ANISOTROPIC BLIND IMAGE RESTORATION

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ABSTRACT

Anisotropic diffusion is proposed as a technique to regularize joint blur identification and image restoration. In comparison with previously proposed space-adaptive regularization methods, it shares the feature of space-adaptive degree of regularization, but it has the unique feature of adapting its direction of regularization to the orientation of edges. Consequently, good restoration quality was observed with both numerically and photographically degraded images.

1. INTRODUCTION

Blind restoration of shift-invariantly blurred images has been a natural extension of linear image restoration due to known blurs (for example [1] and [2]). Most of these methods are based on parametric statistical formulations which use isotropic and shift-invariant models but which are limited to a small number of unknown model parameters. In a recent paper [3] a regularization based deterministic blind restoration method was presented which uses shift-adaptive degree of regularization for both the image and the blur with an isotropic functional form for the regularization. This approach was shown to be effective in restoring images degraded by a point spread function with moderate support. However, these as well as other previous techniques suffered from ringing artifacts due to the isotropic nature of the models and the regularization function.

The success of ringing artifact reduction in image restoration depends mainly on how restoration methods accommodate the piecewise smoothness of the image. This paper builds on the results presented in [4] to give an algorithm for blind restoration using a regularization method which adapts both its degree and orientation of regularization operation to the spatial activities of the image and the blur intensity, and edge orientations. This is achieved by anisotropic diffusion. The resulting restored images clearly show the combined benefits of blind restoration of moderate-sized blur support with significantly-reduced ringing artifacts.

This work was supported by the BMDO/IST program managed by the Office of Naval Research under Contract N00014-92-J-1911

2. FORMULATION

The degradation model for the image is represented by

$$\begin{aligned} g(x,y) &= \int_{\mathcal{D}} d(x,y;s,t)u(x-s,y-t)dsdt \\ &\quad +n(x,y), \quad (x,y)\in\Omega, \end{aligned} \tag{1}$$

where $u(x,y)$, $g(x,y)$, $d(x,y;s,t)$, and $n(x,y)$ represent the original image, observed image, blur operator or point spread function (PSF), and observation noise, respectively. Ω and $\mathcal{D}$ denote the supports of the observed image and the PSF, respectively. The convolution in (1) represents the mechanism that causes an observed image to be blurred. The task is to recover the original image $u(x,y)$ from the observed image $g(x,y)$, when the point spread function is *unknown.*

An anisotropic blind image restoration may be formulated in terms of minimizing the following functional:

$$\begin{aligned} L(\hat{u},\hat{d}) &= \frac{1}{2}\int_{\Omega} e^2(x,y)dxdy \\ &+ \lambda\int_{\Omega}\theta\left(|\nabla\hat{u}(x,y)|\right)dxdy \\ &+ \gamma\int_{\mathcal{D}}\beta\left(|\nabla\hat{d}(x,y)|\right)dxdy \end{aligned} \tag{2}$$

subject to the constraints of

$$\sum_{x\in\mathcal{D}}\hat{d}(x,y)=1, \tag{3}$$

$$\hat{d}(x,y)\geq 0, \quad (x,y)\in\mathcal{D} \tag{4}$$

$$0\leq \min \leq \hat{u}(x,y)\leq \max <\infty, \quad (x,Y)\in\Omega. \tag{5}$$

The $e(\cdot)$ in the first term of (2) is the restoration residual:

$$e(x,y)=g(x,y)-\int_{\mathcal{D}}\hat{d}(s,t)\hat{u}(x-s,y-t)dsdt. \tag{6}$$

which represents the fidelity of the estimate $\hat{u}(\cdot)$ to the 5 observation $g(\cdot)$. Direct minimization of the first term of (2) would lead to excessive noise magnification due to the ill-conditioning of the problem, so two piecewise smoothness constraints (second and third terms) are imposed. The $|\nabla \hat{u}(x,y)|$ in the second term of (2) represents the magnitude of the image gradient at (x,y) and the $\theta(\cdot)$ is an increasing function, so that the minimization of this term represents a decrease in the image gradient, that is, it is a smoothing operation. It is proved in [5, 4] that the minimization of this term leads to anisotropic diffusion, and that if the function $\theta(\cdot)$ is properly designed, the diffusion operation can progress in such a way that the image is smoothed only in the direction of edges and the directional smoothing operation is encouraged at large intensity transitions and discouraged in smooth areas. In addition, the degree of regularization is also adapted to local spatial activities. The minimization of $\beta|\nabla \hat{d}(x,y;s,t)|$ in the third term of (2) plays a similar role for the PSF. The λ and γ are the regularization parameters which control the tradeoff between fidelity to the observation and smoothness of the estimates $\hat{d}$ and $\hat{u}$.

3. ALTERNATING MINIMIZATION

Following arguments presented in [3, 6], the following alternating minimization procedure is used for the minimization of (2):

- Image descent step:
  Fix $\hat{d}(x)$, descend (2) with respect to $\hat{u}(x)$.
- PSF descent step:
  Fix $\hat{u}(x)$, descend (2) with respect to $\hat{d}(x)$.

Let us first consider the image descent step. Since the PSF is fixed for now, this step is nothing but image restoration with a known PSF. Under our current anisotropic regularization formation, this step is, therefore, the same as the method presented in [4] which basically follows the idea of steepest descent:

$$\begin{aligned} \frac{\partial \hat{u}}{\partial t} &= -\nabla_{\hat{u}} L(\hat{d},\hat{u}) \\ &= \int_{\Omega} e(s,t)\hat{d}(s-x,t-y)dsdt \\ &\quad +\lambda \operatorname{div}\left(\theta'(|\nabla \hat{u}|)\frac{\nabla \hat{u}}{|\nabla \hat{u}|}\right). \end{aligned} \tag{7}$$

The $\nabla_{\hat{u}} L(\hat{d},\hat{u})$ in (7) is the gradient of $L(\hat{d},\hat{u})$ with respect to $\hat{u}$ (while $\hat{d}$ is fixed) and the second term in the second equation of (7) is anisotropic diffusion which provides orientation-selective regularization to image restoration. [4, 6].

For the PSF descent step, a similar procedure is followed to give the following formula:

$$\begin{aligned} \frac{\partial \hat{d}}{\partial t} &= -\nabla_{\hat{d}} L(\hat{d},\hat{u}) \\ &= \int_{\Omega} e(s,t)\hat{u}(s-x,t-y)dsdt \\ &\quad +\gamma \operatorname{div}\left(\beta'(|\nabla \hat{d}|)\frac{\nabla \hat{d}}{|\nabla \hat{d}|}\right). \end{aligned} \tag{8}$$

where the $\nabla_{\hat{d}} L(\hat{d},\hat{u})$ is the gradient of $L(\hat{d},\hat{u})$ with respect to $\hat{d}$ (while $\hat{u}$ is fixed) and the second term in the second equation is anisotropic diffusion which provides orientation-selective regularization [4, 6] to blur identification.

4. NUMERICAL SIMULATION

Equations (7) and (8) can be implemented by Euler's forward difference scheme which is equivalent to steepest descent method. In our simulation, however, we discretize the cost function (2) and the gradients $\nabla_{\hat{u}} L(\hat{d},\hat{u})$ and $\nabla_{\hat{d}} L(\hat{d},\hat{u})$, and then employ partial conjugate gradient method to do the minimization. Within each of the image and the PSF descent steps, the conjugate gradient descents are stopped after 10 iterations. Following the derivation presented in [5, 6], the functions $\theta(\cdot)$ and $\beta(\cdot)$ are selected as

$$f(x) = \begin{cases} \frac{x^2}{2T}, & x < T, \\ x, & x \geq T \end{cases} \tag{9}$$

where T is called diffusion coefficient threshold.

Fig. 1 is the cameraman image degraded by a 1×10 linear motion blur and 30 dB (SNR) Gaussian noise. Fig. 2 (top) is the image restored by the space-adaptive algorithm in [7] assuming that the point spread function is *known*. Fig. 2 (middle) is the one by the proposed algorithm assuming that the point spread function is *unknown*. Fig. 2 (bottom) is the identified PSF. The regularization parameters for the image and the PSF are 0.1 and 10^5, respectively. The diffusion coefficient threshold for the image and the PSF are 0.1 and 0.01, respectively.

We now use the anisotropic regularization method to restore the photographically degraded image in Figure 3(top). Since we do not know the support of the PSF, so we run the algorithm assuming a support of 7×7 and 9×9, respectively. The restored images are shown in Fig. 3 (middle and bottom) and the identified PSF's are shown in Fig. 4, respectively. The regularization parameters for the image and the PSF are 0.03 and 10^4, respectively. The diffusion coefficient threshold for the image and the PSF are 0.1 and 10^{-4}, respectively.

Figure 1: Cameraman image as degraded by 1×10 linear motion blur and 30 dB Gaussian noise.

5. CONCLUSION

Anisotropic diffusion was proposed as a technique to regularize joint blur identification and image restoration. In comparison with previously proposed space-adaptive regularization methods, it shares the feature of space-adaptive degree of regularization, but it has the unique feature of adapting its direction of regularization to the orientation of edges. Consequently, good restoration quality was observed with both numerically and photographically degraded images. The proposed method suffers from the problems of local minima and non-uniqueness of solution which are associated with other blind deconvolution methods.

6. REFERENCES

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Figure 2: Top: image restored by a well-known space-adaptive algorithm with *known* PSF. Middle: image restored by the proposed blind algorithm. Bottom: identified PSF (dashed line) in comparison with the real PSF (solid line).

Figure 3: Top: Photographically degraded image. Middle: Restored image when the support is assumed to be 7×7. Bottom: Restored image when the support is assumed to be 9×9.

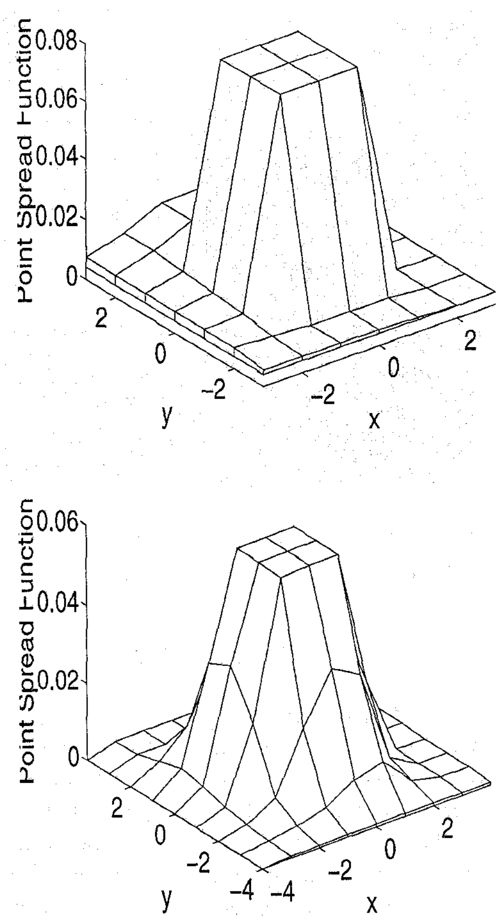

Figure 4: Identified PSF when the support is assumed to be 7×7 (top) and 9×9 (bottom).

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