Problem Chosen	2022	Team Control Number
A	MCM/ICM Summary Sheet	XJ162

Finless Porpoise PVA based on Vortex, ARIMA, GF and CA

Summary

In this paper, we predict the population size of the endangered species, Yangtze Finless Porpoise, based on mathematical models. In the first place, we apply **Vortex model** to calculate the estimated number of population in Swan Oxbow and in all ex situ conservation areas. Then, we predict the size of population after 20 years by developing **ARIMA model, Gray Forecast model and Cellular Automata model**. With other conditions unchanged, the influence of sex ratio over Finless Porpoises in all ex situ protected areas is assessed by using Vortex model, which is measured by population size and genetic diversity. In order to estimate whether the functional extinction and species extinction will occur, we define two kinds of functional extinction, one with only one sex left and the other with less than **200** individuals surviving, whose time of occurrence is predicted by Vortex model and Gray Forecast model. Finally, we test our model's sensitivity by studying the effects of mortality rates over Finless Porpoise population.

For Problem 1 (a), we use Vortex model to predict the number of Finless Porpoise population 20 years later in Swan Oxbow, which is **50**, and in all five ex situ conservation areas, which is **185**. Then we establish ARIMA model, Gray Forecast model and Cellular model to predict the number, which is **62, 49 and 49**.

For Problem 1 (b), we use Vortex model to calculate the size and the genetic diversity, which is defined as the normalization of **GeneDiv** and **nAlleles** and adjust the male-female **sex ratio from 1:9 to 9:1** to evaluate the impact of sex ratio on Finless Porpoise population. It can be discovered that with male proportion growing, the number of population shrinks gradually, and the genetic diversity increases at first, then decreases, approaching its peak when the ratio is **6:4**.

For Problem 2, we define functional extinction as population with less than **200** individuals or with **only one sex**. We assess when the functional extinction occurs based on Gray Forecast model and Vortex model. Based on previous data, Gray Forecast model predicts the number every five years, and finds that the population will become less than 200 in **2082**. In other words, it will functionally extinct after **55 to 60 years**. Because of the deficiency of ex situ conservation, we change the sex ratio of Vortex model, the catastrophe probability and mortality rate. It is predicted by Vortex model that the Finless Porpoises will functionally extinct after **55.1 years or 71.3 years**, and will completely extinct after **95.5 years**. Based on what has been discussed above, we can draw a conclusion that without ex situ conservation, the Finless Porpoises will functionally extinct after **55-71.3 years**.

Finally, sensitivity test is carried out based on Vortex model by setting additional catastrophes or rising the early mortality rates, and the test is measured by population size and the extinction. With additional catastrophes set, the Finless Porpoises will extinct after **45.3 years**. With the early mortality rates risen, the extinction will occur after **34.8 years**. Thus, both circumstances promotes the probability of extinction and brings the extinction forward. What's more, it's found that the increase of early mortality rates influence the development of the population the most.

Keywords: Vortex model, ARIMA time series model, Gray Forecast model, Cellular Automata model, Finless Porpoises in Yangtze River, Population viability analysis.

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1 Introduction

1.1 Problem Restatement

Finless Porpoise is the only freshwater mammal in the Yangtze River at present, which is distributed in the middle and lower reaches of the Yangtze River, Dongting Lake and Poyang Lake, and its population has decreased dramatically in the past 20 years. According to the statistics, the number of Finless Porpoises in the Yangtze River was more than 2,700 in 1991. However, in the year of 2006, there were fewer than 1,800 Finless Porpoises surviving in the area. In 2011, there were probably just over 1,000 of them, and in 2018 there were about 1,012.

In fact, since the 1980s, the ecologists along with the government had explored and developed three conservation strategies: in situ conservation, ex situ conservation and artificial breeding. Among them, ex situ protection, that is, selecting some waters with similar ecological environment to the Yangtze River to establish ex situ protection, is the most direct and effective measure to protect the Yangtze Finless Porpoise.

China has set up five ex-situ protected sites until now, in which more than 150 Yangtze Finless Porpoises are conserved. On September 18, 2021, CCTV reported that the population of the Yangtze Finless Porpoise is growing steadily. The population decline of the Yangtze Finless Porpoise has been curbed, but its critically endangered status remains unchanged.

Based on what has been discussed above, please address the following problems:

1 (a) Establish a mathematical model to predict the population number of Finless Porpoises in five ex situ protected areas after 20 years.
  (b) Explain how the sex ratio of 150 Finless Porpoises in ex situ protected areas affects the population development of Finless Porpoises.

2 Will the Yangtze Finless Porpoise become functionally extinct without ex situ conservation strategies?

3 Based on your analysis, please submit no more than 2 pages of recommendations for the protection of Finless Porpoises to the relevant authorities.

1.2 Overview of Our Work

We develop 4 kinds of models to fully assess the trend in terms of annual change of finless population size in Swan Oxbow, which are furthermore applied on all five ex-situ protected areas to predict the change of population.

Besides, Vortex model, as the most comprehensive and authoritative model widely used in studying small-size populations of creatures, is the core for analyzing the viability of finless porpoise population from different perspectives, especially when the discussion of sex ratio is involved.

For more clear and general information of our work, please see Figure 1.1.

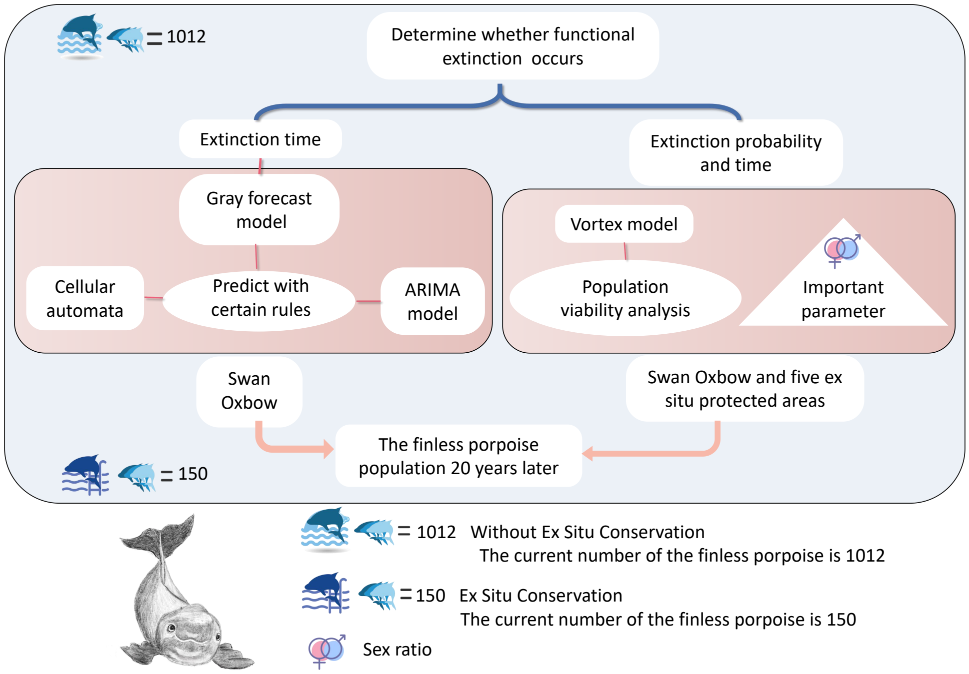


Figure 1.1: Frame Diagram of our work

2 Assumptions and Justifications

These are necessary assumptions for simplifying the model.

1. The initial carrying capacity per unit area of each ex-situ conservation layout is constant;
2. The carrying capacity of Finless Porpoise in Yangtze River is 3000, and there is no large-scale migration of Finless Porpoises in adjacent monitored river segments within a certain period of time(**?**);
3. Assume that carrying capacities for Finless Porpoises decline 10% every year;
4. Under natural conditions, it is assumed that there are two natural disasters affecting the population of finless porpoise, namely, the blasting operation in the Yangtze River channel and the capsizing accident of the ship carrying pesticides in the Yangtze River, which happen once every 10 years. According to the field observation, the reproductive rate and survival rate of the population of Finless Porpoise decreased to 95 percent after the disaster (**?**);
5. Under ex situ conservation conditions, it is assumed that there are following advantages in the reserve environment, and the mortality rate of each age of Finless Porpoise population is greatly reduced, and only one disaster exists;
6. Suppose the mortality rates of male and female Finless Porpoises are significantly different;
7. Assume that no artificial harvest and supplement;
8. Assume that the reproductive system of the Finless Porpoise is polygynous;
9. Assume all Finless Porpoises in Yangtze River belongs to the same population.

3 Notations

Table 3.1: Notation Descriptions(**?**)

Symbol	Definition
r	Innate rate of increase
Stoch r	The mean population growth rate experienced in the simulations, averaged across all years in which the population was extant.
N-extant	Average extant population size
N-all	Average population size
PE	Probability of Extinction
GeneDiv	Genetic Diversity
TE	Time of Extinction(year)
medianTE	If at least 50% of the iterations went extinct, the median time to extinction
SD	Standard Deviation
nAlleles	The mean number of alleles remaining within extant populations (from an original number equal to twice the number of founder individuals)
K	Carrying capacity
N_t	Size of Finless Porpoise population in the year of $1991 + t$

4 Introduction and Results of Models on Problem 1(a)

Considering tremendous cost on massive Finless Porpoise population census, merely six years of data was collected in Swan Oxbow of the Yangtse River during the three decades since 1992. (**?**) Thus, we've applied **Lagrange interpolation** to obtain other years' data in Swan Oxbow.

4.1 Relation between Population Size and Time based on Lagrange Interpolation

Given n distinct real values $x_1, x_2, \cdots, x_n$and n real values $y_1, y_2, \cdots, y_n$ (not necessarily distinct), there is a unique polynomial P with real coefficients satisfying $P(x_i) = y_i$ for $i \in \{1, 2, \cdots, n\}$, such that deg($P$)<n.

The polynomial $P(x)$ is defined as follows:

$$P(x) = \sum_{k=1}^{n} y_k p_k(x), \quad p_k(x) = \frac{(x - x_1) \cdots (x - x_{k-1})(x - x_{k+1}) \cdots (x - x_n)}{(x_k - x_1) \cdots (x_k - x_{k-1})(x_k - x_{k+1}) \cdots (x_k - x_n)}$$

After substituted the number in 1992, 2002, 2005, 2007, 2015 and 2021, the figure of the polynomial is as follows:

According to Lagrange interpolation and the definition of N_t :

$$N_t = P(t) \quad t = 1, \cdots, 30$$

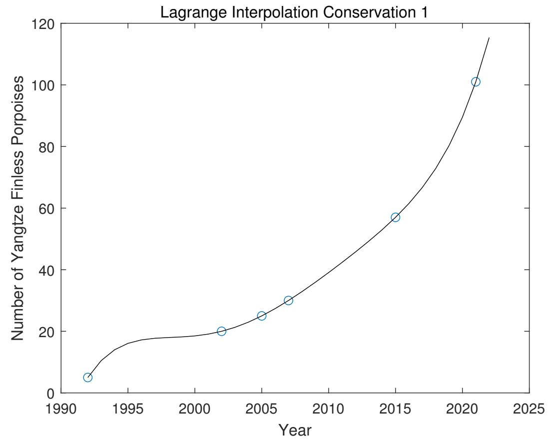


Figure 4.1: Langrange Interpolation Conservation

and the exact numbers are listed below:

Table 4.1: Estimated size of population on year basis from 1992 to 2022

Year	Number	Year	Number	Year	Number
1992	5	2002	20	2012	45.7364
1993	10.4864	2003	21.2919	2013	49.2555
1994	13.9987	2004	22.9647	2014	52.9737
1995	16.0839	2005	25	2015	57
1996	17.2014	2006	27.3612	2016	61.4941
1997	17.7296	2007	30	2017	66.6734
1998	17.9732	2008	32.8636	2018	72.82
1999	18.1701	2009	35.9015	2019	80.2876
2000	18.4981	2010	39.0724	2020	89.5084
2001	19.0819	2011	42.3514	2021	101

4.2 Model I: Vortex model based Finless Porpoise analysis

Primary functional principles of Vortex model are shown in the form of pseudocode Algorithm 1.

Algorithm 1 Vortex model based Finless Porpoise Population Size Prediction Algorithm

Input: features of Finless Porpoises described in table 6.1

for scenario $\leftarrow 1$ to m **do**
  if NumberOfPopulations ≥ 1 **then**
    READ POPULATION and MIGRATION
    for each population, pSource $\leftarrow 1$ to n **do**
      $BreedEV[P] = \sqrt{BreedEV[P]^2 - BreedEV[P] \times EVConcordance}$
      Calculate the parameters $\leftarrow$ LOCAL_EV, CATASTROPHES, MIGRATE, BREED, MORTALITY(p)
    end for
    for each Year $\leftarrow 1$ to l **do**
      $SE = \sqrt{PE \times (1 - PE)}$
      if population extincted **then**
        $YearRecolonized[P] = CurrentYear$
      else
        $TimeToRecolonization[p] = CurrentYear - YearExtinct[p]$
      end if
      if not extinct $\&\&PopulationSize[p], N > CarryingCapacity[p]$ **then**
        for each indiividual **do**
          if $RAND() > \frac{K}{N}$ **then**
            Extinct
          else
            if $N > K$ **then**
              $YearExtinct[p] = CurrentYear$
            end if
            if others **then**
              $TimeToReextinction[p] = CurrrentYear - Recolonization[p]$
            end if
          end if
        end for
      end if
    end for
  end if
end for

Output: The Number of iterated population in each year

By inputting pre-set parameters (see Table 6.1), carrying capacity $K = 101$ and present population size $N_0 = 82$ (both numbers come from news in People's Daily, and 19 Finless Porpoises were moved from Swan Oxbow to other ex situ conservation areas in 2021) in Swan Oxbow, we learn from Vortex model that the population size reaches its peak of **88.84**, and than declines and stabilizes at about **50**.

According to our assumptions 2, we assume the carrying capacity for five ex-situ protected areas $K = 369$. Vortex shows that the number roars to **241.26** in the forth year, and then settles around **185**.

4.3 Model II: Auto Regressive Integrated Moving Average(ARIMA) model

We apply ARIMA model in order to predict the size of Finless Porpoise population in five ex-situ conservation areas after 20 years, whose **algorithm block diagram** is shown as Figure 4.2.

Under regular circumstances, the time series we obtain in the real world has tendency, seasonality and non-stationarity. Thus, it's vital for us to transfer the non-stationary time series to stationary time series and make an assumption that the time series is an Auto Regressive Moving Average(ARMA) series to predict the future data. ARMA series is defined as follows.

$$X_t - \phi_1 X_{t-1} - \cdots - \phi_p X_{t-p} \\ = \epsilon_t - \theta_1 \epsilon_{t-1} - \cdots - \theta_q \epsilon_{t-q}$$

ϵ_1 is a stationary white noise whose average is zero and deviation is σ_ϵ^2;$X_t$is an ARMA series with p and q degree, recorded briefly as $\mathbf{ARMA}(p, q)$ series. Akaike Information Criterion(AIC) is one of the most commonly used criterion to determine the degree of $\mathbf{ARMA}(p, q)$: choose p, q such that

$$\min \mathbf{AIC} = n \ln \hat{\sigma_\epsilon^2} \\ + 2(p + q + 1) \tag{4.1}$$

n is the capacity of sample;$\hat{\sigma_\epsilon^2}$ is the estimation of σ_ϵ^2 relating to p and q. Suppose $p = \hat{p}, q = \hat{q}$, such that equation(4.1) reaches the minimum, than we deem the series is $\mathbf{ARMA}(\hat{p}, \hat{q})$.

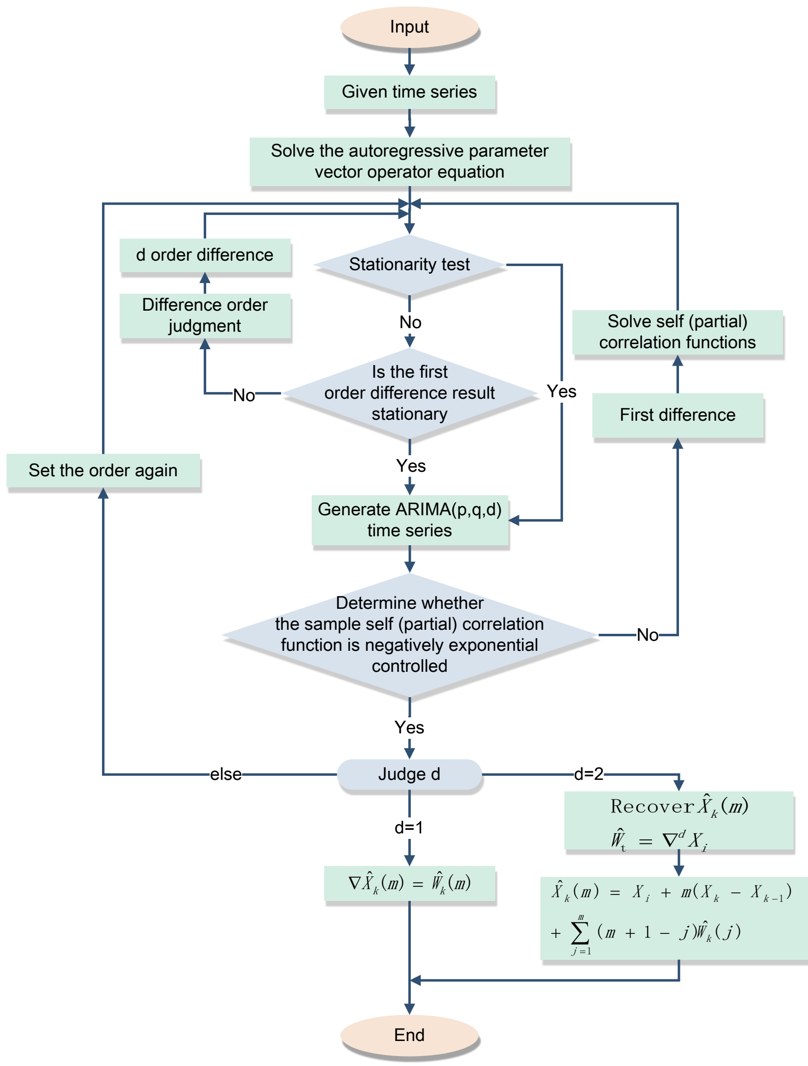


Figure 4.2: Algorithm Block of ARIMA

Suppose $\mathbf{ARMA}(p, q)$ series has an unknown average parameter μ, the model becomes

$$\phi(B)(X_y - \mu) = \theta(B)\epsilon_t,$$

meanwhile, the number of unknown parameters is $k = p + q + 2$, the AIC is: choose p, q such that

$$\min \mathbf{AIC} = n \ln \hat{\sigma_\epsilon^2} + 2(p + q + 2). \tag{4.2}$$

In fact, equations (4.1) and (4.2) have the same minimum point $\hat{p}, \hat{q}$. After that, we usually choose $p = 1, q = 1$ to make parameter estimation over ARMA model.

It's demonstrated that the differential operation can stabilize certain class of non-stationary series. And It's emphasized that stationary test must be conducted previously. Stationary test can be applied by calculating sample autocorrelation function and sample coefficient of partial function.

If the functions are truncated or trending to 0 (meaning being controlled by negative index), than the series belongs to ARMA model.

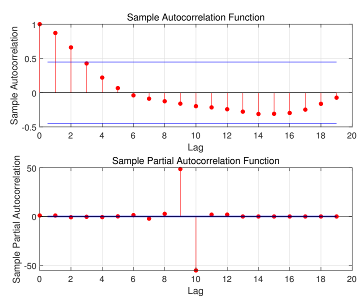
If at least one of the functions above is not truncated or trending to 0, than it's not stationary.

Suppose the series is non-stationary, which can be transformed to a stationary series by d -degree differential operation, denoted as $\mathbf{ARIMA}(p, q, d)$ series,than differentiate the sample by d -degree:

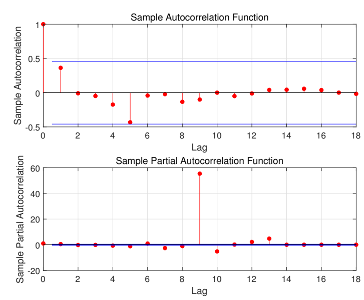
$$W_t = \nabla^d X_t, \quad t = d+1, \ldots, n$$

After that, apply stationary test on W_t and repeat steps above until it becomes a stationary series, Than W_t(which is denoted as X_t) complies ARMA model.

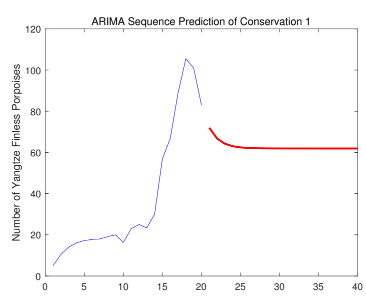
The figures below describe the result of ARIMA model on the time series of the size of Finless Porpoise $N_t, \quad t = 1 \cdots 30$, in which the Figure(4.3(c)) clearly shows that the number will decline and settle around **62** in the future 20 years.



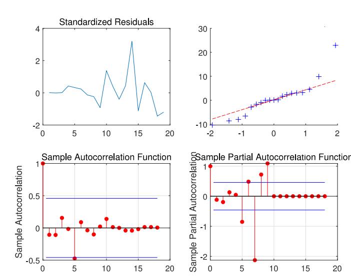
(a) Sample Autocorrelation Function and Sample Partial Autocorrelation Function



(b) Sample Autocorrelation Function and Sample Partial Autocorrelation Function



(c) ARIMA Sequence Prediction of Conservation



(d) Standardaized Residuals and QQ figure

Figure 4.3: Results of ARIMA model

4.4 Model III: Cellular Automata based Population Size Prediction

Cellular automata (CA) is a kind of grid dynamics model with discrete time, space and state, and local spatial interaction and temporal causality, which has the ability to simulate the space-time evolution process of complex system.

Unlike general dynamical models, cellular automata are not determined by strictly defined physical equations or functions, but are composed of rules constructed by a series of models. Any model that satisfies these rules can be regarded as a cellular automata model. Therefore, cellular automata is a general term for a class of models, or a method framework. Its characteristic is that time, space and state are discrete, each variable only takes a finite number of states, and its state change rules are local in time and space.

The block diagram of a typical cellular automata is shown in Figure 4.4, and the simulation result is shown in Figure 4.5, in which we learn that the predicted number of population in Swan Oxbow is **49**.

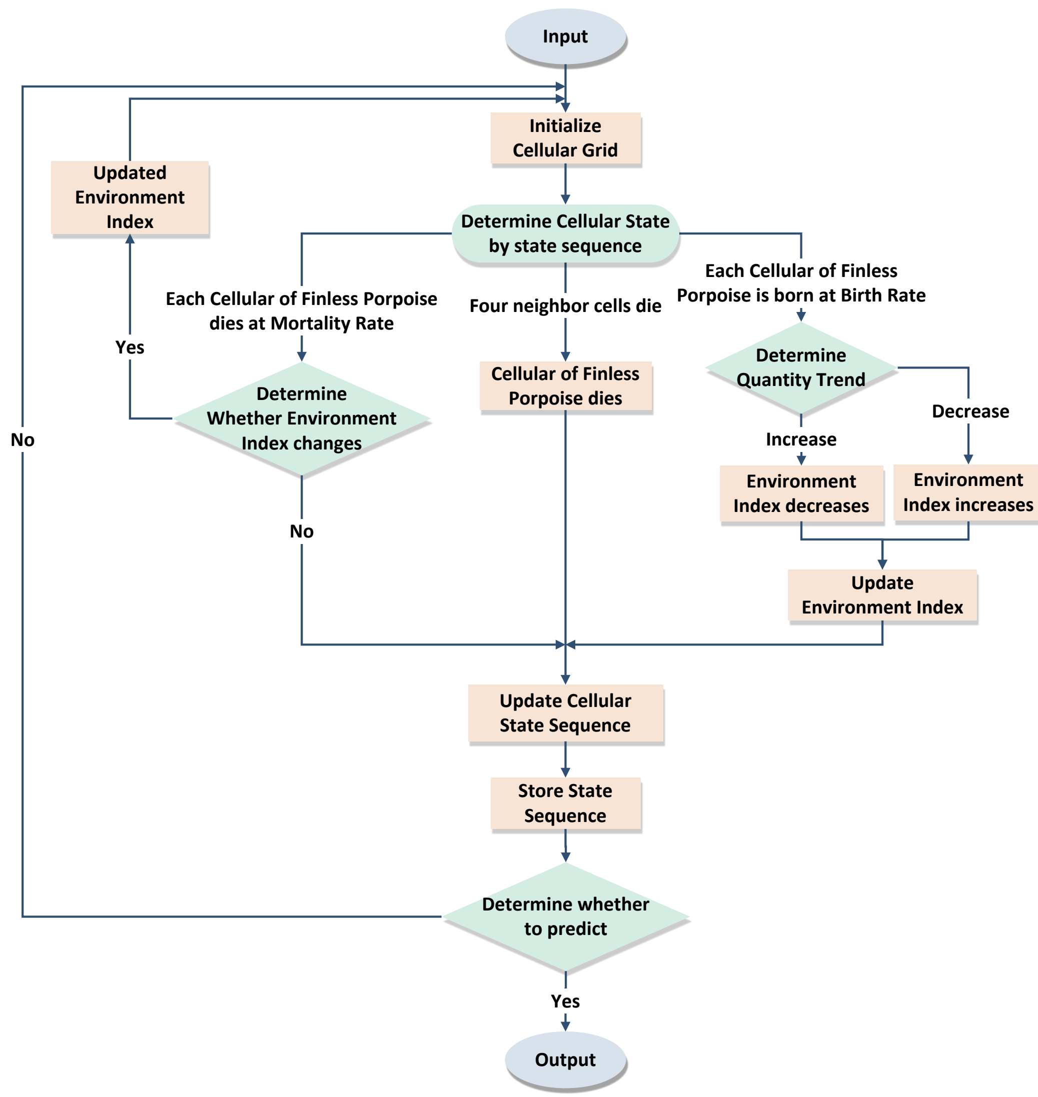
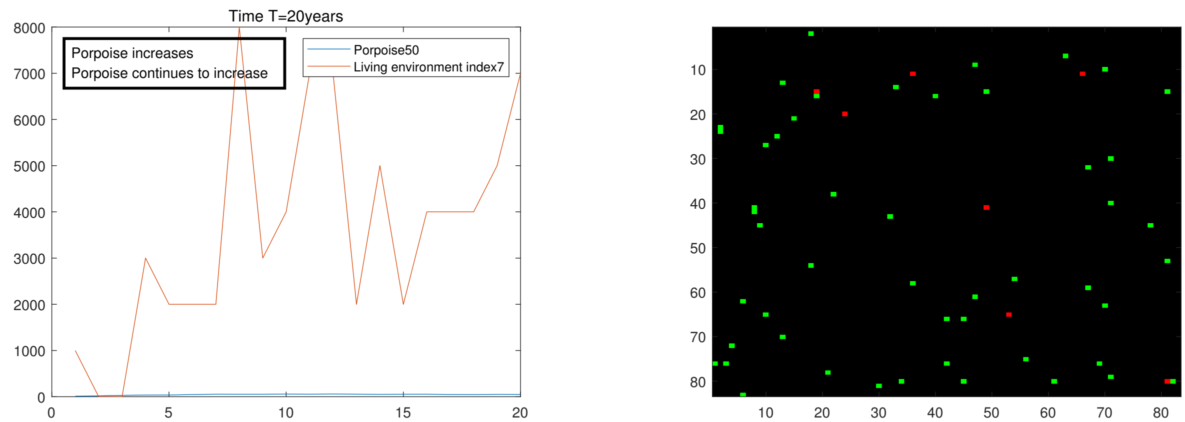


Figure 4.4: Block Diagram of typical Cellular Automata



(a) Living Environment Index and Population (b) Cell Distribution

Figure 4.5: Results of CA Simulating Population in Swan Oxbow

4.5 Model IV: Gray Forecast model

The very heart of Gray Forecast is Gray model, which is used to model and forecast the approximate exponential law after summing up the original data. It is suitable for the prediction scenario with less data. While **Vehulst** model is primarily used to describe procedures with saturation state, that is S-shape procedure, which can be applied in the sphere of biological growth and reproductive prediction. The fundamental is as follows.

Suppose the sample is $x^{(0)} = \{x^{(0)}(1), x^{(0)}(2) \cdots x^{(0)}(n)\}$, and it is accumulated once to generate a (1-AGO) sequence, which is:

$$x^{(1)} = (x^{(1)}(1), x^{(1)}(2), \cdots, x^{(1)}(n))$$

$z^{(1)}$ is the mean-generating sequence of $X^{(1)}$, which is:

$$z^{(1)} = (z^{(1)}(2), z^{(1)}(3), \cdots, z^{(1)}(n)).$$

Then, we call the following equations Gray Verhulst model with parameters a and b:

$$x^{(0)} + az^{(1)} = b(x^{(1)})^2$$

, and we call the following equations the winterization equations of of Gray Verhulst model with t denoting time:

$$\frac{\mathbf{d}x^{(1)}}{\mathbf{d}t} + ax^{(1)} = b(x^{(1)})^2$$

Theorem 1. Suppose the Gray Verhulst model is depicted as above, if $\boldsymbol{u} = [a, b]^T$ is the parameter vector,and

$$\boldsymbol{B} = \begin{bmatrix} -z^{(1)}(2) & (z^{(1)}(2))^2 \\ -z^{(1)}(3) & (z^{(1)}(3))^2 \\ \vdots & \vdots \\ -z^{(1)}(n) & (z^{(1)}(n))^2 \end{bmatrix}, \boldsymbol{Y} = \begin{pmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{pmatrix},$$

then the least square estimation of parameter $\boldsymbol{u}$ satisfies

$$\hat{\boldsymbol{u}} = [\hat{a}, \hat{b}]^T = (\boldsymbol{B}^T \boldsymbol{B})^{-1} \boldsymbol{B}^T \boldsymbol{Y}$$

Theorem 2 Suppose Gray Verhulst model is defined as above, then solution to the winterization equations is

$$x^{(1)}(t) = \frac{\hat{a}x^{(0)}(1)}{\hat{b}x^{(0)}(1) + [\hat{a} - \hat{b}x^{(0)}(1)]e^{\hat{a}t}},$$

and the time response sequence of Gray Verhulst model is

$$\hat{x}^{(1)}(k+1) = \frac{\hat{a}x^{(0)}(1)}{\hat{b}x^{(0)}(1) + [\hat{a} - \hat{b}x^{(0)}(1)]e^{\hat{a}t}},$$

and the reductive formula is

$$\hat{x}^{(0)}(k+1) = \hat{x}^{(1)}(k+1) - \hat{x}^{(1)}(k)$$

After developing basic rules of Gray Forecast model, we deploy it on the analysis of the Finless Porpoise size in future 20 years.

Suppose the number series of Finless Porpoise is

$$N^{(0)} = (N^{(0)}(1), N^{(0)}(2), \cdots, N^{(0)}(n)).$$

Then we accumulate it and generate Verhulst series:

$$N^{(1)} = (N^{(1)}(1), N^{(1)}(2), \cdots, N^{(1)}(n)),$$

Thus, we can get Verhulst differential equations:

$$\frac{\mathbf{d}N^{(1)}}{\mathbf{d}t} + aN^{(1)} = u$$

, in which a and u is defined as above. Further more, we could solve the following equations to get the answer:

$$\boldsymbol{B} = \begin{bmatrix} -\frac{1}{1}[N^{(1)}(1) + N^{(1)}(2)] \\ -\frac{1}{2}[N^{(1)}(1) + N^{(1)}(3)] \\ \vdots \\ -\frac{1}{2}[N^{(1)}(n-1) + N^{(1)}(n)] \end{bmatrix}, \boldsymbol{Y_n} = \begin{pmatrix} N^{(0)}(2) \\ N^{(0)}(3) \\ \vdots \\ N^{(0)}(n) \end{pmatrix},$$

$$N^{(1)}(\hat{k+1}) = [N^{(0)}(T) - \frac{\hat{u}}{\hat{a}}]e^{-\hat{a}k} + \frac{\hat{u}}{\hat{a}} \quad k = 0, 1, \cdots, n$$

After iterating based on Table 4.1, Gray Forecast model comes with a prediction of **49.308** after 20 years in Swan Oxbow.

5 Solution to Problem 1(b) based on Vortex model

Considering the abundant parameter settings in Vortex software among which sex ratio is easy to edit and is of great significance, we decide to further apply Vortex model in order to address the problem 1(b) concerning the effects of sex ratio in five ex-situ conservation zones.

In order to more comprehensively demonstrate the impact of sex ratio, we compare the surviving condition 20 years later with male-female ratio ranging from $1:9$ to $9:1$. We regard the number of population as one important factor, and the Genetic Diversity, which is defined as **the normalization of** $\boldsymbol{GeneDiv + nAlleles}$, as the other. The result is shown in Figure 5.1.

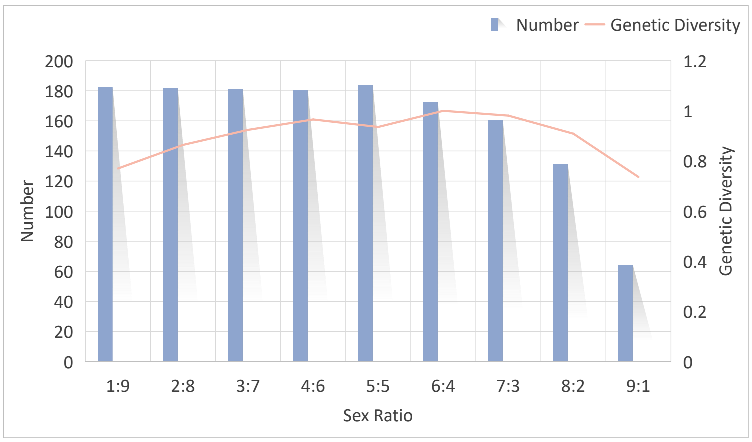


Figure 5.1: Influence of Sex Ratio over Population Size

From the Figure 5.1 could draw a very reasonable conclusion that the larger the male-female ratio is, the more damage it will cause to the survival of whole population, which corresponds with Finless Porpoise's polygyny.

6 Vortex model and Gray Forecast model based Population Prediction without Ex-Situ Conservation

The known **existing wild Finless Porpoise population is 1012** (**?**). According to previous researches, female Finless Porpoises are more vulnerable than the male ones in their early ages, so the ratio of male Finless Porpoises to female ones which are 0-6 years old is $3.7:1$ (**?**), and the ratio of those older than 6 is $2:1$. For simplicity, we perceive that **the ratio of male Finless Porpoises to female ones in all ages** is $\mathbf{2.85:1}$, which is the median of the above two ratios. What's more, we assume that **the carrying capacity** K of Finless Porpoises in the whole Yangtze River is **3000**.

In order to thoroughly discuss the risk of Finless Porpoise extinction, we define two types of **functional extinction** and one type of **species extinction**.

The first type of functional extinction (**Functional Extinction I**) is that there only exists one sex, which means this population is no longer able to reproduce and bond to be extinct in the future.

The second type of functional extinction (**Functional Extinction II**) is that the number of remaining individuals in the population is unable to prevent inbreeding depression which will cause the fading and extinction of the population. We define the number of population that reaches the second functional extinction is **200**.

The **Species Extinction** is defined as no living individual of this population found in this area.

6.1 Result of Vortex model

The result of Vortex model simulation based on previous conditions is shown as Table 6.1, in which the meaning of these parameters is shown in Table 3.1.

Scenario	Species Extinction	Functional Extinction I	Functional Extinction II
nRuns	30	30	30
stoch-r	-0.0239	-0.0165	-0.0120
SD(r)	0.1399	0.1340	0.1359
PE	0.0667	0.1000	0.4667
N-extant	256.00	340.00	427.25
SD(N-ext)	259.57	257.31	214.72
N-all	238.93	306.03	268.37
SD(N-all)	258.75	264.76	236.37
GeneDiv	0.8852	0.9300	0.9621
SD(GD)	0.1257	0.0321	0.0132
nAlleles	31.71	35.59	51.56
SD(nA)	20.83	19.06	16.43
medianTE	0	0	80
meanTE	95.5	71.3	55.1

In a nut shell, from the **meanTE** value in Table 6.1 we can draw a reasonable conclusion that without ex-situ conservation actions, the Finless Porpoises in Yangtze River will suffer **Functional Extinction II after 55.1 years, Functional Extinction II after 71.3 years and Species Extinction after 95.5 years.**

What's shown in Figure 9 describes the variation trends in these scenarios.

6.2 Result of Gray Forecast model

Given the fact that the Gray Forecast model is unable to predict the date of Functional Extinction I, we deploy the model to measure Functional Extinction II.

The Figure 6.2 has explicitly shown that it will take the Finless Porpoise population 55 to 60 years to reach Functional Extinction II, which is unbelievably similar to the conclusion of Vortex model from section 6.1.

Table 6.1: Values inputted into Vortex(**?**)

Times simulated	1000times
Years simulated	100a
Reporting interval	10a
Populations simulated	1
Inbreeding depression (Y/N)	Y
Heterosis or Lethal	H
Lethal equivalents	3.14
EV correlation between reproduction and survival	0.5
EV correlation among populations	0.5
Types of catastrophe	2
Monogamous, Polygynous or Hermaphroditic	P
Female breeding age	4a
Male breeding age	5a
Maximum breeding age	15a
Maximum litter size	1
$P(0)$ (the percent of adult females that breed at low densities when there is no Allee effect)	70%
$P(K)$ (the percent that breed when the population is at carrying capacity)	25%
B	2
A	2
Percent litter size 1	100%
Moralities in different ages	see Table
Catastrophes and influence	see Table
Percent males in breeding pool	70%
Start at stable age distribution (Y/N)?	Y
Trend in K (Y/N) ?	Y
Years of trend	5a

Years of trend	5a
Percent age in K	-10%
Harvest (Y/N)?	N
Supplement (Y/N)?	N
Sex ratio (proportion males) at birth[1]	0.5
Density dependent breeding[2] (Y/N)?	Y

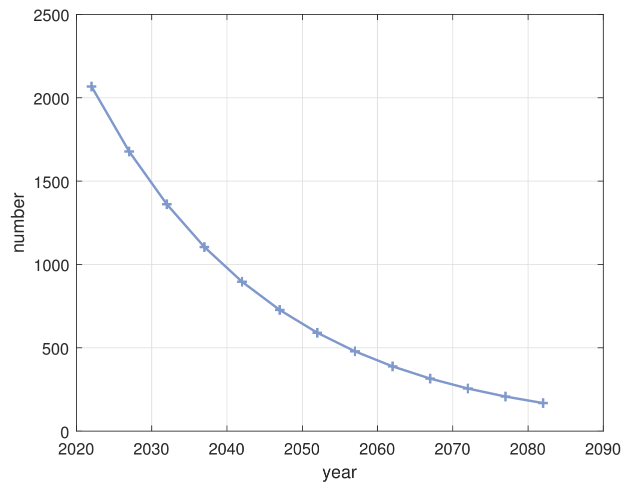


Figure 6.1: Predicted Population Size based on Gray Forecast

7 Sensitivity Test

In order to implement sensitivity test, we make two basic assumptions and study whether the result coincides with our assumptions.

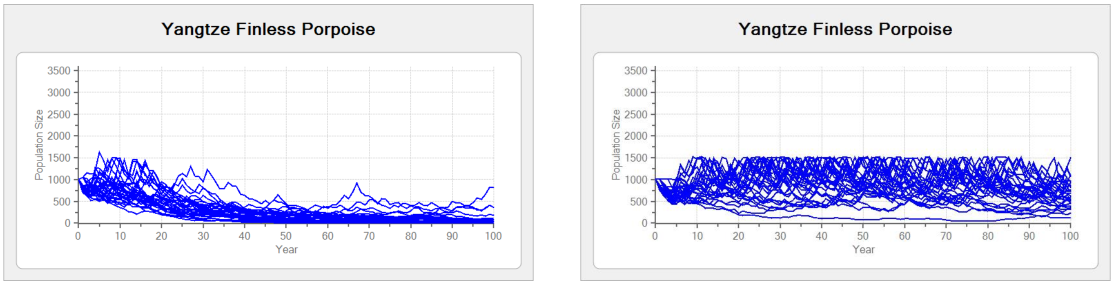
First assumption is that under the influence of human activities, the environment of the Yangtze River will deteriorate further in the future, and the factors threatening the survival of the Finless Porpoise will develop further, which will lead to the increase of the mortality rate of the young population of the Finless Porpoise. More precisely, 30% of Finless Porpoises in their 0 to 2 years old would die due to Environmental Variations.

The second assumption is that as the environment of the Yangtze river deteriorates, there is a possibility that the habitat of Finless Porpoises will shrink further and that factors that were previously unlikely to be a disaster, such as an epidemic outbreak, will become a disaster. Suppose that the probability of an epidemic is 10%, that 10% of individuals will die from the disease, and that 10% of individuals will be severely affected in reproduction.

[1]Modifiable when discussing different scenarios.

[2]$P(N)$ is the percent of females the breed when the population size is N, which can be difined as $P(N) = P(0) - [P(0) - P(K)(\frac{N}{K})^B]\frac{N}{N+A}$

Under assumptions described above, we apply Vortex model to see whether the extinction probability of the Finless Porpoise will be greatly increased.



(a) Number of Finless Porpoise population in higher early death rate

(b) Number of Finless Porpoise population in epidemic

Figure 7.1: Sensitivity Test

From Figure 7.1 and the result of Vortex model, we can see clearly that for the first assumption, which consists of 30% early death, it will take the Finless Porpoises **34.8** years to functionally extinct (type II), which is significantly shorter than the natural situation of 55.1 years; for the second scenario, the functional extinction comes after **45.3** years, which is also less than the natural situation.

8 Evaluation of Model

Strength:

1. With four predictive models established, we predict and analyze the Finless Porpoise population detailedly and comprehensively, which not only can form a contrast, but also verify each other, strengthening the reliability of our result;
2. We apply Vortex model to predict the development of finless porpoise population, which endows us with more professional and authoritative results;
3. Considering the complicated environment faced by Finless Porpoises, we assume various survival circumstances, which makes our models more reasonable;
4. Carrying capacity we assume changes along with time, which is correspond with reality, making our models more practical.

Weakness:

1. Some parameter settings refer to past papers and data, which may be dramatically different from the current living environment of Finless Porpoise, thus it may lead to certain unreliable results.

9 Conclusions

1. (a) According to Vortex model, there will be 50 Finless Porpoises in the population of Swan Oxbow after 20 years, and 185 in all five ex situ conservation areas;

   (b) According to ARIMA model, there will be 62 Finless Porpoises in the population of Swan Oxbow after 20 years;
   (c) According to Cellular Automata model, there will be 49 Finless Porpoises in the population of Swan Oxbow after 20 years;
   (d) According to Gray Forecast model, there will be 49.308 Finless Porpoises in the population of Swan Oxbow after 20 years;

2. According to Vortex model, the higher the male-female ratio is, the less the population size will remain;

3. (a) According to Vortex model, without ex situ conservation, the Finless Porpoises will functionally extinct (II) after 55.1 years, functionally extinct (I) after 71.3 years, and species extinct after 95.5 years;
   (b) According to Gray Forecast model, without ex situ conservation, the Finless Porpoises will functionally extinct (II) after 55 to 60 years.

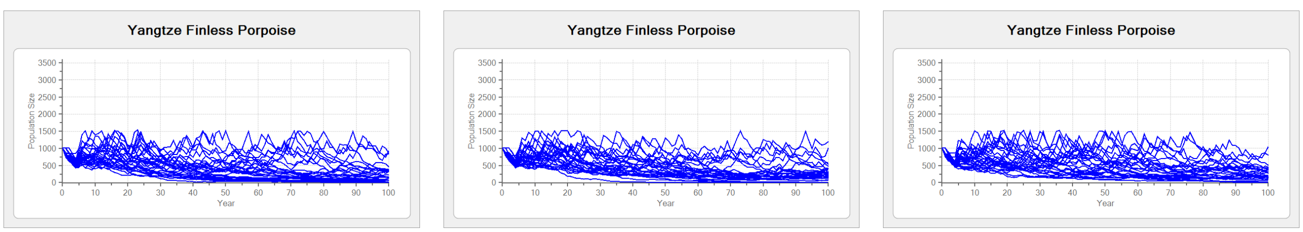
Policy Advice on Finless Porpoise Conservation

To: Relavant Authorities

From: MCM/ICM team XJ162

Date: January 16, 2022

We established four kinds of prediction models to predict the population development of Yangtze Finless Porpoise under ex situ protection and non-ex situ protection, and the advantages and disadvantages under the two circumstances were analyzed. On this basis, the following suggestions for the protection of Finless Porpoise were put forward to relevant departments. About in situ conservation, we found the following findings from the Vortex model. Without ex situ conservation, the Finless Porpoise will be functionally and completely extinct in 55 71.3 years and 95.5 years. The predicted results are as follows:



(a) Variation trend in terms of Species Extinction (b) Variation trend in terms of Functional Extinction I (c) Variation trend in terms of Functional Extinction II

Figure 9.1: Variation Trends in Different Scenarios

According to the Vortex model, this is due to two main reasons :

(1) the mortality of Finless Porpoises increases due to the increased risk of catastrophe without ex situ conservation;

(2) Under natural conditions, the mortality rate of male and female Finless Porpoises is obviously different, so the male-female ratio is usually higher than 1:1, which will not only lead to a decline in population size, but also damage genetic diversity. As a result, the population of Finless Porpoises slowly declines until they become extinct.

In view of the above analysis, we propose the following suggestions:

(1) The disturbance caused by human activities to animals should be reduced and eliminated, such as limiting ship speed, eliminating pollutant discharge and banning illegal fishing. Strict enforcement of illegal fishing should be practiced and rectification of frequent and disorderly shipping should be controlled. During the dry season, frequent and intense human activities are prohibited in nearshore waters, especially illegal fishing using rolling hooks, custom nets and camouflage.

(2) The state should adopt active financial policies to return lakes to nature with more than 10,000 acres of land along the Yangtze River, so as to restore the ecological role of flood storage and fish migration,

and restore the integrity of the Yangtze River habitat. To achieve significant results, the ban on spring fishing on the Yangtze should be extended to important tributaries along the river, and large lakes along the river should be locked and connected to the River all year round.

(3) Replenish a certain number of suitable wild individuals as soon as possible, so that the population can grow rapidly and form a larger effective breeding population while strengthening the protection of genetic diversity.

(4) Introduce a certain number of wild individuals representing different genetic variation, especially females with reproductive potential, from the natural population. The implementation of this strategy not only increases the genetic diversity of ex situ conservation population, but also contributes to the rapid growth of the population to form a larger effective reproductive population, which is ultimately beneficial to the long-term conservation of genetic resources of the endangered population.

In terms of ex situ conservation, we found that the population of Finless Porpoises would eventually remain stable under ex situ conservation. But there are still some problems: (1) the population is small, which could lead to an increased risk of inbreeding, losing more genetic diversity, reducing climate change and disaster resistance ability of offspring, forming a vicious circle, This is obviously bad for the population; (2) Due to the limited area of ex-situ reserve, its environmental carrying capacity is also limited, and its development will be limited when the population number increases close to K value.

Therefore, we offer the following suggestions:

(1) The strategy of ex situ conservation is to make effective breeding plans to avoid inbreeding. Therefore, during the breeding season, the male dolphins with strong breeding ability should be properly separated so that the male dolphins with weak breeding ability can obtain mating opportunities. Of course, before such a step can be taken, it is necessary to make sure that the male involved in the breeding is physically mature, otherwise it will result in reproductive failure during a given mating season.

(2) When a move to reserve population reaches the K value, administrators should take the migration or expansion method of reserve area, always ensure that porpoises number increases in a relatively stable state, because the expanding population is beneficial to increase the genetic diversity of population, and it can also further save dolphins endangered situation;

(3) Increase the number of ex situ conservation areas. In view of the importance of ex situ protection to increase the population number of Finless Porpoise, more ex situ conservation areas should be built and Finless Porpoise can be released into the Yangtze River after the ecosystem of the Yangtze River basin is restored to a better level.

In addition, we suggest that artificial breeding efforts can be intensified. Humans help Finless Porpoises reproduce to increase their numbers. For Finless Porpoise, its natural reproductive capacity is low, the survival rate of young is low, which is an important reason for the extinction of its species. We can copy the conservation methods of mammals such as giant pandas by breeding them in captivity and then releasing them back into nature reserves to survive. This is a good way to increase the population of Finless Porpoises.

The above suggestions are based on the analysis of existing models and combined with the actual situation. We hope to make relevant departments pay more attention to the protection of Finless Porpoise, and take effective measures to implement the protection of Finless Porpoise population, and keep the smiling angel of the Yangtze River together.

Appendices

Input python source:

```
import numpy as np
import math


def predict(data):
    x1 = data.cumsum()
    z = (x1[:len(x1) - 1] + x1[1:]) / 2.0
    B = np.array([-z, z*z]).T
    Y = data[1:]
    u = np.dot(np.dot(np.linalg.inv(np.dot(B.T, B)), B.T), Y)
    a, b = u[0], u[1]
    return [a*data[0]/(b*data[0]+(a-b*data[0])*math.exp(a*i)) for i in range(len(data))]
    # Gray Forecast Model Function


if __name__ == '__main__':
    raw_data = np.loadtxt('conservation_1.txt')
    data = np.array(raw_data)
    # [5.0000, 12.4864, 13.9987,  16.0839, 17.201,
    # 17.7296, 17.9732, 19.0819, 20.0000,  21.2919,
    # 22.9647,  25.0000,  28.3612, 30.0000,  57.0000,
    # 66.6734,  88.8200,  105.5084, 101.0000, 52]
    predict_data = predict(data)  # Prediction
    result = np.ediff1d(predict_data)  # Diminishing
    print('Original result: ', data[1:])
    print('Prediction result: ', result)
    print('Relative error: ', (np.array(result[:len(data)])
                    - np.array(data[1:len(data)])) / np.array(data[1:len(data)]))
```

Input matlab source:

```
clc;
clear;
x = [1992, 2002, 2005, 2007, 2015, 2021]    % Year
y = [5, 20, 25, 30, 57, 101]    % Number of Yangtze Finless Porpoises
xi = 1992:1:2022    % Prediction
yi = lagrange(x, y, xi)     % Lagrange Interpolation
plot(x, y, 'o', xi, yi, 'k')
title('Lagrange Interpolation Conservation 1')
xlabel('Year')
ylabel('Number of Yangtze Finless Porpoises')
```

```
function yy=lagrange(x,y,xx)    % Lagrange Function
m = length(x);
n = length(y);
if m~= n, error('Length of vector x and y should be the same');
end
s = 0;
for i = 1:n
    t = ones(1,length(xx));
```

```
    for j = 1:n
        if j~=i,
            t = t.*(xx - x(j))/(x(i) - x(j))    %Data (x, y) at interpolation point xx
        end
    end
    s = s + t * y(i)
end
yy = s
```

```
clc;
clear;
data = textread('conservation_1.txt');
data=nonzeros(data');
% Remove the zero elements in the order of the original data
r11=autocorr(data);
% Calculate the self correlation coefficient
r12=parcorr(data);
% Calculate partial correlation coefficient
figure
subplot(211),autocorr(data);
subplot(212),parcorr(data);
% The autocorrelation and partial autocorrelation
%of the original data are drawn on a graph
diff=diff(data);
% R11 is positive, not controlled by a negative exponential,
%so calculate the first order difference
r21=autocorr(diff);
% Calculate the self correlation coefficient
r22=parcorr(diff);
% Calculate the partial correlation coefficient
adf=adftest(diff);
% If adf == 1, stable time sequence
figure
subplot(211),autocorr(diff);
subplot(212),parcorr(diff);
% Plot on the same figure, stable time sequence
n=length(diff);
% Caculate the differencial data
k=0;
  for i = 0:3
    for j = 0:3
        if i == 0 & j == 0
            continue
        elseif i == 0
            ToEstMd = arima('MALags', 1:j, 'Constant', 0);
        elseif j == 0
            ToEstMd = arima('ARLags', 1:i, 'Constant', 0);
        else
            ToEstMd = arima('ARLags', 1:i, 'MALags', 1:j, 'Constant', 0);
            % Model structure
        end
        k = k + 1;
        R(k) = i;
        M(k) = j;
        [EstMd,EstParamCov,LogL,info] = estimate(ToEstMd,diff);
```

```
        % Fitness, and estimate model parameter
        numParams = sum(any(EstParamCov));
        [aic(k), bic(k)] = aicbic(LogL,numParams,n);
    end
end
fprintf('R, M, AIC, BIC value: \n%f');
check  = [R',M',aic',bic'];
res=infer(EstMd,diff);
% Verification
figure
subplot(2,2,1)
plot(res./sqrt(EstMd.Variance))
% Standardlized residual
title('Standardized Residuals')
subplot(2,2,2),qqplot(res)
% Fit the hypothesis of normality
subplot(2,2,3),autocorr(res)
subplot(2,2,4),parcorr(res)


% The autocorrelation coefficient rapidly decreases to 0 after 1 order lag,
% and the partial correlation coefficient
%is the same as the self correlation coefficient,
% so p = 1, q = 1
p=input('p = ');
q=input('q = ');
ToEstMd=arima('ARLags', 1:p, 'MALags', 1:q, 'Constant', 0);
 [EstMd, EstParamCov, LogL, info] = estimate(ToEstMd, diff);
 dx_forest = forecast(EstMd, 20, 'Y0', diff);       % 20 years prediction
 x_forest = data(end)+cumsum(dx_forest)
figure
h4 = plot(data, 'b');
hold on
h5 = plot(length(data)+1: length(data)+20, x_forest, 'r', 'LineWidth', 2);
title('ARIMA Sequence Prediction of Conservation 1')
xlabel('Year(since 1992)')
ylabel('Number of Yangtze Finless Porpoises')
hold off
```

```
close;
clear;
clc;
n = 83;     % Porpoise population size
Plight = 0.09;
Pgrowth = 0.0008;
UL = [n1:n-1];
DR = [2:n1];
veg = zeros(n,n);         %Initialization
im = image(cat(3,veg,veg,veg));
mannual = annotation('textbox',[0.1,0.1,0.1,0.1],'LineStyle','-','LineWidth',2,'String','123'
for i = 1 : 20
    sum = (veg(UL,:) == 1) + (veg(:,UL) == 1) + (veg(DR,:) == 1) + (veg(:,DR) == 1);
    % Update the porpoise matrix according to the rules:
    % porpoise = porpoise - reduced porpoise + new porpoise
    veg = 2 * (veg == 2) - ( (veg == 2) & (sum > 0 | (rand(n,n) < Plight)) ) + 2 * ( (veg == (
```

```
    % The meaning of value of veg:
    % empty == 0
    % dying == 1
    % living == 2
    living = find(veg == 2);
    dying = find(veg == 1);
    l_number=length(living);
    number=length(dying);
    Yangtzeporpoise(i) = l_number;
    livingenvironment(i)=number * 1000;
    if (number>=0&&number<=10)
        str1='Porpoise increases';
    elseif (number>10&&number<=100)
        str1='Environment index changes';
    elseif (number>100)
        str1='Porpoise decreases';
    end
    if ((l_number>40000)||(number>=10)) str2='Porpoise decreases sharply';
    elseif (l_number>30000&&l_number<=40000) str2='Porpoise decreases';
    elseif (l_number>20000&&l_number<=30000) str2='Porpoise begins to decrease';
    elseif (l_number>=0&&l_number<=20000) str2='Porpoise continues to increase';
    end
    str=[str1 10 str2];
    set(im, 'cdata', cat(3, (veg == 1), (veg == 2), zeros(n)) )
    drawnow;figure(2);delete(mannual);plot(Yangtzeporpoise);
    hold on
    plot(livingenvironment);
    legend(['Porpoise',num2str(l_number)],['Living environment index',num2str(number)]);
    title(['Time T=',num2str(i),'years']);
    mannual = annotation('textbox',[0.15,0.8,0.1,0.1],'LineStyle','-','LineWidth',2,'String',s
    hold off
    % pause(0.0001)
end
```

```
clc;
clear;
format short g
x0=[2800 2500 2100 1800 1350 1012];
for i=2:6
    x1(1)=x0(1);
    x1(i)= x1(i-1)+x0(i);
end
for i=1:5
    yn(i)=x0(i+1);
end
for i=1:5
    b(i)=(-0.5)*(x1(i)+x1(i+1));
end
for i=1:5
    B(i,1)=b(i);
    B(i,2)=1;
end
c=inv(B'*B)*B'*yn'
a=c(1,1)
u=c(2,1)
```

```
for t=1:6
    x11(t) = u/a + exp(-a*t) * (-u+ 2800*a) / a;
end
x11
syms t
X=u/a+exp(-a*t)*(-u+2800*a)/a;
C=1:15;
y=subs(X,t,C)

z2022=y(2)-y(1)
z2027=y(3)-y(2)
z2032=y(4)-y(3)
z2037=y(5)-y(4)
z2042=y(6)-y(5)
z2047=y(7)-y(6)
z2052=y(8)-y(7)
z2057=y(9)-y(8)
z2062=y(10)-y(9)
z2067=y(11)-y(10)
z2072=y(12)-y(11)
z2077=y(13)-y(12)
z2082=y(14)-y(13)
```