Math Concepts

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November 2, 2016

Complex Numbers

$$c = a + ib = A\exp(i\phi)$$
$$i = \sqrt{-1},\ \exp(i\phi) = \cos\phi + i\sin\phi$$
$$|c| = A = \sqrt{a^2 + b^2}$$
$$\phi = \tan^{-1}(b/a)$$

Impulse Function

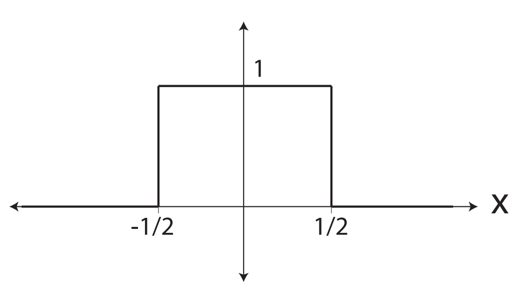
The impulse (or delta) funcion, $\delta(x)$, can be used to represent sampling of a continuous signal, and has the following properties:

1. $\delta(x) = 0$ for all $x \neq 0$
2. $\delta(x) \to \infty$ at $x = 0$
3. $\int_{-\infty}^{\infty} \delta(x)dx = 1$
4. $\delta(ax) = \frac{1}{|a|}\delta(x)$
5. $f(x)\delta(x) = f(0)\delta(x)$
6. $f(x) * \delta(x) = f(x)$

Other Functions

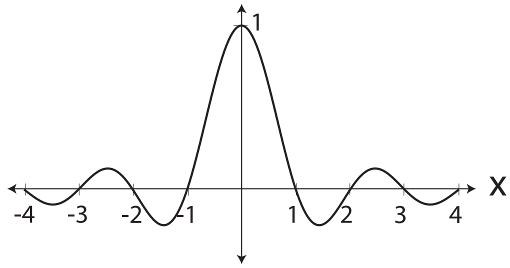
Rectangle function

$$\mathrm{rect}(x) = \sqcap(x) = \begin{cases} 1 & |x| < 1/2 \\ 1/2 & |x| = 1/2 \\ 0 & |x| > 1/2 \end{cases}$$



Sinc

$$\mathrm{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$$



Comb/Shah (Impulse Train)

$$\text{Ш}(x) = \sum_{n=-\infty}^{\infty} \delta(x - n)$$

Convolution

The convolution operation $*$, is defined as:

$$g(x) * h(x) = \int_{-\infty}^{\infty} g(x - \tau)h(\tau)d\tau$$

Fourier Transforms

The Fourier transform, $\mathcal{F}\{\cdot\}$ of a complex-valued function, $f(x)$ is:

$$\mathcal{F}\{f(x)\} = F(k_x) = \int_{-\infty}^{\infty} f(x)\exp(-i2\pi k_x x)dx$$

Inverse Fourier Transform:

$$\mathcal{F}^{-1}\{F(k_x)\} = f(x) = \int_{-\infty}^{\infty} F(k_x)\exp(+i2\pi x k_x)dk_x$$

2-D Fourier Transform

$$F(k_x, k_y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x, y)\exp(-i2\pi(k_x x + k_y y))dxdy$$

N-D Fourier Transform

$$F(\vec{k}) = \int_{-\infty}^{\infty} f(\vec{x})\exp(-i2\pi\vec{k}\cdot\vec{x})d\vec{x}$$

Duality

$$\mathcal{F}\{f(x)\} = F(k_x)$$

$$\mathcal{F}\{F(x)\} = f(-k_x)$$

$$\mathcal{F}\{F(-x)\} = f(k_x)$$

Separable
If $f(x, y) = f_x(x)f_y(y)$, then $\mathcal{F}\{f(x, y)\} = \mathcal{F}\{f_x(x)\}\mathcal{F}\{f_y(y)\}$

Transform Identities and pairs

For $\mathcal{F}\{f(x)\} = F(k_x), \mathcal{F}\{g(x)\} = G(k_x)$:

Function	Fourier Transform
$\delta(x)$	1
1	$\delta(k_x)$
$\mathrm{rect}(x)$	$\mathrm{sinc}(k_x)$
$f(ax)$	$\frac{1}{\|a\|}F(\frac{k_x}{a})$
$a \cdot f(x) + b \cdot g(x)$	$a \cdot F(k_x) + b \cdot G(k_x)$
$f(x-a)$	$\exp(-i2\pi a k_x)F(k_x)$
$\exp(i2\pi a x)f(x)$	$F(k_x - a)$
$f(x) * g(x)$	$F(k_x)G(k_x)$
$f(x)g(x)$	$F(k_x) * G(k_x)$