Ex vivo Magnetic Resonance Diffusion Weighted Imaging in Congenital Heart Disease, an Insight into the Microstructures of Tetralogy of Fallot, Biventricular and Univentricular Systemic Right Ventricle

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Ex vivo Magnetic Resonance Diffusion Weighted Imaging in Congenital Heart Disease,
an Insight into the Microstructures of Tetralogy of Fallot,
Biventricular and Univentricular Systemic Right Ventricle (Tous et al, 2020)
from the Journal of Cardiovascular Magnetic Resonance Imaging. This sequence
was part of a comparison between SE monopolar, SE bipolar, STEAM monopolar,
TRSE and TRSE adjusted to scan long term (~40 years) formalin fixed specimens
with low T2 (~20 ms). The scans were performed on a 3 T Skyra bought in
the early 2010 (43 mT / m gradient amplitude, 180 mT / m / ms slew rate).

TRSE and TRSE adjusted :

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(* modified mathematica from (Mattiello et al, 1995) and (Zubkov et al, 2014) *)
Date : 22/09/2017;
Input :
Known sequence diagram of the diffusion sequence with the timing and gradients' amplitude;
Output : B matrix [ndir, 6] with ndir the number of diffusion directions,
and Bmatrix[1, :] = [Bxx, Byy, Bzz, 2 Bxy, 2 Bxz, 2 Byz];
Goal :
= > Generating the B matrix from the known sequence diagram;
= > determining M0 = F = Integral (g (t)), M1 = Integral (t*g (t)),
M2 = Integral (t^2*g (t)) for the zeroth moment, first moment, second moment;
= > Calculating velocity shift Kv = γ*M1, d = Integral (F^2),
Maxwell gradient moment = Integral (g (t)^2);

method : References :
–' Spin Diffusion Measurements : Spin Echoes in the Presence of a Time –
Dependent Field Gradient' (STEjskal and Tanner, 1965);
–' Tissue Perfusion in Humans Studied by Fourier Velocity Distribution,
Line Scan, and Echo Planar Imaging' (Feinberg, 1990);
– ' Analytical expressions for the b matrix in
NMR diffusion Imaging and Spectroscopy' (Mattiello et al, 1993);
– ' Estimation of the Effective Self Diffusion TEnsor from the NMR Spin Echo' (Basser et al, 1993);
– ' Part II Analytical Calculation of the b Matrix
in Diffusion Imaging' (Mattiello et al, 1995); = > (Mathematica);
– ' Pusled – Field Gradient Nuclear Magnetic Resonance as a tool for
studying translational diffusion : Part 1 Basic Theory' (Price 1997);
– ' The b matrix in diffusion TEnsor echo planar imaging' (Mattiello et al, 1997);
–' Reduction of eddy – current –
induced distortion in diffusion MRI using a twice – refocused spin echo' (Reese, 2003);
–' Handbook of MRI Pulse Sequences (Bernstein, King, Zhou, 2004)'
–' Double spin echo diffusion weighting
with a modified eddy current adjustment' (Finsterbusch, 2010);
– ' Efficient and precise calculation of the b matrix elements in diffusion
weigthed imaging pulse sequences' (Zubkov et al, 2014);
= > (Mathematica);
–' Orthogonalizing crusher and diffusion –
encoding gradients to suppress undesired echo pathways in the twice –
refocused spin echo diffusion sequence' (Nagy, 2014);

"Double–spin–echo diffusion weighting
with a modified eddy current adjustment (Finsterbusch,2010)";
"code adapted from: Pusled–Field Gradient Nuclear Magnetic Resonance as a tool
for studying translational diffusion:Part 1Basic Theory (Price 1997), (Maple)";

```
ClearAll["Global`*"]
```

```
              t
F[g_, ti_] = ∫  g ⅆtd;
              ti
```

```
g1 = 0;
```

```
l1 = 0;
F1 = F[g1, l1];

l2 = t1;
g2 = g;
F2 = Replace[F1, t → l2, All] + F[g2, l2];

l3 = t1 + δ1;
g3 = 0;
F3 = Replace[F2, t → l3, All] +  F[g3, l3];

l4 = t1 + δ1 + d;
g4 = g;
F4 = Replace[F3, t → l4, All] +  F[g4, l4] ;

l5 = t1 + δ1 + d + δ2 ;
g5 = 0;
F5 = Replace[F4, t → l5, All] + F[g5, l5];

l6 = t1 + Δ;
g6 = -g;
F6 = Replace[F5, t → l6, All] + F[g6, l6];

l7 = t1 + Δ + δ11;
g7 = 0;
F7 = Replace[F6, t → l7, All] + F[g7, l7];

l8 = t1 + Δ + δ11 + d;
g8 = -g;
F8 = Replace[F7, t → l8, All] + F[g8, l8];

l9 = t1 + Δ + δ11 + d + δ22;
g9 = 0;
F9 = Replace[F8, t → l9, All] + F[g9, l9];

l10 = TE;

T1 = l3 + d / 2;
T2 = l7 + d / 2;

(* Define the function "f" [=F(tau)] = Integration between the two RF180 *)
f1 = F3;
f2 = F7;

γ = 2. * Pi * 42.5756 * 1 000 000.;
g = 42. * 10^-6;
δ1 = 6020. * 10^-6;
δ2 = 14 040. * 10^-6;
δ11 = 10 970. * 10^-6;
δ22 = 9090. * 10^-6;
TE = 62 500. * 10^-6;
```

```
d = 5440. * 10^-6;
t1 = 3420. * 10^-6;
ϵ = 400. * 10^-6;
δ = δ1 + δ2;
Δ = δ + d + 2 ϵ;

(* Define the integral of F between the two RF180
 with time at 1st RF180 is T1 and time at 2nd RF180 is T2 *)
FT1toT2 = Simplify[Integrate[F3, {t, T1, l4}] + Integrate[F4, {t, l4, l5}] +
     Integrate[F5, {t, l5, l6}] + Integrate[F6, {t, l6, l7}] + Integrate[F7, {t, l7, T2}]];
FT2toTE =  Simplify[Integrate[F7, {t, T2, l8}] + Integrate[F8, {t, l8, l9}] +
     Integrate[F9, {t, l9, TE}]];
FT1toTE = FT1toT2 + FT2toTE;

FINT =  Simplify[
    Integrate[F1, {t, l1, l2}] + Integrate[F2, {t, l2, l3}] +  Integrate[F3, {t, l3, l4}] +
     Integrate[F4, {t, l4, l5}] + Integrate[F, {t, l5, l6}] + Integrate[F6, {t, l6, l7}] +
     Integrate[F7, {t, l7, l8}] + Integrate[F8, {t, l8, l9}] + Integrate[F9, {t, l9, l10}]];

(* Define the integral of F^2 between 0 and 2τ *)
FSQINT  =  Simplify[
    Integrate[F1^2, {t, l1, l2}] + Integrate[F2^2, {t, l2, l3}] +  Integrate[F3^2, {t, l3, l4}] +
     Integrate[F4^2, {t, l4, l5}] + Integrate[F5^2, {t, l5, l6}] + Integrate[F6^2, {t, l6, l7}] +
     Integrate[F7^2, {t, l7, l8}] + Integrate[F8^2, {t, l8, l9}] + Integrate[F9^2, {t, l9, l10}]];

(* with b= Integral[F^2,{t,0,TE}]*)
bfound = Simplify[γ^2 * FSQINT /.
     {(TE - t1) → (δ1 + d + δ2 + δ11 + d + δ22), δ → δ1 + δ2, Δ → δ1 + δ2 + d, δ11 → δ1 + δ2 - δ22}];
(*1/3 g^2 γ^2 (2 (δ1+δ2)^3+3 d (δ1^2+δ22^2))*) (* 801.59*)

(* Stejskal and Tanner*)
bfound =
   Simplify[γ^2 * FSQINT /. {(TE - t1) → (δ1 + d + δ2 + δ11 + d + δ22), δ11 → δ1 + δ2 - δ22, d → 0}] /.
     δ1 + δ2 → δ /. -δ1 - δ2 → -δ;
(* 1/3 g^2 γ^2 δ^2 (-δ+3 Δ) *) (* 801.59*)

bFinsterbusch = Simplify[
    (γ^2 * g^2 * (δ^2 * (Δ - δ/3) - 2 δ * δ2 * d + d * (δ2^2 + δ22^2))) /. δ → δ1 + δ2 /. Δ → δ1 + δ2 + d];
(*1/3 g^2 γ^2 (2 (δ1+δ2)^3+3 d (δ1^2+δ22^2))*) (* 801.59*)

(* Stejskal and Tanner*)
bFinsterbusch = Simplify[(γ^2 * g^2 * (δ^2 * (Δ - δ/3) - 2 δ * δ2 * d + d * (δ2^2 + δ22^2))) /. d → 0];
(* g^2 γ^2 δ^2 (-δ/3+Δ) *) (* 801.59*)
```

Solve the cubic equation of TRSE adjusted;

```
ClearAll["Global`*"]

γ = 2. * Pi * 42.5756 * 1 000 000.;

G = 42. * 10^-6;
d = 5700. * 10^-6; (* time d,
total duration of the crushers + duration of gradient slice refocalisation *)
delay = 6760. * 10^-6;
ϵ = 400. * 10^-6;
Interduration1 = 3420. * 10^-6;
Interduration2 = 6100. * 10^-6;

δ2[t1_] =
  Floor[t1 * Log[Exp[δ / t1] * ((1 + Exp[d / t1]) / (1 + Exp[(δ + d) / t1]))] * 10^6, 10] * 10^-6;
δ22[t2_] = Floor[t2 * Log[Exp[δ / t2] * ((1 + Exp[d / t2]) / (1 + Exp[ (δ + d) / t2]))] * 10^6, 10] *
   10^-6;
bvalue = Simplify[(γ^2 * G^2 * (δ^2 * (Δ - δ / 3) - 2 δ * δ2[t1] * d + d * (δ2[t1]^2 + δ22[t2]^2))) /.
          Δ → δ1 + δ2[t1] + d + 2 ϵ + delay /. δ1 → δ - δ2[t1] /.
      δ → 17 180 * 10^-6 /. t1 → 100 * 10^-3 /. t2 → 30 * 10^-3] ;(* 800.69*)
(* y=ComplexExpand[Re[Solve[bvalue==800.,δ]]] *)

(*TRSE*)
(* b=800 with t1=90ms, t2=15ms, δ=19840us, G=42mT/m,
TE=60.08ms, d=5700.*10^-6ms, delay=0.*10^-6; *)
(* b=800 with t1=30ms, t2=30ms, δ=19440us, G=42mT/m,
TE=59.28ms, d=5700.*10^-6ms, delay=0.*10^-6; *)
(* b=800 with t1=5ms, t2=5ms, δ=18670us, G=42mT/m,
TE=57.74ms, d=5700.*10^-6ms, delay=0.*10^-6; *)

(*TRSE adj*)
(* b=800 with t1=100ms, t2=30ms, δ=17180us, G=42mT/m,
TE=63.64ms, d=5700.*10^-6ms, delay=6760.*10^-6; *)

TE = Simplify[(Interduration1 + δ1 + δ2[t1] + δ11 + δ22[t2] + 2 d + delay + 4 ϵ + Interduration2) /.
            Δ → δ1 + δ2[t1] + d + 2 ϵ + delay /. δ1 → δ - δ2[t1] /. δ11 → δ - δ22[t2] /.
       δ → 17 180 * 10^-6 /. t1 → 100 * 10^-3 /. t2 → 30 * 10^-3] * 10^6;

B[t1_, t2_, δ_] =
  Simplify[(γ^2 * G^2 * (δ^2 * (Δ - δ / 3) - 2 δ * δ2[t1] * d + d * (δ2[t1]^2 + δ22[t2]^2))) /.
      Δ → δ1 + δ2[t1] + d + 2 ϵ + delay /. δ1 → δ - δ2[t1]] * 10^-3;
```

Gradient amplitude and sequence timing.

"code adapted from: 'Efficient and precise calculation of the b matrix elements in diffusion weigthed imaging pulse sequences' (Zubkov et al, 2014); (Mathematica)";

```
ClearAll["Global`*"]

trap[DurationGrad_, RampTime_, AmpGrad_, t_] =
  AmpGrad * Clip[UnitTriangle[2 t / (DurationGrad + RampTime) - 1] *
     (RampTime + DurationGrad) / (2 RampTime)];
```

```
(*defining the trapezoidal gradient pulse*)

TRSE = 1;(* TRSE=0 => TRSEadjusted *)
ndir = 32;
ScaleDiagram = 50.;

(* (our values, 2017) *)
γ = 2. * Pi * 42.5756 * 1000000.;
ϵ = 400. * 10^-6;
shiftADC = 500. * 10^-6;
tReadout = 7700. * 10^-6;
GmaxDiff = 42.;
GmaxCrush = 19.79;
PhaseDispersionCrushers = 6.;
SliceThickness = 4.; (* mm *)
RampCrushers = 240. * 10^-6;

RampGsl180 = 1000. * 10^-6;
RampGrdp = 30. * 10^-6;
RampGpe = 20. * 10^-6;
RampGsrf = ϵ;
RampGro = ϵ;

(* Bernstein et al, handbook of MRI pusle sequences *)
(* Duration of the crushers' gradients according the phse dispersion input. *)
AreaCrushers = PhaseDispersionCrushers * Pi / (γ * 0.000001 * SliceThickness * 0.001);
DurationCrushers = N[Round[AreaCrushers / (GmaxCrush * 0.001)]];

(* For the graph's plot , d and kv ; *)
(*SignDelta={1.,-1.,1.,-1.};*)

(*For the bmatrix calculation;*)
SignDelta = {1., 1., -1., -1.};

(* "Encoding of anisotropic diffusion with tetrahedral gradients: a general
   mathematical diffusion formalism and experiemental results" (Conturo, 1996)*)
(* 4dir *)
GradDiff4 = 1 / Sqrt[3.] * {{1., 1., 1.},
{-1., -1., 1.},
{1., -1., -1.},
{-1., 1., -1.}} // MatrixForm;

(* " Optimal strategies for measuring diffusion in
   anisotropic systems by magnetic resonance imaging" (Jones, 1999)*)
(* we have sorted the gradient encoding scheme to alternate between
 the gradient axis at each new direction*)
(* 6dir *)
GradDiff6 = {{-0.887689, -0.101313, -0.449159},
{0.152552, 0.851204, 0.502175},
{-0.006226, 0.064447, -0.997902},
{0.789559, -0.384929, -0.47794},
{-0.399917, 0.82842, -0.392157},
{0.636679, 0.653135, -0.409945}} // MatrixForm;

(* 32dir:Electrostatic Repulsion scheme *)
GradDiff32 = {{0.978177, -0.099085, -0.182624},
{0.004364, -0.977355, 0.211562},
{0.058008, -0.049572, -0.997085},
{-0.951171, 0.161172, -0.263244},
{0.117967, -0.96576, -0.231065},
{-0.20677, 0.303548, -0.93011},
{-0.944892, -0.293928, -0.144174},
{-0.353468, -0.934011, -0.05181},
{-0.435353, -0.090815, -0.895667},
{0.890215, 0.360105, -0.279001},
{0.519013, -0.854199, -0.031151},
{-0.102942, -0.448113, -0.88803},
{0.841861, -0.525064, -0.124811},
{0.378146, 0.845537, -0.376926},
{0.478308, 0.041353, -0.877218},
{0.801211, 0.063809, -0.59497},
{-0.281684, -0.832306, -0.477411},
{0.231306, 0.428135, -0.873612},
{-0.80002, -0.223072, -0.556963},
{-0.348364, -0.817823, 0.458049},
{0.37987, -0.40282, -0.832727},
{-0.760744, 0.566441, -0.316879},
{0.11871, -0.750307, -0.650344},
{-0.49852, -0.514078, -0.697998},
{0.74759, -0.362889, -0.556256},
{0.029535, -0.733272, 0.679294},
{-0.467151 , 0.549242, -0.692895},
{0.716315, -0.197382, 0.669278},
{0.514837, -0.726299, -0.455448},
{-0.697256, -0.653755, -0.294005},
{-0.680714, -0.715159, 0.15867},
{0.599777, 0.492419, -0.630707}} // MatrixForm;

(* 64dir:Electrostatic Repulsion scheme *)
GradDiff64 = {{-0.997625, -0.026724, 0.063488},
{-0.154722, 0.987867, 0.013398},
{-0.015834, -0.014472, -0.99977},
{0.963101, -0.267540, 0, .029315},
{-0.12564, -0.9854060, 0.114845},
{0.065038, 0.29541300, -0.953153},
{-0.959665, -0.2166720, -0.179153},
{0.006778, -0.955284, -0.295611},
{0.30751, 0.090828, -0.9472},
{0.949992, -0.0861430, 0.300156},
{-0.27859, -0.9485760, -0.150306},
{-0.055576, -0.316879, -0.946836},
{0.927325, -0.21633400, -0.305397},
{-0.132838, 0.93592300, -0.326194},
{-0.3284670, -0.0346480, -0.94388},
{0.926285, 0.117378, -0.358076},
{-0.419397, 0.89762, 0.135587},
{0.2760480, -0.2176970, -0.936165},
{0.923215, 0.3629970, -0.126124},
{0.208215, 0.891081, -0.403263},
{-0.239236, 0.2854840, -0.928044},
{0.83921, -0.379789, 0.389213},
{0.446139, 0.883423, -0.143262},
{-0.370819, -0.344175, -0.862576},
{-0.837437, 0.5327130, -0.12213},
{-0.423944, 0.882753, -0.202533},
{0.218118, -0.5048100, -0.835219},
{-0.835774, -0.104423, -0.539052},
{0.291451, -0.8639870, -0.410588},
{0.304862, 0.464001, -0.831722},
{-0.834875, 0.510044, 0.206975},
{-0.141862, -0.824696, -0.547496},
{-0.020026, 0.576391, -0.816928},
{-0.830952, -0.381428, -0.405009},
{-0.407192, -0.82266700, -0.396753},
{-0.522214, 0.248724, -0.815738},
{0.827364, 0.548564, 0.12061},
{-0.063469 , 0.795508, -0.60261},
{0.580539, -0.068549, -0.811342},
{0.797023, -0.136705, -0.588273},
{0.601245, 0.789791, 0.121382},
{0.539573, 0.252383, -0.803221},
{0.792823, 0.455369, -0.405057},
{-0.378698, 0.766359, -0.518923},
{-0.122014, -0.589755, -0.798312},
{0.768559 , 0.1941, -0.609624},
{0.66151, -0.749911, -0.006173},
{-0.607712, -0.077141, -0.790402},
{-0.762479 , 0.1863260, -0.619604},
{0.137929, -0.7466040, -0.650813},
{0.532848, -0.37023500, -0.760920},
{0.734088, -0.4443650, -0.513473},
{-0.524955, -0.74215100, 0.416694},
{-0.332668 , 0.55742500, -0.760663},
{0.726209, 0.66617000, -0.169821},
{0.611904, -0.7124550, -0.343485},
{-0.637493, -0.37653400, -0.672179},
{0.269452, 0.7119370, -0.648492},
{-0.413285, -0.619494, -0.6674},
{-0.651714, -0.630831, -0.421095},
{-0.645004 , 0.682074, -0.344595},
{0.456964, -0.642255, -0.61538},
{0.576347, 0.522314, -0.6285},
{-0.611822 , 0.50219, -0.611129}} // MatrixForm;
```

```
Switch[ndir, 4, GradDiff = GradDiff4, 6, GradDiff = GradDiff6,
  32, GradDiff = GradDiff32, 64, GradDiff = GradDiff64];

(*"Orthogonalizing crusher and diffusion-encoding gradients to suppress undesired echo
  pathways in the twice-refocused spin echo diffusion sequence(Nagy, 2014)"*)
dir = 1;
While[dir < ndir + 1,
  if [((Abs[GradDiff[[1, dir, 1]]] + Abs[GradDiff[[1, dir, 3]]]) ≠ 0.),
   CoordCrusherX[dir] = -Sign[GradDiff[[1, dir, 1]] * GmaxDiff * 10^2] *
     Sign[GradDiff[[1, dir, 3]] * GmaxDiff * 10^2] *
     (1 - Abs[GradDiff[[1, dir, 1]]] /
        (Abs[GradDiff[[1, dir, 1]]] + Abs[GradDiff[[1, dir, 3]]]));
   CoordCrusherY[dir] = 0.;
   CoordCrusherZ[dir] = 1. - Abs[CoordCrusherX[dir]];
   ,
   CoordCrusherX[dir] = GradDiff[[1, dir, 1]];
   CoordCrusherY[dir] = 0.;
   CoordCrusherZ[dir] = 1. - Abs[CoordCrusherX[dir]];
  ];

  (* (our values, 2017) *)
  subamp[dir] = {Gsl90 → 7.5 * 10^-6, Gsl180 → 6 * 10^-6,
    Gsrf → -8.28 * 10^-6, Gpe → 0. * 10^-6, Grdp → -4.06 * 10^-6, Gro → 1.53 * 10^-6,
    Gcr → GmaxCrush * CoordCrusherX[dir] * 10^-6, Gcp → GmaxCrush * CoordCrusherY[dir] * 10^-6,
    Gcs → GmaxCrush * CoordCrusherZ[dir] * 10^-6, Gdr → GmaxDiff * GradDiff[[1, dir, 1]] * 10^-6,
    Gdp → GmaxDiff * GradDiff[[1, dir, 2]] * 10^-6,
    Gds → GmaxDiff * GradDiff[[1, dir, 3]] * 10^-6};
  dir++];
dir = 1;

(* (our values, 2017) *)
time1 = {Gsl90t → 2 * 1280. * 10^-6 + ϵ,
   Gsl180t → 3440. * 10^-6 + RampGsl180, Grdpt → 1680. * 10^-6 + RampGrdp,
   Gpet → 1700. * 10^-6 + RampGpe, Gsrft → 940. * 10^-6 + RampGsrf,
   Grot → RampGro + shiftADC + tReadout, Crut → DurationCrushers * 10^-6};

If[TRSE == 1,
  time1 = Join[time1, {TE → 62500. * 10^-6}];
  ,
  time1 = Join[time1, {TE → 63640. * 10^-6}];
 ];

Interduration1 = (Max[Gpet + RampGpe, ϵ + Gsrft + RampGsrf] + (Gsl90t - ϵ) / 2.) /. time1;
Interduration2 = (Grdpt + RampGrdp + RampGro + shiftADC + tReadout / 2.) /. time1;

If[TRSE == 1,
  "Reduction of eddy-current-induced distortion
    in diffusion MRI using a twice-refocused spin echo (Reese,2003)";
  (* TRSE *)
  d = ((Gsl180t - RampGsl180) + If[RampGsl180 > Crut, 2 * RampGsl180, 2 * Crut]) /. time1 ;
  DelayinDelta = 0. * 10^-6;
  TimeForDiffusionInTEhalf = (TE / 2 - (2 ϵ + d)) /. time1;
  δ1temp = N[Ceiling[(If[TimeForDiffusionInTEhalf / 2 - Interduration2 < 0,
         0, TimeForDiffusionInTEhalf / 2 - Interduration2]) * 10^6, 10] * 10^-6];
  δ2temp = N[Ceiling[(TimeForDiffusionInTEhalf / 2 + (Interduration2 - Interduration1) / 2) *
       10^6, 10] * 10^-6];
  δ11temp = N[Ceiling[(TimeForDiffusionInTEhalf / 2 - (Interduration2 - Interduration1) / 2) *
       10^6, 10] * 10^-6];
  δ22temp = N[Ceiling[(TimeForDiffusionInTEhalf / 2 - Interduration1) * 10^6, 10] * 10^-6];
  time1 = Join[time1, {δ1 → δ1temp, δ2 → δ2temp, δ11 → δ11temp, δ22 → δ22temp}];
  ,
  "Double-spin-echo diffusion weighting
    with a modified eddy current adjustement(Finsterbusch, 2010)";
  (* TRSE adjusted *)
  d = ((Gsl180t - RampGsl180) + If[RampGsl180 > Crut + 2 RampCrushers,
       2 * RampGsl180, 2 * (Crut + RampCrushers)]) /. time1 ;
  EddyCurrentt1 = 100. * 10^-3;
  EddyCurrentt2 = 30. * 10^-3;
  (*Δ=27.13*10^-3;
  δ=19.00*10^-3;
  DelayinDelta=1630.*10^-6;*)
  Δ = 30.44 * 10^-3;
  δ = 17.18 * 10^-3;
  DelayinDelta = 6760. * 10^-6;

  (* adding a delay in Δ helps to the reduce eddy current*)
  δ2[EddyCurrentt1_] = N[Floor[EddyCurrentt1 * Log[Exp[δ / EddyCurrentt1] *
          ((1 + Exp[d / EddyCurrentt1]) / (1 + Exp[(δ + d) / EddyCurrentt1]))] * 10^6, 10] * 10^-6];
  δ22[EddyCurrentt2_] = N[Floor[EddyCurrentt2 * Log[Exp[δ / EddyCurrentt2] * ((1 + Exp[
              d / EddyCurrentt2]) / (1 + Exp[ (δ + d) / EddyCurrentt2]))] * 10^6, 10] * 10^-6];
  time1 = Join[time1, {δ1 → δ - δ2[EddyCurrentt1], δ2 → δ2[EddyCurrentt1],
     δ11 → δ - δ22[EddyCurrentt2], δ22 → δ22[EddyCurrentt2]}]];

(* Check timing *)
TE1 = N[TE / 2 - ((Gsl90t - ϵ) / 2. + ϵ + Max[Gpet + RampGpe, Gsrft + RampGsrf] + (δ1 + ϵ) + d +
      (δ22 + ϵ) + Grdpt + RampGrdp + RampGro + shiftADC + tReadout / 2.) /. time1];
TE2 = N[TE / 2 - ((δ2 + ϵ) + d + (δ11 + ϵ) + DelayinDelta) /. time1];

time2 = N[
   {t1 → (Gsl90t + ϵ) / 2.,
     t2 → (Gsl90t + ϵ) / 2. + Max[Gpet + RampGpe, Gsrft + RampGsrf], (* Starting δ1 *)
     t3 → (Gsl90t + ϵ) / 2. + Max[Gpet + RampGpe, Gsrft + RampGsrf] + (δ1 + ϵ),
     t4 → (Gsl90t + ϵ) / 2. + Max[Gpet + RampGpe, Gsrft + RampGsrf] + (δ1 + ϵ) + d,
     (* Starting δ2 *)
     t5 → (Gsl90t + ϵ) / 2. + Max[Gpet + RampGpe, Gsrft + RampGsrf] + (δ1 + ϵ) + d + (δ2 + ϵ),
     t6 → (Gsl90t + ϵ) / 2. + Max[Gpet + RampGpe, Gsrft + RampGsrf] +
       (δ1 + ϵ) + d + (δ2 + ϵ) + DelayinDelta, (* Starting δ11 *)
     t7 → (Gsl90t + ϵ) / 2. + Max[Gpet + RampGpe, Gsrft + RampGsrf] +
      (δ1 + ϵ) + d + (δ2 + ϵ) + DelayinDelta + (δ11 + ϵ),
    t8 → (Gsl90t + ϵ) / 2. + Max[Gpet + RampGpe, Gsrft + RampGsrf] + (δ1 + ϵ) +
      d + (δ2 + ϵ) + DelayinDelta + (δ11 + ϵ) + d, (* Starting δ22 *)
    t9 → (Gsl90t + ϵ) / 2. + Max[Gpet + RampGpe, Gsrft + RampGsrf] + (δ1 + ϵ) +
      d + (δ2 + ϵ) + DelayinDelta + (δ11 + ϵ) + d + (δ22 + ϵ),
    t10 → (Gsl90t + ϵ) / 2. + Max[Gpet + RampGpe, Gsrft + RampGsrf] + (δ1 + ϵ) +
      d + (δ2 + ϵ) + DelayinDelta + (δ11 + ϵ) + d + (δ22 + ϵ) + Grdpt + RampGrdp,
    t11 → (Gsl90t + ϵ) / 2. + Max[Gpet + RampGpe, Gsrft + RampGsrf] + (δ1 + ϵ) +
      d + (δ2 + ϵ) + DelayinDelta + (δ11 + ϵ) + d + (δ22 + ϵ) + Grdpt +
      RampGrdp + RampGro + shiftADC + tReadout / 2} /. time1];
```

```
subs[g_] := g /. time1;
```

```
T1 = (t3 + (t4 - t3) / 2.) /. time2;
T2 = (t7 + (t8 - t7) / 2.) /. time2;
```

```
"Double-spin-echo diffusion weighting
    with a modified eddy current adjustment (Finsterbusch, 2010)";
If[TRSE == 1,
  bvalue =
   Simplify[(γ^2 * g^2 * (δ^2 * (Δ - δ / 3) - 2 δ * δ2 * d + d * (δ2^2 + δ22^2))) /. g -> GmaxDiff * 10^-6 /.
      δ → δ1 + δ2 /. Δ → δ1 + δ2 + 2 ϵ + d + DelayinDelta /. time1],
  bvalue = Simplify[(γ^2 * g^2 * (δ^2 * (Δ - δ / 3) - 2 δ * δ2[EddyCurrentt1] * d +
         d * (δ2[EddyCurrentt1]^2 + δ22[EddyCurrentt2]^2))) /.
     g -> GmaxDiff * 10^-6]]; (* bvalue_TRSEadj=800.69 s/mm2*)
(* bvalue_TRSE=801.59 s/mm2*)
```

Defining the TRSE gradient pulse and its integral;

```
FiInt[f_, ll_, ul_] := Integrate[f, {t, ll, ul},
   Assumptions → {ll > 0, ul > 0, offs > 0, wid > RampTime, wid > 0, RampTime > 0}] ;
```

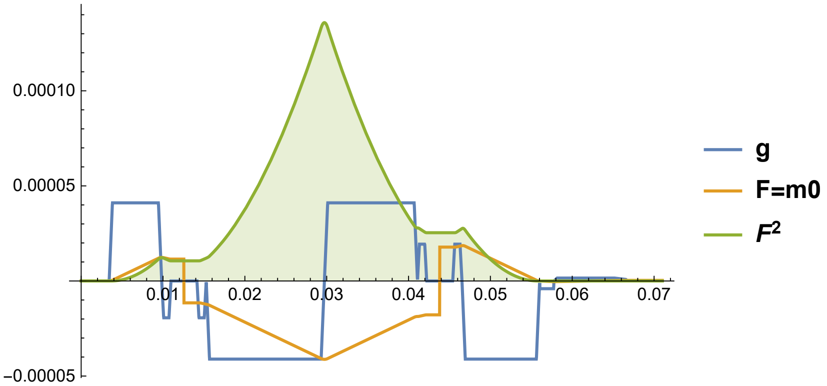
integral;

```
idtrap[DurationGrad_, RampTime_, AmpGrad_, ll_, ul_] =
  Simplify[Refine[FiInt[trap[wid, RampTime, amp, t - ll], ll, ul], Assumptions → {wid > 0,
       RampTime > 0, ul > 0, wid > RampTime}]] /. wid → DurationGrad /. amp → AmpGrad;
```

```
READ;
dir = 1;
While[dir < ndir + 1,
  AmpIntReadAtT1[dir] = subs[idtrap[δ1, ϵ, SignDelta[[1]] * Gdr, t2, T1] +
       idtrap[Crut, RampCrushers, -Gcr, t3, T1]] /. time2 /. subamp[dir];
  AmpIntReadAtT2[dir] = subs[-AmpIntReadAtT1[dir] + idtrap[Crut, RampCrushers,
        -Gcr, t3, T2] + idtrap[δ2, ϵ, SignDelta[[2]] * Gdr, t4, T2] +
       idtrap[δ11, ϵ, SignDelta[[3]] * Gdr, t6, T2] +
       idtrap[Crut, RampCrushers, Gcr, t7, T2]] /. time2 /. subamp[dir];

  Gread[t_, dir] =
   subs[trap[δ1, ϵ, SignDelta[[1]] * Gdr, t - t2] + trap[Crut, RampCrushers, -Gcr, t - t3] +
       trap[Crut, RampCrushers, -Gcr, t - (t4 - (Crut + RampCrushers))] +
       trap[δ2, ϵ, SignDelta[[2]] * Gdr, t - t4] +
       trap[δ11, ϵ, SignDelta[[3]] * Gdr, t - t6] + trap[Crut, RampCrushers, Gcr, t - t7] +
       trap[Crut, RampCrushers, Gcr, t - (t8 - (Crut + RampCrushers))] +
       trap[δ22, ϵ, SignDelta[[4]] * Gdr, t - t8] + trap[Grdpt, RampGrdp, Grdp, t - t9] +
       trap[Grot, RampGro, Gro, t - t10]] /. time2 /. subamp[dir];

  Fread[t_, dir] =
   subs[(1 - UnitStep[t - T1]) * (idtrap[δ1, ϵ, SignDelta[[1]] * Gdr, t2, t] + idtrap[
          Crut, RampCrushers, -Gcr, t3, t]) +
       (UnitStep[t - T1] - UnitStep[t - T2]) * (-AmpIntReadAtT1[dir] +
          idtrap[Crut, RampCrushers, -Gcr, t4 - Crut, t]) +
       (1 - UnitStep[t - T2]) * (idtrap[δ2, ϵ, SignDelta[[2]] * Gdr, t4, t] + idtrap[δ11, ϵ,
          SignDelta[[3]] * Gdr, t6, t] + idtrap[Crut, RampCrushers, Gcr, t7, t]) + UnitStep[
         t - T2] * (-AmpIntReadAtT2[dir] + idtrap[Crut, RampCrushers, Gcr, t8 - Crut, t]) +
       idtrap[δ22, ϵ, SignDelta[[4]] * Gdr, t8, t] + idtrap[Grdpt, RampGrdp, Grdp, t9, t] +
       idtrap[Grot, RampGrdp, Gro, t10, t]] /. time2 /. subamp[dir];
  dir++];
dir = 1;
```

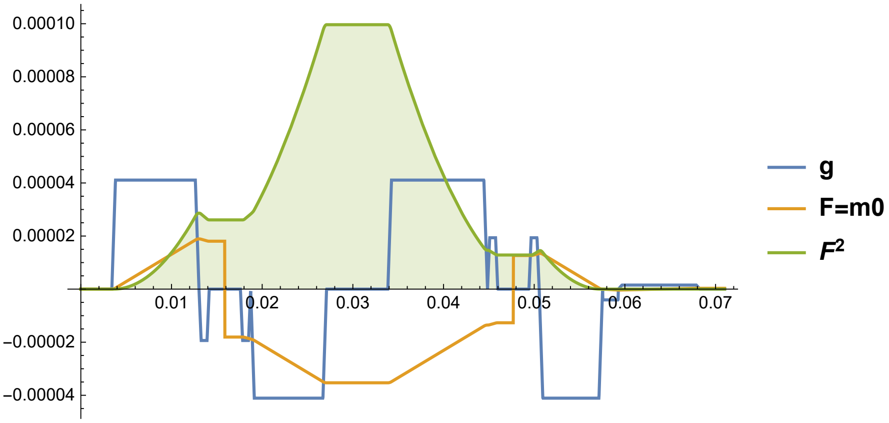
```
dirPlot = 1;
{subs[Gread[t, dirPlot] /. t → TE] /. time2, subs[(γ * Fread[t, dirPlot] /. t → TE)] /.
    time2, subs[(γ * Fread[t, dirPlot] /. t → TE)^2] /. time2} // AbsoluteTiming;

(*Plot[{Gread[t,dirPlot]/.time2,ScaleDiagram*Fread[t,dirPlot]/.time2,
   8(100ScaleDiagram)^2*(Fread[t,dirPlot])^2/.time2},{t,0.*10^-6,71000.*10^-6},
  PlotRange→Full,Filling→{3->Axis},PlotLegends→{"g","F=m0","F^2"}]//AbsoluteTiming*)
```

(* TRSE:



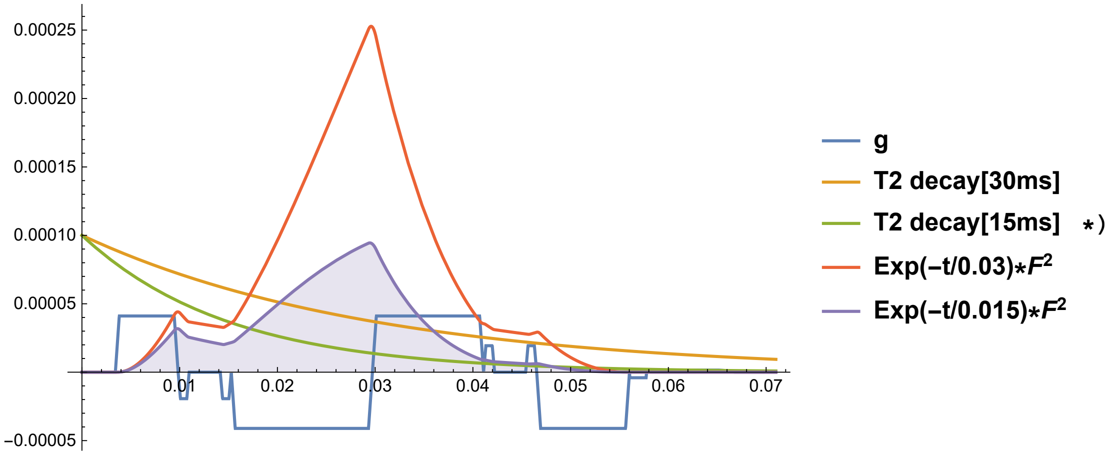
*)

(* TRSEadj:



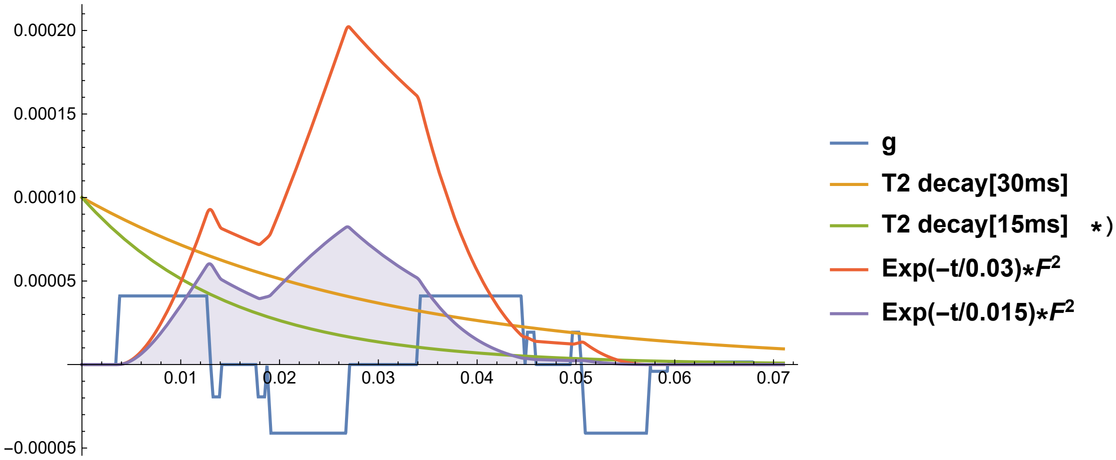
*)

```
dirPlot = 1;
(*Plot[
  {subs[Gread[t,dirPlot]]/.time2/.subamp[dirPlot],Exp[-t/0.030]*10^-4,Exp[-t/0.015]*10^-4,
   40(100ScaleDiagram)^2*subs[Exp[-t/0.03]*Fread[t,dirPlot]^2]/.time2/.subamp[dirPlot],
   40(100ScaleDiagram)^2*subs[Exp[-t/0.015]*Fread[t,dirPlot]^2]/.time2/.subamp[dirPlot]},
  {t,0.*10^-6,71000.*10^-6},PlotRange→Full,Filling→{5->Axis},
  PlotLegends→{"g","T2 decay[30ms]","T2 decay[15ms]",
    "Exp(-t/0.03)*F^2","Exp(-t/0.015)*F^2"}]//AbsoluteTiming*)
(* TRSE:
```



*)

```
(* TRSEadj:
```

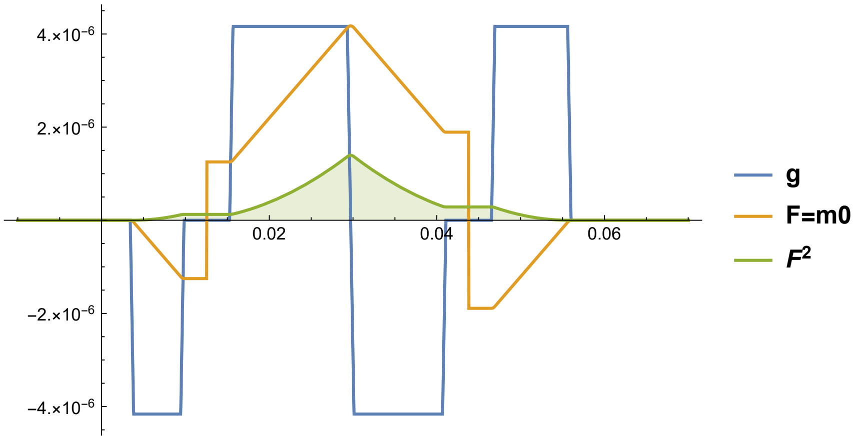


*)

```
PHASE;
dir = 1;
While[dir < ndir + 1,
  AmpIntPhaseAtT1[dir] =
   subs[trap[Gpet, ϵ, Gpe, t - (t1 - ϵ)] + idtrap[δ1, ϵ, SignDelta[[1]] * Gdp, t2, T1] +
       idtrap[Crut, RampCrushers, -Gcp, t3, T1]] /. time2 /. subamp[dir];

  AmpIntPhaseAtT2[dir] =
   subs[-AmpIntPhaseAtT1[dir] + idtrap[Crut, RampCrushers, -Gcp, t3, T2] +
       idtrap[δ2, ϵ, SignDelta[[2]] * Gdp, t4, T2] + idtrap[δ11, ϵ, SignDelta[[3]] * Gdp,
        t6, T2] + idtrap[Crut, RampCrushers, Gcp, t7, T2]] /. time2 /. subamp[dir];

  Gphase[t_, dir] =
   subs[trap[Gpet, ϵ, Gpe, t - (t1 - ϵ)] + trap[δ1, ϵ, SignDelta[[1]] * Gdp, t - t2] +
       trap[Crut, RampCrushers, -Gcp, t - t3] + trap[Crut, RampCrushers, -Gcp,
        t - (t4 - (Crut + RampCrushers))] + trap[δ2, ϵ, SignDelta[[2]] * Gdp, t - t4] +
       trap[δ11, ϵ, SignDelta[[3]] * Gdp, t - t6] + trap[Crut, RampCrushers, Gcp, t - t7] +
       trap[Crut, RampCrushers, Gcp, t - (t8 - (Crut + RampCrushers))] +
       trap[δ22, ϵ, SignDelta[[4]] * Gdp, t - t8]] /. time2 /. subamp[dir];

  Fphase[t_, dir] =
   subs[(1 - UnitStep[t - T1]) * (idtrap[Gpet, ϵ, Gpe, t1 - ϵ, t] + idtrap[δ1, ϵ,
           SignDelta[[1]] * Gdp, t2, t] + idtrap[Crut, RampCrushers, -Gcp, t3, t]) +
       (UnitStep[t - T1] - UnitStep[t - T2]) * (-AmpIntPhaseAtT1[dir] +
          idtrap[Crut, RampCrushers, -Gcp, t4 - Crut, t]) +
       (1 - UnitStep[t - T2]) * (idtrap[δ2, ϵ, SignDelta[[2]] * Gdp, t4, t] + idtrap[δ11, ϵ,
           SignDelta[[3]] * Gdp, t6, t] + idtrap[Crut, RampCrushers, Gcp, t7, t]) + UnitStep[
         t - T2] * (-AmpIntPhaseAtT2[dir] + idtrap[Crut, RampCrushers, Gcp, t8 - Crut, t]) +
       idtrap[δ22, ϵ, SignDelta[[4]] * Gdp, t8, t]] /. time2 /. subamp[dir];
  dir++];
dir = 1;

dirPlot = 1;
{subs[Gphase[t, dirPlot] /. t → TE] /. time2 /. subamp[dirPlot],
   subs[γ * Fphase[t, dirPlot] /. t → TE] /. time2 /. subamp[dirPlot],
   subs[(γ * Fphase[t, dirPlot] /. t → TE)^2] /. time2 /. subamp[dirPlot]} // AbsoluteTiming;
```

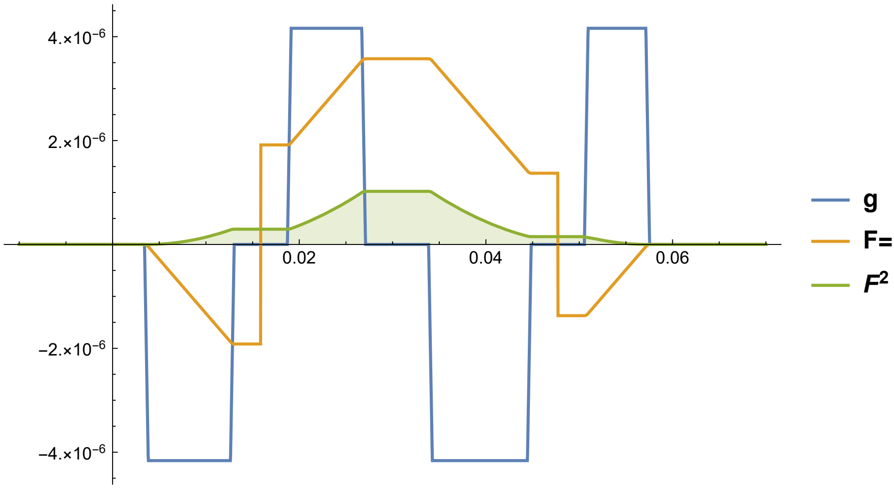
```
dirPlot = 1;
(*Plot[{subs[Gphase[t,dirPlot]]/.time2/.subamp[dirPlot],
   ScaleDiagram*subs[Fphase[t,dirPlot]]/.time2/.subamp[dirPlot],
   8(100ScaleDiagram)^2*subs[(Fphase[t,dirPlot])^2]/.time2/.subamp[dirPlot]},
  {t,-10000.*10^-6,70000.*10^-6},PlotRange→Full,Filling→{3->Axis},
  PlotLegends→{"g","F=m0","F^2"}]//AbsoluteTiming*)
```

(* TRSE:



*)

(* TRSEadj:

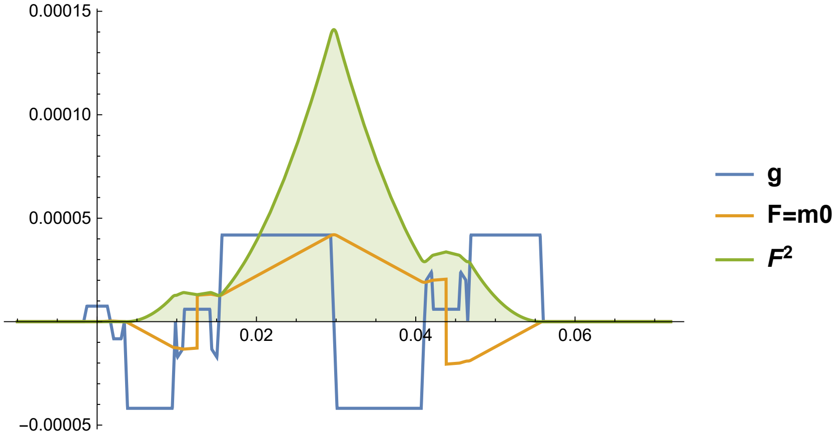


*)

```
SLICE;
dir = 1;
While[dir < ndir + 1,
  (* In TRSE the Flat top of Gsl180 starts when the duration of the crusher ends*)
  (* In TRSE adjusted the Flat top of
   Gsl180 starts when the total time of the crusher ends*)
  If[TRSE == 1,
   AmpIntSliceAtT1[dir] =
    subs[idtrap[Gsl90t, ϵ, Gsl90, t1, T1] / 2. + idtrap[Gsrft, ϵ, Gsrf, t1, T1] + idtrap[δ1,
        ϵ, SignDelta[[1]] * Gds, t2, T1] + idtrap[Crut, RampCrushers, -Gcs, t3, T1] +
       idtrap[Gsl180t, RampGsl180, Gsl180, t3, T1]] /. time2 /. subamp[dir];

   AmpIntSliceAtT2[dir] =
    subs[-AmpIntSliceAtT1[dir] + idtrap[Gsl180t, RampGsl180, Gsl180, T1, T2] + idtrap[Crut,
     RampCrushers, -Gcs, T1, T2] + idtrap[δ2, ϵ, SignDelta[[2]] * Gds, t4, T2] +
    idtrap[δ11, ϵ, SignDelta[[3]] * Gds, t6, T2] + idtrap[Crut, RampCrushers, Gcs,
     t7, T2] + idtrap[Gsl180t, RampGsl180, Gsl180, t7, T2]] /. time2 /. subamp[dir];

Gslice[t_, dir] = subs[trap[Gsl90t, ϵ, Gsl90, t + t1] +
    trap[Gsrft, ϵ, Gsrf, t - t1] + trap[δ1, ϵ, SignDelta[[1]] * Gds, t - t2] +
    trap[Crut, RampCrushers, -Gcs, t - t3] + trap[Gsl180t, RampGsl180, Gsl180, t - t3] +
    trap[Crut, RampCrushers, -Gcs, t - (t4 - (Crut + RampCrushers))] +
    trap[δ2, ϵ, SignDelta[[2]] * Gds, t - t4] + trap[δ11, ϵ, SignDelta[[3]] * Gds, t - t6] +
    trap[Crut, RampCrushers, Gcs, t - t7] + trap[Gsl180t, RampGsl180, Gsl180, t - t7] +
    trap[Crut, RampCrushers, Gcs, t - (t8 - (Crut + RampCrushers))] +
    trap[δ22, ϵ, SignDelta[[4]] * Gds, t - t8]] /. time2 /. subamp[dir];

Fslice[t_, dir] =
  subs[(1 - UnitStep[t - t1]) * idtrap[Gsl90t, ϵ, Gsl90, t1, t + t1] / 2. + UnitStep[t - t1] *
      (idtrap[Gsl90t, ϵ, Gsl90, t1, t + t1] / 2. + idtrap[Gsrft, ϵ, Gsrf, t1, t]) +
     (1 - UnitStep[t - T1]) * (idtrap[δ1, ϵ, SignDelta[[1]] * Gds, t2, t] + idtrap[Crut,
         RampCrushers, -Gcs, t3, t] + idtrap[Gsl180t, RampGsl180, Gsl180, t3, t]) +
     (UnitStep[t - T1] - UnitStep[t - T2]) * (-AmpIntSliceAtT1[dir] + idtrap[Gsl180t,
         RampGsl180, Gsl180, t3, t] - idtrap[Gsl180t, RampGsl180, Gsl180, t3, T1] +
        idtrap[Crut, RampCrushers, -Gcs, t4 - (Crut + RampCrushers), t]) +
     (1 - UnitStep[t - T2]) * (idtrap[δ2, ϵ, SignDelta[[2]] * Gds, t4, t] +
        idtrap[δ11, ϵ, SignDelta[[3]] * Gds, t6, t] + idtrap[Crut, RampCrushers, Gcs,
         t7, t] + idtrap[Gsl180t, RampGsl180, Gsl180, t7, t]) + UnitStep[t - T2] *
      (-AmpIntSliceAtT2[dir] + idtrap[Gsl180t, RampGsl180, Gsl180, t7, t]
        + idtrap[Crut, RampCrushers, Gcs, t8 - (Crut + RampCrushers), t]) +
     idtrap[δ22, ϵ, SignDelta[[4]] * Gds, t8, t]] /. time2 /. subamp[dir];
,
AmpIntSliceAtT1[dir] =
  subs[idtrap[Gsl90t, ϵ, Gsl90, t1, T1] / 2. + idtrap[Gsrft, ϵ, Gsrf, t1, T1] +
     idtrap[δ1, ϵ, SignDelta[[1]] * Gds, t2, T1] + idtrap[Crut,
      RampCrushers, -Gcs, t3, T1] + idtrap[Gsl180t, RampGsl180, Gsl180,
      (t3 + Crut + RampCrushers - RampGsl180), T1]] /. time2 /. subamp[dir];

AmpIntSliceAtT2[dir] =
  subs[-AmpIntSliceAtT1[dir] + idtrap[Gsl180t, RampGsl180, Gsl180, (t3 + Crut +
         RampCrushers - RampGsl180), T2] + idtrap[Crut, RampCrushers, -Gcs, t3, T2] +
     idtrap[δ2, ϵ, SignDelta[[2]] * Gds, t4, T2] + idtrap[δ11, ϵ, SignDelta[[3]] * Gds, t6,
      T2] + idtrap[Crut, RampCrushers, Gcs, (t7 + Crut + RampCrushers - RampGsl180), T2] +
     idtrap[Gsl180t, RampGsl180, Gsl180, t7, T2]] /. time2 /. subamp[dir];

Gslice[t_, dir] = subs[trap[Gsl90t, ϵ, Gsl90, t + t1] + trap[Gsrft, ϵ, Gsrf, t - t1] +
     trap[δ1, ϵ, SignDelta[[1]] * Gds, t - t2] + trap[Crut, RampCrushers, -Gcs, t - t3] +
     trap[Gsl180t, RampGsl180, Gsl180, t - (t3 + Crut + RampCrushers - RampGsl180)] +
     trap[Crut, RampCrushers, -Gcs, t - (t4 - (Crut + RampCrushers))] +
     trap[δ2, ϵ, SignDelta[[2]] * Gds, t - t4] +
     trap[δ11, ϵ, SignDelta[[3]] * Gds, t - t6] + trap[Crut, RampCrushers, Gcs, t - t7] +
     trap[Gsl180t, RampGsl180, Gsl180, t - (t7 + Crut + RampCrushers - RampGsl180)] +
     trap[Crut, RampCrushers, Gcs, t - (t8 - (Crut + RampCrushers))] +
        trap[δ22, ∊, SignDelta[[4]] * Gds, t - t8]] /. time2 /. subamp[dir];

    Fslice[t_, dir] =
     subs[(1 - UnitStep[t - t1]) * (idtrap[Gsl90t, ∊, Gsl90, t1, t + t1] / 2.) + UnitStep[t - t1] *
          (idtrap[Gsl90t, ∊, Gsl90, t1, t + t1] / 2. + idtrap[Gsrft, ∊, Gsrf, t1, t]) +
        (1 - UnitStep[t - T1]) * (idtrap[δ1, ∊, SignDelta[[1]] * Gds, t2, t] +
           idtrap[Crut, RampCrushers, -Gcs, t3, t] + idtrap[Gsl180t,
            RampGsl180, Gsl180, (t3 + Crut + RampCrushers - RampGsl180), t]) +
        (UnitStep[t - T1] - UnitStep[t - T2]) * (-AmpIntSliceAtT1[dir] +
           idtrap[Gsl180t, RampGsl180, Gsl180, (t3 + Crut + RampCrushers - RampGsl180), t] -
           idtrap[Gsl180t, RampGsl180, Gsl180, (t3 + Crut + RampCrushers - RampGsl180), T1] +
           idtrap[Crut, RampCrushers, -Gcs, t4 - (Crut + RampCrushers), t]) +
        (1 - UnitStep[t - T2]) * (idtrap[δ2, ∊, SignDelta[[2]] * Gds, t4, t] + idtrap[
            δ11, ∊, SignDelta[[3]] * Gds, t6, t] + idtrap[Crut, RampCrushers, Gcs, t7, t] +
           idtrap[Gsl180t, RampGsl180, Gsl180, (t7 + Crut + RampCrushers - RampGsl180), t]) +
        UnitStep[t - T2] * (-AmpIntSliceAtT2[dir] + idtrap[Gsl180t, RampGsl180,
            Gsl180, (t7 + Crut + RampCrushers - RampGsl180), t]
           + idtrap[Crut, RampCrushers, Gcs, t8 - (Crut + RampCrushers), t]) +
        idtrap[δ22, ∊, SignDelta[[4]] * Gds, t8, t]] /. time2 /. subamp[dir];
  ];
  dir++];
dir = 1;

dirPlot = 3;
{subs[Gslice[t, dirPlot] /. t → TE] /. time2 /. subamp[dirPlot],
   subs[γ * Fslice[t, dirPlot] /. t → TE] /. time2 /. subamp[dirPlot],
   subs[(γ * Fslice[t, dirPlot] /. t → TE)^2] /. time2 /. subamp[dirPlot]} // AbsoluteTiming;
```

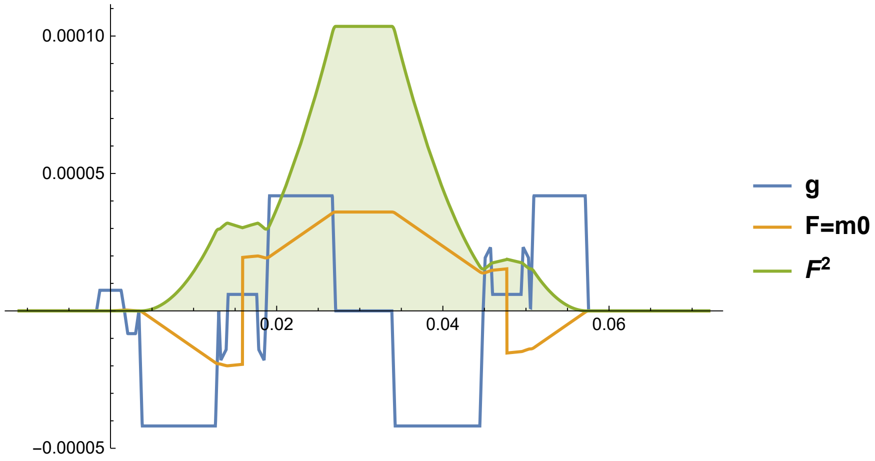
```
(*Plot[{subs[Gslice[t,dirPlot]]/.time2/.subamp[dirPlot],
   ScaleDiagram*subs[Fslice[t,dirPlot]]/.time2/.subamp[dirPlot],
   8(100ScaleDiagram)^2*subs[(Fslice[t,dirPlot])^2]/.time2/.subamp[dirPlot]},
  {t,-11000.*10^-6,72000.*10^-6},PlotRange→Full,Filling→{3->Axis},
  PlotLegends→{"g","F=m0","F^2"}]//AbsoluteTiming*)
```

(* TRSE:



*)

(* TRSEadj:

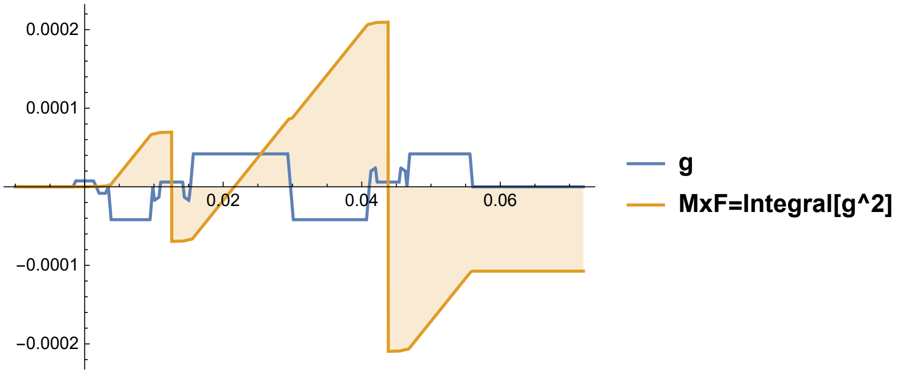


*)

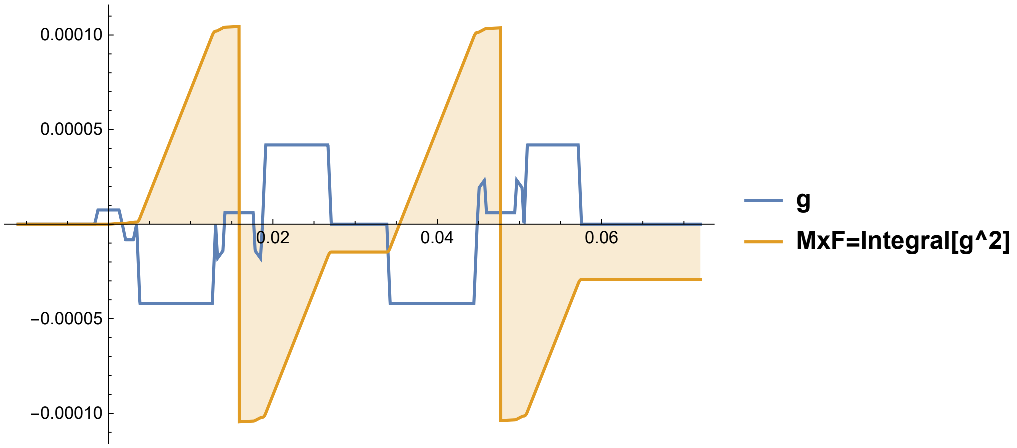
"Maxwell gradient moment= integral(g^2)";

```
dir = 1;
While[dir < ndir + 1,
  AmpMxIntSliceAtT1[dir] =
   subs[ idtrap[Gsl90t, ϵ, Gsl90^2, t1, T1] / 2 + idtrap[Gsrft, ϵ, Gsrf^2, t1, T1] + idtrap[δ1, ϵ,
        (SignDelta[[1]] * Gds)^2, t2, T1] + idtrap[Crut, RampCrushers, (-Gcs)^2, t3, T1] +
       idtrap[Gsl180t, RampGsl180, Gsl180^2, t3, T1]] /. time2 /. subamp[dir];

  AmpMxIntSliceAtT2[dir] =
   subs[-AmpMxIntSliceAtT1[dir] + idtrap[Gsl180t, RampGsl180, Gsl180^2, t3, T1] +
       idtrap[Crut, RampCrushers, (-Gcs)^2, t4 - (Crut + RampCrushers), T2] +
       idtrap[δ2, ϵ, (SignDelta[[2]] * Gds)^2, t4, T2] + idtrap[δ11, ϵ,
        (SignDelta[[3]] * Gds)^2, t6, T2] + idtrap[Crut, RampCrushers, Gcs^2, t7, T2] +
       idtrap[Gsl180t, RampGsl180, Gsl180^2, t7, T2]] /. time2 /. subamp[dir];
```

```
    MxFslice[t_, dir] = (1 - UnitStep[t - t1]) * idtrap[Gsl90t, ϵ, Gsl90^2, t1, t + t1] / 2. +
      (1 - UnitStep[t - T1]) *
       (idtrap[Gsl90t, ϵ, Gsl90^2, t1, t + t1] / 2. + idtrap[Gsrft, ϵ, Gsrf^2, t1, t] + idtrap[δ1,
          ϵ, (SignDelta[[1]] * Gds)^2, t2, t] + idtrap[Crut, RampCrushers, (-Gcs)^2, t3, t] +
         idtrap[Gsl180t, RampGsl180, Gsl180^2, t3, t]) + (UnitStep[t - T1] - UnitStep[t - T2]) *
       (-AmpMxIntSliceAtT1[dir] + idtrap[Gsl180t, RampGsl180, Gsl180^2, t3, t] -
         idtrap[Gsl180t, RampGsl180, Gsl180^2, t3, T1] + idtrap[Crut, RampCrushers, (-Gcs)^2,
          t4 - (Crut + RampCrushers), t] + idtrap[δ2, ϵ, (SignDelta[[2]] * Gds)^2, t4, t] +
         idtrap[δ11, ϵ, (SignDelta[[3]] * Gds)^2, t6, t] + idtrap[Crut, RampCrushers,
          Gcs^2, t7, t] + idtrap[Gsl180t, RampGsl180, Gsl180^2, t7, t]) +
      UnitStep[t - T2] * (-AmpMxIntSliceAtT2[dir] +
         idtrap[Gsl180t, RampGsl180, Gsl180^2, t7, t] - idtrap[Gsl180t, RampGsl180, Gsl180^2,
          t7, T2] + idtrap[Crut, RampCrushers, Gcs^2, t8 - (Crut + RampCrushers), t]) +
      idtrap[δ22, ϵ, (SignDelta[[4]] * Gds)^2, t8, t];
    dir++];
  dir = 1;

  dirPlot = 3;
  {subs[Gslice[t, dirPlot] /. t → TE] /. time2 /. subamp[dirPlot],
    subs[(MxFslice[t, dirPlot] /. t → TE)] /. time2 /. subamp[dirPlot]};
  (*Plot[{subs[Gslice[t,dirPlot]]/.time2/.subamp[dirPlot],
     (50ScaleDiagram)^2*subs[MxFslice[t,dirPlot]]/.time2/.subamp[dirPlot]},
    {t,-11000.*10^-6,72000.*10^-6},PlotRange→Full,Filling→{2->Axis},
    PlotLegends→{"g","MxF=Integral[g^2]"}]//AbsoluteTiming*)

  (* TRSE:
```



```
  *)

  (* TRSEadj:
```



*)

integral (integral);

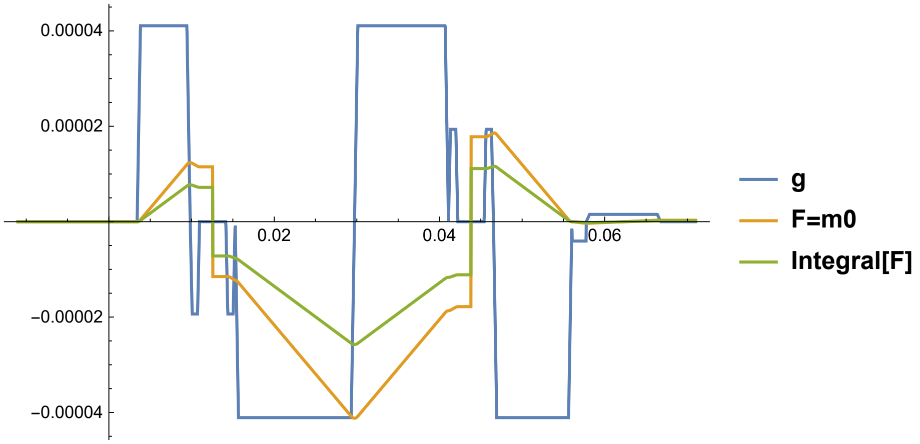
```
i2dtrap[DurationGrad_, RampTime_, AmpGrad_, ll_, ul_, a_, b_] =
  Simplify[Refine[FiInt[idtrap[DurationGrad, RampTime, amp, ll, ul], a, b],
      Assumptions → {wid > 0., a ≥ 0., b > a, b > 0., a < ul < b, ll ≥ 0., RampTime > 0.,
        ul > 0., AmpGrad ≥ 0., wid > RampTime}]] /. wid → DurationGrad /. amp → AmpGrad;
```

READ;

```
dir = 1;
While[dir < ndir + 1,
  Amp2IntReadAtT1[dir] = subs[ i2dtrap[δ1, ϵ, SignDelta[[1]] * Gdr, t2, T1, 0, TE] +
       i2dtrap[Crut, RampCrushers, -Gcr, t3, T1, 0, TE]] /. time2 /. subamp[dir];

  Amp2IntReadAtT2[dir] =
   subs[Abs[Amp2IntReadAtT1[dir]] + Abs[i2dtrap[Crut, RampCrushers, -Gcr, t3, T2, 0, TE] +
        i2dtrap[δ2, ϵ, SignDelta[[2]] * Gdr, t4, T2, 0, TE] +
        i2dtrap[δ11, ϵ, SignDelta[[3]] * Gdr, t6, T2, 0, TE] +
        i2dtrap[Crut, RampCrushers, Gcr, t7, T2, 0, TE]]] /. time2 /. subamp[dir];

  IntFread[t_, dir] =
   (1 - UnitStep[t - T1]) * (i2dtrap[δ1, ϵ, SignDelta[[1]] * Gdr, t2, t, 0, TE] +
       i2dtrap[Crut, RampCrushers, -Gcr, t3, t, 0, TE]) +
    (UnitStep[t - T1] - UnitStep[t - T2]) * (-Amp2IntReadAtT1[dir]
       + i2dtrap[Crut, RampCrushers, -Gcr, t4 - Crut, t, 0, TE]) +
    (1 - UnitStep[t - T2]) * (i2dtrap[δ2, ϵ, SignDelta[[2]] * Gdr, t4, t, 0, TE] +
       i2dtrap[δ11, ϵ, SignDelta[[3]] * Gdr, t6, t, 0, TE] +
       i2dtrap[Crut, RampCrushers, Gcr, t7, t, 0, TE]) +
    UnitStep[t - T2] * (-Sign[-Amp2IntReadAtT1[dir]] * Amp2IntReadAtT2[dir]
       + i2dtrap[Crut, RampCrushers, Gcr, t8 - Crut, t, 0, TE]) +
    i2dtrap[δ22, ϵ, SignDelta[[4]] * Gdr, t8, t, 0, TE] +
    i2dtrap[Grdpt, ϵ, Grdp, t9, t, 0, TE] + i2dtrap[Grot, ϵ, Gro, t10, t, 0, TE];
dir++];
dir = 1;

dirPlot = 1;
```

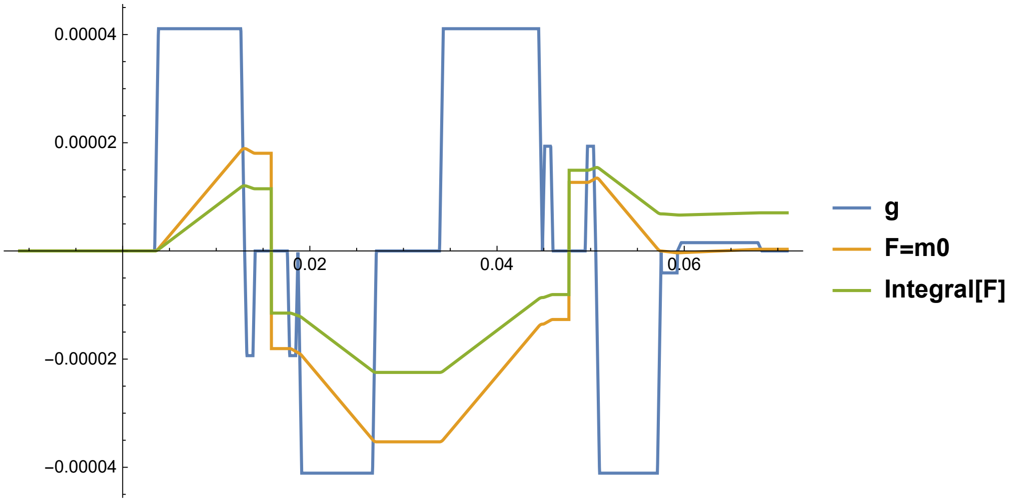
```
(*Plot[{subs[Gread[t,dirPlot]]/.time2/.subamp[dirPlot],
  ScaleDiagram*subs[Fread[t,dirPlot]]/.time2/.subamp[dirPlot],
  10ScaleDiagram*subs[IntFread[t,dirPlot]]/.time2/.subamp[dirPlot]},
 {t,-11000*10^-6,71000*10^-6},PlotRange→Full,PlotLegends→{"g","F=m0","Integral[F]"}]*)
```

(* TRSE:



*)

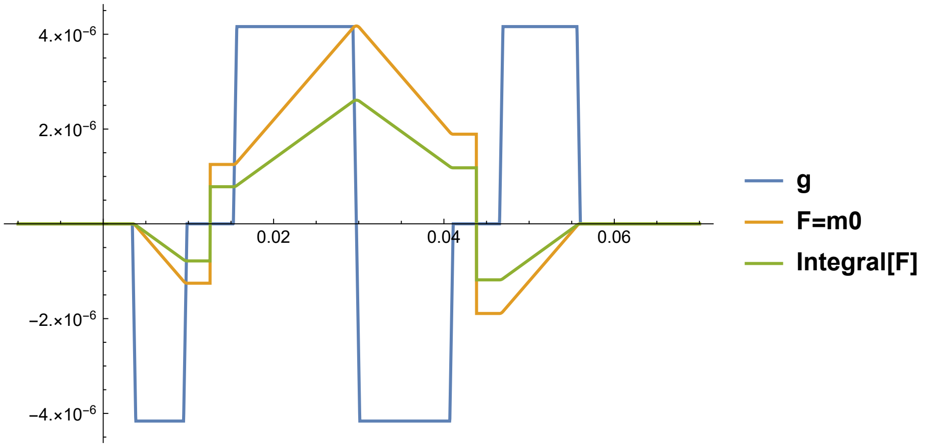
(* TRSEadj:



*)

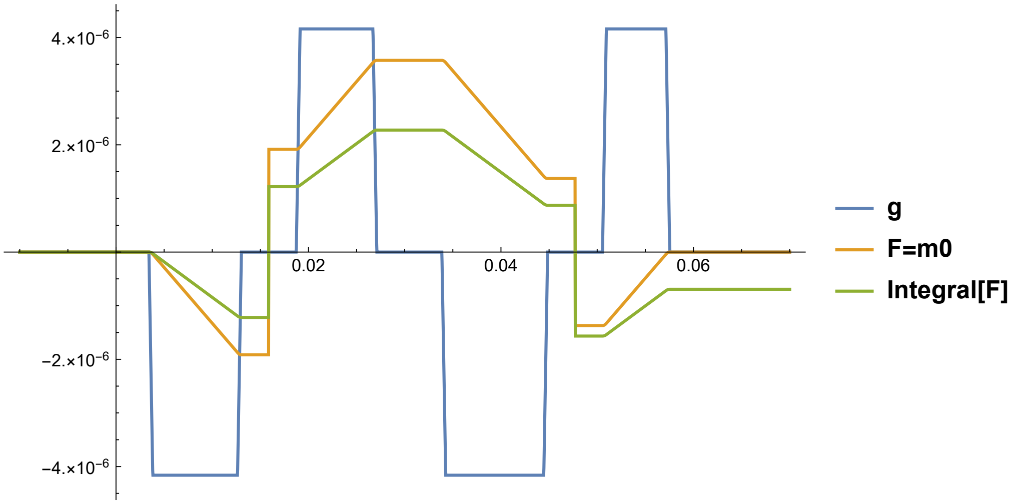
```
PHASE;
dir = 1;
While[dir < ndir + 1,
  Amp2IntPhaseAtT1[dir] = subs[i2dtrap[Gpet, RampGpe, Gpe, t1, T1, 0, TE] +
       i2dtrap[δ1, ϵ, SignDelta[[1]] * Gdp, t2, T1, 0, TE] +
       i2dtrap[Crut, RampCrushers, -Gcp, t3, T1, 0, TE]] /. time2 /. subamp[dir];

  Amp2IntPhaseAtT2[dir] =
   subs[Abs[Amp2IntPhaseAtT1[dir]] + Abs[i2dtrap[Crut, RampCrushers, -Gcp, t3, T2, 0, TE] +
          i2dtrap[δ2, ϵ, SignDelta[[2]] * Gdp, t4, T2, 0, TE] +
          i2dtrap[δ11, ϵ, SignDelta[[3]] * Gdp, t6, T2, 0, TE] +
          i2dtrap[Crut, RampCrushers, Gcp, t7, T2, 0, TE]]] /. time2 /. subamp[dir];

  IntFphase[t_, dir] = (1 - UnitStep[t - T1]) *
     (i2dtrap[Gpet, RampGpe, Gpe, t1, t, 0, TE] + i2dtrap[δ1, ϵ, SignDelta[[1]] * Gdp,
        t2, t, 0, TE] + i2dtrap[Crut, RampCrushers, -Gcp, t3, t, 0, TE]) +
      (UnitStep[t - T1] - UnitStep[t - T2]) * (-Amp2IntPhaseAtT1[dir]
         + i2dtrap[Crut, RampCrushers, -Gcp, t4 - Crut, t, 0, TE]) +
      (1 - UnitStep[t - T2]) * (i2dtrap[δ2, ϵ, SignDelta[[2]] * Gdp, t4, t, 0, TE] +
         i2dtrap[δ11, ϵ, SignDelta[[3]] * Gdp, t6, t, 0, TE] +
         i2dtrap[Crut, RampCrushers, Gcp, t7, t, 0, TE]) +
      UnitStep[t - T2] * (-Sign[-Amp2IntPhaseAtT1[dir]] * Amp2IntPhaseAtT2[dir]
         + i2dtrap[Crut, RampCrushers, Gcp, t8 - Crut, t, 0, TE]) +
      i2dtrap[δ22, ϵ, SignDelta[[4]] * Gdp, t8, t, 0, TE];
   dir++];
dir = 1;

dirPlot = 1;
(*Plot[{subs[Gphase[t,dirPlot]]/.time2/.subamp[dirPlot],
   ScaleDiagram*subs[Fphase[t,dirPlot]]/.time2/.subamp[dirPlot],
   10ScaleDiagram*subs[IntFphase[t,dirPlot]]/.time2/.subamp[dirPlot]},
  {t,-10000*10^-6,70000*10^-6},PlotRange→Full,PlotLegends→{"g","F=m0","Integral[F]"}]*)
```

(* TRSE:



*)

(* TRSEadj:

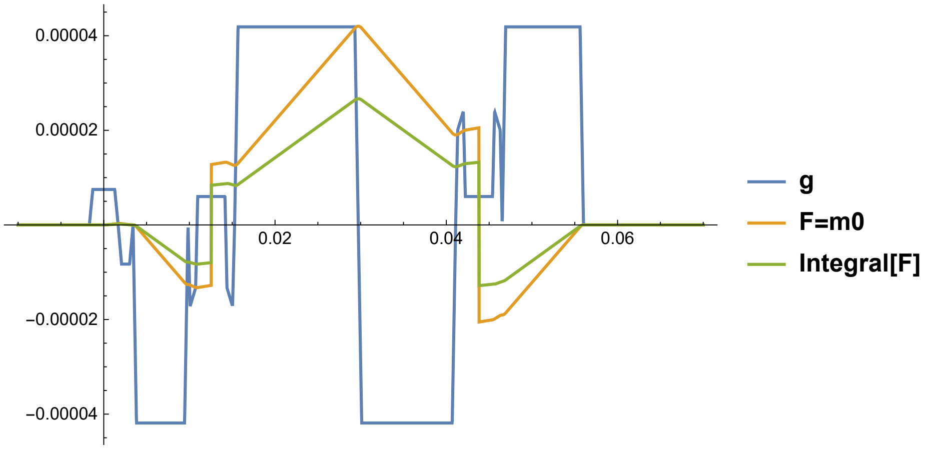


*)

```
SLICE;
dir = 1;
While[dir < ndir + 1,
  Amp2IntSliceAtT1[dir] =
   subs[ i2dtrap[Gsl90t, ϵ, Gsl90, t1, T1, 0, TE] / 2. + i2dtrap[Gsrft, ϵ, Gsrf, t1, T1, 0, TE] +
       i2dtrap[δ1, ϵ, SignDelta[[1]] * Gds, t2, T1, 0, TE] +
       i2dtrap[Crut, RampCrushers, -Gcs, t3, T1, 0, TE] +
       i2dtrap[Gsl180t, RampGsl180, Gsl180, t3, TE, 0, TE] / 2.] /. time2 /. subamp[dir];

  Amp2IntSliceAtT2[dir] =
   subs[Abs[Amp2IntSliceAtT1[dir]] + Abs[i2dtrap[Gsl180t, RampGsl180, Gsl180, t3, TE, 0,
           TE] / 2. + i2dtrap[Crut, RampCrushers, -Gcs, t3, T2, 0, TE] + i2dtrap[δ2, ϵ,
          SignDelta[[2]] * Gds, t4, T2, 0, TE] + i2dtrap[δ11, ϵ, SignDelta[[3]] * Gds,
          t6, T2, 0, TE] + i2dtrap[Crut, RampCrushers, Gcs, t3, T2, 0, TE] +
         i2dtrap[Gsl180t, RampGsl180, Gsl180, t7, T2, 0, TE]]] /. time2 /. subamp[dir];

  IntFslice[t_, dir] =
   (1 - UnitStep[t - t1]) * i2dtrap[Gsl90t, ϵ, Gsl90, t1, t + t1, 0, TE] / 2. +
    UnitStep[t - t1] * (i2dtrap[Gsl90t, ϵ, Gsl90, t1, t + t1, 0, TE] / 2. +
       i2dtrap[Gsrft, ϵ, Gsrf, t1, t, 0, TE]) +
    (1 - UnitStep[t - T1]) * (i2dtrap[δ1, ϵ, SignDelta[[1]] * Gds, t2, t, 0, TE] +
       i2dtrap[Crut, RampCrushers, -Gcs, t3, t, 0, TE] +
       i2dtrap[Gsl180t, RampGsl180, Gsl180, t3, t, 0, TE]) +
    (UnitStep[t - T1] - UnitStep[t - T2]) * (-Amp2IntSliceAtT1[dir] +
       i2dtrap[Gsl180t, RampGsl180, Gsl180, t3, t, 0, TE]
       + i2dtrap[Crut, RampCrushers, -Gcs, t4 - Crut, t, 0, TE]) +
    (1 - UnitStep[t - T2]) * (i2dtrap[δ2, ϵ, SignDelta[[2]] * Gds, t4, t, 0, TE] +
       i2dtrap[δ11, ϵ, SignDelta[[3]] * Gds, t6, t, 0, TE] + i2dtrap[Crut, RampCrushers,
        Gcs, t7, t, 0, TE] + i2dtrap[Gsl180t, RampGsl180, Gsl180, t7, t, 0, TE]) +
    UnitStep[t - T2] * (-Sign[-Amp2IntSliceAtT1[dir]] * Amp2IntSliceAtT2[dir] +
       i2dtrap[Gsl180t, RampGsl180, Gsl180, t7, t, 0, TE] - i2dtrap[Gsl180t, RampGsl180,
        Gsl180, t7, T2, 0, TE] + i2dtrap[Crut, RampCrushers, Gcs, t8 - Crut, t, 0, TE]) +
    i2dtrap[δ22, ϵ, SignDelta[[4]] * Gds, t8, t, 0, TE];
  dir++];
dir = 1;
```

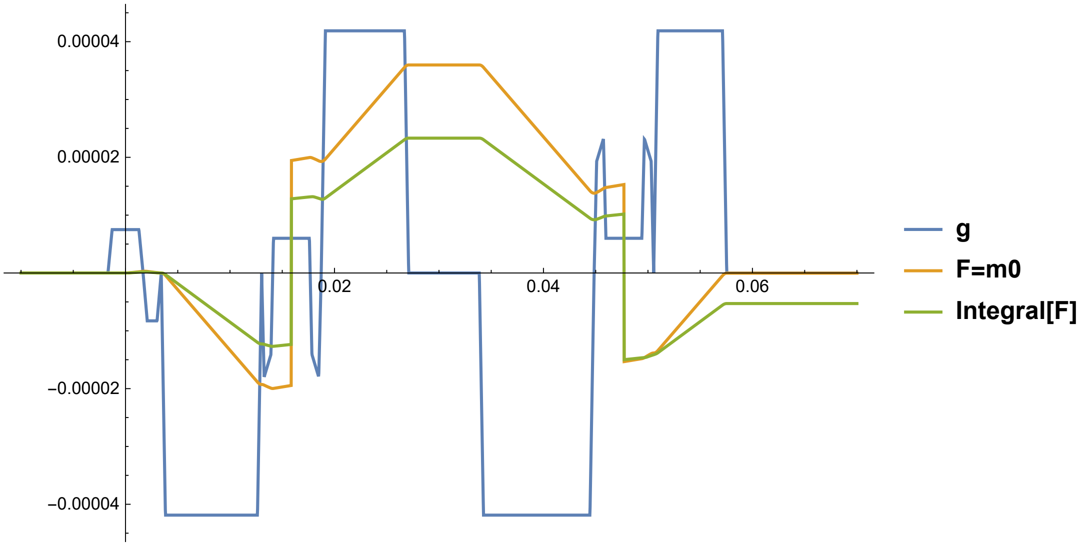
```
dirPlot = 3;
(*Plot[{subs[Gslice[t,dirPlot]]/.time2/.subamp[dirPlot],
  ScaleDiagram*subs[Fslice[t,dirPlot]]/.time2/.subamp[dirPlot],
  10ScaleDiagram*subs[IntFslice[t,dirPlot]]/.time2/.subamp[dirPlot]},
 {t,-10000*10^-6,70000*10^-6},PlotRange→Full,PlotLegends→{"g","F=m0","Integral[F]"}]*)
```

```
(* TRSE:
```



```
*)
```

```
(* TRSEadj:
```

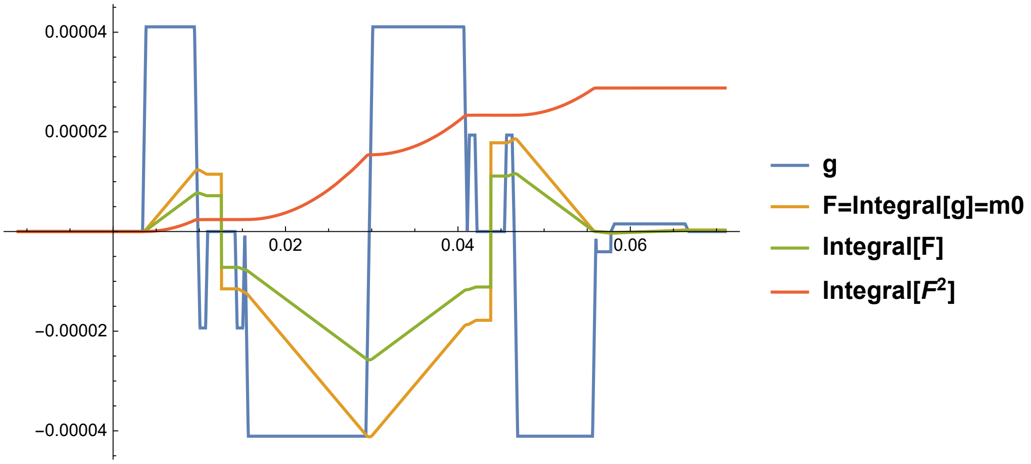


```
*)
```

integral[integral^2];

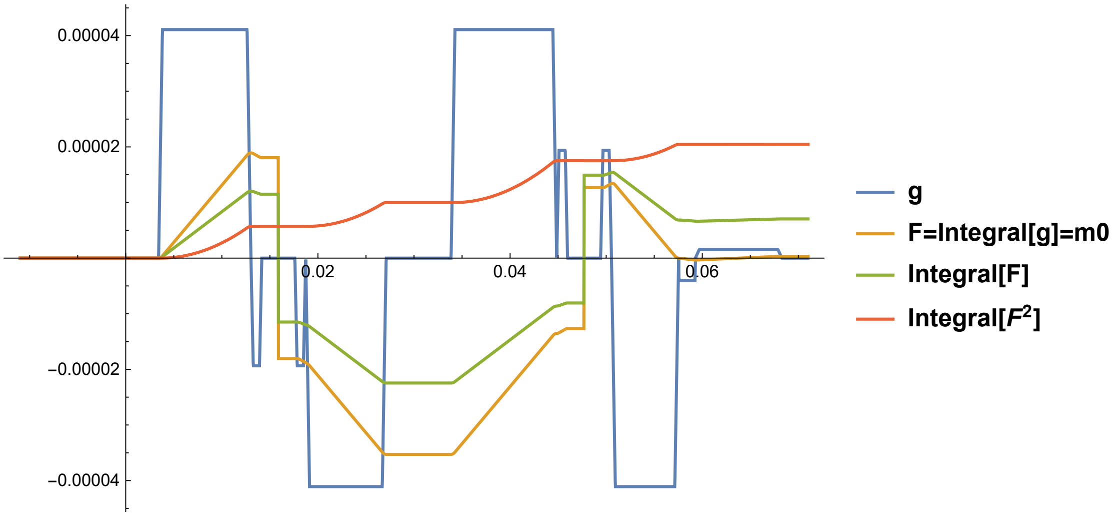
```
iSqidtrap[DurationGrad_, RampTime_, AmpGrad_, ll_, ul_, a_, b_] =
  Simplify[Refine[FiInt[(idtrap[DurationGrad, RampTime, amp, ll, ul])^2, a, b],
      Assumptions → {wid > 0., a ≥ 0., b > a, b > 0., a < ul < b, ll ≥ 0., RampTime > 0.,
        ul > 0., AmpGrad ≥ 0., wid > RampTime}]] /. wid → DurationGrad /. amp → AmpGrad;
dir = 1;
While[dir < ndir + 1,
  AmpIntSqIntReadAtT1[dir] = subs[ iSqidtrap[δ1, ϵ, SignDelta[[1]] * Gdr, t2, T1, 0, TE] +
       iSqidtrap[Crut, RampCrushers, -Gcr, t3, T1, 0, TE]] /. time2 /. subamp[dir];

  AmpIntSqIntReadAtT2[dir] =
    subs[Abs[AmpIntSqIntReadAtT1[dir]] + Abs[iSqidtrap[Crut, RampCrushers, -Gcr,
           t3, T2, 0, TE] + iSqidtrap[δ2, ϵ, SignDelta[[2]] * Gdr, t4, T2, 0, TE] +
          iSqidtrap[δ11, ϵ, SignDelta[[3]] * Gdr, t6, T2, 0, TE] +
          iSqidtrap[Crut, RampCrushers, Gcr, t7, T2, 0, TE]]] /. time2 /. subamp[dir];

   IntSqFread[t_, dir] =
    iSqidtrap[δ1, ϵ, SignDelta[[1]] * Gdr, t2, t, 0, TE] +
     iSqidtrap[Crut, RampCrushers, -Gcr, t3, t, 0, TE] +
     (UnitStep[t - T1] - UnitStep[t - T2]) *
      (iSqidtrap[Crut, RampCrushers, -Gcr, t4 - Crut, t, 0, TE]) +
     iSqidtrap[δ2, ϵ, SignDelta[[2]] * Gdr, t4, t, 0, TE] + iSqidtrap[δ11, ϵ,
      SignDelta[[3]] * Gdr, t6, t, 0, TE] + iSqidtrap[Crut, RampCrushers, Gcr, t7, t, 0, TE] +
     iSqidtrap[Crut, RampCrushers, Gcr, t8 - Crut, t, 0, TE] +
     iSqidtrap[δ22, ϵ, SignDelta[[4]] * Gdr, t8, t, 0, TE] +
     iSqidtrap[Grdpt, RampGrdp, Grdp, t9, t, 0, TE] + iSqidtrap[Grot, ϵ, Gro, t10, t, 0, TE];
dir++];
dir = 1;

dirPlot = 1;
(*Plot[{subs[Gread[t,dirPlot]]/.time2/.subamp[dirPlot],
   ScaleDiagram*subs[Fread[t,dirPlot]]/.time2/.subamp[dirPlot],
   10ScaleDiagram*subs[IntFread[t,dirPlot]]/.time2/.subamp[dirPlot],
   (500ScaleDiagram)^2*subs[IntSqFread[t,dirPlot]]/.time2/.subamp[dirPlot]},
  {t,-11000*10^-6,71000*10^-6},PlotRange→Full,
  PlotLegends→{"g","F=Integral[g]=m0","Integral[F]","Integral[F^2]"}]*)
```

(* TRSE:



*)

(* TRSEadj:

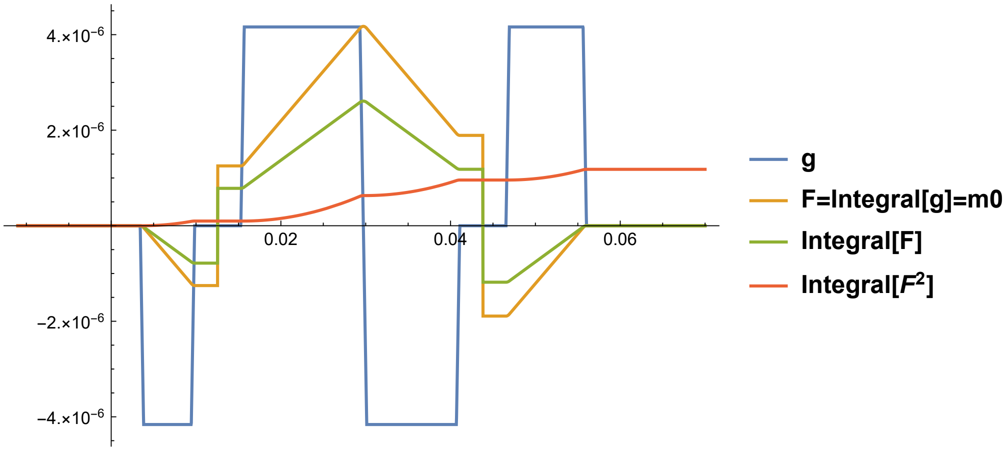


*)

```
PHASE;
dir = 1;
While[dir < ndir + 1,
  AmpIntSqIntPhaseAtT1[dir] = subs[ iSqidtrap[Gpet, ϵ, Gpe, t1, T1, 0, TE] +
       iSqidtrap[δ1, ϵ, SignDelta[[1]] * Gdp, t2, T1, 0, TE] +
       iSqidtrap[Crut, RampCrushers, -Gcp, t3, T1, 0, TE]] /. time2 /. subamp[dir];

  AmpIntSqIntPhaseAtT2[dir] =
   subs[Abs[AmpIntSqIntPhaseAtT1[dir]] + Abs[iSqidtrap[Crut, RampCrushers, -Gcp,
          t3, T2, 0, TE] + iSqidtrap[δ2, ϵ, SignDelta[[2]] * Gdp, t4, T2, 0, TE] +
         iSqidtrap[δ11, ϵ, SignDelta[[3]] * Gdp, t6, T2, 0, TE] +
         iSqidtrap[Crut, RampCrushers, Gcp, t7, T2, 0, TE]]] /. time2 /. subamp[dir];

  IntSqFphase[t_, dir] = iSqidtrap[Gpet, ϵ, Gpe, t1, t, 0, TE] +
    iSqidtrap[δ1, ϵ, SignDelta[[1]] * Gdp, t2, t, 0, TE] +
    iSqidtrap[Crut, RampCrushers, -Gcp, t3, t, 0, TE] +
    (UnitStep[t - T1] - UnitStep[t - T2]) *
     iSqidtrap[Crut, RampCrushers, -Gcp, t4 - Crut, t, 0, TE] +
    iSqidtrap[δ2, ϵ, SignDelta[[2]] * Gdp, t4, t, 0, TE] + iSqidtrap[δ11, ϵ,
     SignDelta[[3]] * Gdp, t6, t, 0, TE] + iSqidtrap[Crut, RampCrushers, Gcp, t7, t, 0, TE] +
    iSqidtrap[Crut, RampCrushers, Gcp, t8 - Crut, t, 0, TE] +
    iSqidtrap[δ22, ϵ, SignDelta[[4]] * Gdp, t8, t, 0, TE];
dir++];
dir = 1;
```

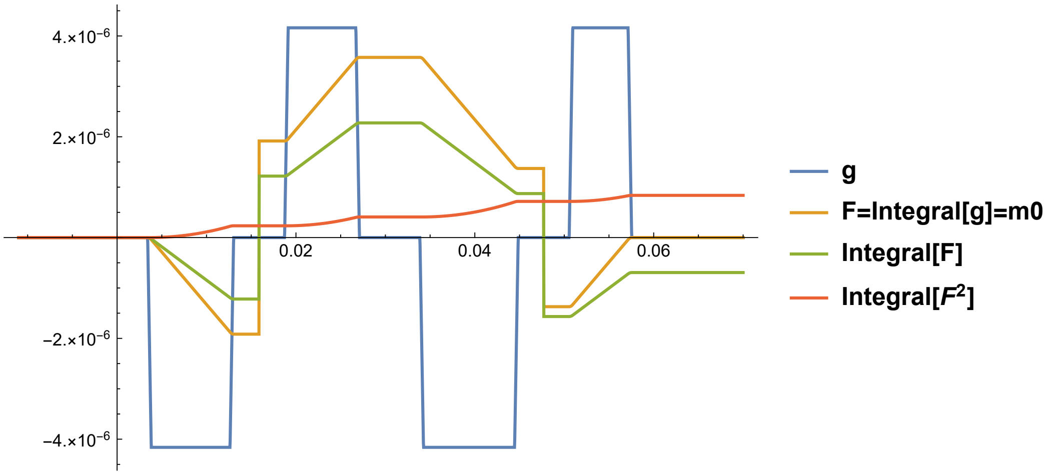
```
dirPlot = 1;
(*Plot[{subs[Gphase[t,dirPlot]]/.time2/.subamp[dirPlot],
  ScaleDiagram*subs[Fphase[t,dirPlot]]/.time2/.subamp[dirPlot],
  10ScaleDiagram*subs[IntFphase[t,dirPlot]]/.time2/.subamp[dirPlot],
  (1000ScaleDiagram)^2*subs[IntSqFphase[t,dirPlot]]/.time2/.subamp[dirPlot]},
 {t,-11000*10^-6,70000*10^-6},PlotRange→Full,
 PlotLegends→{"g","F=Integral[g]=m0","Integral[F]","Integral[F^2]"}]*)
```

(* **TRSE:**



*)

(* **TRSEadj:**

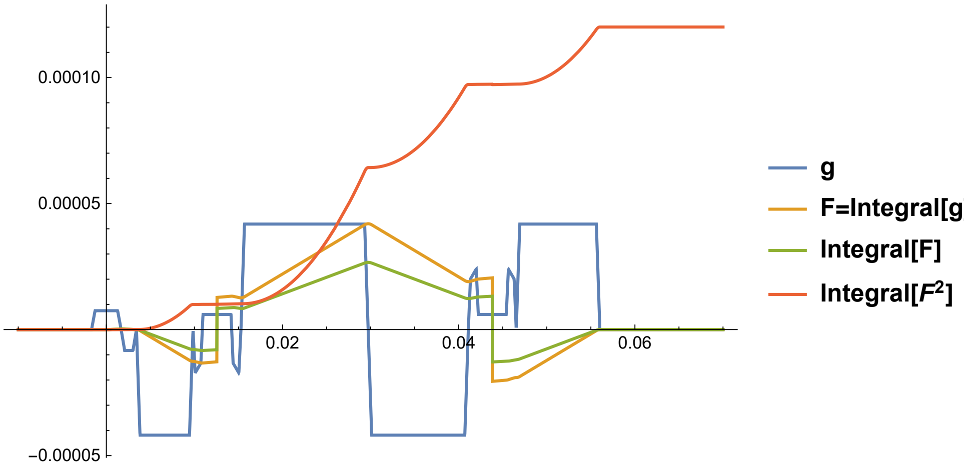


*)

```
SLICE;
dir = 1;
While[dir < ndir + 1,
  AmpIntSqIntSliceAtT1[dir] =
   subs[ iSqidtrap[Gsl90t, ∊, Gsl90, t1, T1, 0, TE] / 2. + iSqidtrap[Gsrft, ∊, Gsrf,
        t1, T1, 0, TE] + iSqidtrap[δ1, ∊, SignDelta[[1]] * Gds, t2, T1, 0, TE] +
       iSqidtrap[Crut, RampCrushers, -Gcs, t3, T1, 0, TE] +
       iSqidtrap[Gsl180t, RampGsl180, Gsl180, t3, TE, 0, TE] / 2.] /. time2 /. subamp[dir];

  AmpIntSqIntSliceAtT2[dir] =
   subs[Abs[AmpIntSqIntSliceAtT1[dir]] + Abs[iSqidtrap[Gsl180t, RampGsl180, Gsl180, t3, TE,
          0, TE] / 2. + iSqidtrap[Crut, RampCrushers, -Gcs, t3, T2, 0, TE] + iSqidtrap[δ2, ∊,
         SignDelta[[2]] * Gds, t4, T2, 0, TE] + iSqidtrap[δ11, ∊, SignDelta[[3]] * Gds,
         t6, T2, 0, TE] + iSqidtrap[Crut, RampCrushers, Gcs, t3, T2, 0, TE] +
        iSqidtrap[Gsl180t, RampGsl180, Gsl180, t7, T2, 0, TE]]] /. time2 /. subamp[dir];

  IntSqFslice[t_, dir] =
   (1 - UnitStep[t - t1]) * iSqidtrap[Gsl90t, ∊, Gsl90, t1, t + t1, 0, TE] / 2. +
    (iSqidtrap[Gsl90t, ∊, Gsl90, t1, t + t1, 0, TE] / 2. +
      iSqidtrap[Gsrft, ∊, Gsrf, t1, t, 0, TE]) +
    (iSqidtrap[δ1, ∊, SignDelta[[1]] * Gds, t2, t, 0, TE] + iSqidtrap[Crut, RampCrushers,
       -Gcs, t3, t, 0, TE] + iSqidtrap[Gsl180t, RampGsl180, Gsl180, t3, t, 0, TE]) +
    (UnitStep[t - T1] - UnitStep[t - T2]) * (iSqidtrap[Gsl180t, RampGsl180, Gsl180,
        t3, t, 0, TE] + iSqidtrap[Crut, RampCrushers, -Gcs, t4 - Crut, t, 0, TE]) +
    (iSqidtrap[δ2, ∊, SignDelta[[2]] * Gds, t4, t, 0, TE] + iSqidtrap[δ11, ∊,
       SignDelta[[3]] * Gds, t6, t, 0, TE] + iSqidtrap[Crut, RampCrushers, Gcs, t7, t, 0, TE] +
      iSqidtrap[Gsl180t, RampGsl180, Gsl180, t7, t, 0, TE]) +
    iSqidtrap[Gsl180t, RampGsl180, Gsl180, t7, t, 0, TE] -
    iSqidtrap[Gsl180t, RampGsl180, Gsl180, t7, T2, 0, TE] +
    iSqidtrap[Crut, RampCrushers, Gcs, t8 - Crut, t, 0, TE] +
    iSqidtrap[δ22, ∊, SignDelta[[4]] * Gds, t8, t, 0, TE];
  dir++];
dir = 1;
```

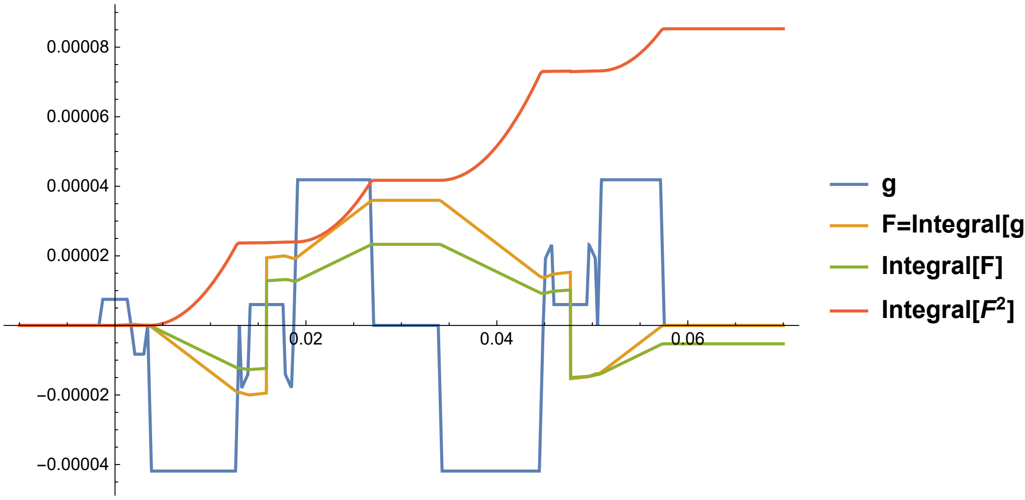
```
dirPlot = 3;
(*Plot[{subs[Gslice[t,dirPlot]]/.time2/.subamp[dirPlot],
  ScaleDiagram*subs[Fslice[t,dirPlot]]/.time2/.subamp[dirPlot],
  10ScaleDiagram*subs[IntFslice[t,dirPlot]]/.time2/.subamp[dirPlot],
  (1000ScaleDiagram)^2*subs[IntSqFslice[t,dirPlot]]/.time2/.subamp[dirPlot]},
 {t,-10000*10^-6,70000*10^-6},PlotRange→Full,
 PlotLegends→{"g","F=Integral[g]=m0","Integral[F]","Integral[F^2]"}]*)
```

(* TRSE:



*)

(* TRSEadj:



*)

"Bmatrix calculation";

```
(*set SignDelta={1.,1.,-1.,-1.};*)
```

f1 and f2 calculation;

```
dir = 1;
While[dir < ndir + 1,
  (F[t_, dir] = {{Fread[t, dir]}, {Fphase[t, dir]}, {Fslice[t, dir]}}) // MatrixForm;

  f1read[dir] = Fread[t, dir] /. t → T1 ;
  f1phase[dir] = Fphase[t, dir] /. t → T1 ;
  f1slice[dir] = Fslice[t, dir] /. t → T1 ;
  (f1[dir] = {{f1read[dir]}, {f1phase[dir]}, {f1slice[dir]}}) // MatrixForm;

  f2read[dir] = Fread[t, dir] /. t → T2 ;
  f2phase[dir] = Fphase[t, dir] /. t → T2 ;
  f2slice[dir] = Fslice[t, dir] /. t → T2 ;
  (f2[dir] = {{f2read[dir]}, {f2phase[dir]}, {f2slice[dir]}}) // MatrixForm;
  dir++];
dir = 1;
```

b calculation;

```
(* with b = γ^2*Integral[ Transpose[F-( (ξ-1)f1 + (β-1)f2].[F-( (ξ-1)f1 + (β-1)f2],
   {t,0,TE}] with ξ=1 at t<T1 and ξ=-1 at t>T1; β=1 at t<T2 and β=-1 at t>T2 *)
dir = 1;
While[dir < ndir + 1,
  b1v[dir] = NIntegrate[F[t, dir].Transpose[F[t, dir]], {t, 0, T1}];
  b2v[dir] =
   NIntegrate[(F[t, dir] - 2 f1[dir]).Transpose[F[t, dir] - 2 f1[dir]], {t, T1, T2}];
  b3v[dir] = NIntegrate[(F[t, dir] - 2 f1[dir] - 2 f2[dir]).
     Transpose[F[t, dir] - 2 f1[dir] - 2 f2[dir]], {t, T2, N[TE /. time1]}];
  dir++];
dir = 1;

bTensor =
  Reap[Do[Sow[γ^2 * (b1v[dir] + b2v[dir] + b3v[dir])], {dir, 1, ndir}]][[2]] // MatrixForm;
bTrace = Transpose[Reap[Do[Sow[Tr[bTensor[[1, 1, dir]]]], {dir, 1, ndir}]][[2]]];
Mean[bTrace]; (* 749.86 *)
StandardDeviation[bTrace]; (* 32.33 *)
Bmatrix =
 Reap[Do[Sow[ {bTensor[[1, 1, dir, 1, 1]], bTensor[[1, 1, dir, 2, 2]], bTensor[[1, 1,
        dir, 3, 3]], 2 bTensor[[1, 1, dir, 1, 2]], 2 bTensor[[1, 1, dir, 1, 3]],
       2 bTensor[[1, 1, dir, 2, 3]]}], {dir, 1, ndir}]][[2]]

(*Switch[ndir,4,Export["C:\\users\\BmatrixTRSEadj_4dir.xlsx",Bmatrix,"XLSX"],
  6,Export["C:\\users\\BmatrixTRSEadj_6dir.xlsx",Bmatrix,"XLSX"],
  32,Export["C:\\users\\BmatrixTRSEadj_32dir.xlsx",Bmatrix,"XLSX"],
  64,Export["C:\\users\\BmatrixTRSEadj_64dir.xlsx",Bmatrix,"XLSX"]];*)
```

Bmatrix display;

```
Bmatrix = TableForm[
   Transpose[Reap[Do[Sow[ {{bTensor[[1, 1, dir, 1, 1]], bTensor[[1, 1, dir, 2, 2]],
          bTensor[[1, 1, dir, 3, 3]], 2 bTensor[[1, 1, dir, 1, 2]], 2 bTensor[[1, 1,
            dir, 1, 3]], 2 bTensor[[1, 1, dir, 2, 3]]}}], {dir, 1, ndir}]][[2]]]];
Bmatrix[[
  1,
  1,
  dir]];
```

(*For the graph's plot and the calculation of the diffusion dependence d, velocity phase shift ϕ(t)=kv . v *)

```
(*"Tissue perfusion in humans studied by fourier velocity
   distribution,line scan,and echo-planar imaging"(Feinberg, 1990)*)
(*set SignDelta={1.,-1.,1.,-1.};*)
d = γ^2 * NIntegrate[F[t, 1].Transpose[F[t, 1]], {t, 0, N[TE /. time1]}];
dTrace = Tr[dslice];
dExp =
  γ^2 * NIntegrate[F[t, 1].Transpose[F[t, 1]] * Exp[-t / 0.030], {t, 0, N[TE /. time1]}] //
   MatrixForm;

kv = TableForm[
   Transpose[Reap[Do[Sow[ {{γ * NIntegrate[t * Gread[t, dir], {t, 0, N[TE /. time1]}],
          γ * NIntegrate[t * Gphase[t, dir], {t, 0, N[TE /. time1]}], γ *
           NIntegrate[t * Gslice[t, dir], {t, 0, N[TE /. time1]}]}}], {dir, 1, 3}]][[2]]]];
```

M0 and M1 and M2 ;

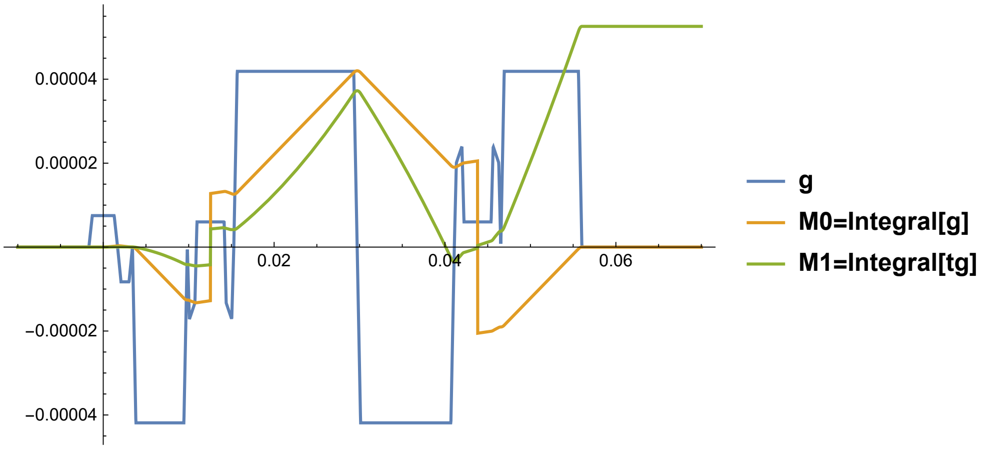
integral M1;

```
iM1dtrap[DurationGrad_, RampTime_, AmpGrad_, ll_, ul_] =
  Simplify[Refine[FiInt[t * trap[wid, RampTime, amp, t - ll], ll, ul], Assumptions → {wid > 0,
       RampTime > 0, ul > 0, wid > RampTime}]] /. wid → DurationGrad /. amp → AmpGrad;
```

```
dir = 1;
While[dir < ndir + 1,
  AmpIntM1SliceAtT1[dir] =
   subs[ iM1dtrap[Gsl90t, ϵ, Gsl90, 0, t1] / 2. + iM1dtrap[Gsrft, ϵ, Gsrf, t1, t1 + Gsrft + ϵ] +
       iM1dtrap[δ1, ϵ, SignDelta[[1]] * Gds, t2, t3] +
       iM1dtrap[Crut, RampCrushers, -Gcs, t3, t3 + Crut + RampCrushers] +
       iM1dtrap[Gsl180t, RampGsl180, Gsl180, t3, T1]] /. time2 /. subamp[dir];

  AmpIntM1SliceAtT2[dir] =
   subs[-AmpIntM1SliceAtT1[dir] + iM1dtrap[Gsl180t, RampGsl180, Gsl180, t3, T1] +
       iM1dtrap[Crut, RampCrushers, -Gcs, t4 - (Crut + RampCrushers), t4] +
       iM1dtrap[δ2, ϵ, SignDelta[[2]] * Gds, t4, t6] + iM1dtrap[δ11, ϵ, SignDelta[[3]] * Gds,
        t6, t7] + iM1dtrap[Crut, RampCrushers, Gcs, t7, t7 + Crut + RampCrushers] +
       iM1dtrap[Gsl180t, RampGsl180, Gsl180, t7, T2]] /. time2 /. subamp[dir];

  M1slice[t_, dir] =
   (1 - UnitStep[t - t1]) * iM1dtrap[Gsl90t, ϵ, Gsl90, t1, t + t1] / 2. + UnitStep[t - t1] *
     (iM1dtrap[Gsl90t, ϵ, Gsl90, t1, t + t1] / 2. + iM1dtrap[Gsrft, ϵ, Gsrf, t1, t]) +
    (1 - UnitStep[t - T1]) * (iM1dtrap[δ1, ϵ, SignDelta[[1]] * Gds, t2, t] + iM1dtrap[Crut,
        RampCrushers, -Gcs, t3, t] + iM1dtrap[Gsl180t, RampGsl180, Gsl180, t3, t]) +
    (UnitStep[t - T1] - UnitStep[t - T2]) * (-AmpIntM1SliceAtT1[dir] + iM1dtrap[Gsl180t,
        RampGsl180, Gsl180, t3, t] - iM1dtrap[Gsl180t, RampGsl180, Gsl180, t3, T1]
       + iM1dtrap[Crut, RampCrushers, -Gcs, t4 - (Crut + 2 RampCrushers), t]) +
    (1 - UnitStep[t - T2]) * (iM1dtrap[δ2, ϵ, SignDelta[[2]] * Gds, t4, t] +
       iM1dtrap[δ11, ϵ, SignDelta[[3]] * Gds, t6, t] + iM1dtrap[Crut, RampCrushers,
        Gcs, t7, t] + iM1dtrap[Gsl180t, RampGsl180, Gsl180, t7, t]) +
    UnitStep[t - T2] * (-AmpIntM1SliceAtT2[dir] +
       iM1dtrap[Gsl180t, RampGsl180, Gsl180, t7, t] - iM1dtrap[Gsl180t, RampGsl180, Gsl180,
        t7, T2] + iM1dtrap[Crut, RampCrushers, Gcs, t8 - (Crut + RampCrushers), t]) +
    iM1dtrap[δ22, ϵ, SignDelta[[4]] * Gds, t8, t];
  dir++];
dir = 1;
```

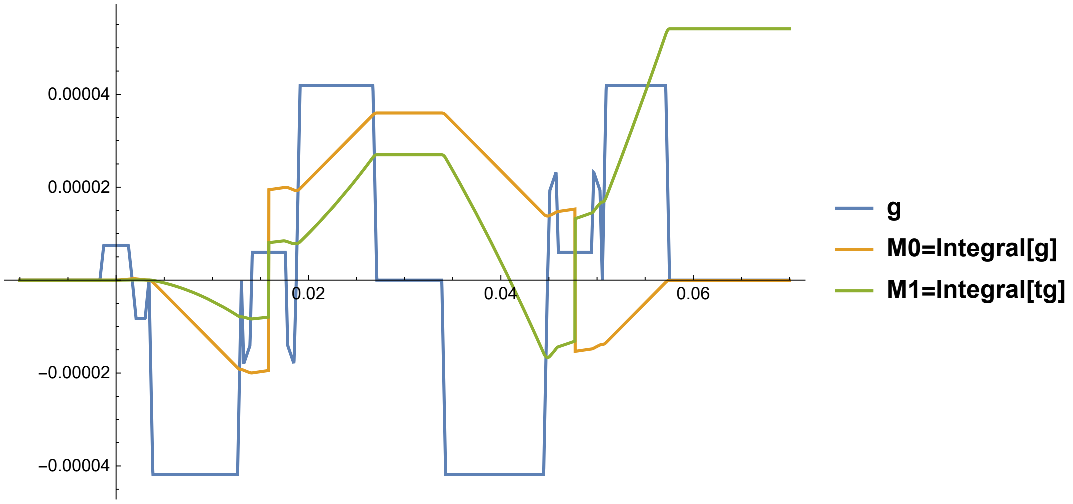
```
dirPlot = 3;
(*Plot[{subs[Gslice[t,dirPlot]]/.time2/.subamp[dirPlot],
  ScaleDiagram*subs[Fslice[t,dirPlot]]/.time2/.subamp[dirPlot],
  50ScaleDiagram*subs[M1slice[t,dirPlot]]/.time2/.subamp[dirPlot]},
 {t,-10000*10^-6,70000*10^-6},PlotRange→Full,
 PlotLegends→{"g","M0=Integral[g]","M1=Integral[tg]"}]*)
```

(* TRSE:



*)

(* TRSEadj:



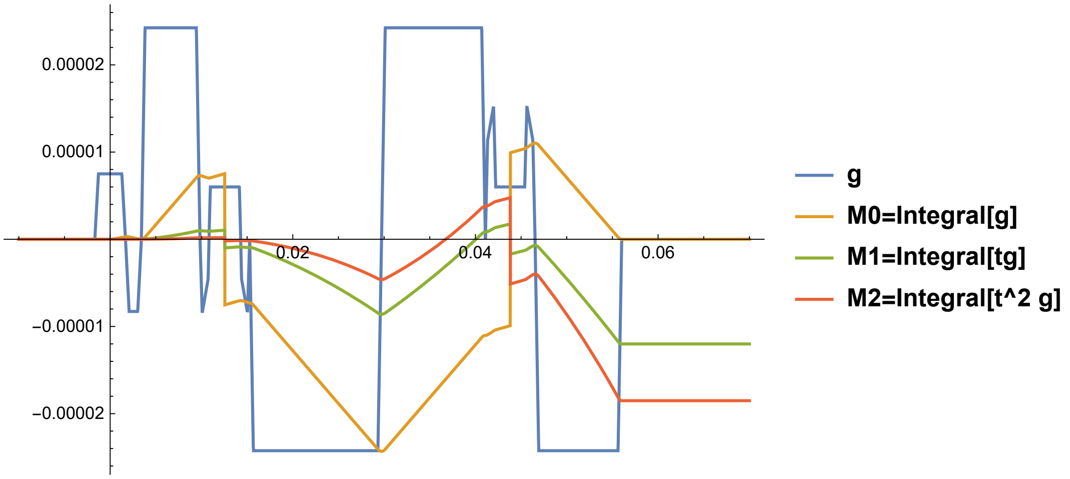
*)

integral M2;

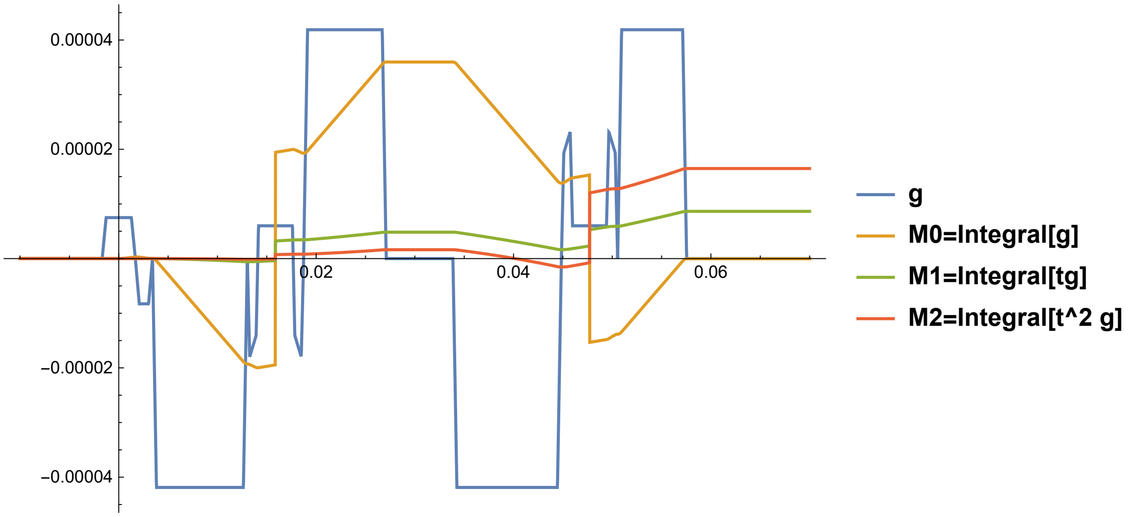
```
iM2dtrap[DurationGrad_, RampTime_, AmpGrad_, ll_, ul_] =
  Simplify[Refine[FiInt[t^2 * trap[wid, RampTime, amp, t - ll], ll, ul], Assumptions → {wid > 0,
        RampTime > 0, ul > 0, wid > RampTime}]] /. wid → DurationGrad /. amp → AmpGrad;
```

```
dir = 1;
While[dir < ndir + 1, AmpIntM2SliceAtT1[dir] =
   subs[ iM2dtrap[Gsl90t, ϵ, Gsl90, 0, t1] / 2. + iM2dtrap[Gsrft, ϵ, Gsrf, t1, t1 + Gsrft + ϵ] +
       iM2dtrap[δ1, ϵ, SignDelta[[1]] * Gds, t2, t3] +
       iM2dtrap[Crut, RampCrushers, -Gcs, t3, t3 + Crut + RampCrushers] +
       iM2dtrap[Gsl180t, RampGsl180, Gsl180, t3, T1]] /. time2 /. subamp[dir];

  AmpIntM2SliceAtT2[dir] =
   subs[AmpIntM2SliceAtT1[dir] + iM2dtrap[Gsl180t, RampGsl180, Gsl180, t3, T1] +
       iM2dtrap[Crut, RampCrushers, -Gcs, t4 - (Crut + RampCrushers), t4] +
       iM2dtrap[δ2, ϵ, SignDelta[[2]] * Gds, t4, t6] + iM2dtrap[δ11, ϵ, SignDelta[[3]] * Gds,
        t6, t7] + iM2dtrap[Crut, RampCrushers, Gcs, t7, t7 + Crut + RampCrushers] +
       iM2dtrap[Gsl180t, RampGsl180, Gsl180, t7, T2]] /. time2 /. subamp[dir];

  M2slice[t_, dir] =
   (1 - UnitStep[t - t1]) * iM2dtrap[Gsl90t, ϵ, Gsl90, t1, t + t1] / 2. + UnitStep[t - t1] *
     (iM2dtrap[Gsl90t, ϵ, Gsl90, t1, t + t1] / 2. + iM2dtrap[Gsrft, ϵ, Gsrf, t1, t]) +
    (1 - UnitStep[t - T1]) * (iM2dtrap[δ1, ϵ, SignDelta[[1]] * Gds, t2, t] + iM2dtrap[Crut,
        RampCrushers, -Gcs, t3, t] + iM2dtrap[Gsl180t, RampGsl180, Gsl180, t3, t]) +
    (UnitStep[t - T1] - UnitStep[t - T2]) * (-AmpIntM2SliceAtT1[dir] + iM2dtrap[Gsl180t,
        RampGsl180, Gsl180, t3, t] - iM2dtrap[Gsl180t, RampGsl180, Gsl180, t3, T1]
       + iM2dtrap[Crut, RampCrushers, -Gcs, t4 - (Crut + 2 RampCrushers), t]) +
    (1 - UnitStep[t - T2]) * (iM2dtrap[δ2, ϵ, SignDelta[[2]] * Gds, t4, t] +
       iM2dtrap[δ11, ϵ, SignDelta[[3]] * Gds, t6, t] + iM2dtrap[Crut, RampCrushers,
        Gcs, t7, t] + iM2dtrap[Gsl180t, RampGsl180, Gsl180, t7, t]) +
    UnitStep[t - T2] * (-AmpIntM2SliceAtT2[dir] +
       iM2dtrap[Gsl180t, RampGsl180, Gsl180, t7, t] - iM2dtrap[Gsl180t, RampGsl180, Gsl180,
        t7, T2] + iM2dtrap[Crut, RampCrushers, Gcs, t8 - (Crut + RampCrushers), t]) +
    iM2dtrap[δ22, ϵ, SignDelta[[4]] * Gds, t8, t];

  dir++];
dir = 1;
```

```
dirPlot = 3;
(*Plot[{subs[Gslice[t,dirPlot]]/.time2/.subamp[dirPlot],
  ScaleDiagram*subs[Fslice[t,dirPlot]]/.time2/.subamp[dir],
  20ScaleDiagram*subs[M1slice[t,dirPlot]]/.time2/.subamp[dir],
  10ScaleDiagram^2*subs[M2slice[t,dirPlot]]/.time2/.subamp[dir]},
 {t,-10000*10^-6,70000*10^-6},PlotRange→Full,
 PlotLegends→{"g","M0=Integral[g]","M1=Integral[tg]","M2=Integral[t^2 g]"}]*)
(* TRSE:
```



```
*)
(* TRSEadj:
```



```
*)
```