PWM outputs, part 2

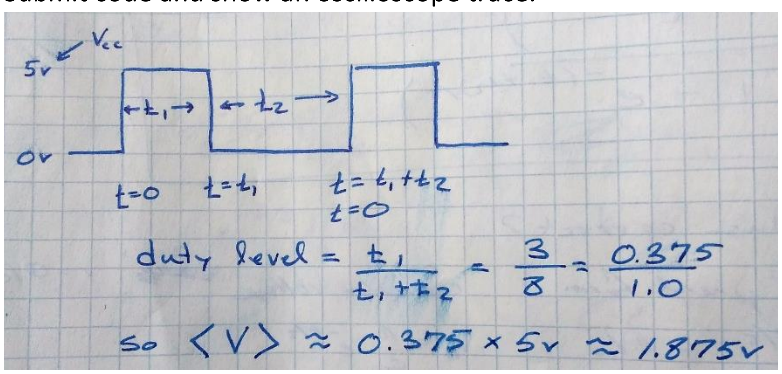
Purpose: Further develop understanding of PWM outputs, smoothing, arbitrary function generation etc.

Due by/before: 2 March

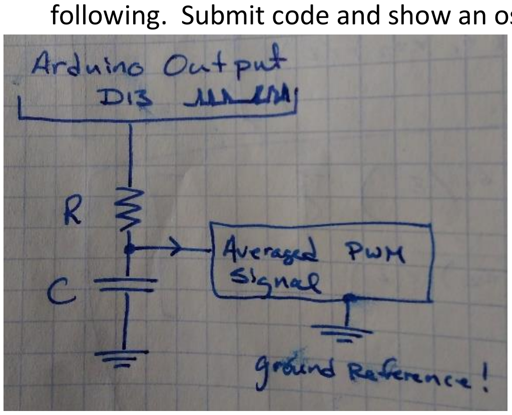
We've talked in class about Pulse-width modulated (PWM) outputs and how they can be used to approximate traditional analog outputs. For this lab, feel free to use your PWM generator (function) from Lab 2, or the example I have online, or the system-supplied analogwrite().

Do 4 of the 5 problems for full credit (ie, do problem 2 or problem 4).

1.
   A. Make sure you can generate a PWM wave with average value of 3.27+/-0.01 volts, measured with a multimeter. Reminder, the PWM function we've been working with is HIGH (5.0v) for t1 and LOW (0v) for t2. The cycle is periodic at a time t=t1+t2. Equivalently, the duty is $d = \frac{t_1}{t_1+t_2} = t_1 * f$, where f is the frequency associated with one PWM wave (not one individual on/off unit.) How many intervals do you need to break the PWM cycle into to achieve +/-0.01v resolution? Submit code and show an oscilloscope trace.



   B. Next, make sure you can generate an oscillating function that is 1.22v for 100ms and 3.77v for 200ms (both +/-0.01v). Note, for this you'll want to use a resistor-capacitor filter like the following. Submit code and show an oscilloscope trace.



2. Now, let's spend some time talking about how the filtering circuit works. When the Arduino PWM is high, the voltage loop for the averaging circuit looks like: $V_{CC} - IR - Q/C = 0$. When

the Arduino PWM signal is low the voltage loop is $-\mathrm{IR} - \mathrm{Q}/\mathrm{C} = 0$. Remember that I is current in the resistor and Q is the charge stored in the capacitor.

a. Assuming $\mathrm{I} = \frac{\mathrm{dQ}}{\mathrm{dt}}$, do the algebra/derivatives to show that the averaged PWM signal should be shaped like: $\mathrm{Q}_1 = \mathrm{A}\ \mathrm{Exp}[-\mathrm{t}/\tau] + \mathrm{B}$ and $\mathrm{Q}_2 = \mathrm{D}\ \mathrm{Exp}[-\mathrm{t}/\tau]$ (note $\mathrm{V}_1 = Q_1/C$ is the averaged signal during t1, $\mathrm{V}_2 = \mathrm{Q}_2/\mathrm{C}$, is the output during t2).

b. The voltage on the capacitor will be continuous in time. So, the following boundary conditions will have to be true: $\mathrm{V}_1(\mathrm{t} = 0) = \mathrm{V}_2(\mathrm{t} = \mathrm{t1} + \mathrm{t2})$ and $\mathrm{V}_1(\mathrm{t} = \mathrm{t1}) = \mathrm{V}_2(\mathrm{t} = \mathrm{t1})$. Use these boundary conditions and the results from a. to show that:

   i. $\tau = RC$

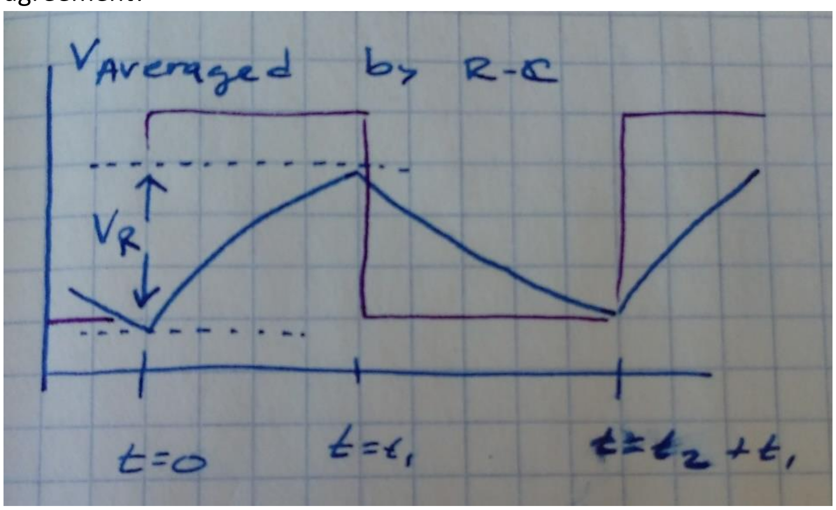
   ii. $\mathrm{B} = \mathrm{CV}_{\mathrm{CC}}$

   iii. $\mathrm{A} = \mathrm{V}_{\mathrm{CC}}\mathrm{C}\left[\frac{Exp[-t_2/\tau]\,(1-Exp[-t_1/\tau])}{(1-Exp[-(t1+t2)/\tau]} - 1\right], \mathrm{or}, \mathrm{V}_{\mathrm{cc}}\mathrm{C}\,\frac{\mathrm{Exp}[-\mathrm{t}_2/\tau]-1}{1-\mathrm{Exp}[-(\mathrm{t}_1+\mathrm{t}_2)/\tau]}$

   iv. $\mathrm{D} = \frac{\mathrm{V}_{\mathrm{CC}}\,C}{\mathrm{Exp}[-\mathrm{t1}/\tau]}\frac{(1-Exp[-t1/\tau])}{(1-Exp[-(t1+t2)/\tau])}, or, V_{cc}C\,\frac{\mathrm{Exp}[\mathrm{t}_1/\tau]-1}{1-\mathrm{Exp}[-(\mathrm{t}_1+\mathrm{t}_2)/\tau]}$

   Note, there are certainly alternate equivalent expressions for A and D.

c. Build the PWM+filter circuit with known R, C, t1, and t2 values and capture the filtered output via an oscilloscope for a duration of $\approx 2(\mathrm{t1} + \mathrm{t2})$. Compare the oscilloscope output to a plot (in Mathematica or similar) for V1 and V2 above. There should be good agreement!



d. The "Ripple Voltage," V_{R}, in this PWM scheme is the total variation in output voltage in the averaged PWM signal. Using V_1 and V_2 above, create an algebraic expression for this ripple voltage, V_{R}. Then, using several different t1, t2, R, and C values, test the accuracy of your prediction for V_{R}.

e. Application Question: if you hold R and C constant, are there t1 or t2 values that make the ripple voltage larger? (application, are the PWM levels at which this averaging routing makes a noisy signal? Or, if t1+t2 is fixed, are there t1 or t2 values that make the averaging routing a clean, smooth signal?)

f. Application Question: If you want the ripple voltage to be no more than 10% of the ideal output, $V_{CC}\frac{t1}{t1+t2}$, what constraint should be imposed on R and C? Back up your answer with data from lab.

3. Use the averaged PWM output to create a function that will play a (musical) note of frequency f. A musical note can be a sine wave, $V(t) = Sin[2\pi f t]$. The pitch (high-ness) of the note is dictated by its frequency, f, where $f = 1/T$, and T is the period, or "length in time" of one cycle of the sine wave. A SEEED Grove buzzer is an easy way to play this note. Here's a list of Piano key frequencies, https://en.wikipedia.org/wiki/Piano_key_frequencies Submit code and show an oscilloscope trace.

4. Use your musical note function to play a song on a speaker. We have low power buzzers in the SEEED Grove kits, or you can use a speaker, but you might need to use an op-amp (LF411) to scale the output. Happy Birthday or Edelweiss are nice choices. Submit code and record a video and share it with me via mediaspace.

5. To conserve energy, a person decides to dim the lights in the house by adding a regulating resistor in series with a lamp. For now, assume that the resistance of the regulating resistor is 5 times the resistance of the lamp. How much power overall was saved by the entire circuit? How much more power is absorbed by the regulating resistor then by the lamp? Why would a PWM circuit be a better choice?