Problem Chosen	2022 MCM/ICM Summary Sheet	Team Control Number
E		**2215432**

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Summary

Keywords:

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1 Introduction

1.1 Problem Restatement

Considering that the carbon dioxide can be sequestered in both forests and wooden products, it's reasonable that more carbon will be stored by forests with the appropriate combination of the regrowth of younger forests and the wooden products. Thus, forest managers are ought to deliberate about the balance between the value of forests as living tress to grow and absorb the carbon and the value of forests harvested as wooden products. What's more, the forest managers should not only consider the factors about forests such as type and age of forests, geography, topography, and benefits and lifespan of forest products, but also the conservation and diversity of wild species, recreational uses and cultural considerations.

1. Design a carbon sequestration model to calculate the amount of carbon dioxide sequestered by a forest and its products, which also determines what kind of manage plan is most efficient at sequestering carbon.

2. Develop a decision model consisting of various ways that forests are valued (including carbon sequestration) for forest managers to understand the best use of a forest. Consider the following questions. What is the scope of the management plan that your decision model might suggest? Are there any conditions that the forests should not be harvested? Whether there is a transition point between management plans applicable to all forests? How can the characteristics of a particular forest and its location be used to determine transition points between management plans?

3. Apply your models to various forests. Identify a forest that your decision model would suggest the inclusion of harvesting in its management plan. How much carbon dioxide can be sequestered by this forest and its products in 100 years? What kind of forest management plan should be carried out for this forest? Why it's best? The best management plan is assumed to include a harvest interval of 10 years longer than current forest practices discussing strategies for transitioning from existing to new schedules in a manner sensitive to the needs of forest managers and all those who use forests.

4. Some people think that we should never cut down any trees, but you've determined the forest which includes harvest in its management. Write a one-to-two-page non-technical newspaper articles to explain why your analysis including in the management of the forest logging, rather than it remaining the same. Finally, your ariticle should convince local community that it's the best decision for their forest.

1.2 Overview of Our Work

2 Assumptions and Justifications

These are necessary assumptions for simplifying the model.

1.

3 Notations

Table 3.1: Notation Descriptions

Symbol	Definition
DBH	Diameter at Breast Height
β_b	Conversion factor of coniferous trees
β_c	Conversion factor of broad leaf trees
n	Total iteration years
λ	Harvest rate
M	Sequestered carbon mass in wooden products
$\mathrm{CarbonMass}_{\mathrm{f}}$	Sequestered carbon mass in forests
ϕ	Proportion in total product
s	Scrap rate
m	Decomposition rate
K	Carrying Capacity
N	Species population
r	Population Growth Rate
α, β	Coefficient of Competition

4 Model I

4.1 Cellular Automata based Population Size Prediction

Cellular automata (CA) is a kind of grid dynamics model with discrete time, space and state, and local spatial interaction and temporal causality, which has the ability to simulate the space-time evolution process of complex system.

Unlike general dynamical models, cellular automata are not determined by strictly defined physical equations or functions, but are composed of rules constructed by a series of models. Any model that satisfies these rules can be regarded as a cellular automata model. Therefore, cellular automata is a general term for a class of models, or a method framework. Its characteristic is that time, space and state are discrete, each variable only takes a finite number of states, and its state change rules are local in time and space.

The block diagram of a typical cellular automata is shown in Figure 4.1, and the simulation result is shown in Figure 4.2, in which we learn that the under the natural circumstances, the forestry population size will reach and remain around 2100 per hectare.

4.2 Binary Timber Volume and Logistic based Carbon Sequestration Model

In order to clearly demonstrate the amount of carbon sequestered by forests and their products, we decide to consider the trunk volume of living trees as an estimation of total Volume of trees that are

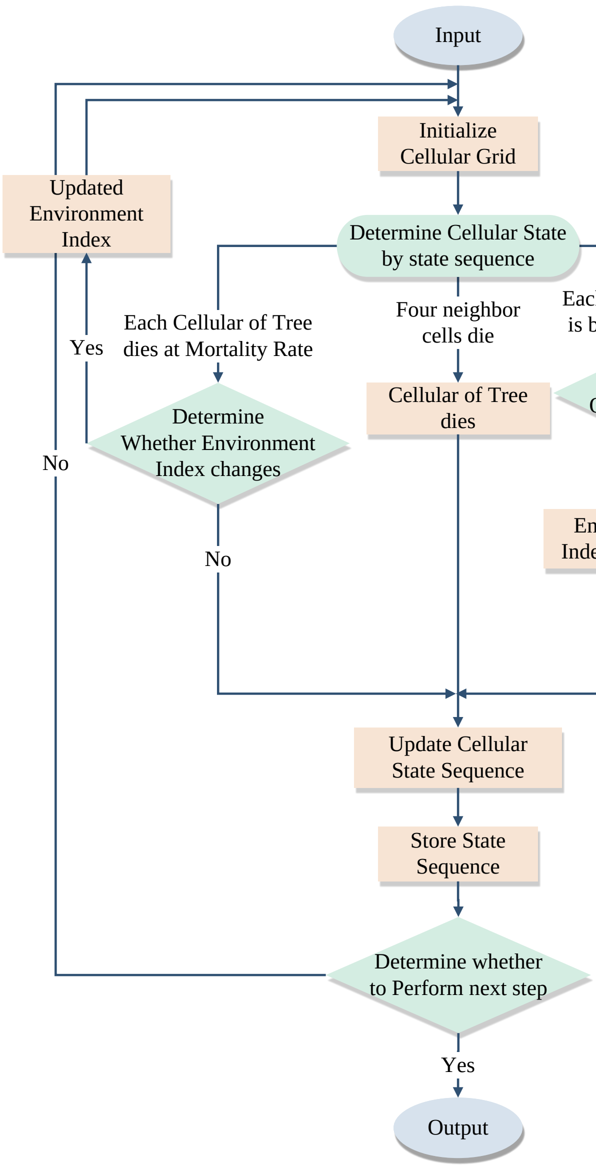


Figure 4.1: Block Diagram of typical Cellular Automata

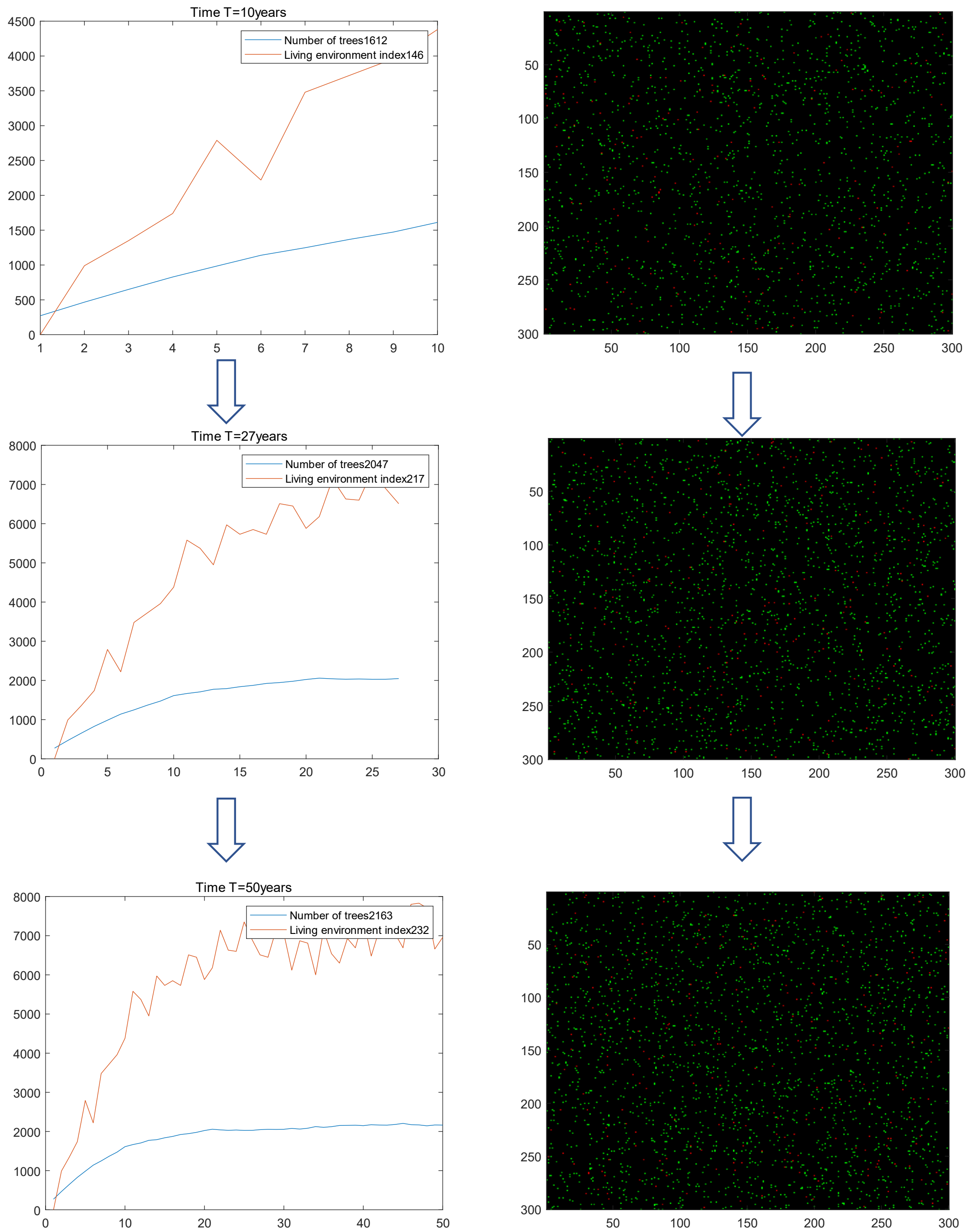


Figure 4.2: Result of Cellular Automation

able to store carbon (Wang Yan, 2009).

For the living trees, we apply Binary Timber Volume Regression model (Luo Qibang, 1992) to depict their growth as shown in Algorithm 1. It is significant to point out that β_b and β_c are two **Conversion Factors** from Timber Volume to Carbon Mass sequestered. Besides, the mass of carbon sequestered in soil and leaves can be deemed as a constant which merely depends on the area of coniferous and broad-leaved forests (Deren, 2011).

As for the wooden products, we consider w kinds of wooden products, each kind, which decomposes at the rate of $m_i (i = 1, \cdots, w)$, accounts for ϕ_i of the total product mass (Birdsey et al., 2006).

Note that the **harvest rate** λ and the product proportion ϕ are flexible regarding the circumstances, which are also the core of our forest management. What's more, we hold the view that the volume of wooden products derive from the volume of harvested trees with a certain **Scrap Rate** s. The complete idea is shown in Algorithm 2.

The Scrap Rate is determined as following descriptions, which is also shown in Figure 4.3 on the right. There are two manufacturing methods of Round Wood harvested from the forests.

One way is to trim the wood neatly into slabs in different shapes so that they can occupy most of the volume of the wood, which will be processed into softwood plywood, panels and board. Meanwhile, the scrap will be manufactured into miscellaneous products. It can approach the utilization rate of 81.3%.

The other kind of craft is to separate the bark and turn it into paper, and the separated log can be processed into softwood and hardwood. The utilization rate of this craft is able to reach almost 100%.

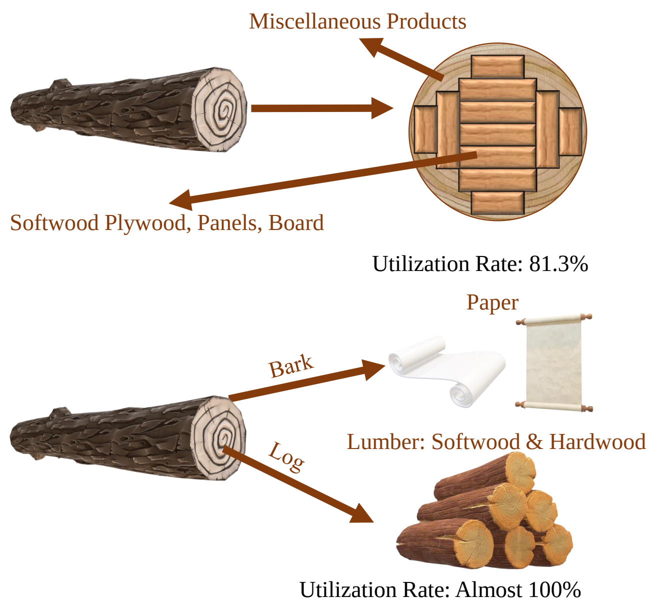


Figure 4.3: Utilization Rate of Round Wood

We apply our model over a hypothetic forestry land with 10000 hectares, in which the broad leaf trees takes one half and the other half is coniferous trees. Two primary decision factors, which are harvest rate and product proportion, are discussed by conducting sensitivity test.

The sensitivity test of Harvest Rate is conducted by calculating the Carbon Dioxide sequestered by the living trees and the wooden products every 1% of harvest rate from 1% to 30%, which can be seen in Figure 4.4(a).

The sensitivity test of Product Proportion is carried out by determining the Carbon sequestered by wooden products under different types of product proportion arrangement. The proportion of products with higher Decomposition Rate increases from Type 1 to Type 5, which means in Type 5, most of the wood is consumed to manufacture high Decomposition Rate products. The result is shown in Figure 4.4(b).

Algorithm 1 Binary Timber Volume Regression of Carbon Prediction Algorithm

Input: Measurement of DBH, Height of tree (h), conversion factor β_c (coniferous) and β_b (broad leaf), Harvest rate $\lambda(t)$, Carrying Capacity K

for x in enumarate $(\mathrm{DBH}, \mathrm{h})$ **do**
  for k $\leftarrow 1$ to m **do** $P_k(x) = \frac{1}{2^k k!} \frac{d^k}{dx_k^k}(x^2-1)^k$
    for $l \leftarrow 1$ to m **do**
      Calculate $\int_{-1}^{1} P_k(x)P_l(x)dx \leftarrow$ Integral of Legendre Polynomials
      if $k = l$ **then** $\int_{-1}^{1} P_k(x)P_l(x) = \frac{2x}{2i+1}$
      else $\int_{-1}^{1} P_k(x)P_l(x) = 0$
      end if
    end for
  end for
end for
Find $(\mathrm{DBH}, \mathrm{h})^{\mathrm{T}}$ to minimize $J = \int_a^b [f(\mathrm{DBH}) + \mathrm{g(h)} - 2\mathrm{(x)}]^2 \mathrm{dx}$
for $t \leftarrow 1$ to n **do**
  Regression of DBH and h
  $V_0 \leftarrow$ Initiate
  $V_t = a\mathrm{DBH}^{\mathrm{b}}\mathrm{h}^{\mathrm{c}} \times \mathrm{Area} \times \mathrm{Density}(\lambda, \mathrm{t})$
end for
$Area = Area_b + Area_c$
$\mathrm{CarbonMass_f} = (\beta_c V_c + \beta_b V_b) + 9.6 \times Area_c \times \frac{\mathrm{Density}(\lambda,\mathrm{t})}{K} + 3.4 \times Area_b \times \frac{\mathrm{Density}(\lambda,\mathrm{t})}{K} + 70 \times Area$

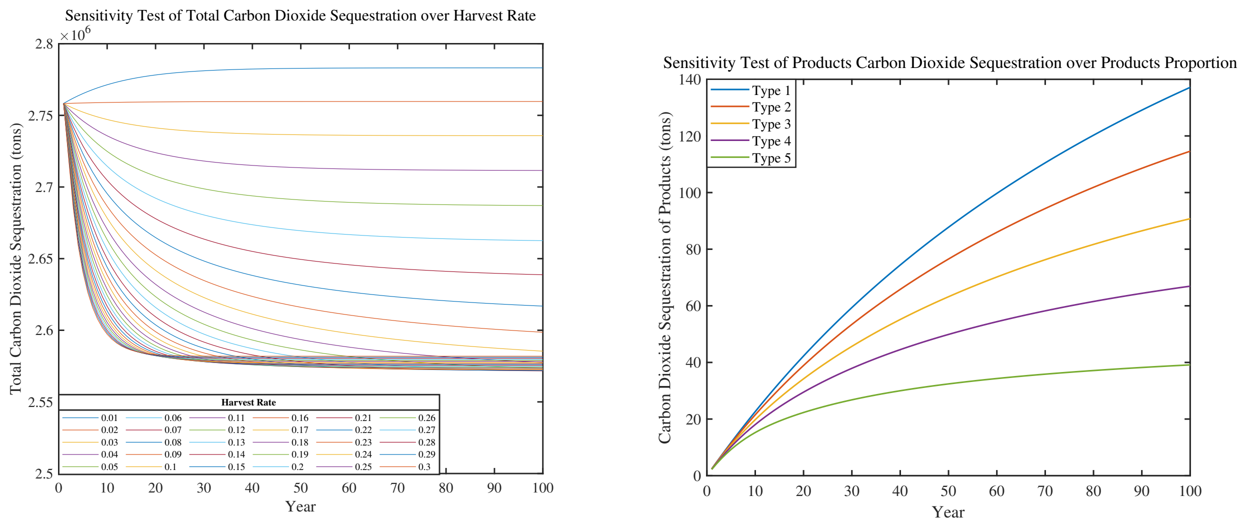
Output: Carbon Dioxide Quantity of Forest = $\frac{44}{12} \times \mathrm{CarbonMass_f}$

Algorithm 2 RBF Neural Network Fitting of wooden products for carbon sequestration Algorithm

Input: ϕ, s, m, w as kinds wooden products, Standardized CarbonMass and V in Algorithm1

for $t \leftarrow 1$ to n **do**
  for $i \leftarrow 1$ to hidden_dim **do**
    $y_0 \leftarrow Initiate$
    $\hat{y_t} = \hat{y_{t-1}} + \phi_{it} V_i(t)$
    $c_i \leftarrow$ Sample CarbonMass
    $\sigma_i \leftarrow$ Z-score Normalization
    $V_i(t) = e^{-\frac{||t-c_i||^2}{2\sigma_i^2}}$
  end for
  $M_0 \leftarrow Initiate$
  for $j \leftarrow 1$ to w **do**
    $M_t = M_{t-1} + (1-s)\frac{\mathrm{Density}(\lambda,\mathrm{t-1}) - \mathrm{Denstity}(\lambda,\mathrm{t})}{\mathrm{Density}(\lambda,\mathrm{t-1})}(\beta_c V_c(t-1) + \beta_b V_b(t-1)) \times (\phi_j(t) m_j(t))$
  end for
end for

Output: Carbon Dioxide Quantity of Wooden Products = $\frac{44}{12} \times M_t$



(a) (b)

Figure 4.4: Results of sensitivity test of Harvest Rate and Product Proportion

5 Model II

In order to comprehensively consider the multiple social interests, we add several indexes which can fully demonstrate the impact of Forests over local society.

The indexes are divided into two aspects, namely **Carbon Sink** and **Cultural Value**.

5.1 Carbon Sink

Carbon Sink includes the following indexes: **Carbon Credit Benefit and Cost Analysis(CVal), Wooden Product Proportion, Harvest Rate** and **Interspecific Competition**, which will exert influence on carbon sink in this Forest.

CVal is a spreadsheet tool to evaluate the direct benefits and costs of Carbon Sequestration contracts for managed forests developed by E.M. (Ted) Bilek, Peter Becker and Tim McAbee. By inputting a series of parameters of the Forests such as tract size, CO_2 sequestration rate and Carbon price, CVal will automatically come up with Average cost of trading, Net trading benefit, Internal Rate of Return and other important economic indexes.

Wooden Product Proportion and Harvest Rate are fully discussed above, which will not be repeated again.

Interspecific Competition is calculated by **Lotka-Volterra Model**, which is used to describe ecology evolution. Suppose there are two species competing for surviving spaces, namely Douglas Fir and Pinus Densiflora, whose population sizes are N_1 and N_2, population growth rates are r_1 and r_2, carrying capacity K_1 and K_2 and coefficients of competition over each other are α and β. Thus, based on the Logistic growth model, the population of Douglas Fir can be described as

$$\frac{dN_1}{dt} = r_1 \cdot N_1 \cdot (\frac{K_1 - N_1 - \alpha N_2}{K_1})$$

, in which α is defined as a coefficients of competition of Douglas Fir over Pinus Densiflora, representing that each Pinus Densiflora will occupy the surviving spaces of α Douglas Firs. Similarly, the population of Pinus Densiflora can be described as

$$\frac{dN_2}{dt} = r_2 \cdot N_2 \cdot (\frac{K_2 - N_2 - \alpha N_1}{K_2})$$

Thus, there are four possible results for these two kinds of trees.

(1) Douglas Fir wins, and Pinus Densiflora gets sidelined;

(2) Pinus Densiflora wins, and Douglas Fir gets sidelined;

(3) Douglas Fir and Pinus Densiflora coexist stably;

(4) Douglas Fir and Pinus Densiflora coexist erratically.

Whichever wins, the intensity of intraspecific competition will decline while the intensity of interspecific competition will roar. If both species suffer high level of intraspecific competition as well as low interspecific competition, they will stably coexist, otherwise it leads to unstable coexistence.

Result of the competition between Douglas Fir and Pinus Densiflora is shown in Figure 5.1, while the thermodynamic diagram of competition intensity between these two species can be seen in Figure 5.2.

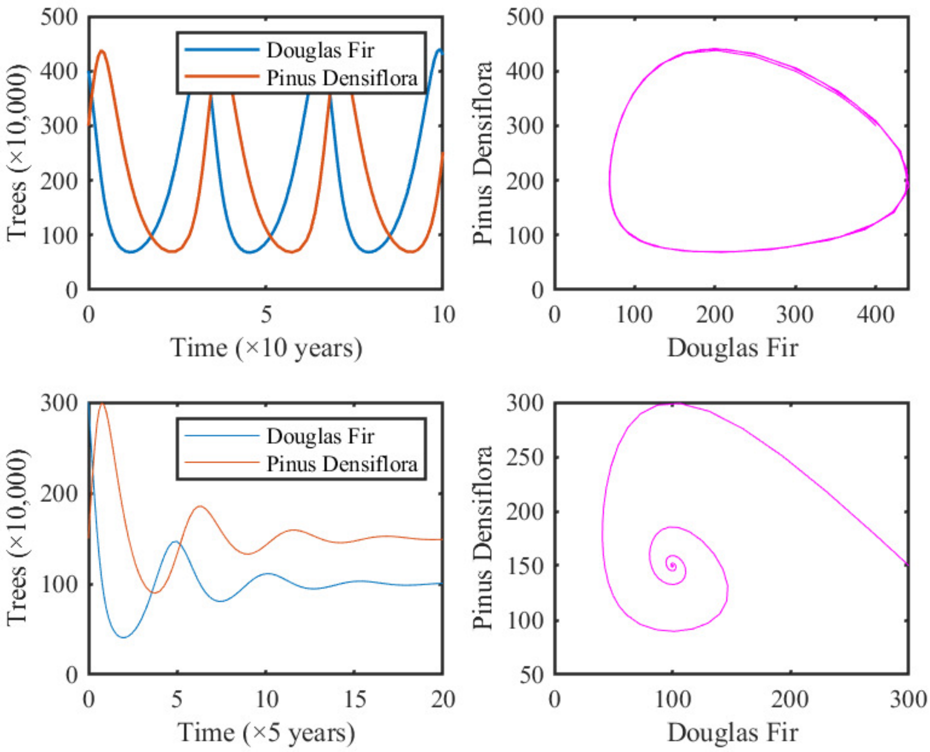


Figure 5.1: Interspecific Competition Result of Douglas Fir and Pinus Densiflora

5.2 Cultural Value

In order to further demonstrate the relationship between the five indexes and Cultural Value, we abstract three concepts, namely Biodiversity, Development of Forest and Forest Influence, from

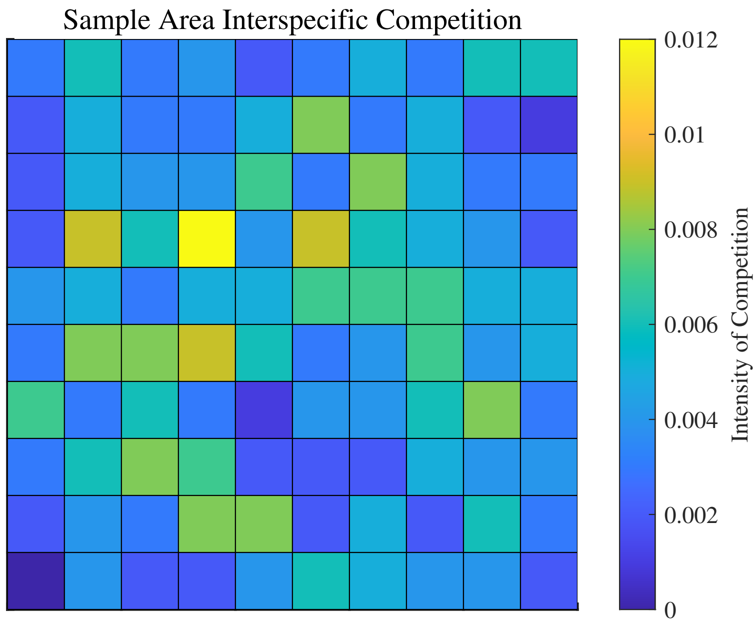


Figure 5.2: Thermodynamic Diagram of Competition

the indexes. The structure of indexes related to Cultural Value can be clearly drawn from Figure 5.3, in which Proportion of Forest and Size of Forest signify the population the Forest can influence, and Volume of Deadwood ,"an element of biodiversity and, to some extent, an indicator of forest management sustainability that deserves to be taken into account in inventories, especially at the national level"(Rondeux & Sanchez, 2010), along with the Forest Hierarchy, which provides arboreal animals with more habitats, determines the Biodiversity of the Forest.

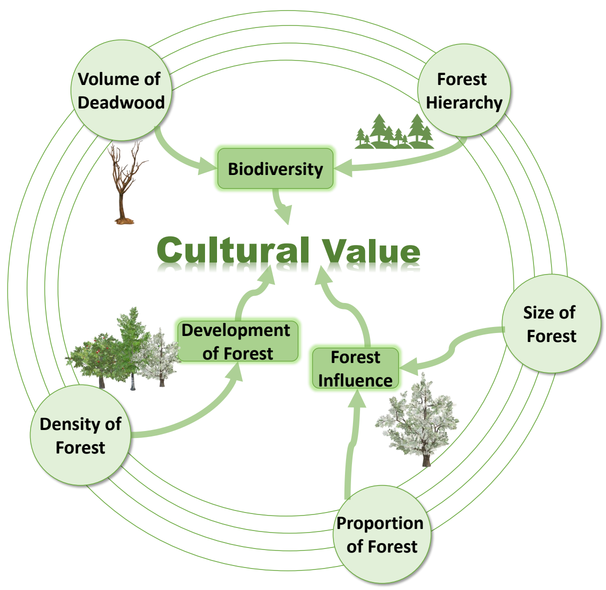
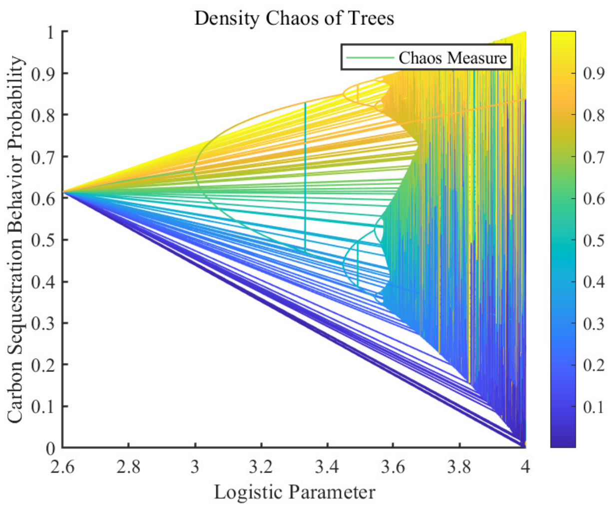


Figure 5.3: Influencing indexes of Forest Culture Value

6 Sensitivity Test

We assume that the density of the forest population follows the Logistic Model, and the **Carrying Capacity of the trees** is K. From the Carbon Sequestration probability predicted on the right we can draw a reasonable conclusion that the Logistic Model will result into chaos after 3.6 rounds of harvests.



7 Evaluation of Model

Strength:

8 Conclusions

Policy Advice on Finless Porpoise Conservation

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Date: February 20, 2022

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Appendices