98

Name: Adam Stammer

CS 435 Exam #2
October 23, 2019

1. [25 points] Consider the language $L_1 = \{a^{2i+1}b^i : i \geq 0\}$.

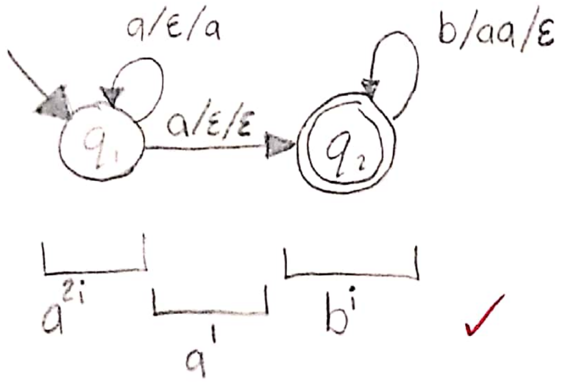
   (a) Write a context-free grammar for L_1.

   $a^{2i}\, a\, b^i$
   twice as many a's as b
   with an a in the middle
   empty string not accepted

   $S \to aaSb$
   $S \to a$

   $\boxed{S \to aaSb \mid a}$ ✓

   (b) Construct a push-down automaton for L_1.



✓

(c) Prove that L_1 is not regular.

$$a^{2i+1} b^i \rightarrow a^{2i} a b^i$$

Let's say $w = a^{2k} a b^k$. Since $|xy| \leq k$ we know that y falls under a^{2k} $\therefore$ $y = a^p$ where $p \geq 1$.

If we pump this up 1 we see that

$w = a^{2k+\cancel{1}} p a b^k$ which is no longer a part of L_1, thus L_1 is not regular.

-1

aaba
aabb

i	j	
0	only 0	→ ε
1	1 or 0	→ ab or aa
2	2 or less	→ aaba, aabb, aaaa
3		
4		
5		

2. [25 points] Consider the language $L_2 = \{a^i b^j a^{i-j} : i \geq j \geq 0\}$.

(a) Write a context-free grammar for L_2.

Empty String accepted

$S \rightarrow \varepsilon$

$S \rightarrow aXb$ //if we do a*b we cant put a*a in between

3 → aaabbb
aaabba
aaabaa
aaaaaa

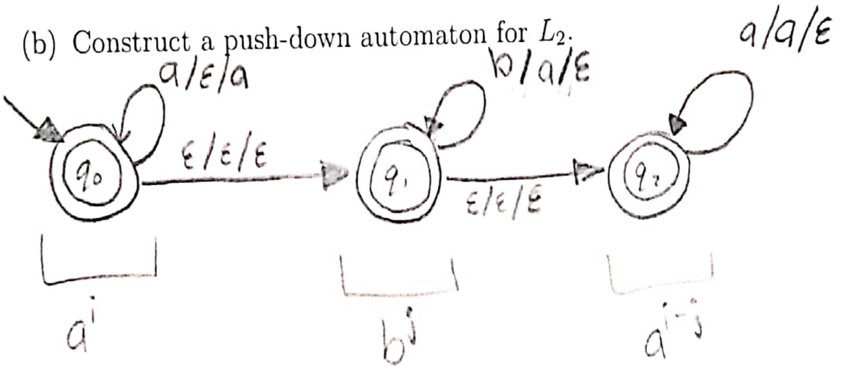
$S \rightarrow aSa$

$X \rightarrow aXb$

$X \rightarrow \varepsilon$

✓

(b) Construct a push-down automaton for L_2.



✓

(c) Prove that L_2 is not regular.

$a^i b^j a^{i-j} : i \geq j \geq 0$

→ $w = a^k b^k a^{k-k} = a^k b^k$

Since $|xy| \leq k$ we know that y falls under a^k

So $y = a^p$ where $p \geq 1$.

Let's pump down 1 so $w = a^{k-p} b^k \to w = a^{k-1} b^k$.

There are now fewer prefixing a's than there are b's and this is not in L_2, thus L_2 is not regular.

✓

3. [25 points] Consider the language $L_3 = \{a^i b^j a^k : j = max(i, k)\}$.

(a) Write a context-free grammar for L_3.

Empty string accepted.

2 cases (i max and k max)

$S \to \varepsilon$

$S \to aSba$ $S \to abTa$

$S \to aIb$ $T \to bKa$

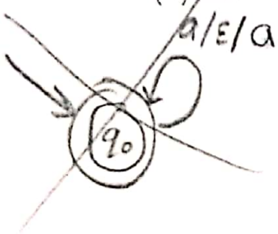
I is max   k is max

$I \to \varepsilon$ $K \to \varepsilon$

$I \to aIb$ $K \to bKa$

This doesn't feel right

(b) Construct a push-down automaton for L_3.



a and b here were not working here but L_3 is not context free so they shouldn't work anyway.

(c) Prove that L_3 is not regular.

$a^i b^j a^k \; : \; j = \max(i, k)$

case $i > k$: $w = a^c b^c a^{c-1}$. Since $|xy| \le c$ y must fall under a^c, so $y = a^p$. Let's pump y up 1

so $w = a^{c+p} b^c a^{c-1}$

$w = a^{c+1} b^c a^{c-1}$ which is not a part of L_3.

case $i < k$: $w = a^c b^{c+1} a^{c+1}$. Since $|xy| \le c$ y must fall under a^c, so $y = a^p$. Let's pump y up 2

So $w = a^{c+p} b^{c+1} a^{c+1}$

$w = a^{c+2} b^{c+1} a^{c+1}$ which is not a part of L_3

Since all cases had a pumped example that fell out of L_3, L_3 is not regular.

4. [25 points] Consider the language $L_4 = \{(ab)^i(ab)^i : i \geq 0\}$

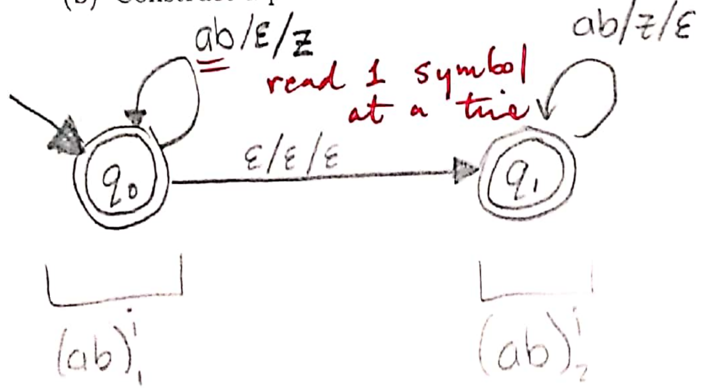
(a) Write a context-free grammar for L_4. Empty string accepted

$S \to abSab$

$S \to \varepsilon$

✓

(b) Construct a push-down automaton for L_4.



✓

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(c) Prove that L_4 is not regular.

$(ab)^i (ab)^i$

Let's use $w = (ab)^k (ab)^k$. Since $|xy| \leq k$

we know that y falls under the first ab^k.

so $y = (ab)^p$, $p \geq 1$. $\to$ $w = (ab)^{k+p} (ab)^k$

Let's pump up 1 $\to$ $w = (ab)^{k+1} (ab)^k$

$k+1 \neq k$, we now have an odd # of (ab)

which is not a part of L_4, thus L_4 is not regular.

10/10

Name: Adam Stammer

CS 435 Quiz #6
October 2, 2019

1. Is the language $L = \{a^i b^j : i, j \geq 0 \text{ and } i + j = 5\}$ regular? Prove your answer.

Yes. This regular. $i+j=5$ and $i,j \geq 0$ is a very finite set.
This means we could build an FSM for this language.

In this case, we know that $|w| = 5$ where $w \in L$ ~~[illegible]~~ With our language also limited such that no a follows a b, this language is very finite.

✓

2. Is the language $L = \{a^i b^j : i, j \geq 0 \text{ and } i - j = 5\}$ regular? Prove your answer.

No. It is not. We can see that $i-j=5$. which means that for every accepted string, $i=j+5$.

✓

So lets use $w = a^{k+5} b^k$.
Since $|xy| \leq k$ we know that y falls under a^{k+5} so $y = a^p$ where $1 \leq p \leq k$

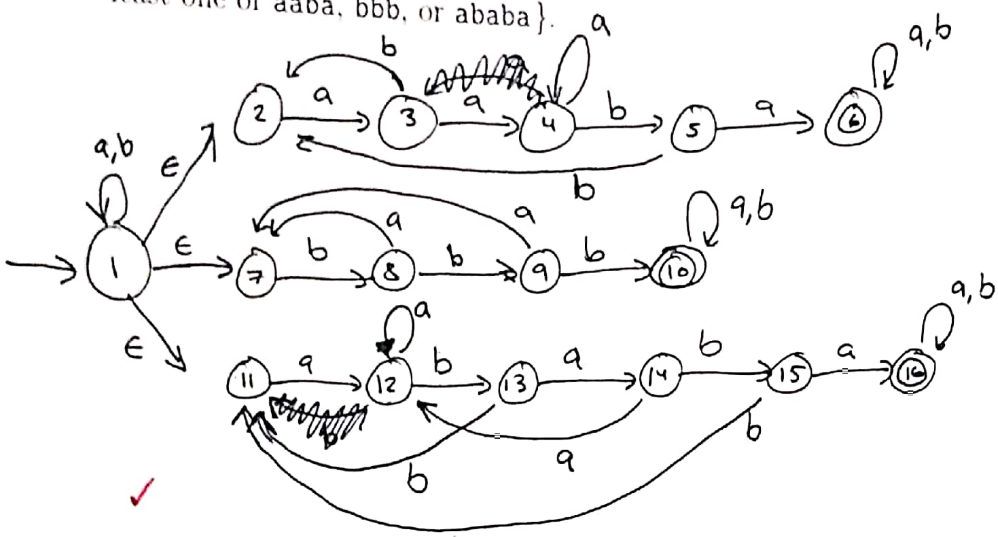
We can say that $w = a^{k+5+p} b^k$.
If we pump out we get $w = a^{k+5-p} b^k$
which shows that $k+5-p$ is not $k+5$ since $p \geq 1$.
So $w = a^{k+5-p} b^k \notin L$, thus L is not regular.

10/10

Name: Adam Stammer

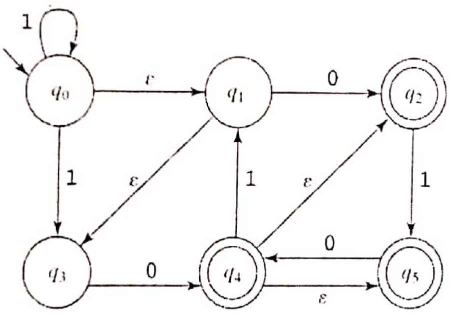
CS 435 Quiz #2
September 11, 2019

1. Build a nondeterministic FSM for the language $L = \{w \in \{a, b\}^* : w$ contains at least one of aaba, bbb, or ababa$\}$.



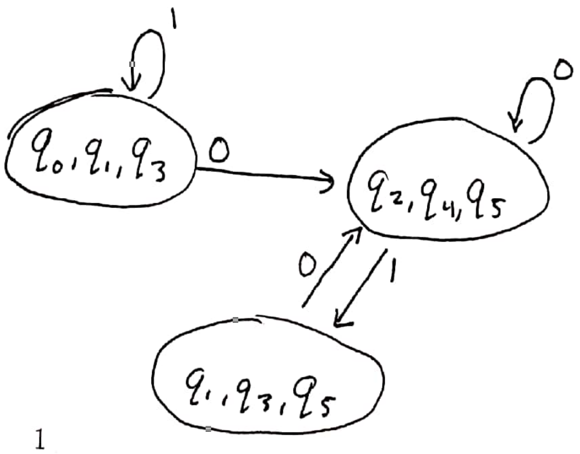
✓

2. Construct an equivalent deterministic FSM for the following NDFSM:



eps(q_0) = $\{q_0, q_1, q_3\}$
(q_1) = $\{q_1, q_3\}$
(q_2) = $\{q_2\}$
(q_3) = $\{q_3\}$
(q_4) = $\{q_4, q_5, q_2\}$
(q_5) = $\{q_5\}$ ✓

$\{q_0, q_1, q_3\}$	0	$\{q_2, q_4, q_5\}$
	1	$\{q_0, q_1, q_3\}$ ←
$\{q_2, q_4, q_5\}$	0	$\{q_2, q_4, q_5\}$ ←
	1	$\{q_1, q_3, q_5\}$
$\{q_1, q_3, q_5\}$	0	$\{q_2, q_4, q_5\}$
	1	$\{ \}$ //dead State

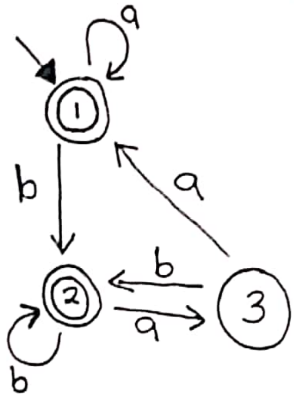


10/10

Name: Adam Stammer

CS 435 Quiz #1
September 3, 2019

Build a deterministic FSM for the language $L = \{w \in \{a, b\}^* : w$ does not end in ba$\}$.



empty string ✓
ends in any >0 number of b's ✓
ends in at least 2 a's ✓

9/10

Name: Adam Stammer

CS 435 Quiz #3
September 18, 2019

1. Write a regular expression for the language $L = \{w \in \{a,b\}^* : w \text{ does not end in ba}\}$.

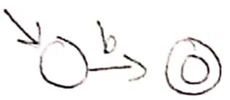
aa ✓
ab ✓
~~ba~~
bb ✓

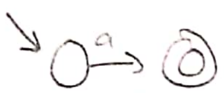
$(a \cup b)^*(b \cup aa)$

What about ε, a?

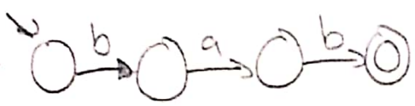
-1

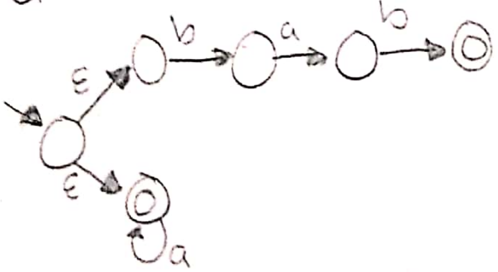
2. Construct an FSM to accept the language generated by the regular expression bab $\cup$ a*.

b:


a:


a^*:


bab:


bab $\cup$ a^*:


✓

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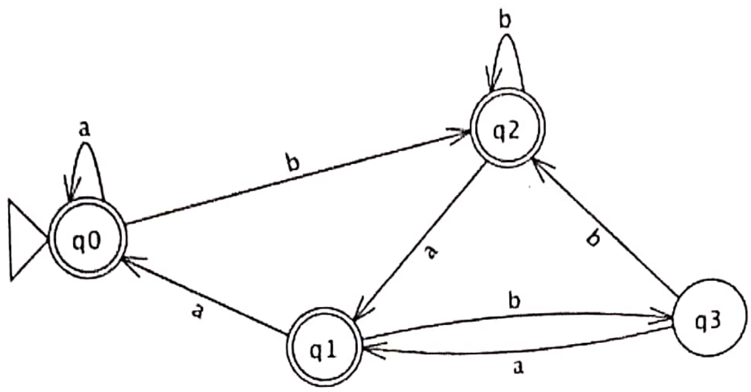
Name: Adam Stanmer

CS 435 Exam #1
September 25, 2019

1. [7 points] Give a clear English description of the language accepted by the following DFSM:



$a^* b b^*$ ab

bab

Any strings of a, b, and c that does not end in bab.

✓

I'm saying that L_{1n} does not have to equal L_{2n}
So L_2 could be redefined as $L_2 = \{0^m 1^m : m \geq 1\}$

2. [6 points] Let $L_1 = \{0^n 1^{n+1} : n \geq 0\}$ and let $L_2 = \{0^n 1^n : n \geq 1\}$. For each of the following strings, state whether or not it is an element of $L_1 L_2$.

(a) ϵ

No, at minimum n values $L_1 L_2$ would be 101

(b) 1

No. L_1 has at least 1 1 and L_1 has at least 1 zero followed by 1 1
So $L_1 L_2 = 101$

(c) 101

Yes. See above

(d) 10011

Yes $L_{1n} = 0$ and $L_{2n} = 2$

(e) 0110101

$0^n 1^{n+1} 0^m 1^{m+1}$

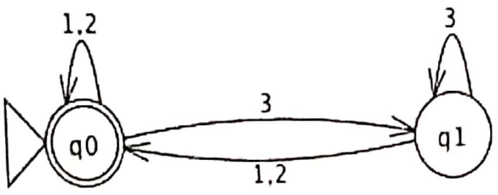
No does not allow for 3 transitions from 0 to 1

(f) 0011101

Yes $L_{1n} = 2$ and $L_{2n} = 1$

good

3. [6 points] For each of the following strings, state whether or not it is accepted by the following DFSM:



cannot end in 3

(a) ϵ Yes

(b) 123

No

(c) 12321

Yes

(d) 1123321

Yes ✓

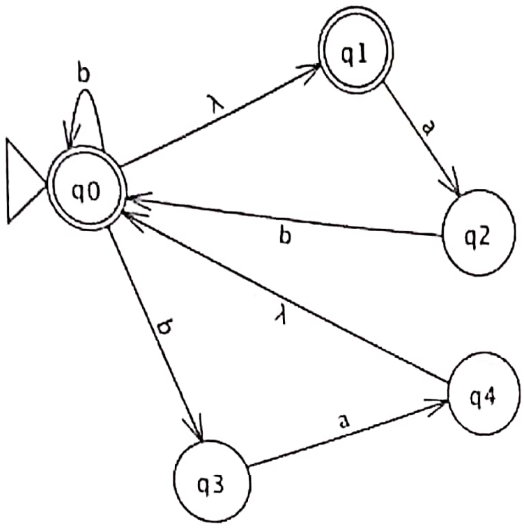
(e) 22133212

Yes

(f) 123123123

No

4. [6 points] For each of the following strings, state whether or not it is accepted by the following NDFSM (where λ is an ϵ):



(a) ϵ

Yes

(b) ba

No

(c) $aabb$

No

(d) $baba$

No

-2

(e) $baab$

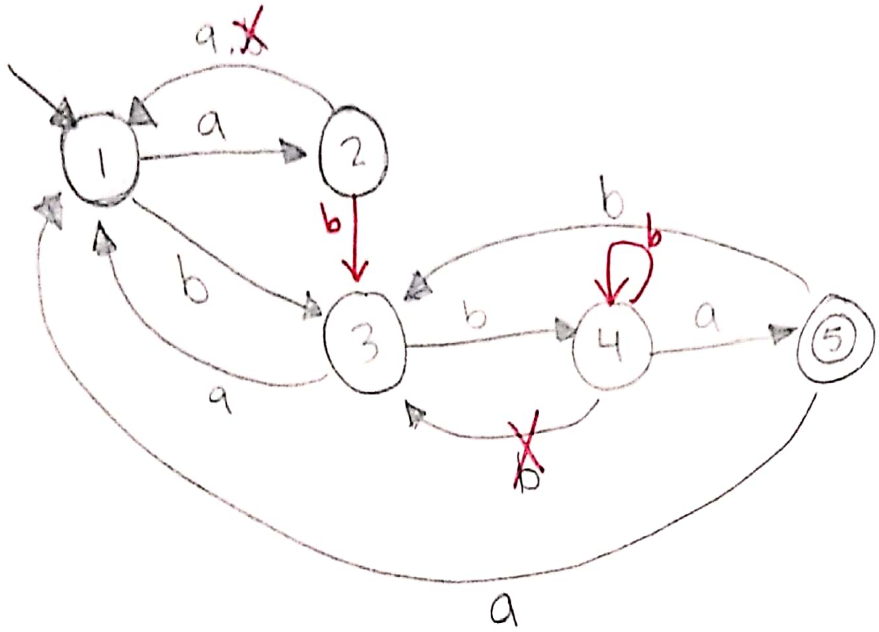
Yes

(f) $bbaa$

No

5. [10 points] Build a DFSM for the language $L = \{w \in \{a, b\}^* : w \text{ ends in } \mathtt{bba}\}$.

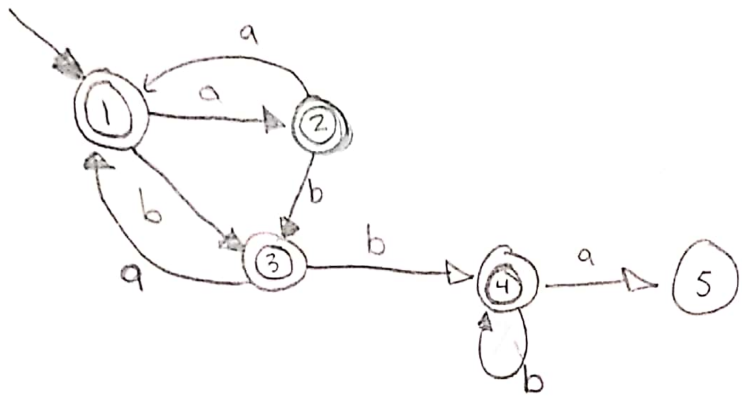
No empty string accepted



-4

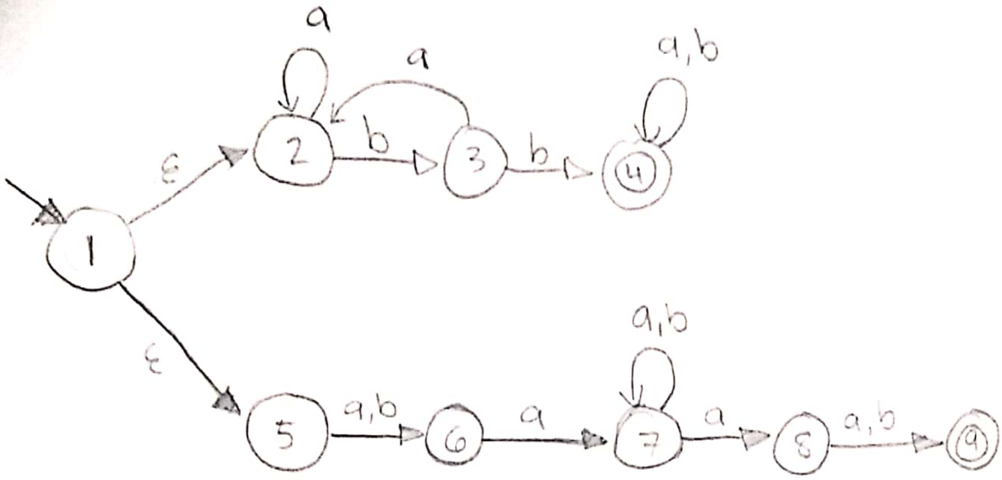
6. [10 points] Build a DFSM for the language $L = \{w \in \{a, b\}^* : w$ does not contain the substring bba$\}$.

empty string accepted



✓

7. *[10 points]* Build a NDFSM for the language $L = \{w \in \{a, b\}^* :$ either w contains bb or both the second character and the second from the last character of w is a$\}$.



also aa

OK

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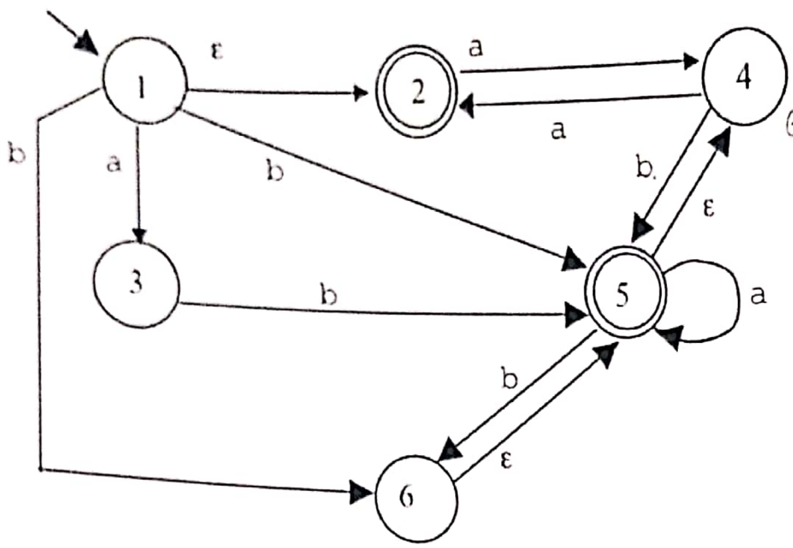
8. [15 points] Construct an equivalent DFSM for the following NDFSM:



esp(1) = 1,2
2 = 2
3 = 3
4 = 4
5 = 5,4
6 = 6,5,4

1,2
a {3,4}
b {5,6,4} -

3,4
a {2}
b {5,4}

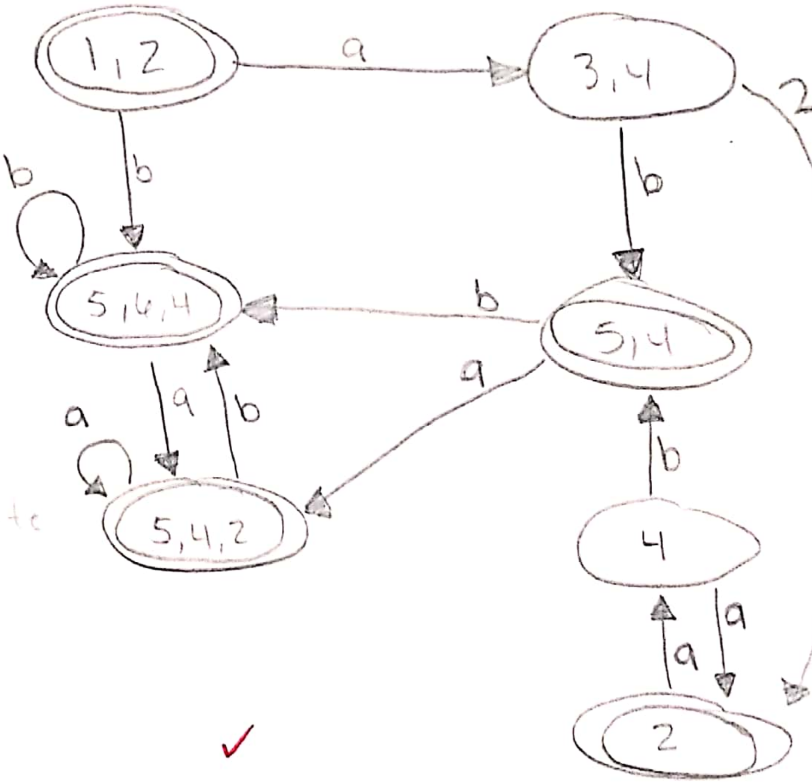
5,6,4
a {5,4,2}=
b {5,4,6} -

2
a {4}
b {} // dead state

5,4
a {5,4,2}=
b {6,5,4} -

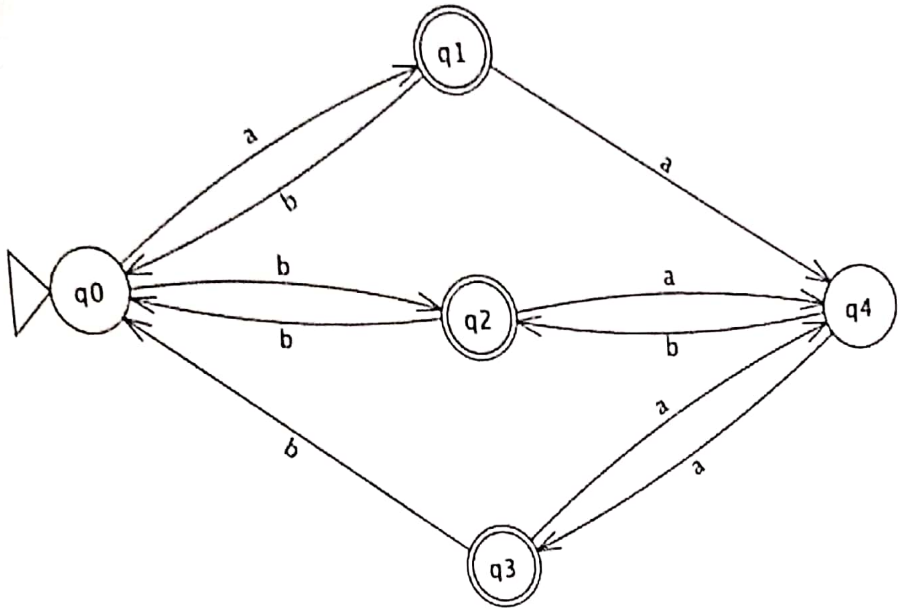
5,4,2
a {5,4,2}=
b {5,4,6} -

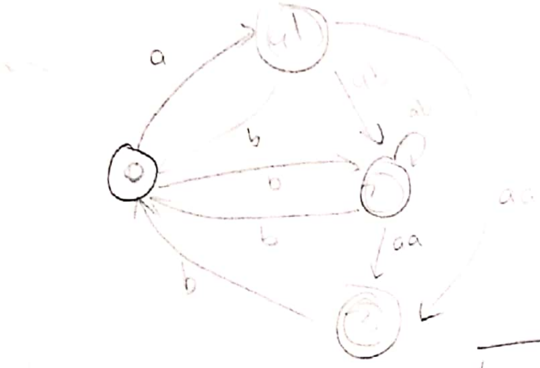
4
a {2}
b {5,4}

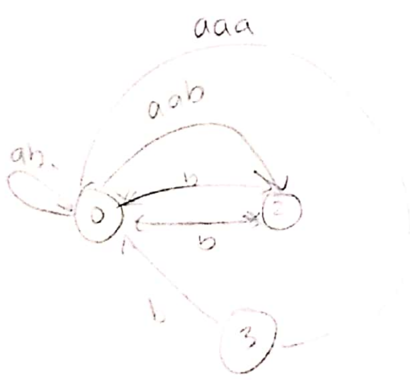


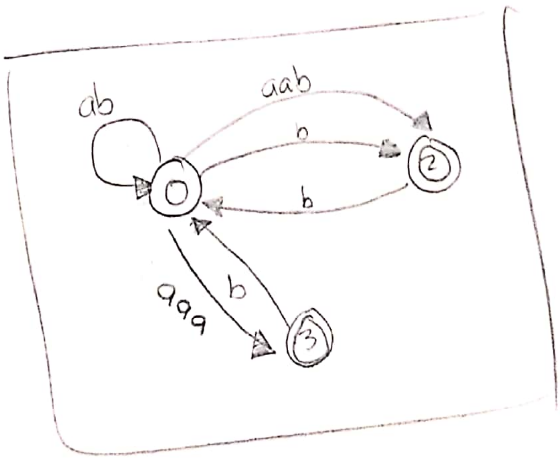
✓

9. [15 points] Construct a minimal DFSM for the following DFSM:









−15

10. [10 points] Write a regular expression for each of the following languages:

(a) $L = \{w \in \{a,b\}^* : w \text{ contains } \mathtt{aba} \text{ as a substring}\}$

✓ $\{a \cup b\}^* aba \{a \cup b\}^*$

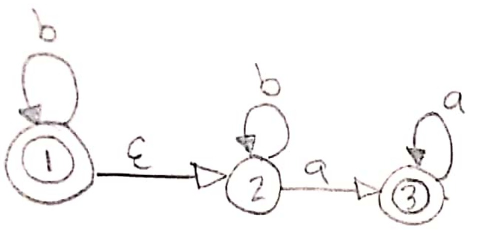
(b) $L = \{w \in \{a,b\}^* : w \text{ does not contain } \mathtt{bb} \text{ as a substring}\}$

ab
aa
bb
ba

-1 $\{aa \cup ba\}^*$

a, ab?

11. [5 points] Give a NDFSM that captures the same language as the regular expression $b^* \cup b^*aa^*$



✓

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10/10

Name: Adam Stammer

CS 435 Quiz #5
October 9, 2019

1. Consider the following grammar for language L:

$$S \rightarrow aS \mid aSbS \mid \epsilon$$

(a) List five strings that are in L.

✓ ε, a, ab, aab, ~~ε,~~ aaab

(b) List five strings that are not in L.

✓ b, ba, bab, bbb, abb

Each grouping of b's must be preceded by a grouping of a's at least equal in length

(c) Indicate whether or not L is regular. Prove your answer.

Not regular

$w = a^k b^k$

✓ Pump b up such that $w = a^k b^{k+1}$. We now have more b's than a's, thus our string is not accepted. L is not regular.

$y = a^p \quad 1 \le p \le K$

Pump a down such that $w = a^{k-1} b^k$. We now have 1 less a's than b's, thus our string is not accepted and L is not regular

right idea

2. Show a context-free grammar for the language $BalDelim$ = {w : where w is a string of delimeters: (,), [,], {, }, that are properly balanced}.

S → (S) | [S] | {S} | SS | ε

({S, (,), [,], {, }}, {(,), [,], {, }}, {S → (S) | [S] | {S} | SS}, S)