Notebook Screen Capture

Problem 1

The random variable w is defined in terms of the random variables x, y, and z, to be

$$\begin{aligned}\mathbf{w} &= \mathbf{x}\mathbf{y} + \mathbf{z}\\ &= \mathbf{v} + \mathbf{z}\end{aligned}$$

The input rv are assumed to be mutually independent, with $x\, U(-1,1)$, $y\, U(-1,1)$, and $z\, U(0,1)$.

Find the theoretical pdf . Start by first finding the pdf on $\mathbf{v} = \mathbf{x}\mathbf{y}$ using the fact that for independent $\mathbf{x}$ and $\mathbf{y}$,

$$\begin{aligned}f_v(v) &= \int_{-\infty}^{\infty} \frac{1}{w} f_x(w) f_y\Big(\frac{v}{w}\Big)\, dw\\ &= \int_{-\infty}^{\infty} \frac{1}{w} f_y(w) f_x\Big(\frac{v}{w}\Big)\, dw\end{aligned}$$

Then find the pdf on the sum $\mathbf{w} = \mathbf{v} + \mathbf{z}$ from a convolution. Note that the rv $\mathbf{v}$ and $\mathbf{z}$ are also independent. Why?

Theoretical Analysis

```python
def pdf_proj1_w(w):
    """
    fw = pdf_proj1_w(w)
    Function plot the pdf of w = x*y + z where x~U(-1,1), y~U(-1,1), and
    z~U(0,1).

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    """

    fw = zeros_like(w)

    for k, wk in enumerate(w):
        if wk >= -1 and wk <= 0:
            fw[k] = -1/2*(wk*log(-wk)-wk-1)
        elif wk > 0 and wk <= 1:
            fw[k] = 1/2*(1 + (wk-1)*log(1-wk) - wk*log(wk))
        elif wk > 1 and wk <= 2:
            fw[k] = 1/2*(2 - wk + (wk-1)*log(wk-1))
        else:
            fw[k] = 0
    return fw
```