Advanced brain imaging methods: Tutorial 0

3rd February 2014

1 Linear algebra: warm up

1.1 Adding noise to an image

Say that for some reason you want to add normally distributed noise to an image. Given an image defined by the matrix A and the matrix B containing normally distributed noise, what would be the matrix C of the image to which this noise has been added ?

$$A = \begin{pmatrix} 136 & 37 & 45 & 158 \\ 28 & 69 & 78 & 55 \\ 227 & 75 & 196 & 170 \\ 212 & 34 & 23 & 125 \end{pmatrix} ; B = \begin{pmatrix} -1 & 6 & 2 & 12 \\ -14 & -6 & -4 & -1 \\ -3 & 1 & 3 & -15 \\ -13 & -4 & 5 & -6 \end{pmatrix}$$

1.2 Rigid body transformation: translations

The transformation matrix of an image is the matrix that allows you to say what are the coordinates [x,y,z] in world space of the voxel with indices [i,j,k] in voxel space.

What is the matrix B that will translate an image by 15 mm along the x dimension, -58 mm along the y dim and 65 mm along the z dimension?

Given the transformation matrix A below, what would be the values of the new transformation matrix once the translation performed by has been applied?

$$A = \begin{pmatrix} 5 & 0 & 0 & 125 \\ 0 & 2 & 0 & 118 \\ 0 & 0 & 1 & 10 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Given this new transformation matrix, what would be the real world coordinates of the voxel with indices [10,25,45] and for any voxel with indices [i,j,k]?

1.3 Rigid body transformation: rotations

What is the matrix C that will rotate an image by 90 degrees around the x axis (pitch)?

Given the transformation matrix A above, what would be the values of the new transformation matrix after this rotation has been applied?

$$A = \begin{pmatrix} 5 & 0 & 0 & 125 \\ 0 & 2 & 0 & 118 \\ 0 & 0 & 1 & 10 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

1.4 Rigid body transformation: order of transformation

For the point with coordinates [14,58,23], what would be its coordinates if:

1. you applied the translation defined by B then the rotation defined by C,
2. you applied the rotation defined by C and then the translation defined by B?

2 General linear model

In class we have seen how to find the equation that defines the ordinary least-square estimate of the β value of the following model:

$$y = \beta x + \varepsilon$$

For each value of x_i, the residual error (ε_i) is given by the difference between the predicted value of the model ($\hat{y}_i = \beta x_i$) and the actual empirical value (y_i). We want to minimize the sum of the squared residual:

$$\sum_{i=1}^{N} \varepsilon_i^2 = \sum_{i=1}^{N} (y_i - \hat{y}_i)^2 = \sum_{i=1}^{N} (y_i - \beta x_i)^2 = \sum_{i=1}^{N} (y_i^2 - 2\beta x_i y_i + \beta^2 x_i^2)$$

At the value of β that minimize this function, we know that its derivative with respect to β will be equal to 0. So to find β, we derived this function with respect to β, and rearranged this derivative to have the value of β expressed in terms of x and y.

Let's now try to do the same with this model:

$$y = \beta_1 x + \beta_0 + \varepsilon$$

Do this by the following steps:

1. Write down the sum of the squared residual of this model.
2. Derive this function with respect to β_1 and rearranged this derivative to have the value of β_1 expressed in terms of x, y and β_0.
3. Do the same with respect to β_0.
4. Substitute β_0 in the function found in step 2 by the solution of step 3.

3 Design matrices

The General Linear Model described by $Y = X\beta + \varepsilon$. Define the design matrices (X) of the following:

1. a two-sample t-test with 2 subjects in group A and 3 subjects in B,
2. a paired t-test with 5 subjects with two conditions a and b,
3. an ANOVA with three groups of subjects and 3 subjects in each group,
4. a repeated-measures ANOVA for 3 subjects and with three within-subject levels (a, b, c).

Here are some advices:

- Do not panic!
- For 1:
  1. write for each subject the equation that describes that subject's y in terms of:
     - the average of the group A (μ_A) times a certain weighting factor,
     - the average of the group B (μ_B) times a certain weighting factor,
     - an error term ε specific to that subject.
  2. go from the equation form to the matrix form, by putting together a) the y of all subjects, b) the weighting factors, c) the 2 averages and d) the error terms.
  3. the design matrix is the one that contains the weighting factors.
- For 2, the process will the same as for 1 but you will have to describe each subject's y in each condition in terms of:
  - the average of the condition a (μ_a) times a certain weighting factor,
  - the average of the condition b (μ_b) times a certain weighting factor,
  - as many subject specific variable (τ) as there are subjects each time multiplied by a certain weighting factor,
  - an error term specific to that subject and that condition.
- For 3 and 4, you can geneleralize by remembering that a two sample t-test is in fact an ANOVA with only two groups and that a paired t-test is in fact a repeated-measures ANOVA with only two levels.