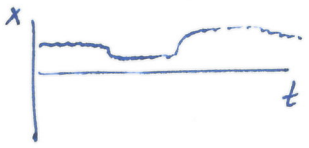
2020 - March - 2

Data = x

$x \in [-12, 12]$ volts



What is average x?

$$\langle x \rangle = \frac{1}{N} \sum_{i=1}^{N} x_i$$

N Measurements; sample "i"

What is standard deviation?

Standard wiggle
tolerance
uncertainty, etc in measurement?

$$\text{uncertainty} = \frac{1}{N} \sum_{i=1}^{N} x_i - \langle x \rangle$$

bad b/c avge is zero!

$$\text{uncertainty} = \frac{1}{N} \sum_{i=1}^{N} |x_i - \langle x \rangle|$$

bad b/c absolute value is hard to work w/ mathematically?

Standard deviation σ (or variance)

$$\sigma^2 = \frac{1}{N} \sum_{i=1}^{N} (x_i - \langle x \rangle)^2$$

- positive definite
- Easy to compute Integral average for a PDF
- Standard (100+ yrs old)

but

if you have $N = 1000$ measurements, you have 2 loops of $N = 1000$ size to compute σ

What's a workaround?

$$S\_x = \sum_{i=1}^{N} x_i$$

and

$$S\_x2 = \sum_{i=1}^{N} (x_i)^2$$

where

$$\langle x \rangle = S\_x / N$$

$$\langle x^2 \rangle = S\_x2 / N$$

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how is this good?

$$\sigma^2 = \frac{1}{N} \sum_{i=1}^{N} (x_i - \langle x \rangle)^2$$

$$= \frac{1}{N} \sum_{i=1}^{N} (x_i^2 - 2\langle x \rangle x_i + \langle x \rangle^2)$$

$$= \frac{1}{N} \sum_{i=1}^{N} x_i^2 - \frac{2\langle x \rangle}{N} \sum_{i=1}^{N} x_i + \langle x \rangle^2$$

↓ how?

$$= \langle x^2 \rangle - 2\langle x \rangle^2 + \langle x \rangle^2$$

so then

$$\sigma^2 = \langle x^2 \rangle - \langle x \rangle^2$$

or

$$\sigma = \sqrt{\langle x^2 \rangle - \langle x \rangle^2}$$

Note, this does not Require any array of values to be stored.

- this is a way of having a good measurement over a time period

$\langle x \rangle \pm \sigma$ Note only as good as the float you're storing the sum in!

eg #s are typically 100.

after 10,000 numbers, the sum looks like

$100 \times 1000 = 1{,}00\ 0{,}000$

7 digits of precision

if next number added is

100.235

these digits are not included b/c of the roundoff implicit in the way arduino stores floats.

> Modern computer systems avoid ↑ should?

this problem - see also Numerical analysis. (Math class @ WSU)