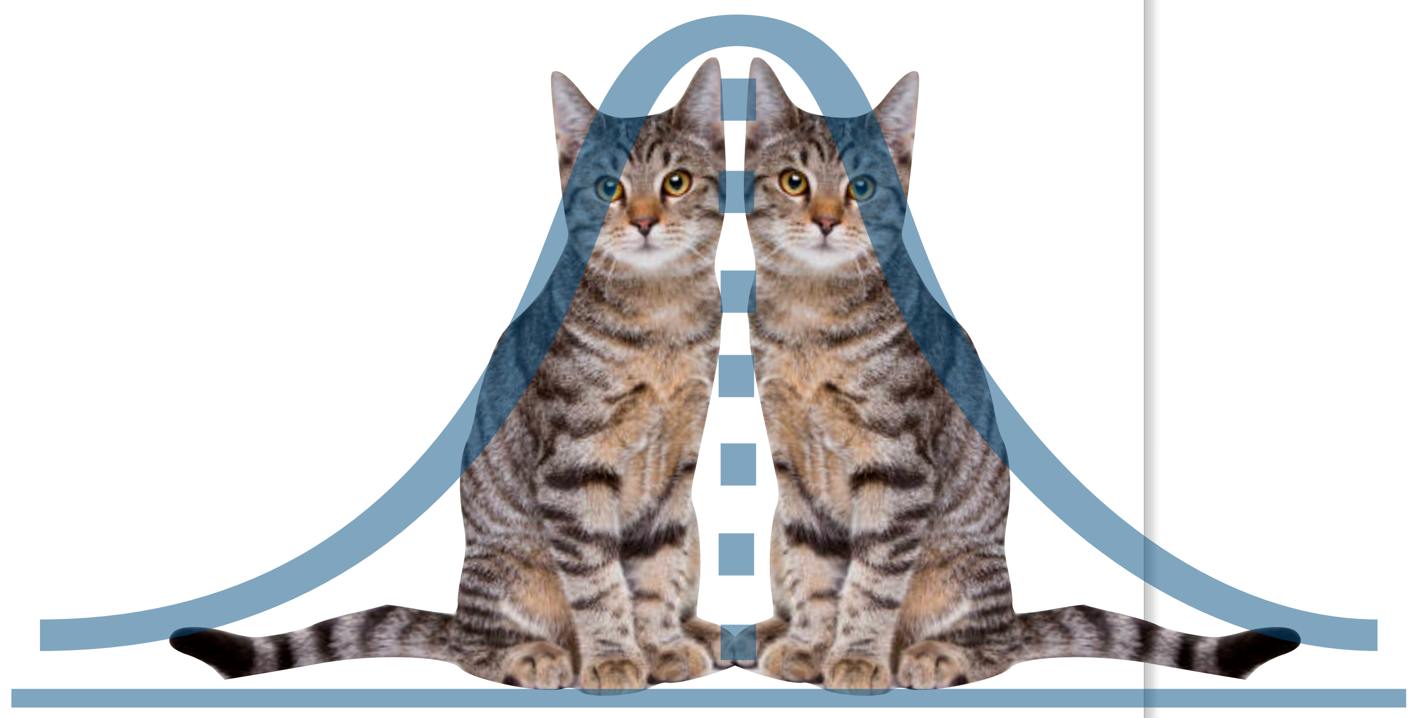
Hypothesis testing - steps

- **Hypotheses** H_0 & H_a
  - State the hypotheses (aka the 2 possible outcomes of the test)
- **Model** 
  - Check your assumptions (use the right tool for the job)
- **Mechanics** 
  - Formula time! (or JMP time)
- **Conclusions** 
  - Read the p-value - "If p is low, H_0 must go"
    - If $p <$ alpha (aka "level of significance") reject null hypothesis; there is evidence supporting H_a
    - If $p >$ alpha (aka "level of significance") fail to reject null hypothesis; there is not enough evidence supporting H_a

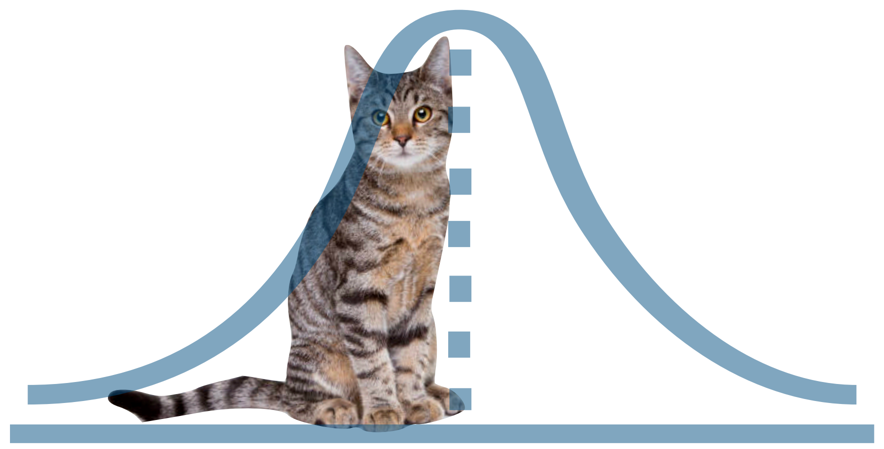
Hypothesis testing - hypotheses

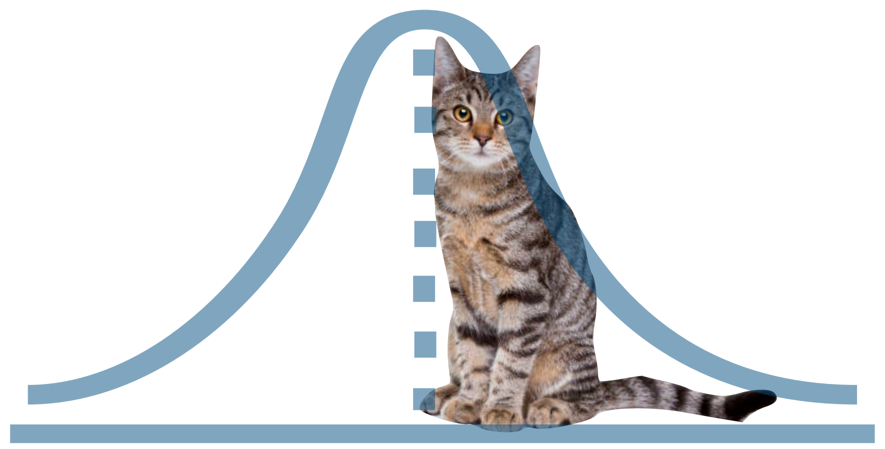
"We reject the hypothesis that the population mean is 0."

- Example "null" hypotheses (H_0):
  - $\mu = 10$ "the true population mean is 10"
  - $p = 0.4$ "the true population proportion is 0.4"
  - $\mu_a - \mu_b = 0$ "the true difference in population means is 0"



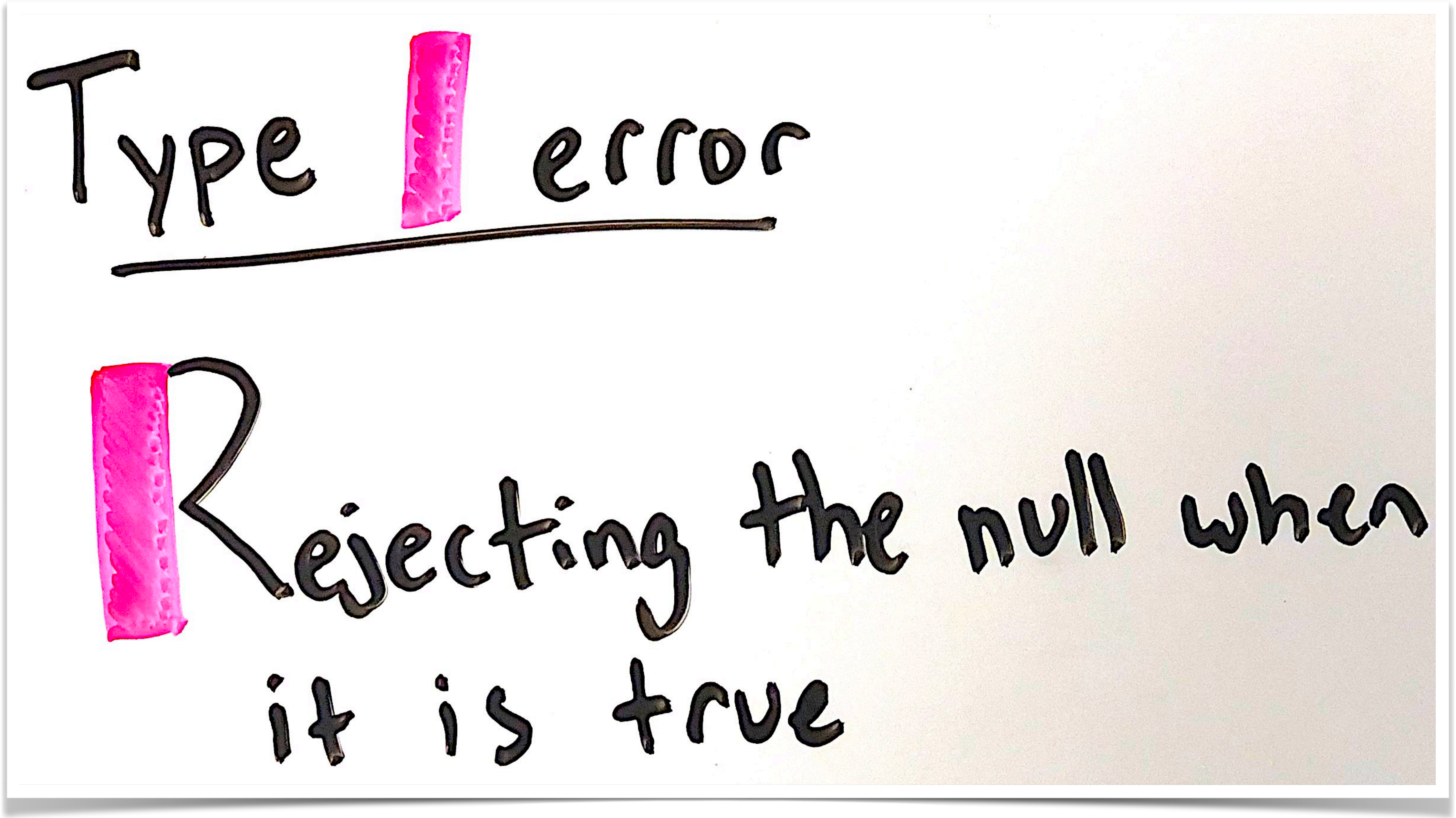
- Example "alternative" hypotheses (H_a):
  - "Two-tailed" tests - use both "tails" of distribution to measure probability
    - $\mu \neq 10; \quad p \neq 0.4; \quad \mu_a - \mu_b \neq 0$
  - "One-tailed" tests - use only 1 "tail" of distribution to measure probability
    - $\mu > 10; \quad p > 0.4; \quad \mu_a - \mu_b > 0$
    - $\mu < 10; \quad p < 0.4; \quad \mu_a - \mu_b < 0$

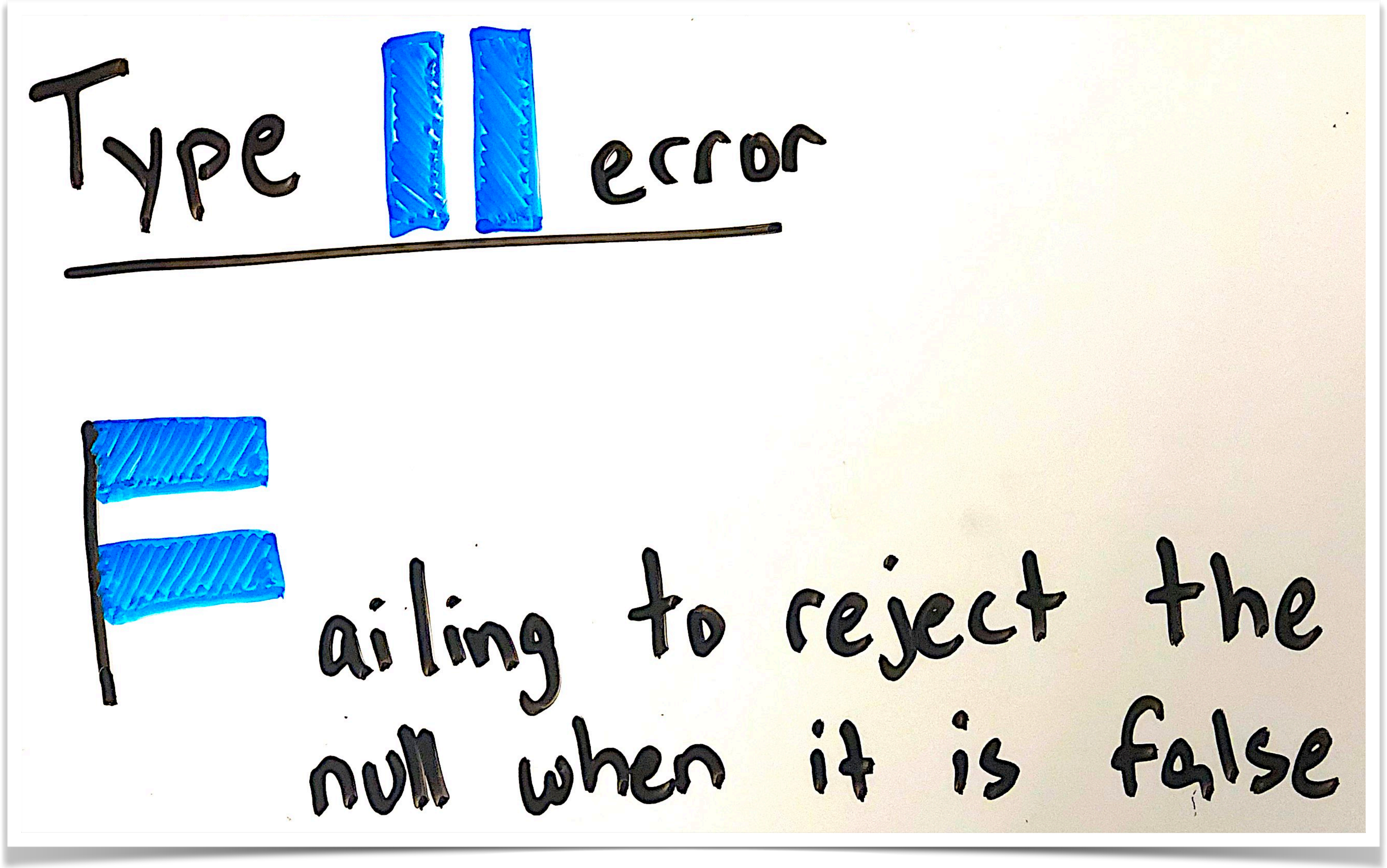




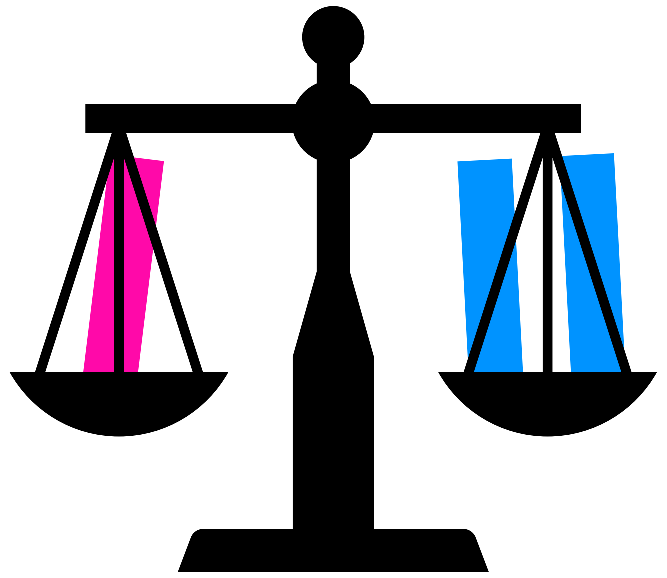
- **Tests and Confidence Intervals will agree*** 🤝
  - *requires alpha and confidence to match (eg *alpha = 0.05 & 95% CI* or *alpha = 0.01 and 99% CI*)
- **If the test rejects a possible value -> CI won't have that value in it's range**
  - If we reject null that mean is 10 then the CI's range will not include 10, maybe (5, 8)
  - If we fail to reject that the mean is 10 then the CI's range will include 10, maybe (7, 13)

Hypothesis testing - errors





- **The probability of making a Type I is denoted as α** (aka alpha or "level of significance")
  - When $p < \alpha$ we're saying "the probability of this happening is very low if the null hypothesis is indeed true", but rare events can just happen 🤷‍♂️ and this can lead to type I errors
- **The probability of making Type II is denoted as β** (aka beta)
- Managing the errors is a balancing act
  - If you lower P(Type I) you raise P(Type II)
  - If you raise P(Type I) you lower P(Type II)

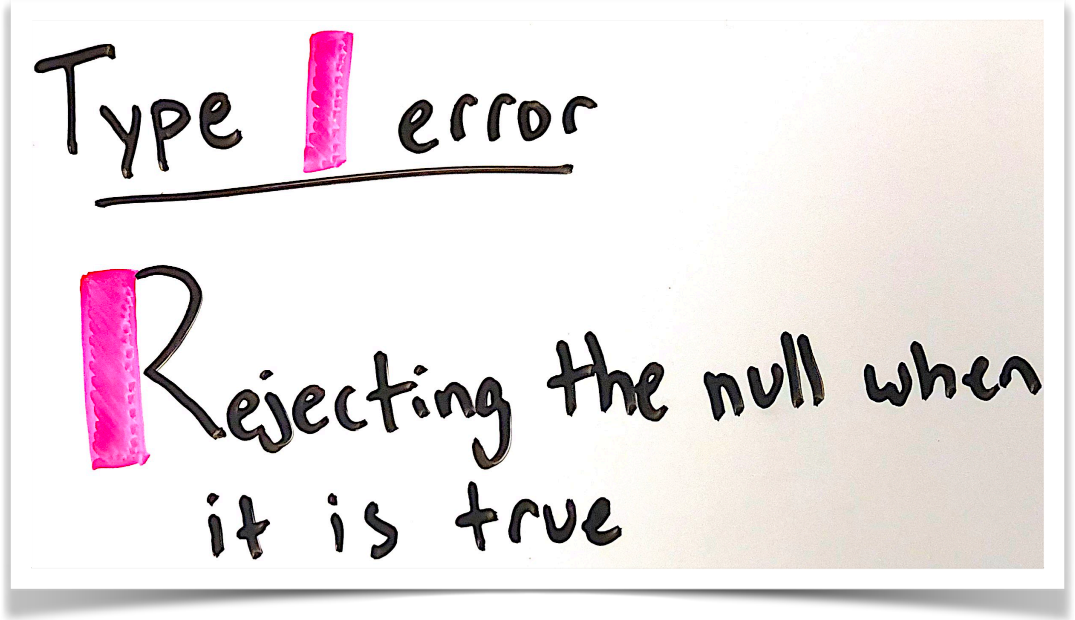


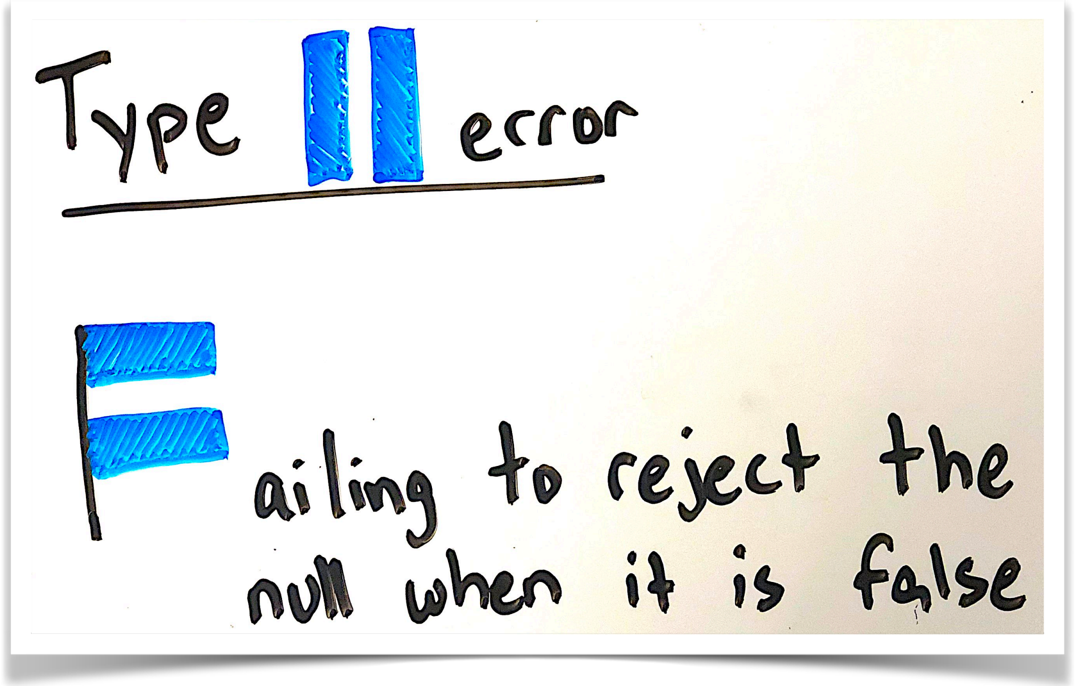
Hypothesis test utility belt

If p is low H_0 must go!

test	data	example null (fail to reject null if p-value > alpha)	example alts (reject null if p-value < alpha)	p-value comes from
Proportion z-test	1 categorical var w/2 categories	$p = 0.5$	$p \neq 0.5$ $p > 0.5$ $p < 0.5$	normal dist (z)
1-sample t-test	1 numeric var	$\mu = 10$	$\mu \neq 10$ $\mu > 10$ $\mu < 10$	student's t-dist (t_{df})
2-sample t-test	1 numeric var & 1 categorical var w/2 categories	$\mu_{group1} - \mu_{group2} = 0$	$\mu_{group1} - \mu_{group2} \neq 0$ $\mu_{group1} - \mu_{group2} > 0$ $\mu_{group1} - \mu_{group2} < 0$	student's t-dist (t_{df})
Chi-square test	2 categorical vars	Counts are independent	Counts are not independent	Chi-Square dist (χ^2_{df})

1. Hypotheses
2. Model
3. Mechanics
4. Conclusions





Make sure to review the assumptions of each!

- For all tests: Random & 10%
- For prop z test: Success/failure
- For t-tests: nearly normal (both groups for 2 sample) & independence (for 2 sample)
- For chi-square: expected cell frequency