Price Contagion in Bail-ins and Bail-outs

Zhiyang Cai, Zidong Liu, Chengjunyi Zheng

Abstract

We study the impact of outside assets price linkages on the bail-in and bail-out strategies played by the government and the banks to inject liquidity to the distressed banks under a financial shock. In the sequential game, the government gives bail-in proposals which the solvent banks choose to accept or decline. In case of liquidity shortage, the banks need to liquidate their outside assets, affecting the asset prices and the book value of other banks. In this project, the subgame perfect equilibrium under asset price linkages is built, and the sparse, dense and centralized interbank networks are studied. According to our analysis, the liquidation of the banks imposes a negative demand shock, leading to the asset price drops. This decreases the book values of the banks and increases the social welfare loss. Hence subsidized bail-in and no intervention strategies are less preferable for the social planner, since in these scenarios, the liquidation of the banks, if any, can be higher under price contagion. Moreover, when the financial shock impacts highly capitalized banks closely in a sparse network, price contagion can lead to heavier losses because of the concentration of liquidation of certain assets held by these banks.

Keywords: Price Contagion, Demand Shock, Ring Network

1. Introduction

Financial institutions are linked to each other via unsecured contractual liabilities as well as outside assets price linkages. On one hand, financial institutions might run into distress and default on its bilateral agreements, failing to repay their liabilities fully to other financial institutions and thus influencing the solvency of other institutions. On the other hand, since the outside assets of the financial institutions are linked to each other by their market prices, influence on others might occur when a financial institution operates on its assets by affecting its market price.

When a large shock hits a financial institution, it may need to liquidate its outside assets to stay solvent. If this liquidation is in a relatively large amount compared to the total amount of the assets in the market, the operation might greatly influence the price of the assets and thus affecting the book value of other asset holders. Moreover, when liquidating outside assets is not sufficient for the institution to commit its liability obligations, the default of the bank might lead to the distress of other banks and thus causing a default cascade. This forces the government to make some intervention. When a financial network is faced with a shock, the government generally has three options of reaction: (i) let the default cascade happen without any intervention, (ii) inject liquidity to help the defaulting banks to stay solvent by itself, (iii) make proposals to the relevant banks to support the defaulting banks cooperatively. On the other hand, the solvent banks are able to choose from accepting or declining the government proposals, based on its own utility under different scenarios.

In this project, a game model is built to study how government and financial institutions interact under a shock given different structures of financial network, which is built based on the interbank liabilities and outside assets on their balance sheets. The model is built based on the Bail-ins and Bail-outs game model proposed by Bernard, Capponi and Stiglitz (2017)[1], combining the Price Contagion model proposed by Capponi and Larsson (2015)[2]. The relationship between the liquidation and the price contagion via the asset demand shock is also considered, and the relationship is built up based on the basic supply and demand curves, see Mishkin (2007)[3].

Our analysis focuses on how the price contagion of outside assets influences the subgame perfect equilibrium established in the Bail-ins and Bail-outs model, assuming that the liquidation behaviors in the game is the main source of the demand shock of the assets, which lead to the contagious change in the asset prices and the book values of the banks. These changes are considered by both the government and the banks to react strategically in the game. Additional to the study on the sparse and dense network learnt in the previous Bail-ins and Bail-outs game model, we also studied the Core-Periphery network structure, which has been considered as a general alternative of financial network. According to our analysis, the liquidation of the banks imposes a negative demand shock, leading to the asset price drops. Hence subsidized bail-in and no intervention strategies are less preferable for the social planner, since in these scenarios, the liquidation of the banks, if any, can be higher under price contagion. Moreover, when the financial shock impacts highly capitalized banks closely in a sparse network, price contagion can lead to heavier losses because of the concentration of liquidation of certain assets held by these banks

2. Model construction

In this sector, we derived the price change due to the liquidation of banks' non-interbank assets and re-derive the equations related to bank's equity and welfare loss under three possible types of strategies: (i) A public bail-out, (ii) A subsidized bail-in (iii) No intervention. Before re-deriving the equation, I set up the notations and variables.

e_i: Non-interbank assets

c: Net cash balance

L^{ij}: Liability of bank j to i. We denote $L^j = \sum_{i=1}^{n} L^{ij}$ as the total liability of bank j to other bank i.

π^{ij}: $\pi^{ij} = L^{ij}/L^j$ if $L^j \neq 0$

A^i: Asset receivable. $A^i = \sum_{j=1}^{n} \pi^{ij} L^j$

α: Liquidation fraction

β: Fraction could be recovered from the interbank assets,

Fundamental default: $\mathcal{F} := \{i \mid L^i > c^i + \alpha e^i + A^i\}$

Default in the clearing equilibrium: $\mathcal{D} := \{i \mid \hat{p}^i < L^i\}$

Contagious default: $\mathcal{C} := \mathcal{D} \backslash \mathcal{F}$

Banks remaining solvent: $\mathcal{S} := \{i \mid \hat{p}^i = L^i\}$

Banks rejecting the social planner's subsidized proposal: $\mathcal{A} := \{i \in \mathcal{S} \bigcup \mathcal{C} \mid a^i = 0\}$

2.1. Demand shock

Suppose there are totally K non-interbank assets. Each bank i holds these K non-interbank asset with total assets value $e_i = \sum_{j=1}^{K} e_i^j$.

When liquidating the assets to raise the desired amount of cash flow, we consider two strategies for the company to choose from, as discussed by Capponi and Larsson (2015)[2]. One is the fixed relative exposure policy, in which the fraction of liquidation values over different assets depend on their relative size held by the bank, i.e.

$$\alpha_t^{ji} = \frac{e_t^{ji}}{e_t^i} = \frac{e_t^{ji}}{\sum_{j=1}^{K} e_t^{ji}}$$

.

Another alternative is liquidity-based strategy, in which the faction of liquidation values over different assets depends on the price impact ratios of the assets, i.e.

$$\alpha_i^j = \frac{\gamma_k e_t^{k,nb}}{\sum_{j=1}^K \gamma_j e_t^{j,nb}}$$

Where according to Capponi and Larsson (2015)[2], S_t is computed by

$$S_t = HM\mathrm{Diag}(\lambda \circ e_t^{tot})M^T$$

In this formulation, H is the diagonal matrix with the price impact ratios on the diagonal, and e_t^{tot} is the vector containing the total asset value of each bank. The matrix M contains the fraction of total asset value held in asset k by bank i, that is $\frac{e_t^{ji}}{\sum_{j=1}^K e_t^{ji}}$

Given that each bank i needs to liquidate l_i non-interbank assets, total asset quantities should be sold for asset j by banks are $\frac{\sum_{i=1}^N \alpha_t^{ji} l_t^i}{P_t^j}$. This quantity of asset j will be absorbed by non-banking sector. The action of selling assets will lead to negative demand shock ($\Delta Z < 0$).

$$\frac{\Delta p_t}{p_t} = (I - S_t)^{-1} \frac{\Delta Z_t}{p_t} \tag{1}$$

Here, we assume $\Delta R_t^i = 0$ for all banks. After we get Δp_t^j, total extra loss for bank i due to price decreasing is

$$EL_i = \sum_{j=1}^K \Delta p_t^j (Q_t^{ji} - \Delta Q_t^{ji})$$

$$= \sum_{j=1}^K \Delta p_t^j \frac{\alpha_t^{ji} e_t^i - \alpha_t^{ji} l_t^i}{p_t^j}$$

Suppose $\frac{\Delta Z_t^j}{p_t} = \frac{\Delta A_t^j}{A_t^j}$. Here A_t^j is the total asset j value in market at time t. ΔA_t^j is the total asset j value needed to be sold by banking sector due to liquidation at time t. We notice that Δp_t is roughly equal to ΔZ_t according to multiple simulation of equation (1). This makes $\frac{\Delta Z_t^j}{p_t} = -\frac{\Delta A_t^j}{A_t^j}$ quite reasonable. When $\Delta A_t^j = A_t^j$, it will leads to $\Delta p_t^j = -p_t^j$. The asset will be worthless.

$$\Delta A_t^j = \sum_{i=1}^N \alpha_t^{ji} l_t^i$$

Hence,

$$\frac{\Delta Z_t^j}{p_t^j} = -\frac{\Delta A_t^j}{A_t^j} = \frac{\sum_{i=1}^N \alpha_t^{ji} l_t^i}{\sum_{i=1}^N \alpha_t^{ji} e_t^i}$$

$$\frac{\Delta Z_t}{p_t} = [\frac{\Delta Z_t^1}{p_t^1}, \frac{\Delta Z_t^2}{p_t^2}, ..., \frac{\Delta Z_t^K}{p_t^K}]$$

Extra loss for firm i

$$EL(l_t^i) = \sum_{j=1}^{K} \Delta p_t^j \frac{\alpha_t^{ji} e_t^i - \alpha_t^{ji} l_t^i}{p_t^j}$$

$$= \sum_{j=1}^{K} \left((I - S_t)^{-1} (\frac{\Delta Z_t}{p_t})^T \right)^i (\alpha_t^{ji} e_t^i - \alpha_t^{ji} l_t^i) \tag{2}$$

2.2. Clearing equilibrium

Liquidation decision l^i and clearing payment p^i remain the same as the condition with no price contagious.

$$l^i = \min(\frac{1}{\alpha}(L^i - c^i - \sum_{j=1}^{n} \pi^{ij} p^j), \sum_{j=1}^{K} e_i^j)$$

$$p^i = \begin{cases} L^i & \text{if } c^i + \alpha l^i + \sum_{j=1}^{n} \pi^{ij} p^j \geq L^i \\ c^i + \alpha l^i + \beta \sum_{j=1}^{n} \pi^{ij} p^j & \text{Other} \end{cases}$$

Senior creditors loss

$$s^i = \left(c^i + \alpha l^i + \beta \sum_{j=1}^{n} \pi^{ij} p^j \right)^-$$

Value of bank i's equity in a clearing equilibrium (l, p) is

$$V^i(l, p) := (\pi p + c + e - (1 - \alpha) l - p - EL(l))^i \mathbb{1}(p^i \geq L^i)$$

Deadweight losses due to default costs

$$w(l, p) = \sum_{i=1}^{n} [(1 - \alpha) l^i + EL(l^i)] + (1 - \beta) \sum_{i \in \mathcal{D}(l,p)} (\pi p)^i + \sum_{i \in \mathcal{D}(l,p)} s^i$$

Value of bank i after the realization of the shock but before the liquidation of its outside assets

$$V_0^i := c^i + e^i + A^i - L^i$$

2.3. Public Bail-outs

The shortfall social planner chooses to cover

$$B = \begin{cases} ||L^{\mathcal{F}} - c^{\mathcal{F}} - A^{\mathcal{F}}||_1 - \alpha||e^{\mathcal{F}}|| & \alpha \geq 0.5 \\ ||L^{\mathcal{F}} - c^{\mathcal{F}} - A^{\mathcal{F}}|| & \alpha < 0.5 \end{cases}$$

For each bank $i \in \mathcal{S} \bigcup \mathcal{C}$, $l_*^i = \min\{\frac{1}{\alpha}(L^i - c^i - A^i)^+, e^i\}$ denote the minimal amount that bank i has to liquidate even if the fundamentally defaulting banks are being rescued.
The equity value of each bank $i \in \mathcal{S} \bigcup \mathcal{C}$ in a public bailout is

$$V_P^i = \begin{cases} V_0^i - (1-\alpha)l_*^i - EL(l_*^i) & \alpha \geq 0.5 \\ V_0^i + \alpha l_*^i & \alpha < 0.5 \end{cases}$$

Welfare lose

$$w_p = B + \min(\alpha, 1-\alpha)||l_*||_1 + \mathbb{1}(\alpha \geq 0.5)(\sum_{i=1}^{N} EL(l_*^i))$$

2.4. Subsidized Bail-ins

When social planner proceeds with the proposed bail-in, the equity value of each bank $i \in \mathcal{S} \bigcup \mathcal{C}$ equals

$$V_R^i(b, \lambda, a) = \begin{cases} V_0^i - b^i + \lambda^i - (1-\alpha)l^i(b^i - \lambda^i) - EL(l^i(b^i - \lambda^i)) & \text{if } i \in A^c \\ V_0^i - (1-\alpha)l_*^i - EL(l_*^i) & \text{if } i \in A \end{cases}$$

The welfare losses are equal to

$$w_R(b, \lambda, a) = b^0 + ||b^A||_1 + ||\lambda^{A^c}||_1 + (1-\alpha)(||l^{A^c}(b-\lambda)||_1 + ||l_*^{FUA}||_1) + ||EL^{A^c}(l^{A^c}(b-\lambda))||_1 + ||EL^{FUA}(l_*^{A^c})||_1$$

Cumulative losses in bank i's interbank assets that arise as a result of a default cascade is $\zeta^i := \pi^{i,\mathcal{D}}(L^{\mathcal{D}} - \hat{p}^{\mathcal{D}})$. The loss in interbank assets that are absorbed by any bank $i \in \mathcal{S} \bigcup \mathcal{C}$ equals $\xi^i = \min(\zeta^i, V_0^i - (1-\alpha)e^i)$.
The maximum amount a bank $i \in \mathcal{S} \bigcup \mathcal{C}$ is willing to contribute without liquidating its outside is $\eta^i := \min(\zeta^i, (V_0^i - e^i)^+)$ The amount of debt b_R^i that bank i has to by from the defaulting company and subsidy λ_R^i it will receive show as follows:

$$b_R^i = \begin{cases} \xi^i & \text{if } \alpha \geq 0.5 \\ \eta^i & \text{if } \alpha < 0.5 \end{cases} \qquad \lambda_R^i = \begin{cases} 0 & \text{if } \alpha \geq 0.5 \\ \alpha l_*^i & \text{if } \alpha < 0.5 \end{cases}$$

$$b_R^0 = B + \alpha ||e^{\mathcal{F}}||_1 \mathbb{1}(\alpha < 0.5) - ||b_R^{\mathcal{S} \bigcup \mathcal{C}}||_1$$

The largest incentive compatible contribution of any bank i to a bail-in is

$$\nu^i = \begin{cases} \xi^i - (1-\alpha)(l^i(\xi^i) - l_*^i) + (ES_i(l_*^i) - ES_i(l^i(\xi^i))) & \text{if } \alpha \geq 0.5 \\ \eta^i & \text{if } \alpha < 0.5 \end{cases}$$

Let $i_1, i_2, .., i_{\mathcal{S} \bigcup \mathcal{C}}$ be a non-increasing ordering of banks in $\mathcal{S} \bigcup \mathcal{C}$ according to ν^i so that $\nu^{i_1} \geq \nu^{i_2} \geq ... \geq \nu^{i_{\mathcal{S} \cup \mathcal{C}}}$. When $w_N \leq w_P$, $w_* = \min(w_{\{i_1, i_2, .., i_m\}}, w_N - \nu^{i_{m+1}})$, where $m := \min(k | w_{\{i_1, i_2, .., i_m\}} < w_N)$

2.5. No intervention

Without intervention, the equity value of each bank is $V_N^i = 0$ for $i \in \mathcal{F}$ and

$$V_N^i = V_0^i - \xi^i - EL(l^i(\xi^i)) - (1-\alpha) l^i(\xi^i)$$

The welfare losses are equals to

$$w_N = \sum_{i=1}^{n} (V_0^i - V_N^i) + ||s^{\mathcal{D}}||_1$$

3. Numerical Analysis

Under the condition shown in table (Appendix A), fundamental default bank only includes bank 1 in both structured networks. We will discussed three cases (complete networks, R_1 an R_2). Each bank is identical in complete networks. Fundamental default bank (bank 1) will directly impact low capitalized bank 2 in R_1 network and it will directly impact high capitalized bank 11 in R_2 network. We will study the case with no price contagion, price contagion under fixed relative exposure strategy and price contagion under liquidity-based allocation strategy.

3.1. Price Contagion Impact over the Complete Network

In this chapter the price contagion impact over the complete network, that is a "densely connected" structure. Different values of recovery rate ? and net cash holding c^1 are set and under a certain condition, the welfare loss as well as the subgame perfect equilibrium outcome is shown as below.

From Figure B.1 we see that price contagion increased the welfare loss significantly under the high shock and high recovery rate conditions. This is because the shock is amplified by the price contagion, and given a higher recovery rate, the government would rather let the banks to liquidate their assets to stay solvent, leading to a more severe asset price drop.

From Figure B.2 we see that given the price contagion, scenarios under which no intervention is the optimal choice have become fewer, which are represented by the red areas on the graph. Since the price drop would amplify the distress over banks and reduce the book values of the banks, no intervention, or the default cascade is reasonably less preferable and the government has more incentive to bail out.

Noting that the results from the liquidity-based allocation strategy and the fixed relative exposure strategy are very similar. This shows that the price contagions are having impact on the game in a similar way, no matter how the banks apply their liquidation strategies. In other words, the difference in the strategies are not amplified by the welfare loss process.

3.2. Price Contagion Impact over the Ring 1 Network

The conclusions from Figure B.3 are similar to the previous chapter. For Figure B.4, we observe that both the no intervention (in red) area and the bail in (in gray) area shrink for price contagion situations. This is intuitive since bail in and intervention mean that more liquidation amount, in any, are required for the banks, and the loss is amplified by the price contagion.

3.3. Price Contagion Impact over the Ring 2 Network

The conclusions from Figure B.5 are similar to the previous chapters. For Figure B.6, we observe that both the no intervention (in red) area and the bail in (in gray) area shrink for price contagion situations. This phenomenon follows the same reasoning as discussed in the previous chapters. Additionally, in the R_2 structure, no intervention outcome is strictly dominated. This is because the default cascade loss is concentrated to the highly capitalized bank linked to the defaulting bank, centralizing both the bail in pressure and the liquidation of assets held by this highly capitalized bank. Hence the price contagion effect might become severe and lead to higher welfare loss.

3.4. Welfare Loss Comparison over Different Structures

In this chapter, we would like to see which kind of network structure has the lowest social welfare loss, that is, most preferable from the social planner's perspective. The network structures studied are the same as those from the previous chapters, including the complete network and the sparse networks (R_1 and R_2). Over a range of recovery rates α and net cash holdings c^1, we can compare the most preferable structure under different situations, i.e. no price contagion, price contagion under liquidity-based allocation strategy and price contagion under fixed relative exposure strategy. The results are shown in Figure B.8 and Figure B.9 in Appendix.

From Figure B.8, we can see that the social welfare losses considering price contagion are generally higher than no price contagion, no matter which liquidation strategy is applied by the banks. This follows the intuition since all the banks have a non-negative outside assets liquidation value under the pay off scenario, which leads to a negative demand shock on all assets. This demand shock leads to the price drop of assets, and thus reducing the book values of the banks and leading to extra loss.

From Figure B.9, we can see the main difference brought by the price contagion is that under a high rate and a high net cash holdings (the upper right part of the graph, the most preferable network structure changes to R_1 from R_2). The interpretation is that under R_1 structure, more banks will suffer from a default cascade, giving them more incentive to accept the bail-in proposal. Hence liquidation is split over more assets from different banks with relatively lower amounts. While under the R_2 structure, the high capitalized bank linked to the defaulting bank has the most incentive to accept the bail-in proposal, hence the liquidation is concentrated in this bank and a limited number of assets with higher amounts. This would have a greater impact on the price change of the assets, thus increasing the extra loss. Also note that the results of price contagion given different liquidation strategies are very similar, as the same patterns observed in the analysis from the previous chapters.

4. Conclusion

Bernard, Capponi and Stiglitz (2017) projected a framework to analyze the behaviors of the social planner and banks when rescuing the defaulting banks to avoid shock amplification and a large default cascade. Particularly, a game theoretical model which regards the banks as active players is established to investigate the interactive outcome. In their strategy, a large amount of asset may need to be liquidated and then introduce a negative shock to financial market as well, which will cause the changes of asset prices, transmit the shocks to other entities holding the same assets and eventually affect the value of the assets and the welfare loss. Therefore, the price contagion should also be considered by the social planner and the banks when making strategies and decisions.

In our project, a price contagion model proposed by Capponi and Larsson (2015) is adopted to investigate the liquidation-caused shocks in the financial market. The influence of the negative shocks is amplified and propagated across asset classes by a systemicness matrix S and eventually lead to the fluctuations of asset prices as the spread of distress in financial market.

Then the strategy framework proposed by social planners and the reactions of banks are adjusted with the price fluctuations. The value of the banks and the welfare loss are updated with an extra loss EL as the result of the feedback of asset market after the liquidation. In addition, we model the outside assets each bank holds as portfolios consisting of multiple assets. Then the fixed relative exposure strategy or liquidity-based strategy is applied by the banks to form the investment strategy.

In the end, we review the examples of three different provides by Bernard, Capponi and Stiglitz (2017) with the updated strategy framework. Compared with the result of the original model where prices keep steady, the optimal welfare loss with the price contagion spread in the asset market are higher no matter what outside asset liquidation strategy (fixed relative exposure strategy or liquidity-based strategy) is adopted by the banks, which indicates the negative effect of the suddenly enlarged liquidation on the whole bank networks. As a result, when the recovery ratio is small (less than 0.5 when the social planner decides to cover the whole shortfall rather than forcing the defaulting banks to do liquidation), public bailout performs like a dominant strategy since it requires smallest size of liquidation and thus minimizes the risk of negative price contagion as well as the social welfare loss. On the other hand, when the negative shock in financial market and the price fluctuation are considered in this strategy framework, no intervention seems obviously less favorable because it is very likely that large default cascade will induce greater shock not only to the bank network itself but also to the asset market through the large amount of required liquidation behavior.

For the further work, the role played by the interbank asset recovery rate in this game and the outcome might be worth investigating. In addition, more network topologies close to the reality could be take in to account when analyzing the outcome strategy framework.

Reference

[**1**]: Bernard, B., Capponi, A., & Stiglitz, J. E. (2017). Bail-Ins and Bail-Outs: Incentives, Connectivity, and Systemic Stability.

[**2**]: Capponi, A., & Larsson, M. (2015). Price contagion through balance sheet linkages. Review of Asset Pricing Studies, rav006.

[**3**]: Mishkin, F. S. (2007).The economics of money, banking, and financial markets. Pearson education.

Appendix A. Bank information

Bank information			
Bank	L	c	e
1	1.5	c^1	0.4
2,...,6	1	0	0.15
7,...,11	1	0	1.2

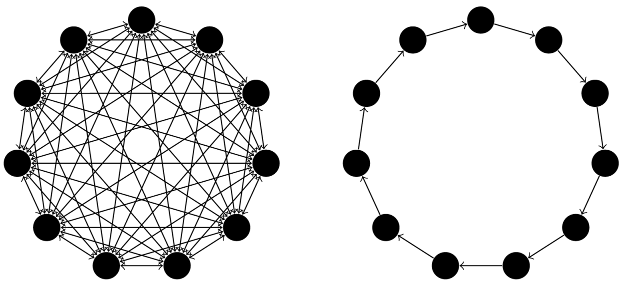
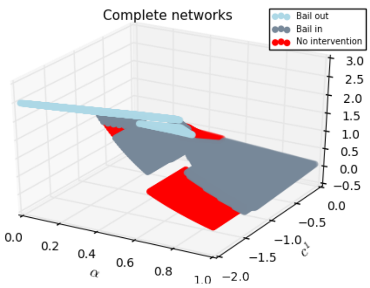
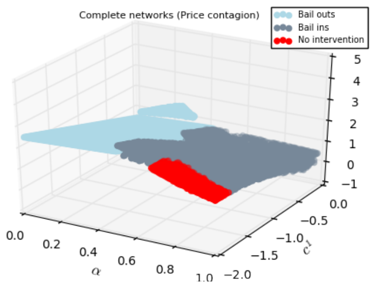


Figure A.1: Complete Networks and Ring Networks

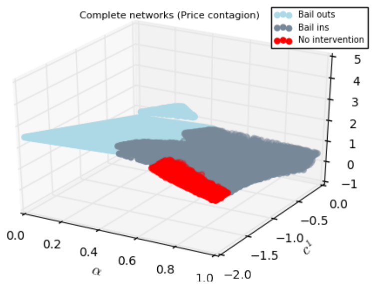
Appendix B. Welfare loss under different network structure



(a) Complete Networks

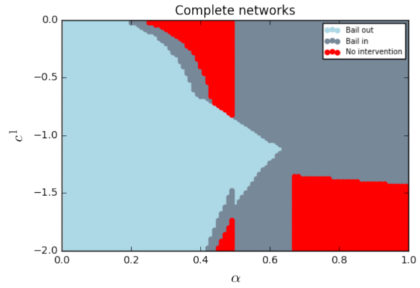


(b) Price contagion under fixed relative exposure strategy

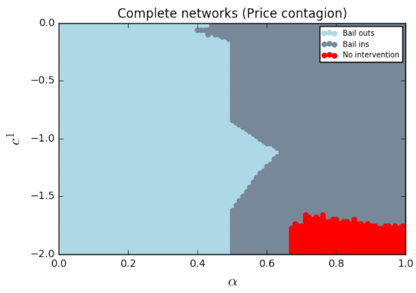


(c) Price contagion under liquidity-based allocation strategy

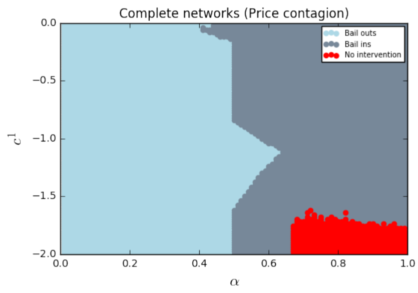
Figure B.2: **Complete Networks welfare loss**



(a) Complete Networks

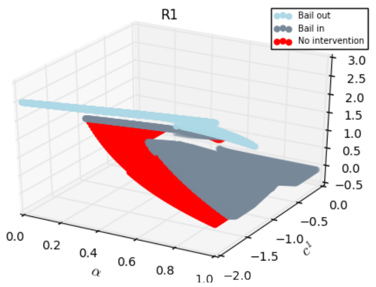


(b) Price contagion under fixed relative exposure strategy

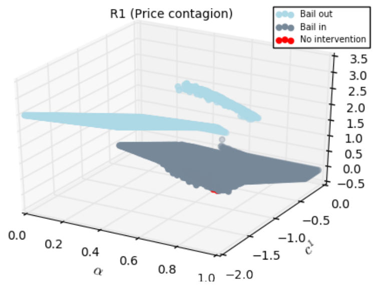


(c) Price contagion under liquidity-based allocation strategy

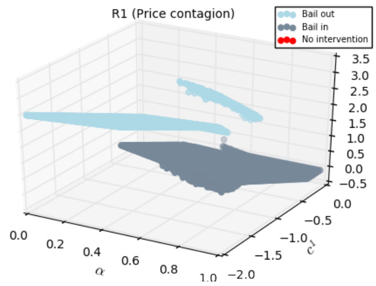
Figure B.3: **Complete Networks strategy distribution**



(a) R_1 Networks

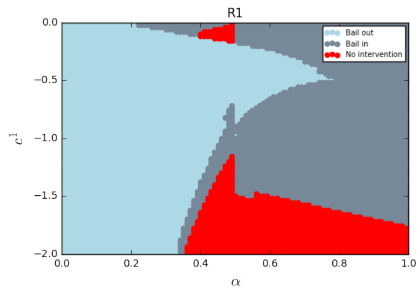


(b) Price contagion under fixed relative exposure strategy

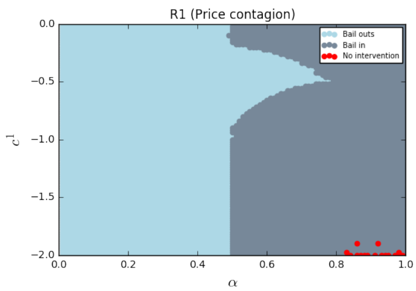


(c) Price contagion under liquidity-based allocation strategy

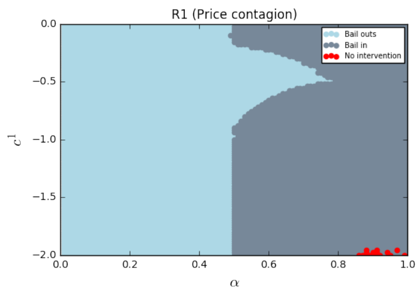
Figure B.4: R_1 **Ring Networks welfare loss**



(a) R_1 Networks

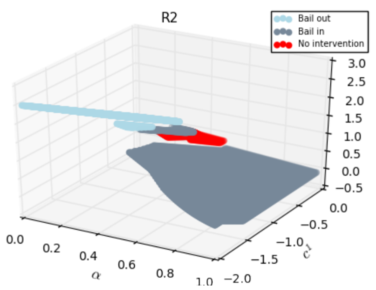


(b) Price contagion under fixed relative exposure strategy

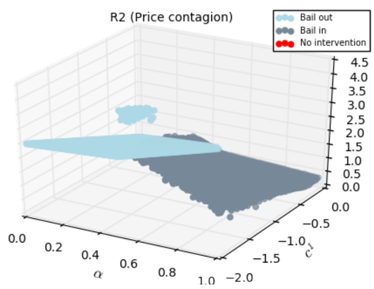


(c) Price contagion under liquidity-based allocation strategy

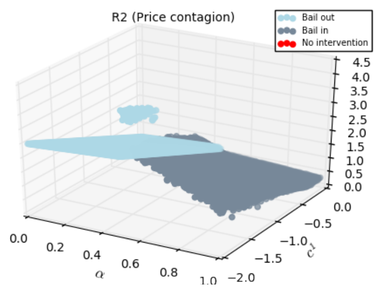
Figure B.5: R_1 **Ring Networks strategy distribution**



(a) R_2 Networks

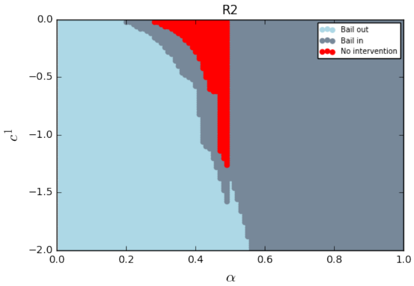


(b) Price contagion under fixed relative exposure strategy

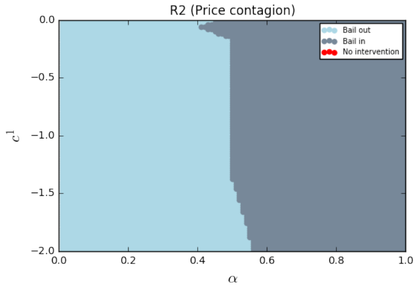


(c) Price contagion under liquidity-based allocation strategy

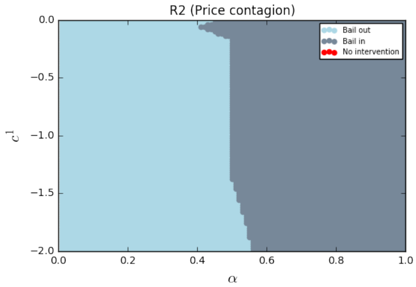
Figure B.6: R_2 **Ring Networks welfare loss**



(a) R_2 Networks

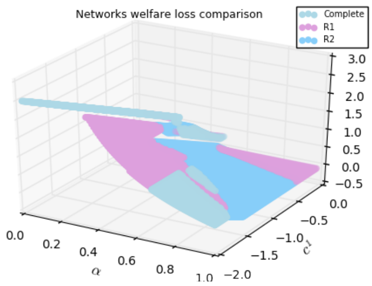


(b) Price contagion under fixed relative exposure strategy

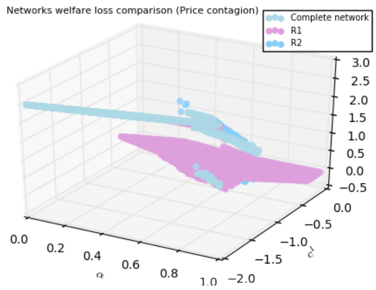


(c) Price contagion under liquidity-based allocation strategy

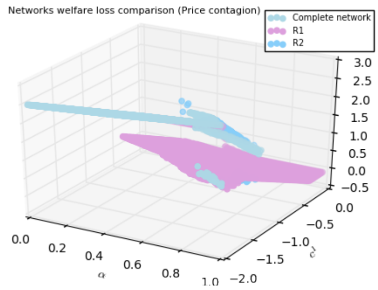
Figure B.7: R_2 **Ring Networks strategy distribution**



(a) No price contagion

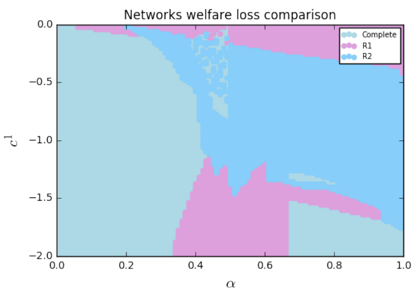


(b) Price contagion under fixed relative exposure strategy

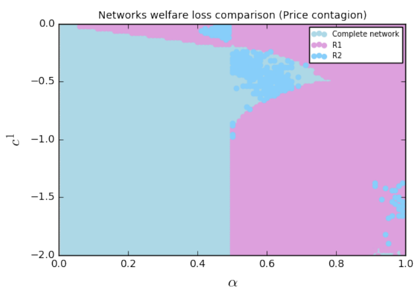


(c) Price contagion under liquidity-based allocation strategy

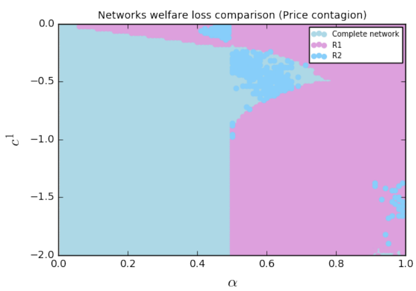
Figure B.8: **Comparison of minimum welfare loss under different situations**



(a) No price contagion



(b) Price contagion under fixed relative exposure strategy



(c) Price contagion under liquidity-based allocation strategy

Figure B.9: **Comparison of minimum welfare loss under different situations**