

Problem Chosen

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Summary Sheet

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Summary

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1 Introduction

1.1 Problem Restatement

Considering that the carbon dioxide can be sequestered in both forests and wooden products, it's reasonable that more carbon will be stored by forests with the appropriate combination of the regrowth of younger forests and the wooden products. Thus, forest managers are ought to deliberate about the balance between the value of forests as living tress to grow and absorb the carbon and the value of forests harvested as wooden products. What's more, the forest managers should not only consider the factors about forests such as type and age of forests, geography, topography, and benefits and lifespan of forest products, but also the conservation and diversity of wild species, recreational uses and cultural considerations.

1. Design a carbon sequestration model to calculate the amount of carbon dioxide sequestered by a forest and its products, which also determines what kind of manage plan is most efficient at sequestering carbon.
2. Develop a decision model consisting of various ways that forests are valued (including carbon sequestration) for forest managers to understand the best use of a forest. Consider the following questions. What is the scope of the management plan that your decision model might suggest? Are there any conditions that the forests should not be harvested? Whether there is a transition point between management plans applicable to all forests? How can the characteristics of a particular forest and its location be used to determine transition points between management plans?
3. Apply your models to various forests. Identify a forest that your decision model would suggest the inclusion of harvesting in its management plan. How much carbon dioxide can be sequestered by this forest and its products in 100 years? What kind of forest management plan should be carried out for this forest? Why it's best? The best management plan is assumed to include a harvest interval of 10 years longer than current forest practices discussing strategies for transitioning from existing to new schedules in a manner sensitive to the needs of forest managers and all those who use forests.
4. Some people think that we should never cut down any trees, but you've determined the forest which includes harvest in its management. Write a one-to-two-page non-technical newspaper articles to explain why your analysis including in the management of the forest logging, rather than it remaining the same. Finally, your ariticle should convince local community that it's the best decision for their forest.

1.2 Overview of Our Work

2 Assumptions and Justifications

These are necessary assumptions for simplifying the model.

- 1.

3 Notations

Table 3.1: Notation Descriptions

Symbol	Definition
DBH	Diameter at Breast Height
β_b	Conversion factor of coniferous trees
β_c	Conversion factor of broad leaf trees
n	Total iteration years
λ	Harvest rate
M	Sequestered carbon mass in wooden products
CarbonMass _f	Sequestered carbon mass in forests
ϕ	Proportion in total product
s	Scrap rate
m	Decomposition rate
K	Carrying Capacity
N	Species population
r	Population Growth Rate
α, β	Coefficient of Competition
p	Rotation Years

4 Model I

4.1 Cellular Automata based Population Size Prediction

Cellular automata (CA) is a kind of grid dynamics model with discrete time, space and state, and local spatial interaction and temporal causality, which has the ability to simulate the space-time evolution process of complex system.

Unlike general dynamical models, cellular automata are not determined by strictly defined physical equations or functions, but are composed of rules constructed by a series of models. Any model that satisfies these rules can be regarded as a cellular automata model. Therefore, cellular automata is a general term for a class of models, or a method framework. Its characteristic is that time, space and state are discrete, each variable only takes a finite number of states, and its state change rules are local in time and space.

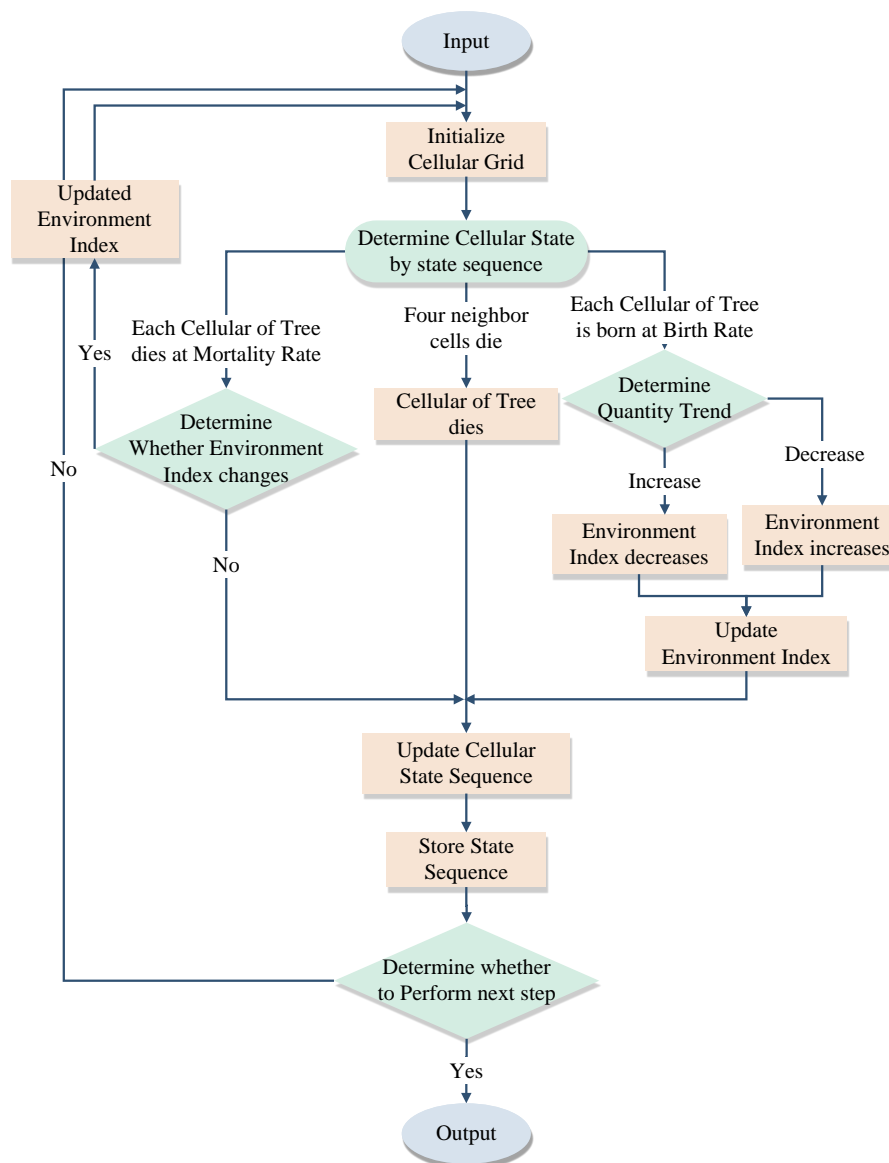


Figure 4.1: Block Diagram of typical Cellular Automata

The block diagram of a typical cellular automata is shown in Figure 4.1, and the simulation result is

shown in Figure 4.2, in which we learn that the under the natural circumstances, the forestry population size will reach and remain around 2100 per hectare.

4.2 Binary Timber Volume and Logistic based Carbon Sequestration Model

In order to clearly demonstrate the amount of carbon sequestered by forests and their products, we decide to consider the trunk volume of living trees as an estimation of total Volume of trees that are able to store carbon (Wang Yan, 2009).

For the living trees, we apply Binary Timber Volume Regression model (Luo Qibang, 1992) to depict their growth as shown in Algorithm 1. It is significant to point out that β_b and β_c are two **Conversion Factors** from Timber Volume to Carbon Mass sequestered. Besides, the mass of carbon sequestered in soil and leaves can be deemed as a constant which merely depends on the area of coniferous and broad-leaved forests (Deren, 2011).

As for the wooden products, we consider w kinds of wooden products, each kind, which decomposes at the rate of $m_i (i = 1, \dots, w)$, accounts for ϕ_i of the total product mass (Birdsey et al., 2006). Note that the **harvest rate** λ and the product proportion ϕ are flexible regarding the circumstances, which are also the core of our forest management. And the harvest will be conducted every p years, which is **Rotation**.

What's more, we hold the view that the volume of wooden products derive from the volume of harvested trees with a certain **Scrap Rate** s . Suppose the harvested wood is stored in a well-organized warehouse and the storage will become products in next p years, which means $\frac{1}{p}$ of all the storage will become products every year. The complete idea is shown in Algorithm 2.

We apply our model over a hypothetical forestry land with 10000 hectares, in which the broad leaf trees takes one half and the other half is coniferous trees. Two primary decision factors, which are harvest rate and product proportion, are discussed by conducting sensitivity test.

The sensitivity test of Harvest Rate is conducted by calculating the Carbon Dioxide sequestered by the living trees and the wooden products every 1% of harvest rate from 1% to 30%, which can be seen in Figure 4.4(a).

The sensitivity test of Product Proportion is carried out by determining the Carbon sequestered by wooden products under different types of product proportion arrangement. The proportion of products with higher Decomposition Rate increases from Type 1 to Type 5, which means in Type 5, most of the wood is consumed to manufacture high Decomposition Rate products. The result is shown in Figure 4.4(b).

Algorithm 1 Binary Timber Volume Regression of Carbon Prediction Algorithm

Input: Measurement of DBH, Height of tree (h), conversion factor β_c (coniferous) and β_b (broad leaf), Harvest rate $\lambda(t)$, Carrying Capacity K , Rotation year p

```

1: for x in enumerate (DBH, h) do
2:   for k  $\leftarrow$  1 to m do  $P_k(x) = \frac{1}{2^k k!} \frac{d^k}{dx^k} (x^2 - 1)^k$ 
3:     for l  $\leftarrow$  1 to m do
4:       Calculate  $\int_{-1}^1 P_k(x) P_l(x) dx \leftarrow$  Integral of Legendre Polynomials
5:       if k = l then  $\int_{-1}^1 P_k(x) P_l(x) = \frac{2x}{2i+1}$ 
6:       else  $\int_{-1}^1 P_k(x) P_l(x) = 0$ 
7:       end if
8:     end for
9:   end for
10:  Find (DBH, h)T to minimize  $J = \int_a^b [f(\text{DBH}) + g(h) - 2(x)]^2 dx$ 
11: for t  $\leftarrow$  1 to n do
12:   if t mod p  $\equiv$  0 then
13:     Regression of DBH and h
14:      $V_0 \leftarrow$  Initiate
15:      $V_t = a \text{DBH}^b h^c \times \text{Area} \times \text{Density}(\lambda, t)$ 
16:   end if
17: end for
18:  $\text{Area} = \text{Area}_b + \text{Area}_c$ 
19:  $\text{CarbonMass}_f = (\beta_c V_c + \beta_b V_b) + 0.02 \times \text{Area}_c \times \frac{\text{Density}(\lambda, t)}{K} + 0.001 \times \text{Area}_b \times \frac{\text{Density}(\lambda, t)}{K} + 70 \times \text{Area}$ 
Output: Carbon Dioxide Quantity of Forest =  $\frac{44}{12} \times \text{CarbonMass}_f$ 

```

Algorithm 2 RBF Neural Network Fitting of wooden products for carbon sequestration Algorithm

Input: ϕ, s, m, w as kinds wooden products, Standardized CarbonMass and V in Algorithm1, Rotation year p

```

1: for t  $\leftarrow$  1 to n do
2:   for i  $\leftarrow$  1 to hidden_dim do
3:      $y_0 \leftarrow$  Initiate
4:      $\hat{y}_t = y_{t-1} + \phi_{it} V_i(t)$ 
5:      $c_i \leftarrow$  Sample CarbonMass
6:      $\sigma_i \leftarrow$  Z-score Normalization
7:      $V_i(t) = e^{-\frac{\|t - c_i\|^2}{2\sigma_i^2}}$ 
8:   end for
9:    $M_0 \leftarrow$  Initiate
10:   $c = t - (t \bmod p) + 1$ 
11: for j  $\leftarrow$  1 to w do
12:    $M_t = M_{t-1} + (1 - s) \lambda \times \frac{1}{p} (\beta_c V_c(c) + \beta_b V_b(c)) \times \phi_j(t) m_j(t)$ 
13: end for
14: end for
Output: Carbon Dioxide Quantity of Wooden Products =  $\frac{44}{12} \times M_t$ 

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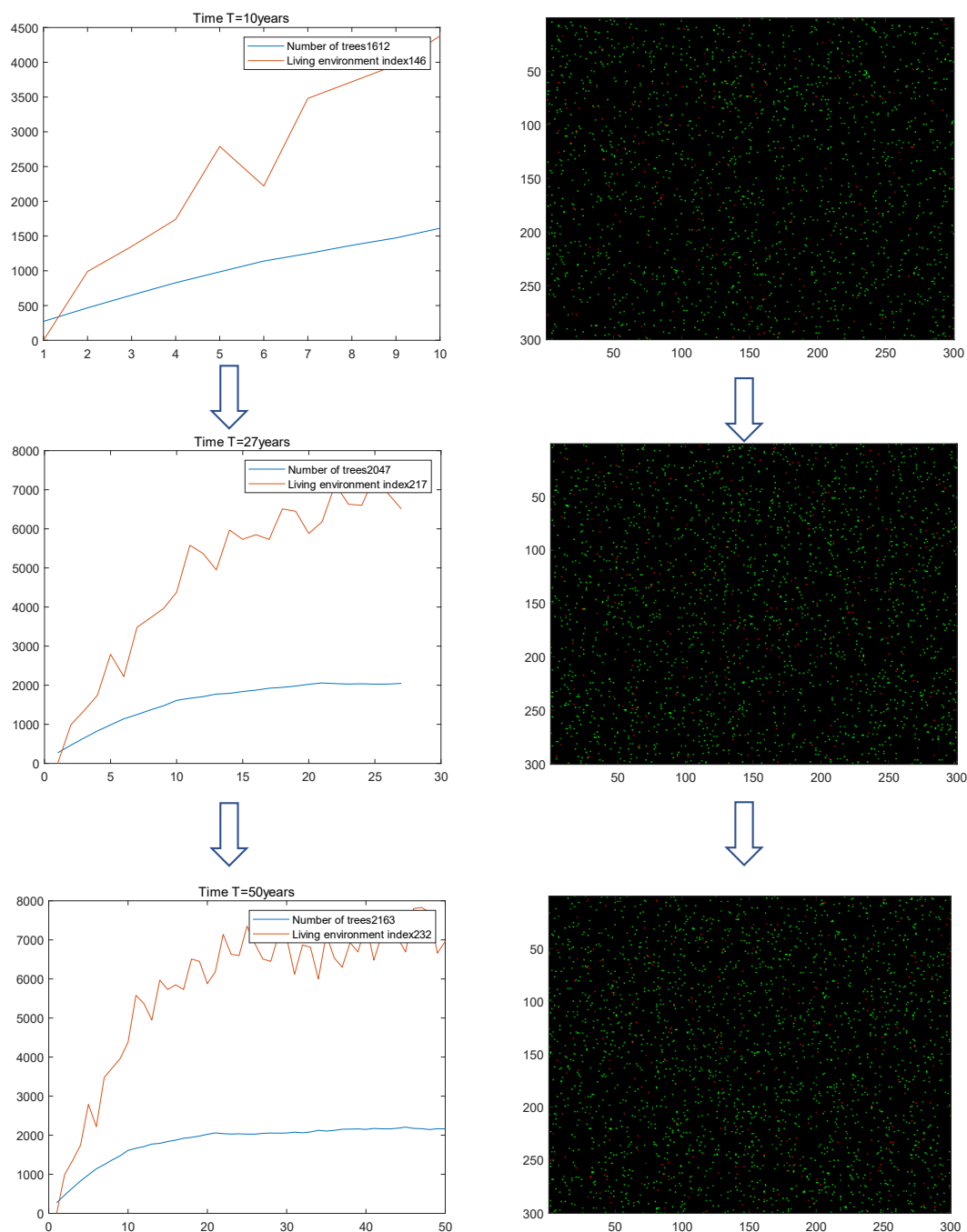


Figure 4.2: Result of Cellular Automation

5 Model II

Figure 5.1 clearly demonstrates the principle of the carbon circle and roles that Forestry Species play in the circle, which helps us to understand the necessary procedure to manage Forestry Carbon Sequestration.

In order to comprehensively consider the multiple social interests, we add several indexes which can fully demonstrate the impact of Forests over local society.

The indexes are divided into two aspects, namely **Carbon Sink** and **Cultural Value**.

The Scrap Rate is determined as following descriptions, which is also shown in Figure 4.3 on the right. There are two manufacturing methods of Round Wood harvested from the forests.

One way is to trim the wood neatly into slabs in different shapes so that they can occupy most of the volume of the wood, which will be processed into softwood plywood, panels and board. Meanwhile, the scrap will be manufactured into miscellaneous products. It can approach the utilization rate of 81.3%.

The other kind of craft is to separate the bark and turn it into paper, and the separated log can be processed into softwood and hardwood. The utilization rate of this craft is able to reach almost 100%.

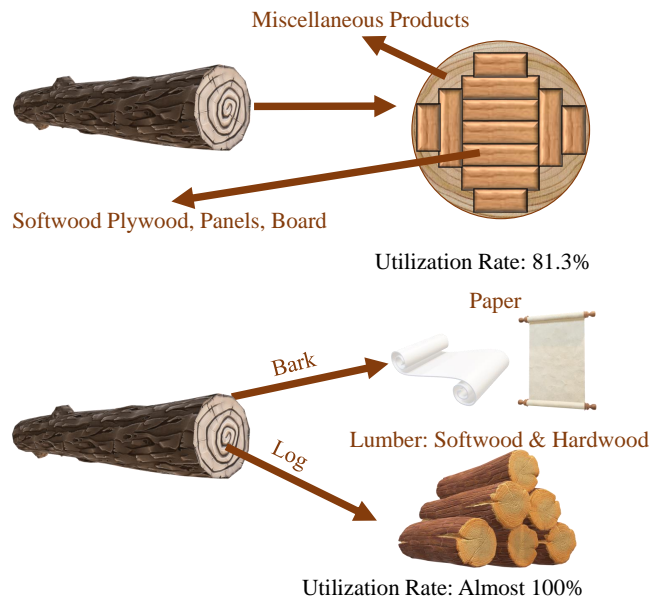


Figure 4.3: Utilization Rate of Round Wood

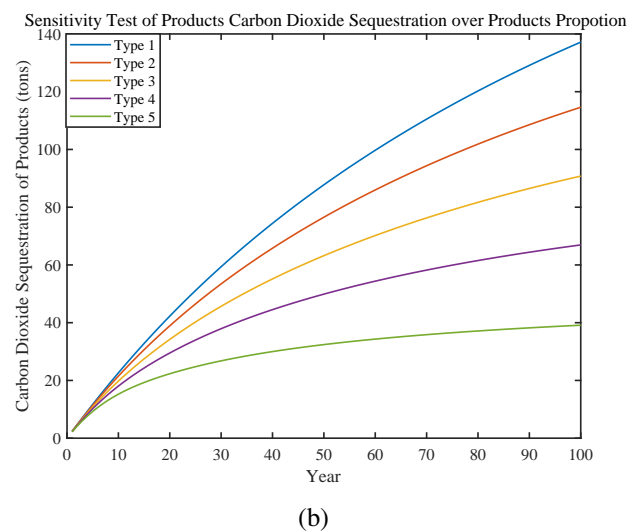
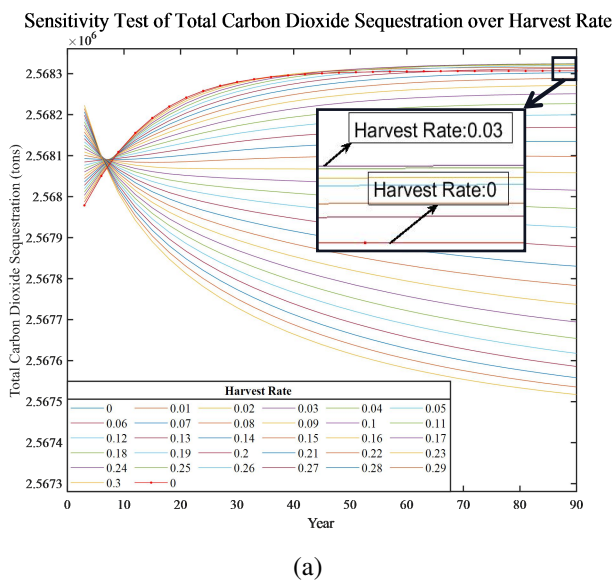


Figure 4.4: Results of sensitivity test of Harvest Rate and Product Proportion

5.1 Carbon Sink

Carbon Sink includes the following indexes: **Carbon Credit Benefit and Cost Analysis(CVal)**, **Wooden Product Proportion**, **Harvest Rate** and **Interspecific Competition**, which will exert influence on carbon sink in this Forest.

CVal is a spreadsheet tool to evaluate the direct benefits and costs of Carbon Sequestration contracts for managed forests developed by E.M. (Ted) Bilek, Peter Becker and Tim McAbee. By inputting a series of parameters of the Forests such as tract size, CO₂ sequestration rate and Carbon price, CVal will automatically come up with Average cost of trading, Net trading benefit, Internal Rate of Return and other important economic indexes.

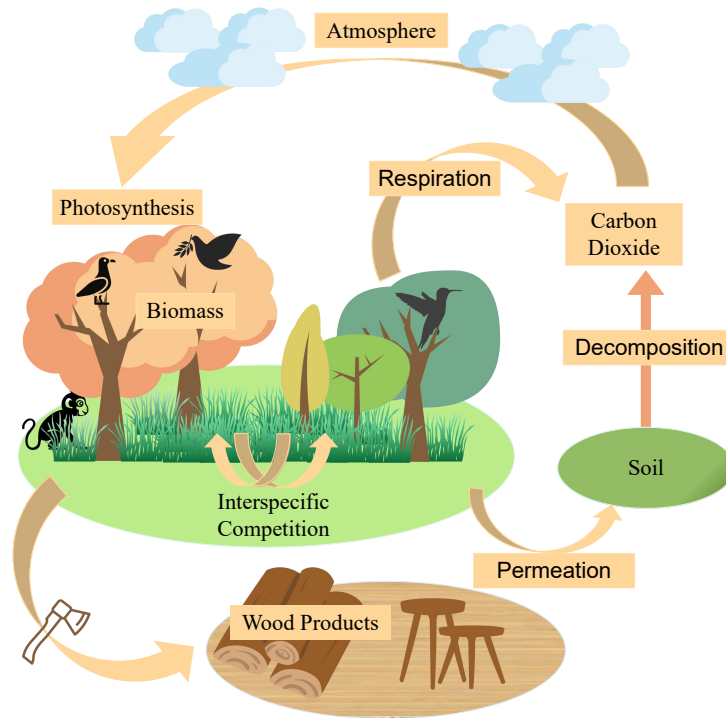


Figure 5.1: Carbon Circle in Forest Ecosystem

Wooden Product Proportion and Harvest Rate are fully discussed above, which will not be repeated again.

Interspecific Competition is calculated by **Lotka-Volterra Model**, which is used to describe ecology evolution. Suppose there are two species competing for surviving spaces, namely Douglas Fir and Pinus Densiflora, whose population sizes are N_1 and N_2 , population growth rates are r_1 and r_2 , carrying capacity K_1 and K_2 and coefficients of competition over each other are α and β . Thus, based on the Logistic growth model, the population of Douglas Fir can be described as

$$\frac{dN_1}{dt} = r_1 \cdot N_1 \cdot \left(\frac{K_1 - N_1 - \alpha N_2}{K_1} \right)$$

, in which α is defined as a coefficients of competition of Douglas Fir over Pinus Densiflora, representing that each Pinus Densiflora will occupy the surviving spaces of α Douglas Firs. Similarly, the population of Pinus Densiflora can be described as

$$\frac{dN_2}{dt} = r_2 \cdot N_2 \cdot \left(\frac{K_2 - N_2 - \alpha N_1}{K_2} \right)$$

Thus, there are four possible results for these two kinds of trees.

- (1) Douglas Fir wins, and Pinus Densiflora gets sidelined;
- (2) Pinus Densiflora wins, and Douglas Fir gets sidelined;
- (3) Douglas Fir and Pinus Densiflora coexist stably;
- (4) Douglas Fir and Pinus Densiflora coexist erratically.

Whichever wins, the intensity of intraspecific competition will decline while the intensity of interspecific competition will roar. If both species suffer high level of intraspecific competition as well as low interspecific competition, they will stably coexist, otherwise it leads to unstable coexistence.

Lotka-Volterra Model can be improved as follows. Suppose a second-order system can be described as the ODE below:

$$\frac{d^2x}{dt^2} + a_1(x, \frac{dx}{dt})\frac{dx}{dt} + a_0(x, \frac{dx}{dt})x = 0, \quad \ddot{x} = f(x, \dot{x})$$

Suppose $x = x_1$, $\frac{dx}{dt} = x_2$, then

$$\begin{aligned} \frac{dx_2}{dx_1} &= \frac{f(x_1, x_2)}{x_2} \\ \begin{cases} \frac{dx_1}{dt} &= x_2 \\ \frac{dx_2}{dt} &= f(x_1, x_2) \end{cases} \end{aligned}$$

Suppose $x_1(t)$ denotes the position of a mass point and $x_2(t)$ denotes the velocity of the mass point, then the solution of the equations depicts the movement of a point in a 2-dimension plane, and the figure of the solution is called **Phase Diagram** in physics.

Apply what we discuss above over the previous equations we can draw the improved Model:

$$\begin{cases} \frac{dr}{dt} = 2(1 - \frac{r}{R}) - \lambda r f, r(0) = r_0 \\ \frac{df}{dt} = -f + \lambda r f, f(0) = f_0 \end{cases}$$

Result of the competition between Douglas Fir and Pinus Densiflora is shown in Figure 5.2, while the thermodynamic diagram of competition intensity between these two species can be seen in Figure 5.3.

5.2 Cultural Value

In order to further demonstrate the relationship between the five indexes and Cultural Value, we abstract three concepts, namely Biodiversity, Development of Forest and Forest Influence, from the indexes. The structure of indexes related to Cultural Value can be clearly drawn from Figure 5.4, in which Proportion of Forest and Size of Forest signify the population the Forest can influence, and Volume of Deadwood, "an element of biodiversity and, to some extent, an indicator of forest management sustainability that deserves to be taken into account in inventories, especially at the national level" (Rondeux & Sanchez, 2010), along with the Forest Hierarchy, which provides arboreal animals with more habitats, determines the Biodiversity of the Forest. The Volume of Deadwood of the same type of forest at a similar latitude shares a similar constant (Smith et al., 2007). Size and Proportion of Forest can be searched and estimated on map websites, while the Forest Hierarchy is easy to search.

6 Application

We apply ARIMA model in order to predict the size of Finless Porpoise population in five ex-situ conservation areas after 20 years, whose **algorithm block diagram** is shown as Figure 6.1.

Under regular circumstances, the time series we obtain in the real world has tendency, seasonality and non-stationarity. Thus, it's vital for us to transfer the non-stationary time series to stationary time series

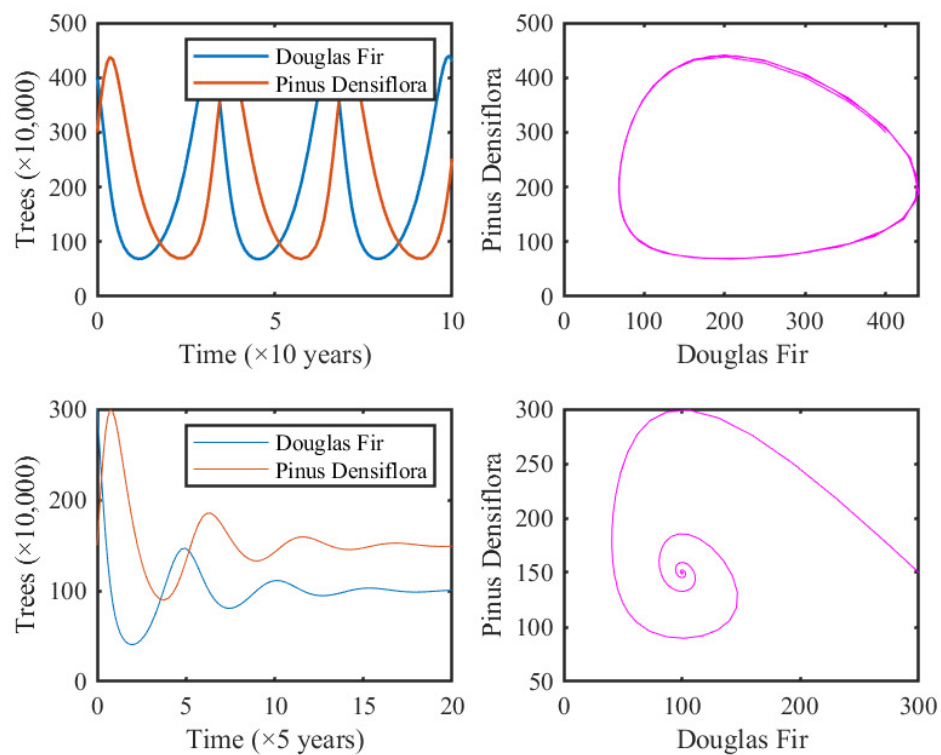


Figure 5.2: Interspecific Competition Result of Douglas Fir and Pinus Densiflora

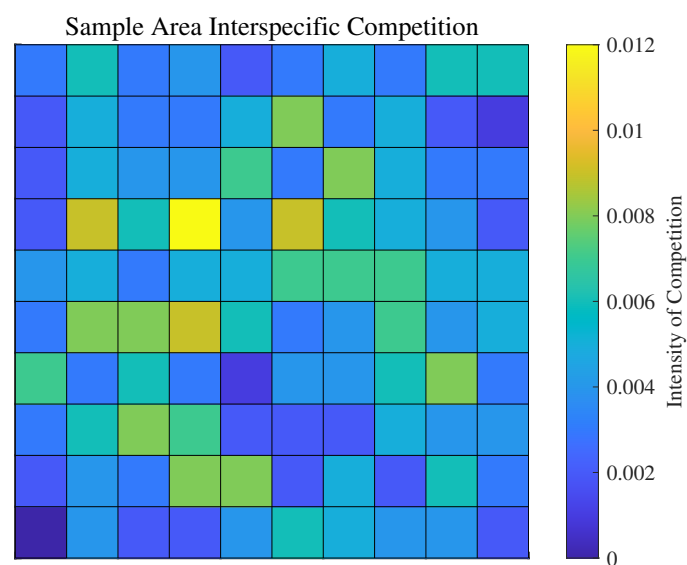


Figure 5.3: Thermodynamic Diagram of Competition

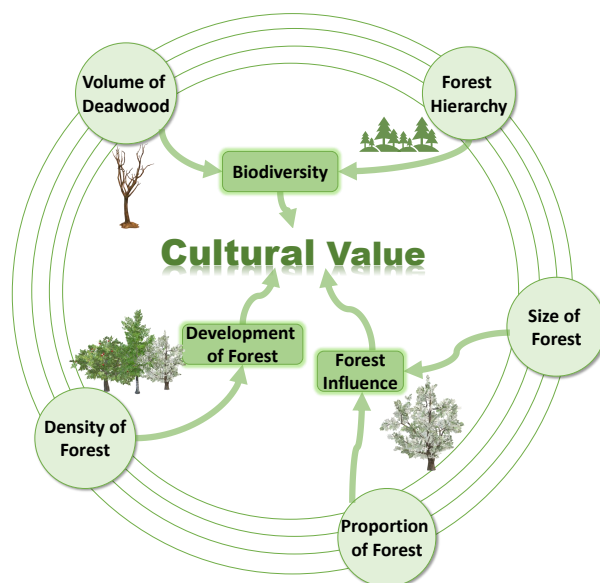


Figure 5.4: Influencing indexes of Forest Culture Value

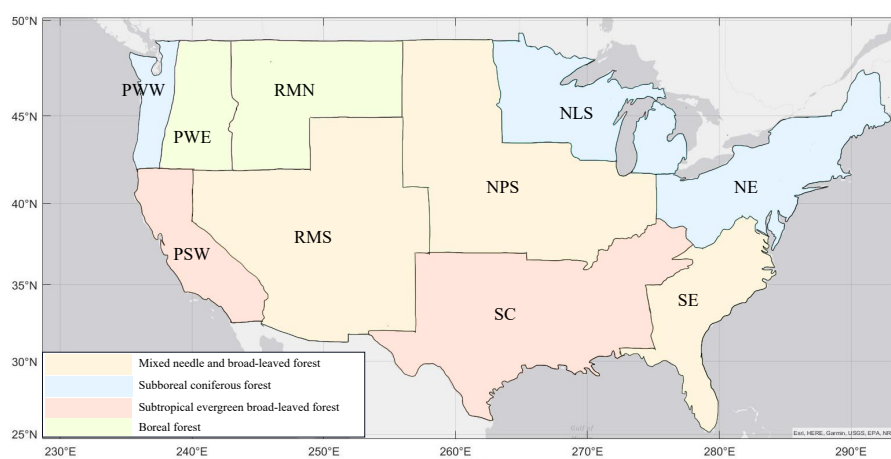


Figure 5.5: Forest division in the United States

and make an assumption that the time series is an Auto Regressive Moving Average(ARMA) series to predict the future data. ARMA series is defined as follows.

$$X_t - \phi_1 X_{t-1} - \cdots - \phi_p X_{t-p} = \epsilon_t - \theta_1 \epsilon_{t-1} - \cdots - \theta_q \epsilon_{t-q}$$

ϵ_1 is a stationary white noise whose average is zero and deviation is σ_ϵ^2 ; X_t is an ARMA series with p and q degree, recorded briefly as $\text{ARMA}(p, q)$ series. Akaike Information Criterion(AIC) is one of the most commonly used criterion to determine the degree of $\text{ARMA}(p, q)$: choose p, q such that

$$\min \text{AIC} = n \ln \hat{\sigma}_\epsilon^2 + 2(p + q + 1) \quad (6.1)$$

n is the capacity of sample; $\hat{\sigma}_\epsilon^2$ is the estimation of σ_ϵ^2 relating to p and q . Suppose $p = \hat{p}, q = \hat{q}$, such that equation(6.1) reaches the minimum, then we deem the series is $\text{ARMA}(\hat{p}, \hat{q})$.

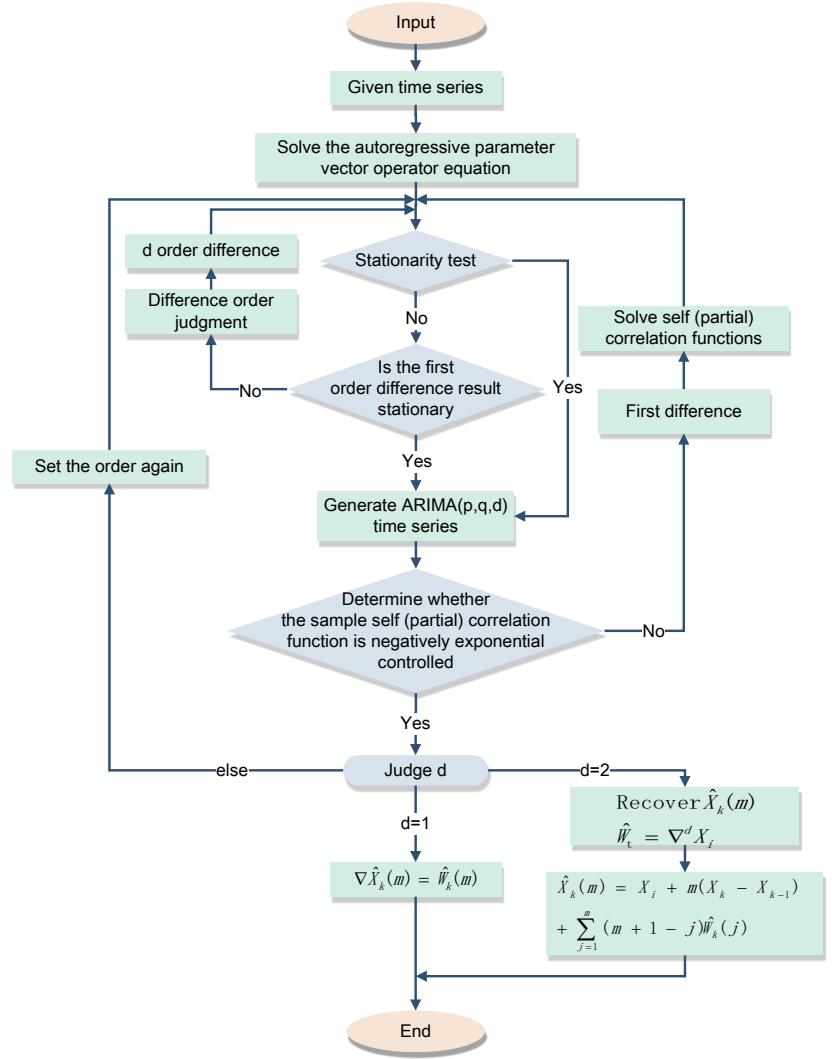


Figure 6.1: Algorithm Block of ARIMA

Suppose $\text{ARMA}(p, q)$ series has an unknown average parameter μ , the model becomes

$$\phi(B)(X_y - \mu) = \theta(B)\epsilon_t,$$

meanwhile, the number of unknown parameters is $k = p + q + 2$, the AIC is: choose p, q such that

$$\min \text{AIC} = n \ln \hat{\sigma}_\epsilon^2 + 2(p + q + 2). \quad (6.2)$$

In fact, equations (6.1) and (6.2) have the same minimum point \hat{p}, \hat{q} . After that, we usually choose $p = 1, q = 1$ to make parameter estimation over ARMA model.

It's demonstrated that the differential operation can stabilize certain class of non-stationary series. And It's emphasized that stationary test must be conducted previously. Stationary test can be applied by calculating sample autocorrelation function and sample coefficient of partial function.

If the functions are truncated or trending to 0 (meaning being controlled by negative index), then the series belongs to ARMA model.

If at least one of the functions above is not truncated or trending to 0, then it's not stationary.

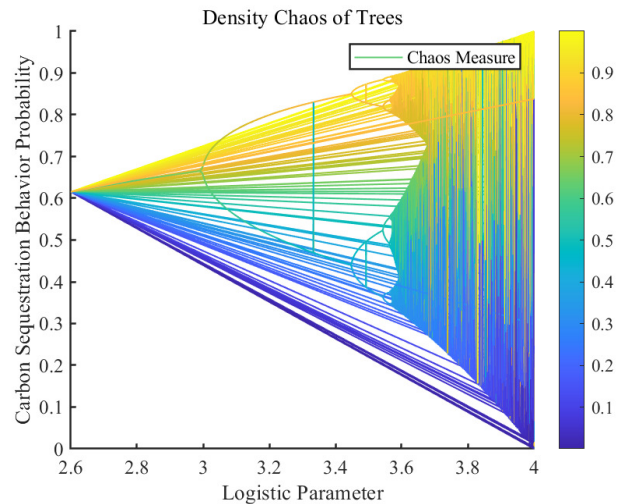
Suppose the series is non-stationary, which can be transformed to a stationary series by d -degree differential operation, denoted as $\text{ARIMA}(p, q, d)$ series, then differentiate the sample by d -degree:

$$W_t = \nabla^d X_t, \quad t = d + 1, \dots, n$$

After that, apply stationary test on W_t and repeat steps above until it becomes a stationary series, Then W_t (which is denoted as X_t) complies ARMA model.

7 Sensitivity Test

We assume that the density of the forest population follows the Logistic Model, and the **Carrying Capacity of the trees** is K . From the Carbon Sequestration probability predicted on the right we can draw a reasonable conclusion that the Logistic Model will result into chaos after 3.6 rounds of harvests.



8 Evaluation of Model

Strength:

9 Conclusions

POLICY ADVICE ON FINLESS PORPOISE CONSERVATION

To: Relavant Authorities

From: MCM/ICM team 2215432

Date: February 21, 2022

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Appendices