Price Contagion in Bail-ins and Bail-outs

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Presentation Outline

Price contagion impact on shock

- Financial institutions are linked to each other via unsecured contractual liabilities as well as outside assets price linkages.
- When a large shock hits a financial institution, it may need to liquidate its outside assets to stay solvent. If this liquidation is in a relatively large amount compared to the total amount of the assets in the market, the operation might greatly influence the price of the assets and thus affecting the book value of other asset holders.
- Our analysis focuses on how the price contagion of outside assets influences the subgame perfect equilibrium established in the Bail-ins and Bail-outs model, assuming that the liquidation behaviors in the game is the main source of the demand shock of the assets, which lead to the contagious change in the asset prices and the book values of the banks.

Model Construction

- Demand shock
- Equation re-derivation
  - Clearing equilibrium
  - Public Bail-outs
  - Subsidized Bail-ins
  - No intervention

Demand shock I

- Suppose there are totally K non-interbank assets. Each bank i holds these K non-interbank asset with total assets value $e^i = \sum_{k=1}^{K} e^{ki}$.
- When liquidating the assets to raise the desired amount of cash flow, we consider two strategies for the company to choose from, as discussed by Capponi and Larsson (2015). One is the fixed relative exposure policy, in which the fraction of liquidation values over different assets depend on their relative size held by the bank, i.e.

$$\alpha_t^{ki} = \frac{e_t^{ki}}{e_t^i} = \frac{e_t^{ki}}{\sum_{k=1}^{K} e_t^{ki}}$$

- Another alternative is liquidity-based strategy, in which the faction of liquidation values over different assets depends on the price impact ratios of the assets, i.e.

$$\alpha_t^{ki} = \frac{\gamma_k e_t^{k,nb}}{\sum_{k=1}^{K} \gamma_j e_t^{k,nb}}$$

Demand shock II

- According to Capponi and Larsson (2015)[2], Z is computed by

$$S_t = HM\text{Diag}(\lambda \circ e_t^{tot})M^T$$

  In this formulation, H is the diagonal matrix with the price impact ratios on the diagonal, and e_t^{tot} is the vector containing the total asset value of each bank. The matrix M contains the fraction of total asset value held in asset k by bank i, that is $\frac{e_t^{ki}}{\sum_{k=1}^{K} e_t^{ki}}$

- Given that each bank i needs to liquidate l_i non-interbank assets, total asset quantities should be sold for asset k by banks are $\frac{\sum_{i=1}^{N} \alpha_t^{ki} l_t^i}{P_t^k}$. This quantity of asset k will be absorbed by non-banking sector. The action of selling assets will lead to negative demand shock ($\Delta Z < 0$).

$$\frac{\Delta p_t}{p_t} = (I - S_t)^{-1} \frac{\Delta Z_t}{p_t}$$

Demand shock III

- After we get Δp_t^k, total extra loss for bank i due to price decreasing is

$$EL_i = \sum_{k=1}^{K} \Delta p_t^j (Q_t^{ki} - \Delta Q_t^{ki})$$

$$= \sum_{k=1}^{K} \Delta p_t^k \frac{\alpha_t^{ki} e_t^i - \alpha_t^{ki} l_t^i}{p_t^k}$$

- Suppose $\frac{\Delta Z_t^k}{p_t} = \frac{\Delta A_t^k}{A_t^k}$. Here A_t^k is the total asset j value in market at time t. ΔA_t^k is the total asset j value needed to be sold by banking sector due to liquidation at time t.

Demand shock IV

- Hence,

$$\frac{\Delta Z_t^k}{p_t^j} = \frac{\Delta A_t^k}{A_t^j} = \frac{\sum_{i=1}^N \alpha_t^{ki} l_t^i}{\sum_{i=1}^N \alpha_t^{ki} e_t^i}$$

$$\frac{\Delta Z_t}{p_t} = [\frac{\Delta Z_t^1}{p_t^1}, \frac{\Delta Z_t^2}{p_t^2}, \ldots, \frac{\Delta Z_t^K}{p_t^K}]$$

- Extra loss for firm i

$$EL(l_t^i) = \sum_{k=1}^K \Delta p_t^k \frac{\alpha_t^{ki} e_t^i - \alpha_t^{ji} l_t^i}{p_t^k}$$

$$= \sum_{k=1}^K \left((I - S_t)^{-1} (\frac{\Delta Z_t}{p_t})^T \right)^k (\alpha_t^{ki} e_t^i - \alpha_t^{ki} l_t^i)$$

Clearing Equilibrium I

- Liquidation decision l^i and clearing payment p^i remain the same as the condition with no price contagious.

$$l^i = \min(\frac{1}{\alpha}(L^i - c^i - \sum_{j=1}^{n} \pi^{ij} p^j), \sum_{k=1}^{K} e_t^{ki})$$

$$p^i = \begin{cases} L^i & \text{if } c^i + \alpha l^i + \sum_{j=1}^{n} \pi^{ij} p^j \geq L^i \\ c^i + \alpha l^i + \beta \sum_{j=1}^{n} \pi^{ij} p^j & \text{Other} \end{cases}$$

- Senior creditors loss

$$s^i = \left(c^i + \alpha l^i + \beta \sum_{j=1}^{n} \pi^{ij} p^j \right)^{-}$$

Clearing Equilibrium II

- Value of bank i's equity in a clearing equilibrium (l, p) is

$$V^i(l,p) := (\pi p + c + e - (1-\alpha)l - p - EL(l))^i \mathbb{1}(p^i \geq L^i)$$

- Deadweight losses due to default costs

$$w(l,p) = \sum_{i=1}^{n} [(1-\alpha)l^i + EL(l^i)] + (1-\beta) \sum_{i \in \mathcal{D}(l,p)} (\pi p)^i + \sum_{i \in \mathcal{D}(l,p)} s^i$$

- Value of bank i after the realization of the shock but before the liquidation of its outside assets

$$V_0^i := c^i + e^i + A^i - L^i$$

Public Bail-outs

- The shortfall social planner chooses to cover

$$B = \begin{cases} ||L^{\mathcal{F}} - c^{\mathcal{F}} - A^{\mathcal{F}}||_1 - \alpha||e^{\mathcal{F}}|| & \alpha < 0.5 \\ ||L^{\mathcal{F}} - c^{\mathcal{F}} - A^{\mathcal{F}}|| & \alpha \geq 0.5 \end{cases}$$

- For each bank $i \in \mathcal{S} \bigcup \mathcal{C}$, $l_*^i = \min\{\frac{1}{\alpha}(L^i - c^i - A^i)^+, e^i\}$ denote the minimal amount that bank i has to liquidate even if the fundamentally defaulting banks are being rescued.
- The equity value of each bank $i \in \mathcal{S} \bigcup \mathcal{C}$ in a public bailout is

$$V_P^i = \begin{cases} V_0^i - (1-\alpha)l_*^i - EL(l_*^i) & \alpha < 0.5 \\ V_0^i + \alpha l_*^i & \alpha \geq 0.5 \end{cases}$$

- Welfare lose

$$w_p = B + \min(\alpha, 1-\alpha)||l_*||_1 + \mathbb{1}(\alpha \geq 0.5)(\sum_{i=1}^{N} EL(l_*^i))$$

Subsidized Bail-ins I

- When social planner proceeds with the proposed bail-in, the equity value of each bank $i \in \mathcal{S} \bigcup \mathcal{C}$ equals

$$V_R^i(b, \lambda, a) = \begin{cases} V_0^i - b^i + \lambda^i - (1-\alpha) l^i(b-\lambda) - EL(l^i(b^i - \lambda^i)) & \text{if } i \in A^c \\ V_0^i - (1-\alpha) l_*^i - EL(l_*^i) & \text{if } i \in A \end{cases}$$

- The welfare losses are equal to

$$w_R(b, \lambda, a) = b^0 + ||b^A||_1 + ||\lambda^{A^c}||_1 + ||EL^{A^c}(l^{A^c}(b-\lambda))||_1 + ||EL^{FUA}(l_*^{A^c})||_1$$
$$+(1-\alpha)(||l^{A^c}(b-\lambda)||_1 + ||l_*^{FUA}||_1)$$

Subsidized Bail-ins II

- Cumulative losses in bank i's interbank assets that arise as a result of a default cascade is
$$\zeta^i := \pi^{i,\mathcal{D}}(L^{\mathcal{D}} - \hat{p}^{\mathcal{D}})$$
- The loss in interbank assets that are absorbed by any bank $i \in \mathcal{S} \bigcup \mathcal{C}$ equals
$$\xi^i = \min(\zeta^i, V_0^i - (1-\alpha)e^i)$$
- The maximum amount a bank $i \in \mathcal{S} \bigcup \mathcal{C}$ is willing to contribute without liquidating its outside is
$$\eta^i := \min(\zeta^i (V_0^i - e^i)^+)$$
- The amount of debt b_R^i that bank i has to by from the defaulting company and subsidy λ_R^i it will receive show as follows:

$$b_R^i = \begin{cases} \xi^i & \text{if } \alpha \geq 0.5 \\ \eta^i & \text{if } \alpha < 0.5 \end{cases} \quad \lambda_R^i = \begin{cases} 0 & \text{if } \alpha \geq 0.5 \\ al_*^i & \text{if } \alpha < 0.5 \end{cases}$$
$$b_R^0 = B + \alpha ||e^{\mathcal{F}}||_1 \mathbb{1}(\alpha < 0.5) - ||b_R^{\mathcal{S} \cup \mathcal{C}}||_1$$

Subsidized Bail-ins III

- The largest incentive compatible contribution of any bank i to a bail-in is

$$\nu^i = \begin{cases} \xi^i - (1-\alpha)(\hat{l}^i - l_*^i) + (ES_i(l_*^i) - ES_i(\hat{l}^i)) & \text{if } i \in A^c \\ \eta^i & \text{if } i \in A \end{cases}$$

- Let $i_1, i_2, .., i_{\mathcal{S} \bigcup \mathcal{C}}$ be a non-increasing ordering of banks in $\mathcal{S} \bigcup \mathcal{C}$ according to ν^i so that $\nu^{i_1} \geq \nu^{i_2} \geq ... \geq \nu^{i_{\mathcal{S} \cup \mathcal{C}}}$.
- When $w_N \leq w_P$,

$$w_* = \min(w_{\{i_1, i_2, .., i_m\}}, w_N - \nu^{i_{m+1}})$$

where $m := \min(k | w_{\{i_1, i_2, .., i_m\}} < w_N)$

No Intervention

- Without intervention, the equity value of each bank is $V_N^i = 0$ for $i \in \mathcal{F}$ and

$$V_N^i = V_0^i - \xi^i - EL(l^i(\xi^i)) - (1-\alpha)l^i(\xi^i)$$

- The welfare losses equals to

$$w_N = \sum_{i=1}^{n} (V_0^i - V_N^i) + ||s^{\mathcal{D}}||_1$$

Background information

- Under the condition shown in table, fundamental default bank only includes bank 1 in both structured networks.
- We will discuss three cases (complete networks, R_1 an R_2). Each bank is identical in complete networks.
- Fundamental default bank (bank 1) will directly impact low capitalized bank 2 in R_1 network and it will directly impact high capitalized bank 11 in R_2 network.

Bank information			
Bank	L	c	e
1	1.5	c^1	0.4
2,...,6	1	0	0.15
7,...,11	1	0	1.2

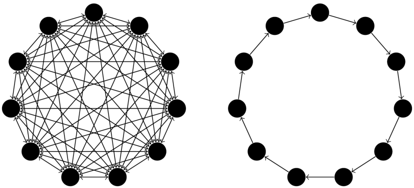
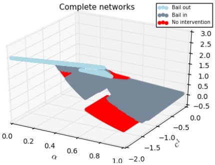


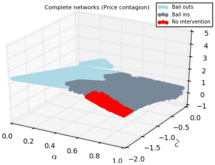
Figure: Complete Networks and Ring Networks

Complete Networks I

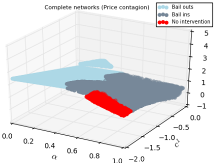
- Price contagion increased the welfare loss significantly under the high shock and high recovery rate conditions. This is because the shock is amplified by the price contagion, and given a higher recovery rate, the government would rather let the banks to liquidate their assets to stay solvent, leading to a more severe asset price drop



(a) Complete Networks



(b) Price contagion under fixed relative exposure strategy

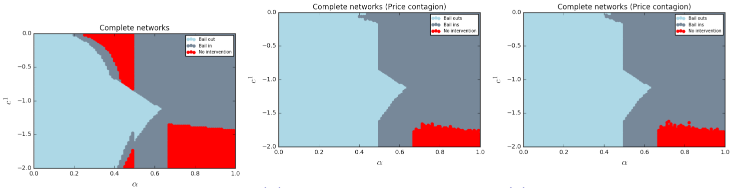


(c) Price contagion under liquidity-based allocation strategy

Figure: Complete Networks welfare loss

Complete Networks II

- Given the price contagion, scenarios under which no intervention is the optimal choice have become fewer, which are represented by the red areas on the graph.
- Since the price drop would amplify the distress over banks and reduce the book values of the banks, no intervention, or the default cascade is reasonably less preferable and the government has more incentive to bail out.
- Liquidity-based allocation strategy and the fixed relative exposure strategy are very similar. This shows that the price contagions are having impact on the game in a similar way, no matter how the banks apply their liquidation strategies.



(a) Complete Networks

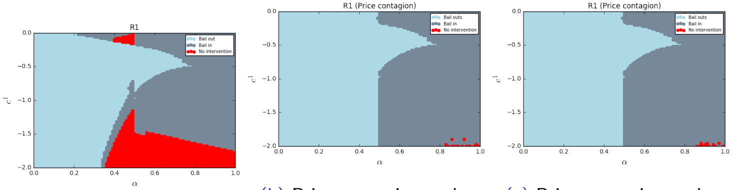
(b) Price contagion under fixed relative exposure strategy

(c) Price contagion under liquidity-based allocation strategy

Figure: Complete Networks strategy distribution

Ring 1 I

- Both the no intervention (in red) area and the bail in (in gray) area shrink for price contagion situations. This is intuitive since bail in and intervention mean that more liquidation amount, in any, are required for the banks, and the loss is amplified by the price contagion.



(a) R_1 Networks

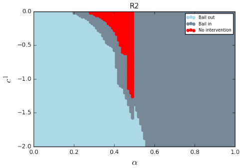
(b) Price contagion under fixed relative exposure strategy

(c) Price contagion under liquidity-based allocation strategy

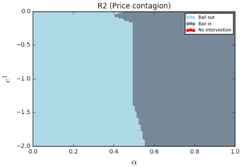
Figure: R_1 Ring Networks strategy distribution

Ring 2 I

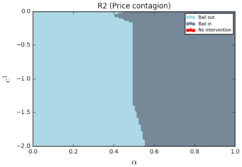
- Both the no intervention (in red) area and the bail in (in gray) area shrink for price contagion situations. This phenomenon follows the same reasoning as discussed in the previous chapters.
- In the R_2 structure, no intervention outcome is strictly dominated.



(a) R_2 Networks



(b) Price contagion under fixed relative exposure strategy



(c) Price contagion under liquidity-based allocation strategy

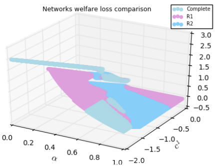
Figure: R_2 Ring Networks strategy distribution

Welfare Loss Comparison over Different Structures I

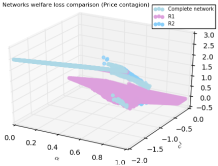
- We would like to see which kind of network structure has the lowest social welfare loss, that is, most preferable from the social planner's perspective.
- The network structures studied are the same as those from the previous chapters, including the complete network and the sparse networks (R_1 and R_2).
- Over a range of recovery rates α and net cash holdings c^1, we can compare the most preferable structure under different situations, i.e. no price contagion, price contagion under liquidity-based allocation strategy and price contagion under fixed relative exposure strategy.

Welfare Loss Comparison over Different Structures II

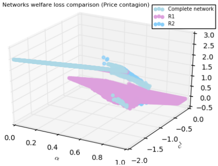
- The social welfare losses considering price contagion are generally higher than no price contagion, no matter which liquidation strategy is applied by the banks. This follows the intuition since all the banks have a non-negative outside assets liquidation value under the pay off scenario, which leads to a negative demand shock on all assets. This demand shock leads to the price drop of assets, and thus reducing the book values of the banks and leading to extra loss.



(a) No price contagion



(b) Price contagion under fixed relative exposure strategy

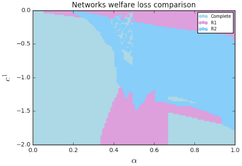


(c) Price contagion under liquidity-based allocation strategy

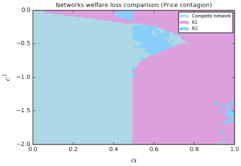
Figure: Comparison of minimum welfare loss under different situations

Welfare Loss Comparison over Different Structures III

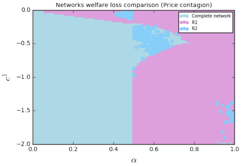
- The main difference brought by the price contagion is that under a high rate and a high net cash holdings (the upper right part of the graph, the most preferable network structure changes to R_1 from R_2).
- Under R_1 structure, more banks will suffer from a default cascade, giving them more incentive to accept the bail-in proposal. Hence liquidation is split over more assets from different banks with relatively lower amounts.
- Under R_2 structure, the high capitalized bank linked to the defaulting bank has the most incentive to accept the bail-in proposal, hence the liquidation is concentrated in this bank and a limited number of assets with higher amounts.



(a) No price contagion



(b) Price contagion under fixed relative exposure strategy



(c) Price contagion under liquidity-based allocation strategy

Figure: Comparison of minimum welfare loss under different situations

Conclusion

- Compared with the result of the original model where prices keep steady, the optimal welfare loss with the price contagion spread in the asset market are higher no matter what outside asset liquidation strategy (fixed relative exposure strategy or liquidity-based strategy) is adopted by the banks, which indicates the negative effect of the suddenly enlarged liquidation on the whole bank networks.
- When the recovery ratio is small ($\alpha < 0.5$), public bailout performs like a dominant strategy since it requires smallest size of liquidation and thus minimizes the risk of negative price contagion as well as the social welfare loss.
- When the negative shock in financial market and the price fluctuation are considered in this strategy framework, no intervention seems obviously less favorable because it is very likely that large default cascade will induce greater shock not only to the bank network itself but also to the asset market through the large amount of required liquidation behavior.