Problem Chosen
E

2022
MCM/ICM
Summary Sheet

Team Control Number
2215432

123

Summary

Carbon Sequestration is one of the fundamental ways to recycle Carbon on the Earth where greenhouse effect floods. Forests, as an essential part of ecosystem, are in whopping capacity of Carbon Sequestration.

In order to better measure the competence of Carbon Sequestration of "Earth's Lung", a **Cellular Automata Model** of natural forest succession is applied to simulate the change of population density under natural conditions, which can enter a stable stage when the **Living Environment Index** reaches **232**. Meanwhile, we develop **Binary Timber Volume Carbon Sequestration Regression Model (BTVCSR)** to determine the Mass of Carbon sequestered during a period of time in the future. Sensitivity test of the Carbon Mass is carried out to find that the Subboreal coniferous forest would sequester **256.83** tons of Carbon Dioxide per hectare per year under an appropriate Harvest Rate of around 3% , which ranks the highest. Furthermore, we compare different proportions of 7 kinds of wooden products and predict the Mass of Carbon Sequestered in those products over 100 years via **Radial Basis Function Neural Network**, which will reach around **140** tons of Carbon Dioxide per hectare per year after 100 years. Our results not only coincide with reality, but also provide Forest Managers with scientific Forest Management Strategy.

In the meantime, social value of Forests is of great significance. We develop **Forest Carbon Sequestration Multidimensional Evaluation Model (FCSME)**, which is divided into **Ecological Value** and **Cultural Value**, to better understand the role Forests play both in Ecosystem and human Society. Through establishing improved **Differential Equations of Carbon Sink Value under Multi-Tree Species Competition**, we obtain **Intensity Thermodynamic of Sample Area Interspecific Competition**, in which the highest intensity index reaches 0.012 , representing the Ecological Value of Competition and Carbon Sink. What's more, we develop a **Comprehensive Evaluation Model of Cultural Value** based on **Forest Hierarchy, Volume of Deadwood, Proportion of Forest over Administrative Region, Forest Density** and **Forest Area**, which is applicable to 4 kinds of typical classes of Forests in 9 Regions of United States. By combining Ecological Value with Cultural Value, we figure out that Subtropical Evergreen Broad-leaved Forest ranks the highest with **0.993638 (no-harvest)** and **0.98369 (harvest)**.

Application on real Forests demonstrates the correctness and robustness of our Model. First of all, we apply **Autoregressive Integrated Moving Average (ARIMA)** model to predict the intensity of competition. After that, by selecting and calculating 10000 hectares of Subboreal Forests consisting of Douglars Fir and Pinus Densiflora as primary types of trees in Washington State, we find that the Forest will sequester **261.3** tons of Carbon Dioxide per hectare per year, whose gap from reality is merely 8.1%.

Keywords:

Contents

1 Introduction

1.1 Problem Restatement

Considering that the carbon dioxide can be sequestered in both forests and wooden products, it's reasonable that more carbon will be stored by forests with the appropriate combination of the regrowth of younger forests and the wooden products. Thus, forest managers are ought to deliberate about the balance between the value of forests as living tress to grow and absorb the carbon and the value of forests harvested as wooden products. What's more, the forest managers should not only consider the factors about forests such as type and age of forests, geography, topography, and benefits and lifespan of forest products, but also the conservation and diversity of wild species, recreational uses and cultural considerations.

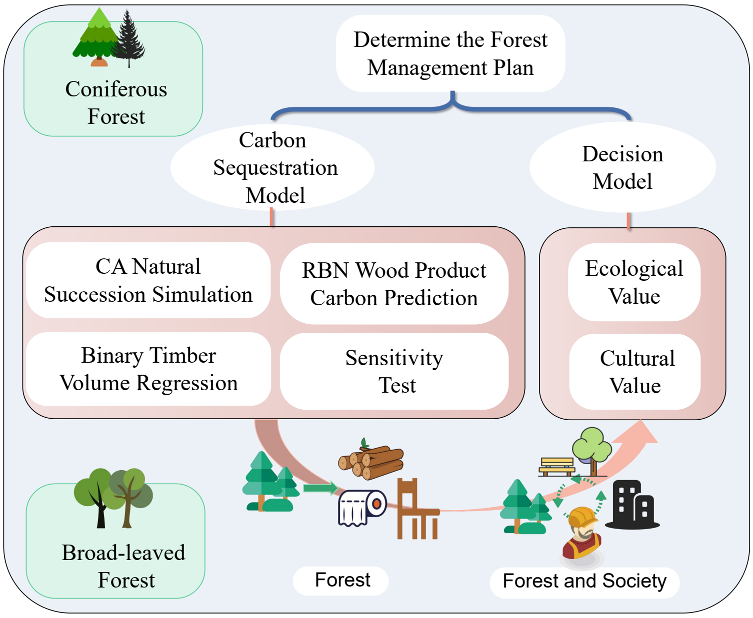
1. Design a carbon sequestration model to calculate the amount of carbon dioxide sequestered by a forest and its products, which also determines what kind of manage plan is most efficient at sequestering carbon.
2. Develop a decision model consisting of various ways that forests are valued (including carbon sequestration) for forest managers to understand the best use of a forest. Consider the following questions. What is the scope of the management plan that your decision model might suggest? Are there any conditions that the forests should not be harvested? Whether there is a transition point between management plans applicable to all forests? How can the characteristics of a particular forest and its location be used to determine transition points between management plans?
3. Apply your models to various forests. Identify a forest that your decision model would suggest the inclusion of harvesting in its management plan. How much carbon dioxide can be sequestered by this forest and its products in 100 years? What kind of forest management plan should be carried out for this forest? Why it's best? The best management plan is assumed to include a harvest interval of 10 years longer than current forest practices discussing strategies for transitioning from existing to new schedules in a manner sensitive to the needs of forest managers and all those who use forests.
4. Some people think that we should never cut down any trees, but you've determined the forest which includes harvest in its management. Write a one-to-two-page non-technical newspaper articles to explain why your analysis including in the management of the forest logging, rather than it remaining the same. Finally, your ariticle should convince local community that it's the best decision for their forest.

1.2 Overview of Our Work

2 Assumptions and Justifications

These are necessary assumptions for simplifying the model.

1.



3 Notations

Table 3.1: Notation Descriptions

Symbol	Definition
DBH	Diameter at Breast Height
β_b	Conversion factor of coniferous trees
β_c	Conversion factor of broad leaf trees
n	Total iteration years
λ	Harvest rate
M	Sequestered carbon mass in wooden products
$\mathrm{CarbonMass_f}$	Sequestered carbon mass in forests
ϕ	Proportion in total product
s	Scrap rate
m	Decomposition rate
K	Carrying Capacity
N	Species population
r	Population Growth Rate
α, β	Coefficient of Competition
p	Rotation Years

4 Binary Timber Volume Carbon Sequestration Regression Model

4.1 Cellular Automata based Population Size Prediction

Cellular automata (CA) is a kind of grid dynamics model with discrete time, space and state, and local spatial interaction and temporal causality, which has the ability to simulate the space-time evolution process of complex system.

Unlike general dynamical models, cellular automata are not determined by strictly defined physical equations or functions, but are composed of rules constructed by a series of models. Any model that satisfies these rules can be regarded as a cellular automata model. Therefore, cellular automata is a general term for a class of models, or a method framework. Its characteristic is that time, space and state are discrete, each variable only takes a finite number of states, and its state change rules are local in time and space.

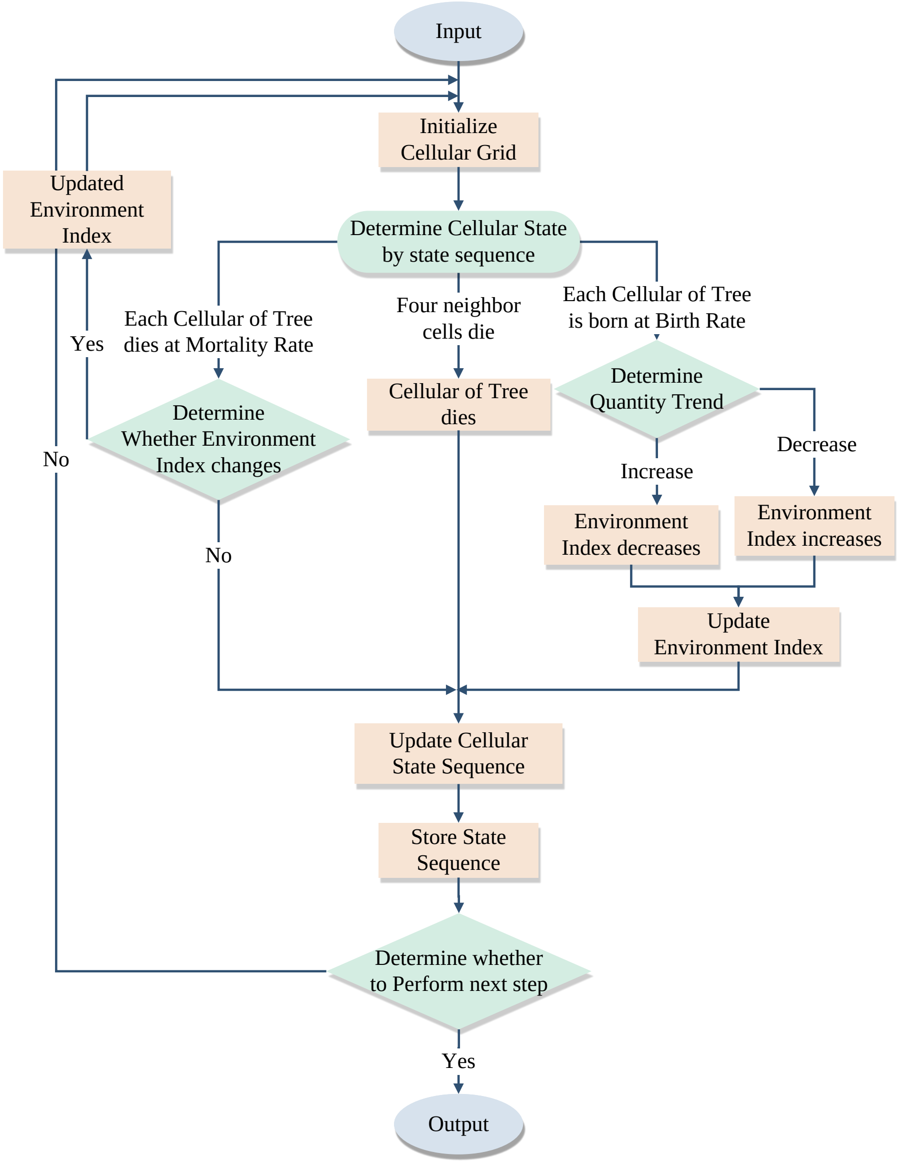


Figure 4.1: Block Diagram of typical Cellular Automata

The block diagram of a typical cellular automata is shown in Figure 4.1, and the simulation result is shown in Figure 4.2, in which we learn that the under the natural circumstances, the forestry population size will reach and remain around 2100 per hectare.

4.2 Binary Timber Volume and Logistic based Carbon Sequestration Model

In order to clearly demonstrate the amount of carbon sequestered by forests and their products, we decide to consider the trunk volume of living trees as an estimation of total Volume of trees that are able to store carbon (Yan & Xing'e, 2009).

For the living trees, we apply Binary Timber Volume Regression model (Qibang & Hui, 1992) to depict their growth as shown in Algorithm 1. It is significant to point out that β_b and β_c are two **Conversion Factors** from Timber Volume to Carbon Mass sequestered. Besides, the mass of carbon sequestered in soil and leaves can be deemed as a constant which merely depends on the area of coniferous and broad-leaved forests (Deren, 2011).

As for the wooden products, we consider w kinds of wooden products, each kind, which decomposes at the rate of $m_i(i = 1, \cdots, w)$, accounts for ϕ_i of the total product mass (Birdsey et al., 2006). Note that the **harvest rate** λ and the product proportion ϕ are flexible regarding the circumstances, which are also the core of our forest management. And the harvest will be conducted every p years, which is **Rotation**.

What's more, we hold the view that the volume of wooden products derive from the volume of harvested trees with a certain **Scrap Rate** s. Suppose the harvested wood is stored in a well-organized warehouse and the storage will become products in next p years, which means $\frac{1}{p}$ of all the storage will become products every year. The complete idea is shown in Algorithm 2.

4.3 Results and Sensitivity Test

We apply our model over a hypothetic forestry land with 10000 hectares, in which the broad leaf trees takes one half and the other half is coniferous trees. Two primary decision factors, which are harvest rate and product proportion, are discussed by conducting sensitivity test. Detailed parameter settings can be found in Appendix.

The sensitivity test of Harvest Rate is conducted by calculating the Carbon Dioxide sequestered by the living trees and the wooden products every 1% of harvest rate from 1% to 30%, which can be seen in Figure 4.5(a). Compared with no harvest at all, which is denoted as red line with dots on it, Total Carbon Sequestration under **the Harvest Rate of 0.03** ranks higher after 90 years , reaching **256.83 tons of Carbon Dioxide per hectare per year.** Meanwhile, it's obvious that as the Harvest Rate growing, the Total Carbon Sequestration increases gently, bypassing those with lower Harvest Rate. Besides, if the Harvest Rate is too high, Total Carbon Sequestration reduces as time passes.

The sensitivity test of Product Proportion is carried out by determining the Carbon sequestered by wooden products under different types of product proportion arrangement. The proportion of products with higher Decomposition Rate increases from Type 1 to Type 5, which means in Type 5, most of the wood is consumed to manufacture high Decomposition Rate products.The detailed proportion is shown in Appendix Table .1 and the result is shown in Figure 4.5(b). It is apparent that with Harvest Rate unchanged, the higher the proportion of products with low Composition Rate, the higher Carbon is sequestered. **Type 5** leads to around **140 tons of Carbon Dioxide Sequestration per hectare per year.**

Algorithm 1 Binary Timber Volume Regression of Carbon Prediction Algorithm

```
Input: Measurement of DBH, Height of tree ($h$), conversion factor $\beta_c$ (coniferous) and $\beta_b$ (broad leaf), Harvest rate $\lambda(t)$, Carrying Capacity $K$, Rotation year $p$
for x in enumarate $(\mathrm{DBH}, \mathrm{h})$ do
    for k $\leftarrow 1$ to $m$ do $P_k(x) = \frac{1}{2^k k!}\frac{d^k}{dx_k^k}(x^2-1)^k$
        for $l \leftarrow 1$ to $m$ do
            Calculate $\int_{-1}^{1} P_k(x)P_l(x)dx \leftarrow$ Integral of Legendre Polynomials
            if $k = l$ then$\int_{-1}^{1} P_k(x)P_l(x) = \frac{2x}{2i+1}$
            else$\int_{-1}^{1} P_k(x)P_l(x) = 0$
            end if
        end for
    end for
end for
Find $(\mathrm{DBH}, \mathrm{h})^{\mathrm{T}}$ to minimize $J = \int_a^b [f(\mathrm{DBH}) + \mathrm{g(h)} - 2\mathrm{(x)}]^2 \mathrm{dx}$
for $t \leftarrow 1$ to $n$ do
    if $t \mod p \equiv 0$ then
        Regression of DBH and $h$
        $V_0 \leftarrow$ Initiate
        $V_t = a\mathrm{DBH^b h^c} \times \mathrm{Area} \times \mathrm{Density}(\lambda, \mathrm{t})$
    end if
end for
$Area = Area_b + Area_c$
$\mathrm{CarbonMass_f} = (\beta_c V_c + \beta_b V_b) + 0.02 \times Area_c \times \frac{\mathrm{Density}(\lambda,\mathrm{t})}{K} + 0.001 \times Area_b \times \frac{\mathrm{Density}(\lambda,\mathrm{t})}{K} + 70 \times Area$
Output: Carbon Dioxide Quantity of Forest = $\frac{44}{12} \times \mathrm{CarbonMass_f}$
```

Algorithm 2 RBF Neural Network Fitting of wooden products for carbon sequestration Algorithm

```
Input: $\phi, s, m, w$ as kinds wooden products, Standardized CarbonMass and $V$ in Algorithm1, Rotation year $p$
for $t \leftarrow 1$ to n do
    for $i \leftarrow 1$ to hidden_dim do
        $y_0 \leftarrow Initiate$
        $\hat{y_t} = \hat{y_{t-1}} + \phi_{it} V_i(t)$
        $c_i \leftarrow$ Sample CarbonMass
        $\sigma_i \leftarrow$ Z-score Normalization
        $V_i(t) = e^{-\frac{||t-c_i||^2}{2\sigma_i^2}}$
    end for
    $M_0 \leftarrow Initiate$
    $c = t - (t \mod p) + 1$
    for $j \leftarrow 1$ to $w$ do
        $M_t = M_{t-1} + (1-s)\lambda \times \frac{1}{p}(\beta_c V_c(c) + \beta_b V_b(c)) \times \phi_j(t) m_j(t)$
    end for
end for
Output: Carbon Dioxide Quantity of Wooden Products = $\frac{44}{12} \times M_t$
```

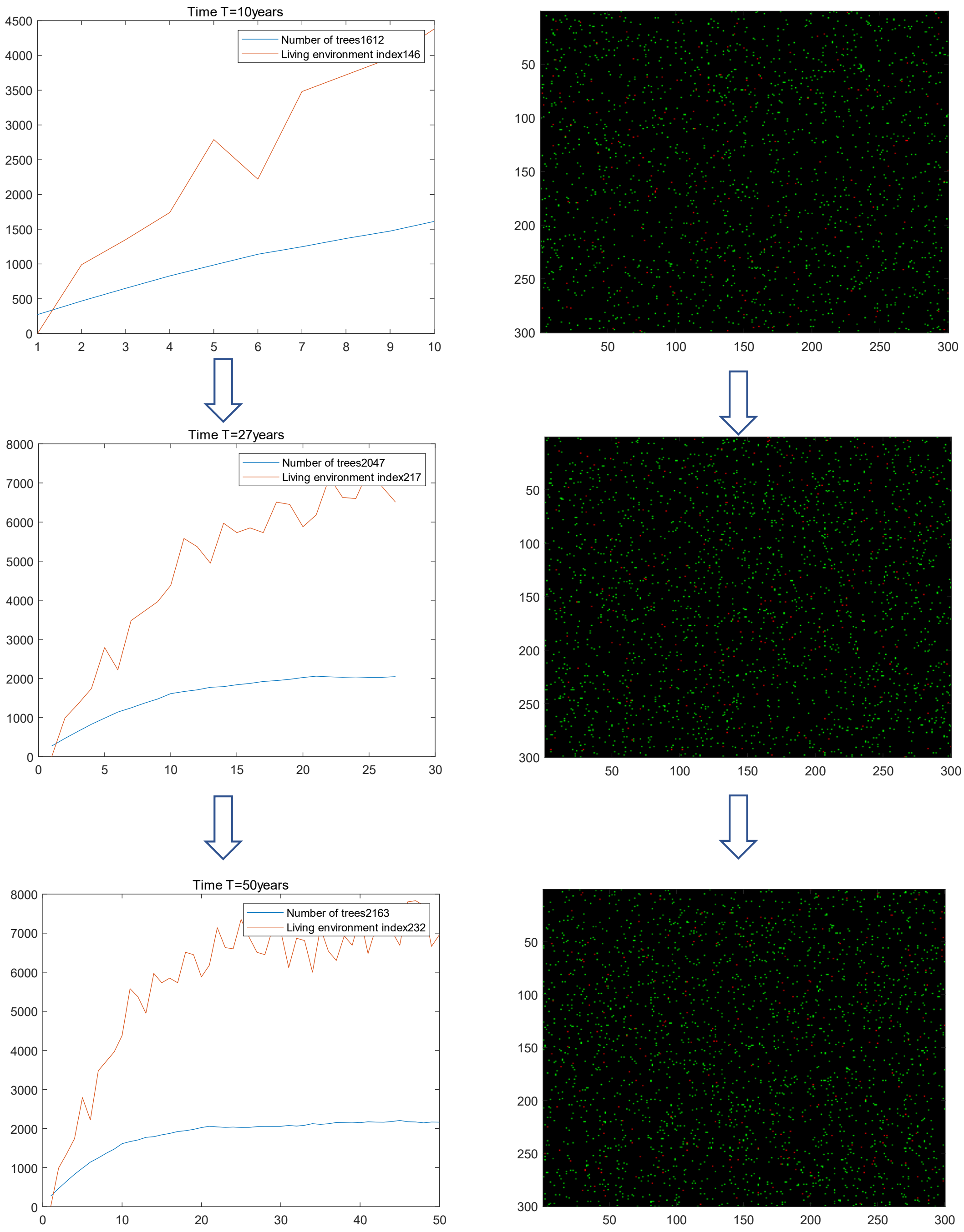


Figure 4.2: Result of Cellular Automation

4.4 Suggestions of Forest Management

From what we have discussed above we can draw a very reasonable conclusion that there exists an appropriate Harvest Rate, which is 0.03 in our hypothetic Forest, that can help the Forest sequester more Carbon Dioxide after 100 years, and the Forest managers should urge more manufactures of anticorrosive wooden products rather than those with lower Decomposition Rate.

What's more, there are some other factors we should take into consideration when managing the forests. For example, the law mandates several formulae to be applied to calculate the minimum diameters at which species can be harvested sustainably (Lumet et al., 1993). Besides, data availability,

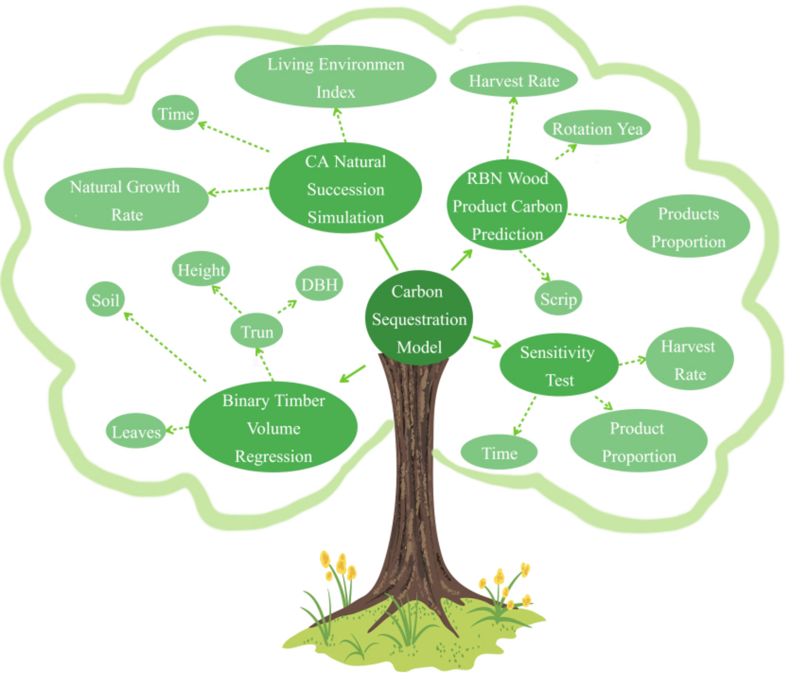


Figure 4.3: Mind Map of Carbon Sequestration Model

The Scrap Rate is determined as following descriptions, which is also shown in Figure 4.4 on the right. There are two manufacturing methods of Round Wood harvested from the forests.

One way is to trim the wood neatly into slabs in different shapes so that they can occupy most of the volume of the wood, which will be processed into softwood plywood, panels and board. Meanwhile, the scrap will be manufactured into miscellaneous products. It can approach the utilization rate of 81.3%.

The other kind of craft is to separate the bark and turn it into paper, and the separated log can be processed into softwood and hardwood. The utilization rate of this craft is able to reach almost 100%.

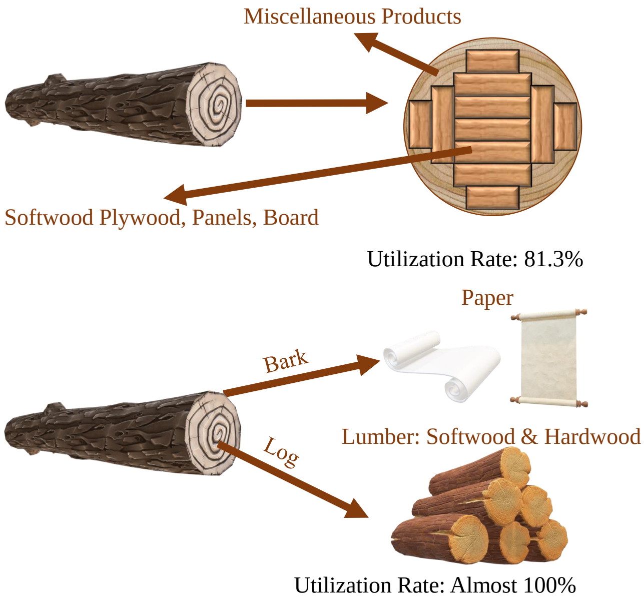
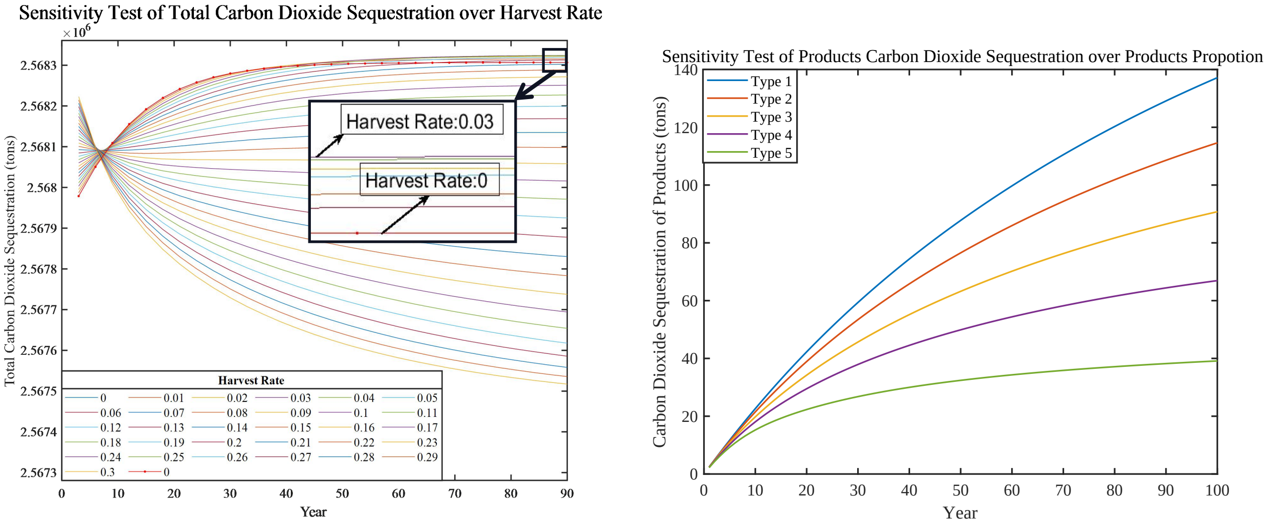


Figure 4.4: Utilization Rate of Round Wood

checking and assessment could help countries to plan and manage their forests, by giving concerned ministries the capacity to plan ahead, react to market shocks, help companies when needed (possibly by also playing with costs and tax burdens) - or even sanctioning them when parameters are not in line with the expected values - and eventually better managing the overall forest sector (Cerutti et al., 2017).



(a) (b)

Figure 4.5: Results of sensitivity test of Harvest Rate and Product Proportion

5 Model II

In order to comprehensively consider the multiple social interests, we add several indexes which can fully demonstrate the impact of Forests over local society.

The indexes are divided into two aspects, namely **Ecological Value** and **Cultural Value**, and Ecological Value weighs **4.123** times than Cultural Value (Smith et al., 2007).

5.1 Ecological Value

Figure 5.1 clearly demonstrates the principle of the carbon circle and roles that Forestry Species play in the circle, which helps us to understand the necessary procedure to manage Forestry Carbon Sequestration.

Ecological Value includes the following indexes: **Carbon Credit Benefit and Cost Analysis(CVal), Wooden Product Proportion, Harvest Rate** and **Interspecific Competition**, which will exert influence on ecological value in this Forest.

CVal is a spreadsheet tool to evaluate the direct benefits and costs of Carbon Sequestration contracts for managed forests developed by E.M. (Ted) Bilek, Peter Becker and Tim McAbee. By inputting a series of parameters of the Forests such as tract size, CO_2 sequestration rate and Carbon price, CVal will automatically come up with Average cost of trading, Net trading benefit, Internal Rate of Return and other important economic indexes.

Wooden Product Proportion and Harvest Rate are fully discussed above, which will not be repeated again.

Interspecific Competition is calculated by **Lotka-Volterra Model**, which is used to describe ecology evolution. Suppose there are two species competing for surviving spaces, namely Douglas Fir and Pinus Densiflora, whose population sizes are N_1 and N_2, population growth rates are r_1 and r_2, carrying capacity K_1 and K_2 and coefficients of competition over each other are α and β. Thus, based on the Logistic growth model, the population of Douglas Fir can be described as

$$\frac{dN_1}{dt} = r_1 \cdot N_1 \cdot (\frac{K_1 - N_1 - \alpha N_2}{K_1})$$

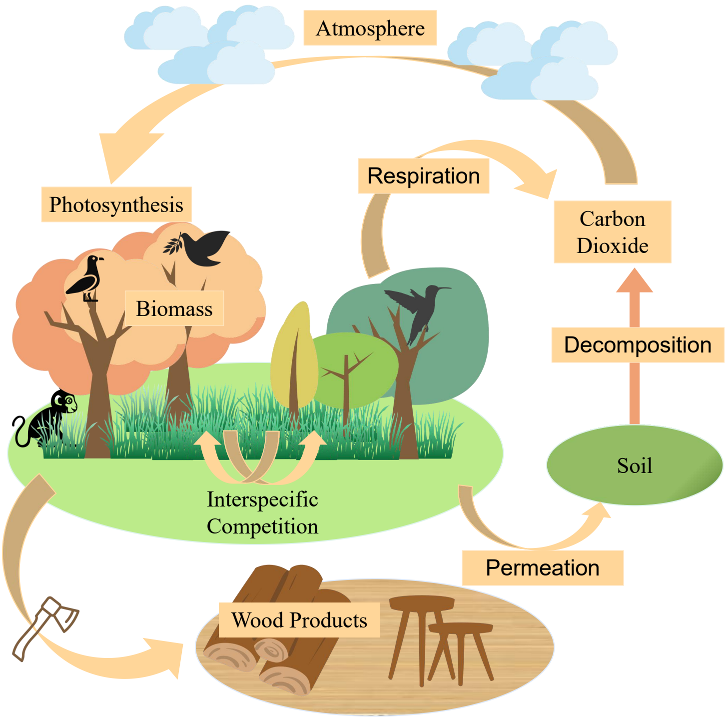


Figure 5.1: Carbon Circle in Forest Ecosystem

, in which α is defined as a coefficients of competition of Douglas Fir over Pinus Densiflora, representing that each Pinus Densiflora will occupy the surviving spaces of α Douglas Firs. Similarly, the population of Pinus Densiflora can be described as

$$\frac{dN_2}{dt} = r_2 \cdot N_2 \cdot (\frac{K_2 - N_2 - \beta N_1}{K_2})$$

Thus, there are four possible results for these two kinds of trees.

(1) Douglas Fir wins, and Pinus Densiflora gets sidelined;

(2) Pinus Densiflora wins, and Douglas Fir gets sidelined;

(3) Douglas Fir and Pinus Densiflora coexist stably;

(4) Douglas Fir and Pinus Densiflora coexist erratically.

Whichever wins, the intensity of intraspecific competition will decline while the intensity of interspecific competition will roar. If both species suffer high level of intraspecific competition as well as low interspecific competition, they will stably coexist, otherwise it leads to unstable coexistence.

Lotka-Volterra Model can be improved as follows. Suppose a second-order system can be described as the ODE below:

$$\frac{d^2x}{dt^2} + a_1(x, \frac{dx}{dt})\frac{dx}{dt} + a_0(x, \frac{dx}{dt})x = 0, \quad \ddot{x} = f(x, \dot{x})$$

Suppose $x = x_1$, $\frac{dx}{dt} = x_2$, then

$$\frac{dx_2}{dx_1} = \frac{f(x_1, x_2)}{x_2}$$

$$\begin{cases} \frac{dx_1}{dt} = x_2 \\ \frac{dx_2}{dt} = f(x_1, x_2) \end{cases}$$

Suppose $x_1(t)$ denotes the position of a mass point and $x_2(t)$ denotes the velocity of the mass point, then the solution of the equations depicts the movement of a point in a 2-dimension plane, and the figure of the solution is called **Phase Diagram** in physics.

Apply what we discuss above over the previous equations we can draw the improved Model:

$$\begin{cases} \frac{dr}{dt} = 2(1 - \frac{r}{R}) - \lambda r f, r(0) = r_0 \\ \frac{df}{dt} = -f + \lambda r f, f(0) = f_0 \end{cases}$$

Result of the competition between Douglas Fir and Pinus Densiflora is shown in Figure 5.2, while the thermodynamic diagram of competition intensity between these two species can be seen in Figure 5.3. In Figure 5.2 we can see clearly that there are two possible results of the interspecific competition. One is that they co-exist peacefully with their population stabilized, while the other is that they compete fiercely, whose population fluctuate dramatically.

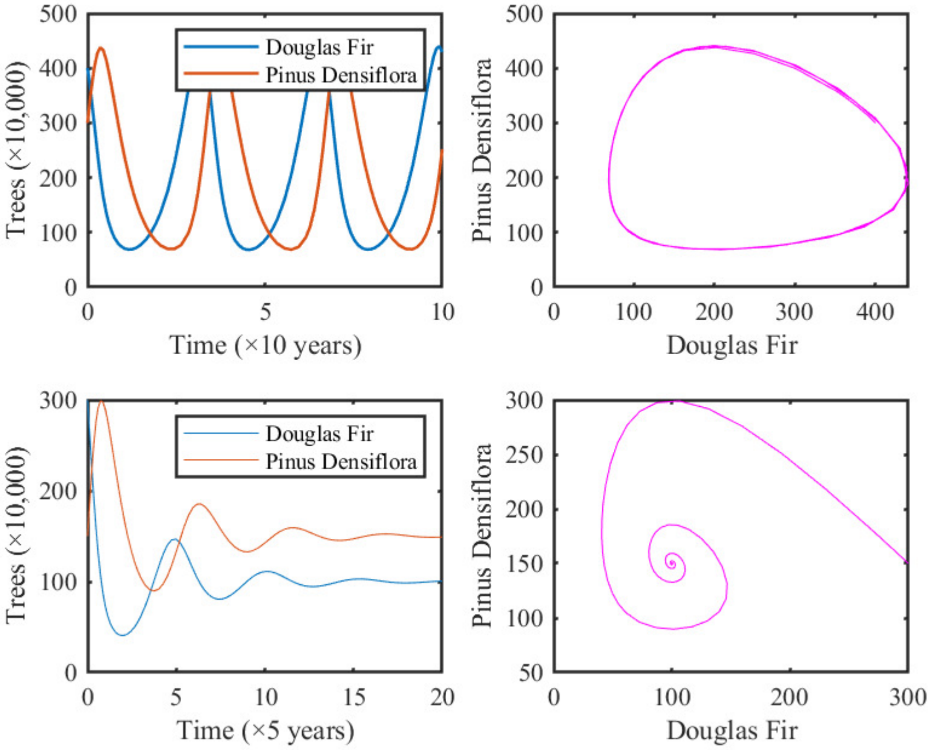


Figure 5.2: Interspecific Competition Result of Douglas Fir and Pinus Densiflora

5.2 Cultural Value

In order to further demonstrate the relationship between the five indexes and Cultural Value, we abstract three concepts, namely Biodiversity, Development of Forest and Forest Influence, from the indexes. The structure of indexes related to Cultural Value can be clearly drawn from Figure 5.4, in which Proportion of Forest and Size of Forest signify the population the Forest can influence, and Volume of Deadwood ,"an element of biodiversity and, to some extent, an indicator of forest

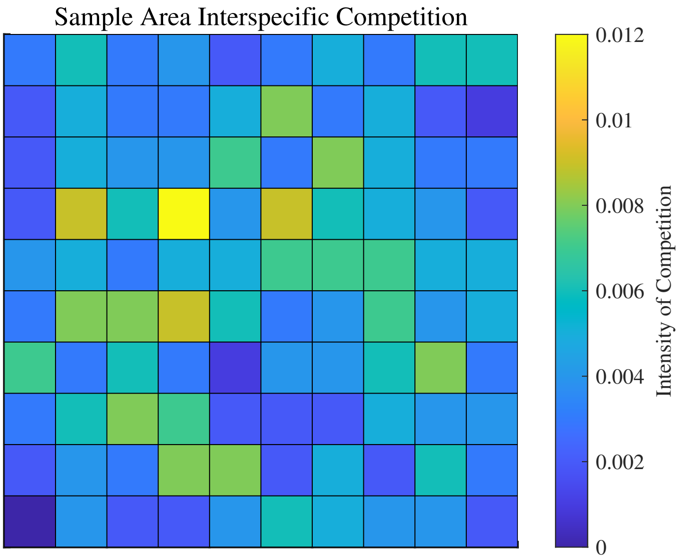


Figure 5.3: Thermodynamic Diagram of Competition

management sustainability that deserves to be taken into account in inventories, especially at the national level"(Rondeux & Sanchez, 2010), along with the Forest Hierarchy, which provides arboreal animals with more habitats, determines the Biodiversity of the Forest. The Volume of Deadwood of the same type of forest at a similar latitude shares a similar constant (Smith et al., 2007). Size and Proportion of Forest can be searched and estimated on map websites, while the Forest Hierarchy is easy to search.

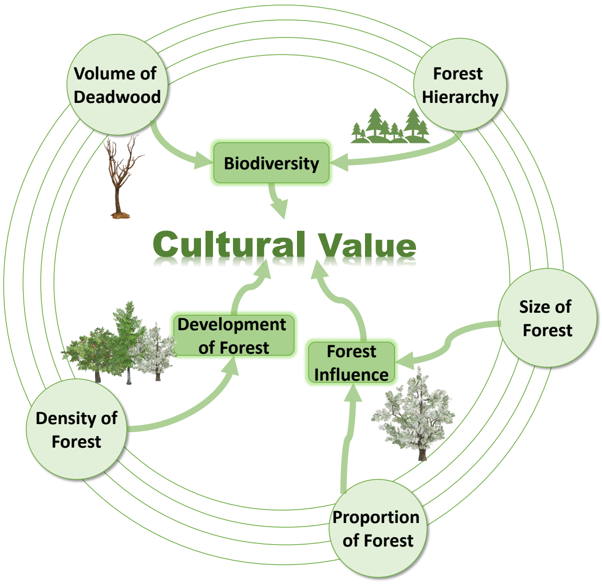


Figure 5.4: Influencing indexes of Forest Culture Value

5.3 Result and Discussion

According to Forestry region distribution of the United States in Figure 5.5, we choose four typical kinds of Forests to test our evaluation model, which are Theropencedrymion, Subboreal coniferous forest, Subtropical evergreen broad-leaved forest and Boreal forest. We search and calculate the index values of these four typical Forests, and come up with their ultimate score by the method below.

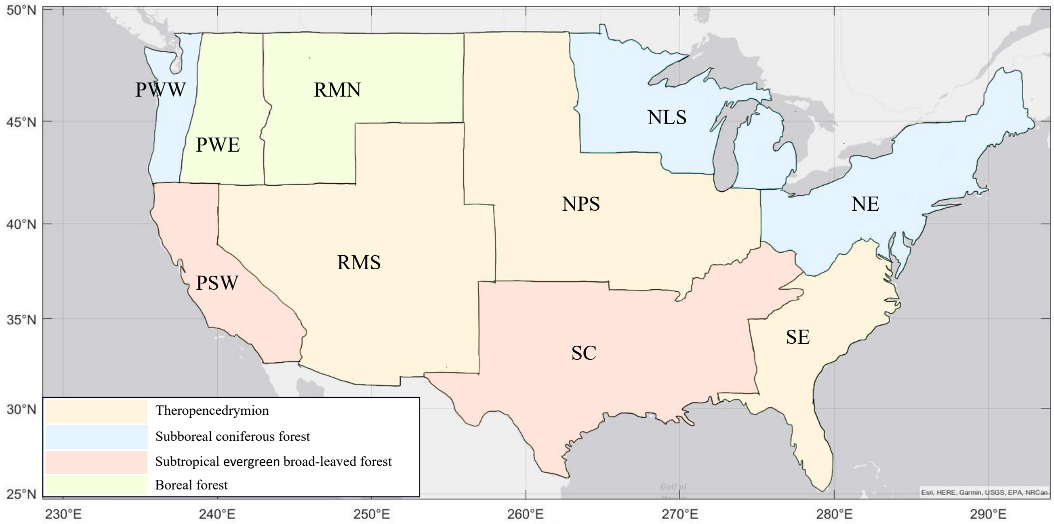


Figure 5.5: Forest division in the United States

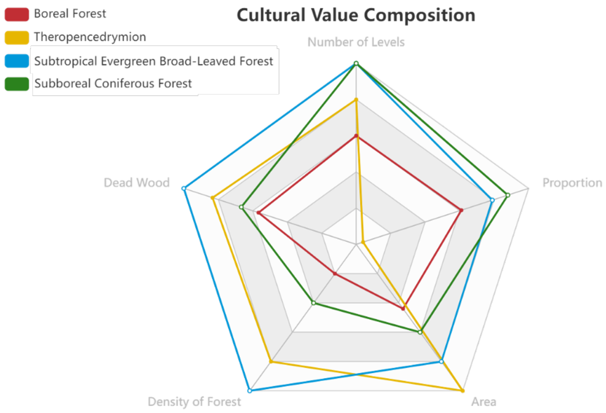


Figure 5.6: Radar Map of Cultural Value

The indexes of Ecological Value are **Z-score Standardaized** and weighted summed, representing Ecological Value score of this kind of Forest. The indexes of Culture Value are measured by **Technique for Order Preference by Similarity to an Ideal Solution (TOPSIS)**, whose basic principle is to sort the indexes by detecting the distance between the evaluation object and the worst solution of the optimal solution. If the evaluation object is closest to the optimal solution and furthest away from the worst solution, it is the best. Otherwise, it is not optimal in which all index values of the optimal solution reach the optimal value of all evaluation indexes and all index values of the worst solution reach the worst value of all evaluation indexes. The composition of Cultural Value is shown in Figure 5.6.

Then the scores of Ecological Value and Cultural Value are weighted added:

$$\mathrm{TotalScore} = \frac{4.123}{5.123} \times \mathrm{CarbonSink} + \frac{1}{5.123}\mathrm{CulturalValue}$$

The final scores are shown in Table 5.1, in which we can see that the Subtropical Evergreen Broad-leaved Forest ranks the highest concerning the indexes above. There are some other interesting findings in this table. For instance, both Theropencedrymion and Subboreal coniferous Forest approach high score in Ecological Value while their Cultural Values are relatively low. However, Boreal Forests are of great significance in Cultural Value, but it doesn't help much on creating Ecological Value.

In order to figure out the conditions when Harvest Rate is 0 is recommended, we calculated the final scores of the four Typical Forests again without Harvest, which is shown in Table 5.2. In comparison

Table 5.1: Scores of Four typical Forests

Types of Forests	Total Score	Ecological Value	Cultural Value
Boreal Forest	0.58902	0.535529	0.809565
Theropencedrymion	0.832925	0.892329	0.588001
Subtropical Evergreen Broad-leaved Forest	0.993638	0.992332	0.999022
Subboreal Coniferous Forest	0.863477	0.932925	0.577142

with Table 5.1, it's obvious that the all the three scores of Boreal Forest, Theropencedrymion and Subboreal coniferous Forest increase without harvest.

What's more, Theropencedrymion suffers severe Interspecific Competition without harvest; CVal of Boreal Forest increases dramatically without harvest; Subboreal coniferous Forest will rank higher in Culture Value if it's not harvested at all.

Table 5.2: Scores of Four typical Forests without Harvest

Types of Forests	Total Score	Ecological Value	Cultural Value
Boreal Forest	0.704714	0.678393	0.813238
Theropencedrymion	0.846885	0.908237	0.593933
Subtropical Evergreen Broad-leaved Forest	0.98369	0.986493	0.972135
Subboreal Coniferous Forest	0.917332	0.947289	0.79382

By further observing index values we can obtain several conditions in which banning harvest is beneficial to in a general basis:

1. Forests with **low CVal**, such as Boreal Forests;
2. Forests with **small Deadwood Volume**, such as Subboreal Coniferous Forest;
3. Forests with **low Forestry Density**.

5.4 Forestry Management Recommendations

6 Application of Model

So as to further reveal future Carbon Sequestration and provide managers with more accurate and scientific management plan, we test our Model on another forest abstracted from the forest in Washington State, which consists of Douglas Fir and Pinus Densiflora. Assume that their Height and DBH are the same and they occupy 10000 hectares in total.

6.1 Autoregressive Integrated Moving Average model

We apply ARIMA model in order to predict the Coefficients of Competition between Douglas Fir and Pinus Densiflora whose **algorithm block diagram** is shown as Figure 6.1.

Under regular circumstances, the time series we obtain in the real world has tendency, seasonality and non-stationarity. Thus, it's vital for us to transfer the non-stationary time series to stationary time series and make an assumption that the time series is an Auto Regressive Moving Average(ARMA) series to predict the future data. ARMA series is defined as follows.

$$X_t - \phi_1 X_{t-1} - \cdots - \phi_p X_{t-p} = \epsilon_t - \theta_1 \epsilon_{t-1} - \cdots - \theta_q \epsilon_{t-q}$$

ϵ_1 is a stationary white noise whose average is zero and deviation is σ_ϵ^2;$X_t$is an ARMA series with p and q degree, recorded briefly as $\textbf{ARMA}(p,q)$ series. Akaike Information Criterion(AIC) is one of the most commonly used criterion to determine the degree of $\textbf{ARMA}(p,q)$: choose p,q such that

$$\min \textbf{AIC} = n \ln \hat{\sigma}_\epsilon^2 + 2(p+q+1) \tag{6.1}$$

n is the capacity of sample;$\hat{\sigma}_\epsilon^2$ is the estimation of σ_ϵ^2 relating to p and q. Suppose $p=\hat{p}, q=\hat{q}$, such that equation(6.1) reaches the minimum, than we deem the series is $\textbf{ARMA}(\hat{p},\hat{q})$.

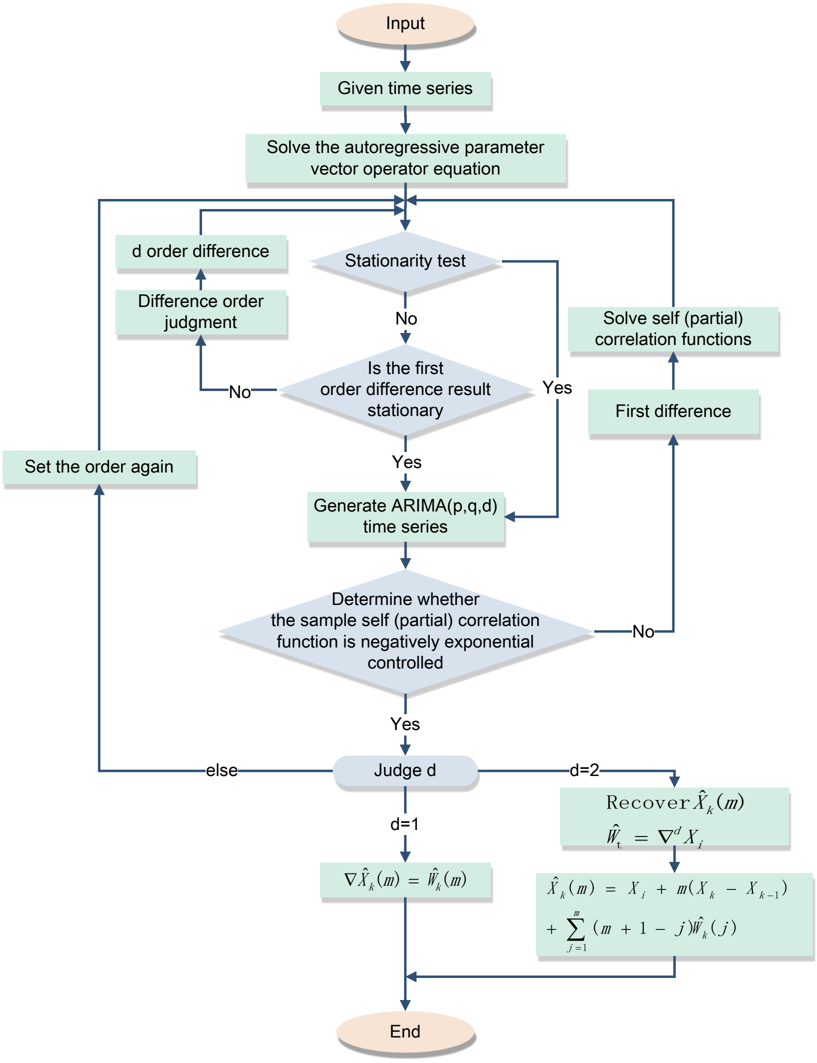


Figure 6.1: Algorithm Block of ARIMA

Suppose $\textbf{ARMA}(p,q)$ series has an unknown average parameter μ, the model becomes

$$\phi(B)(X_y - \mu) = \theta(B)\epsilon_t,$$

meanwhile, the number of unknown parameters is $k = p+q+2$, the AIC is: choose p,q such that

$$\min \textbf{AIC} = n \ln \hat{\sigma}_\epsilon^2 + 2(p+q+2). \tag{6.2}$$

In fact, equations (6.1) and (6.2) have the same minimum point $\hat{p}, \hat{q}$. After that, we usually choose $p=1, q=1$ to make parameter estimation over ARMA model.

It's demonstrated that the differential operation can stabilize certain class of non-stationary series. And It's emphasized that stationary test must be conducted previously. Stationary test can be applied by calculating sample autocorrelation function and sample coefficient of partial function.

If the functions are truncated or trending to 0 (meaning being controlled by negative index), than the series belongs to ARMA model.

If at least one of the functions above is not truncated or trending to 0, than it's not stationary.

Suppose the series is non-stationary, which can be transformed to a stationary series by d -degree differential operation, denoted as $\mathbf{ARIMA}(p, q, d)$ series,than differentiate the sample by d -degree:

$$W_t = \nabla^d X_t, \quad t = d + 1, \ldots, n$$

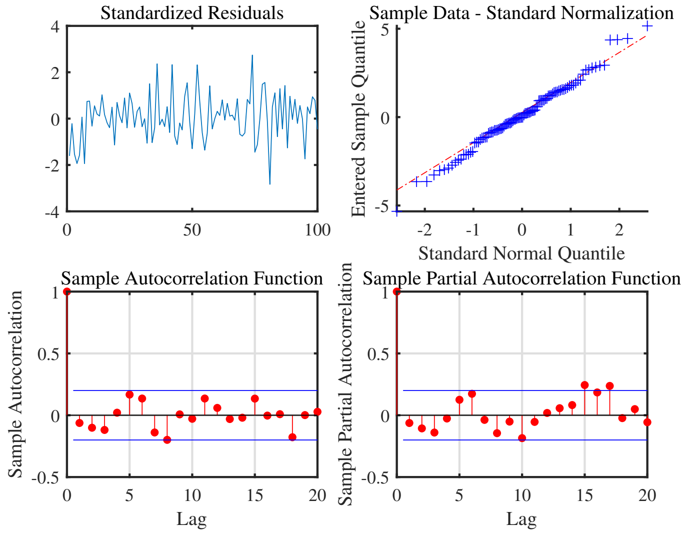


Figure 6.2: Residual, Coefficient of autocorrelation and cross correlation of ARIMA

After that, apply stationary test on W_t and repeat steps above until it becomes a stationary series, Than W_t(which is denoted as X_t) complies ARMA model.
From the result of ARIMA we can see that the predicted Coefficient of Competition between Douglas Fir and Pinus Densiflora α and β is $\mathbf{12.78}$ and $\mathbf{11.23}$ in the future 100 years. Residual, Coefficient of Autocorrelation and Cross Coefficient of ARIMA is shown in Figure 6.2.

6.2 Results and Sensitivity Test

Under the Rotation year $p = 3$, we combine our first Model of Carbon Sequestration with Lotka-Volterra Model to further simulate the change rule of Carbon Sequestration over 100 years. As is shown in Figure 6.3, the Carbon Sequestration will approach $\mathbf{2.613} \times \mathbf{10^6}$ tons under the Harvest Rate of 0.05.

To explore the error between our Model and the real statistics, we apply i-Tree, which is a state-of-the-art, peer-reviewed software suite from the USDA Forest Service that provides urban and rural forestry analysis and benefits assessment tools. The i-Tree tools can help strengthen forest management and advocacy efforts by quantifying forest structure and the environmental benefits that trees provide.

We input the area we select in Washington State into i-Tree with the Harvest Rate of 0.05 , and the result is shown in Table .2 in Appendix. The prediction of Carbon Sequestration from i-Tree is $\mathbf{2.35197} \times \mathbf{10^6}$ tons. The **error rate** is merely $\mathbf{8.9}\%$, meaning that our result of $p = 3$ and Harvest Rate is 0.05 is one of the best management plans.

In order to protect interests of society during the period time of transition from Harvest Rate of 3% to 13%, we test the sensitivity of Carbon Dioxide Sequestration in Rotation Year of 1, 3, 5, 10, 13,

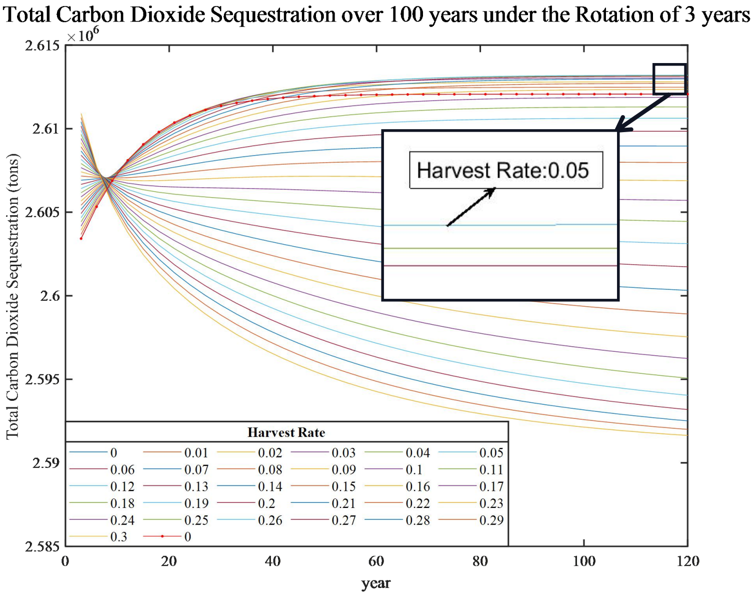


Figure 6.3: Carbon Sequestration of $p = 3$

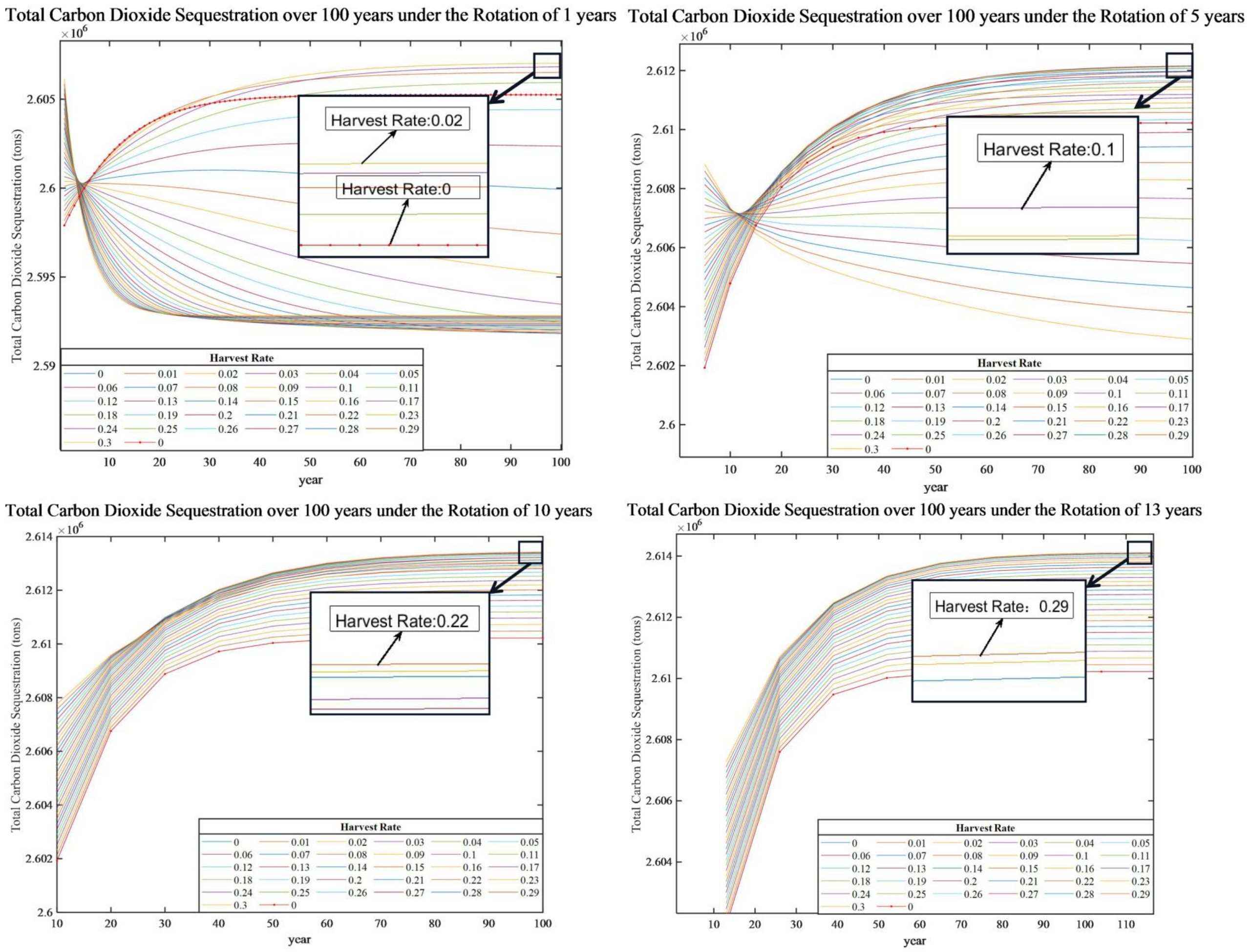
which can be seen in Figure 6.4. It's interesting that with the Rotation Years increase, the Harvest Rate under which the Forest sequesters the most Carbon Dioxide after 100 years increases, too. If the harvest is conducted every 13 years, then the Harvest Rate should be 0.29, which may cause severe change to the forest on a short-term basis, but will accumulate and lead to more Carbon sequestered.

6.3 Discussion and Management Plans

7 Evaluation of Model

Strength:

8 Conclusions


Total Carbon Dioxide Sequestration over 100 years under the Rotation of 1 years

Total Carbon Dioxide Sequestration over 100 years under the Rotation of 5 years

Total Carbon Dioxide Sequestration over 100 years under the Rotation of 10 years

Total Carbon Dioxide Sequestration over 100 years under the Rotation of 13 years

Figure 6.4: Sensitivity Test of different Rotation Year

Harvesting Trees is Good For All!

To: Relavant Authorities

From: MCM/ICM team 2215432

Date: February 21, 2022

References

Birdsey, R., Pregitzer, K., & Lucier, A. (2006). Forest carbon management in the united states. *Journal of Environmental Quality*, *35*.

Cerutti, P. O., Suryadarma, D., Nasi, R., Forni, E., Medjibe, V., Delion, S., & Bastin, D. (2017). The impact of forest management plans on trees and carbon: Modeling a decade of harvesting data in cameroon. *Journal of Forest Economics*, *27*, 1-9.

Deren, Y. (2011). A brief introduction to estimation methods of forest biomass and carbon sequestration. *Journal of Inner Mongolia Forestry*(10), 1.

Lumet, F., Forni, E., Laurent, D., & Maitre, H. (1993). Etude des modalites d'exploitation du bois en liaison avec une gestion durable des forêts tropicales humides–quatrième et dernière etude de cas: Le cameroun. *Rapport d'étude*.

Qibang, L., & Hui, N. (1992). A study on standard volume dynamic model. *Forest Research*, *5*(3), 8.

Rondeux, J., & Sanchez, C. (2010). Review of indicators and field methods for monitoring biodiversity within national forest inventories. core variable: Deadwood. *Environmental monitoring and assessment*, *164*(1), 617–630.

Smith, J. E., Heath, L. S., & Nichols, M. C. (2007). Us forest carbon calculation tool: forest-land carbon stocks and net annual stock change. *United States Department of Agriculture, Forest Service*.

Yan, W., & Xing'e, L. (2009). Advances in carbon sequestration in wood. *World Forestry Research*, *22*(1), 54-58.

Appendices

Parameter Settings of hypothetic Forest

$a = 0.00006303; D1 = 0.25; D2 = 0.35; b = 1.8218; H1 = 25; H2 = 10; c = 1.0282;$

$N1 = 5000; N2 = 5000; s = 0.001;$

$d = [0.1259842520.0314960630.2519685040.5039370080.0629921260.0157480310.007874016];$

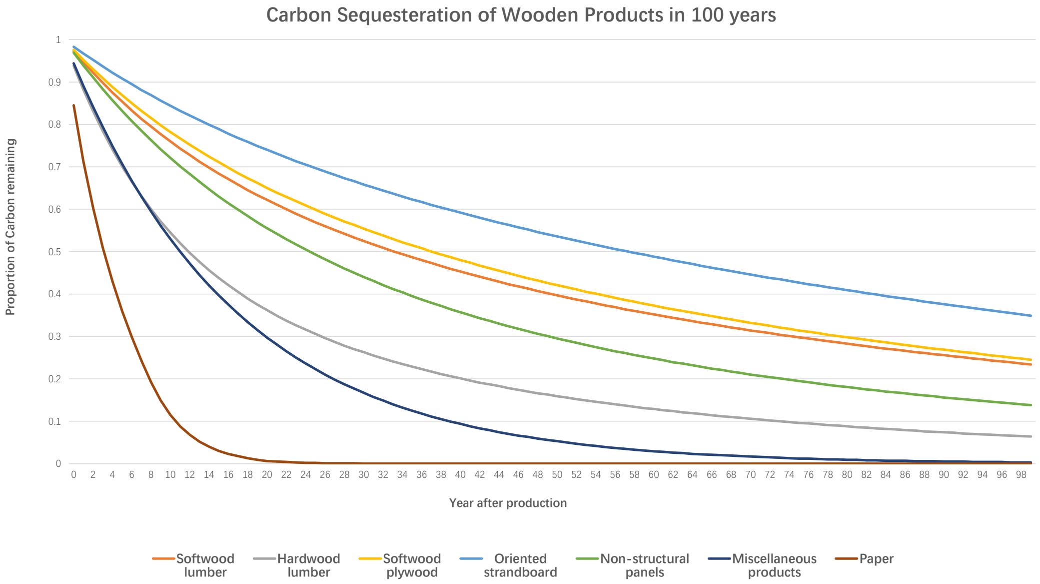


Table .1: Proportion of Products in different Types

Types of wooden production	**Type1**	**Type2**	**Type3**	**Type4**	**Type5**
Softwood lumber	0.125984252	0.178571429	0.142857143	0.107142857	0.031496063
Hardwood lumber	0.031496063	0.107142857	0.142857143	0.178571429	0.125984252
Softwood plywood	0.251968504	0.214285714	0.142857143	0.071428571	0.015748031
Oriented strandboard	0.503937008	0.25	0.142857143	0.035714286	0.007874016
Non-structural panels	0.062992126	0.142857143	0.142857143	0.142857143	0.062992126
Miscellaneous products	0.015748031	0.071428571	0.142857143	0.214285714	0.251968504
Paper	0.007874016	0.035714286	0.142857143	0.25	0.503937008

Table .2: Result of i-Tree

Year	Products(C)	Landfill(C)	Stored(C)	Energy(C)	Emissions(C)	Total(C)
0	105.543	0	105.543	48.085	29.434	288.605
10	47.065	22.474	69.539	68.35	45.174	252.602
20	30.58	27.544	58.125	73.748	51.142	241.139
30	24.084	28.868	52.952	75.524	54.54	235.968
40	19.916	29.601	49.517	76.66	56.975	232.669
50	16.846	30.15	46.996	77.299	58.72	230.011
60	14.462	30.746	45.208	77.665	60.142	228.223
70	12.627	31.206	43.833	77.848	61.334	226.848
80	11.022	31.666	42.688	78.031	62.297	225.704
90	9.826	32.078	41.905	78.031	63.123	224.963
100	8.77	32.449	41.219	78.031	63.812	224.281
Average	23.874	28.269	52.143	74.774	56.137	235.197