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PUMA 560 robot manipulator kinematic table :

joint	θ	d	a	α
1	θ_1	0	0	-90°
2	θ_2	0	a_2	0°
3	θ_3	d_3	a_3	90°
4	θ_4	d_4	0	-90°
5	θ_5	0	0	90°
6	θ_6	0	0	0°

$$A_n = \begin{bmatrix} c\theta_n & -s\theta_n c\alpha_n & s\theta_n s\alpha_n & a_n c\theta_n \\ s\theta_n & c\theta_n c\alpha_n & -c\theta_n s\alpha_n & a_n s\theta_n \\ 0 & s\alpha_n & c\alpha_n & d_n \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_1 = \begin{bmatrix} C_1 & 0 & -S_1 & 0 \\ S_1 & 0 & C_1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_2 = \begin{bmatrix} C_2 & -S_2 & 0 & a_2C_2 \\ S_2 & C_2 & 0 & a_2S_2 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_3 = \begin{bmatrix} C_3 & 0 & S_3 & a_3C_3 \\ S_3 & 0 & -C_3 & a_3S_3 \\ 0 & 1 & 0 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_4 = \begin{bmatrix} C_4 & 0 & -S_4 & 0 \\ S_4 & 0 & C_4 & 0 \\ 0 & -1 & 0 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_5 = \begin{bmatrix} C_5 & 0 & S_5 & 0 \\ S_5 & 0 & -C_5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_6 = \begin{bmatrix} C_6 & -S_6 & 0 & 0 \\ S_6 & C_6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T = A_1 \times A_2 \times A_3 \times A_4 \times A_5 \times A_6 = \begin{bmatrix} n_x & O_x & a_x & P_x \\ n_y & O_y & a_y & P_y \\ n_z & O_z & a_z & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$n_x = C_1\left[C_{23}(C_4C_5C_6 - S_4S_6) - S_5C_6S_{23}\right] - S_1(S_4C_5C_6 + C_4S_6)$$

$$n_y = S_1\left[C_{23}(C_4C_5C_6 - S_4S_6) - S_5C_6S_{23}\right] + C_1(S_4C_5C_6 + C_4S_6)$$

$$n_z = -S_{23}(C_4C_5C_6 - S_4S_6) - S_5C_6C_{23}$$

$$O_x = -C_1\left[C_{23}(C_4C_5S_6 + S_4C_6) - S_{23}S_5S_6\right] + S_1(S_4C_5S_6 - C_4C_6)$$

$$O_y = -S_1\left[C_{23}(C_4C_5S_6 + S_4C_6) - S_5S_6S_{23}\right] - C_1(S_4C_5S_6 - C_4C_6)$$

$$O_z = S_{23}(C_4C_5S_6 + S_4C_6) + S_5S_6C_{23}$$

$$a_x = C_1(C_4S_5C_{23} + C_5S_{23}) - S_1S_4S_5$$

$$a_y = S_1(C_4S_5C_{23} + C_5S_{23}) + C_1S_4S_5$$

$$a_z = -C_4S_5S_{23} + C_5C_{23}$$

$$P_x = C_1(d_4S_{23} + a_3C_{23} + a_2C_2) - S_1d_3$$

$$P_y = S_1(d_4S_{23} + a_3C_{23} + a_2C_2) + C_1d_3$$

$$P_z = d_4C_{23} - a_3S_{23} - a_2S_2$$

① $A_1^{-1} \cdot T = A_2 \times A_3 \times A_4 \times A_5 \times A_6 = {}^1T = \begin{bmatrix} C_1 n_x + S_1 n_y & C_1 O_x + S_1 O_y & C_1 a_x + S_1 a_y & C_1 P_x + S_1 P_y \\ -n_z & -O_z & -a_z & -P_z \\ -S_1 n_x + C_1 n_y & -S_1 O_x + C_1 O_y & -S_1 a_x + C_1 a_y & -S_1 P_x + C_1 P_y \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$$= \begin{bmatrix} C_{23}(C_4C_5C_6 - S_4S_6) - S_5C_6S_{23} & -C_{23}(C_4C_5S_6 + S_4C_6) + S_{23}S_5S_6 & C_4S_5C_{23} + C_5S_{23} & d_4S_{23} + a_3C_{23} + a_2C_2 \\ S_{23}(C_4C_5C_6 - S_4S_6) + S_5C_6C_{23} & -S_{23}(C_4C_5S_6 + S_4C_6) - S_5S_6C_{23} & C_4S_5S_{23} - C_5C_{23} & -d_4C_{23} + a_3S_{23} + a_2S_2 \\ S_4C_5C_6 + C_4S_6 & -S_4C_5S_6 + C_4C_6 & S_4S_5 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

② $A_2^{-1} \cdot A_1^{-1} \cdot T = A_3 \times A_4 \times A_5 \times A_6 = {}^2T = \begin{bmatrix} C_2 & S_2 & 0 & -a_2 \\ -S_2 & C_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_1 & S_1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ -S_1 & C_1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times T = \begin{bmatrix} C_1C_2 & S_1C_2 & -S_2 & -a_2 \\ -C_1S_2 & -S_1S_2 & -C_2 & 0 \\ -S_1 & C_1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times T$

$$= \begin{bmatrix} C_3(C_4C_5C_6 - S_4S_6) - S_3S_5C_6 & -C_3(C_4C_5S_6 + S_4C_6) + S_3S_5S_6 & C_3C_4S_5 + S_3C_5 & d_4S_3 + a_3C_3 \\ S_3(C_4C_5C_6 - S_4S_6) + C_3S_5C_6 & -S_3(C_4C_5S_6 + S_4S_6) - C_3S_5S_6 & S_3C_4S_5 - C_3C_5 & -d_4C_3 + a_3C_3 \\ S_4C_5C_6 + C_4S_6 & -S_4C_5S_6 + C_4C_6 & S_4S_5 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

③ $A_3^{-1} \cdot A_2^{-1} \cdot A_1^{-1} \cdot T = A_4 \times A_5 \times A_6 = {}^3T = \begin{bmatrix} C_4C_5C_6 - S_4S_6 & -C_4C_5S_6 - S_4C_6 & C_4C_5 & 0 \\ S_4C_5C_6 + C_4S_6 & -S_4C_5S_6 + C_4S_6 & S_4C_5 & 0 \\ -S_5C_6 & S_5S_6 & C_5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

④ $A_4^{-1} \cdot A_3^{-1} \cdot A_2^{-1} \cdot A_1^{-1} \cdot T = A^5 \times A^6 = {}^4T = \begin{bmatrix} C_5C_6 & -C_5S_6 & S_5 & 0 \\ S_5C_6 & -S_5S_6 & -C_5 & 0 \\ S_6 & C_6 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

⑤ $A_5^{-1} \cdot A_4^{-1} \cdot A_3^{-1} \cdot A_2^{-1} \cdot A_1^{-1} \cdot T = A^6 = {}^5T = \begin{bmatrix} C_6 & -S_6 & 0 & 0 \\ S_6 & C_6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$.

Jacobian: $dx = J dq$

$$\begin{bmatrix} {}^{T_6}d_x \\ {}^{T_6}d_y \\ {}^{T_6}d_z \\ {}^{T_6}\delta_x \\ {}^{T_6}\delta_y \\ {}^{T_6}\delta_z \end{bmatrix} = \begin{bmatrix} {}^{T_6}d_{1x} & \cdots & {}^{T_6}d_{6x} \\ {}^{T_6}d_{1y} & \cdots & \vdots \\ {}^{T_6}d_{1z} & \cdots & \vdots \\ {}^{T_6}\delta_{1x} & \cdots & \vdots \\ {}^{T_6}\delta_{1y} & \cdots & \vdots \\ {}^{T_6}\delta_{1z} & \cdots & {}^{T_6}\delta_{6z} \end{bmatrix} \begin{bmatrix} dq_1 \\ dq_2 \\ dq_3 \\ dq_4 \\ dq_5 \\ dq_6 \end{bmatrix}$$

$$^{T_6}d_i = \begin{bmatrix} n \times [({}^{i-1}\delta_i \times p) + {}^{i-1}d_i] \\ o \times [({}^{i-1}\delta_i \times p) + {}^{i-1}d_i] \\ a \times [({}^{i-1}\delta_i \times p) + {}^{i-1}d_i] \end{bmatrix} = \begin{bmatrix} p_x n_y - n_x p_y \\ p_x o_y - o_x p_y \\ p_x a_y - a_x p_y \end{bmatrix}$$

$$^{T_6}\delta_i = \begin{bmatrix} n \times {}^{i-1}\delta_i \\ o \times {}^{i-1}\delta_i \\ a \times {}^{i-1}\delta_i \end{bmatrix} = \begin{bmatrix} n_z \\ o_z \\ a_z \end{bmatrix}$$

① let $^0T_6 = [n\ o\ a\ p] \rightarrow \frac{\partial ^0T}{\partial \theta_1} = [Td_{1x}\ Td_{1y}\ Td_{1z}\ T\delta_{1x}\ T\delta_{1y}\ T\delta_{1z}]^T$

$Td_{1x} = p_x n_y - n_x p_y = (d_4 S_{23} + a_3 C_{23} + a_2 C_2)(S_4 C_5 C_6 + C_4 S_6) - d_3 [C_{23}(C_4 C_5 C_6 - S_4 S_6) - S_5 C_6 S_{23}]$

$Td_{1y} = p_x o_y - o_x p_y = -(d_4 S_{23} + a_3 C_{23} + a_2 C_2)(S_4 C_5 S_6 - C_4 C_6) + d_3 [C_{23}(C_4 C_5 C_6 + S_4 C_6) - S_5 S_6 S_{23}]$

$Td_{1z} = p_x a_y - a_x p_y = (d_4 S_{23} + a_3 C_{23} + a_2 C_2) S_4 S_5 - d_3 (C_4 S_5 C_{23} + C_5 S_{23})$

$T\delta_{1x} = n_z = -S_{23}(C_4 C_5 C_6 - S_4 S_6) - S_5 C_6 C_{23}$

$T\delta_{1y} = o_z = S_{23}(C_4 C_5 S_6 + S_4 C_6) + S_5 S_6 C_{23}$

$T\delta_{1z} = a_z = -C_4 S_5 S_{23} + C_5 C_{23}$

② let $^1T_6 = [n\ o\ a\ p] \rightarrow \frac{\partial T}{\partial \theta_2} = [Td_{2x}\ Td_{2y}\ Td_{2z}\ T\delta_{1x}\ T\delta_{1y}\ T\delta_{1z}]^T$

$Td_{2x} = p_x n_y - n_x p_y = (C_4 C_5 C_6 - S_4 S_6)(d_4 + a_2 S_3) + S_5 C_6 (a_3 + a_2 C_3)$

$Td_{2y} = p_x o_y - o_x p_y = -(C_4 C_5 S_6 + S_4 C_6)(d_4 + a_2 S_3) - S_5 S_6 (a_3 + a_2 C_3)$

$Td_{2z} = p_x a_y - a_x p_y = C_4 S_5 d_4 - a_3 C_5 + a_2 (C_4 S_5 S_3 - C_3 C_5)$

$T\delta_{2x} = n_z = S_4 C_5 C_6 + C_4 S_6$

$T\delta_{2y} = o_z = -S_4 C_5 S_6 + C_4 C_6$

$T\delta_{2z} = a_z = S_4 S_5$

③ let $^2T_6 = [n\ o\ a\ p] \rightarrow \frac{\partial T}{\partial \theta_3} = [Td_{3x}\ Td_{3y}\ Td_{3z}\ T\delta_{3x}\ T\delta_{3y}\ T\delta_{3z}]^T$

$Td_{3x} = p_x n_y - n_x p_y = d_4 (C_4 C_5 C_6 - S_4 S_6) + a_3 S_5 C_6$

$Td_{3y} = p_x o_y - o_x p_y = -a_3 S_5 S_6 - d_4 (C_4 C_5 S_6 + S_4 C_6)$

$Td_{3z} = p_x a_y - a_x p_y = d_4 C_4 S_5 - a_3 C_5$

$T\delta_{3x} = n_z = S_4 C_5 C_6 + C_4 S_6$

$T\delta_{3y} = o_z = -S_4 C_5 S_6 + C_4 C_6$

$T\delta_{3z} = a_z = S_4 S_5$

④ let ${}^{3}T_6 = [n\ o\ a\ p] \rightarrow \frac{\partial T}{\partial \theta_4} = \left[{}^{T}d_{4x}\ {}^{T}d_{4y}\ {}^{T}d_{4z}\ {}^{T}\delta_{4x}\ {}^{T}\delta_{4y}\ {}^{T}\delta_{4z}\right]^T$

${}^{T}d_{4x} = p_x n_y - n_x p_y = 0$, ${}^{T}d_{4y} = p_x o_y - o_x p_y = 0$, ${}^{T}d_{4z} = p_x a_y - a_x p_y = 0$

${}^{T}\delta_{4x} = n_z = -S_5 C_6$, ${}^{T}\delta_{4y} = o_z = S_5 S_6$, ${}^{T}\delta_{4z} = C_5$

⑤ let ${}^{4}T_6 = [n\ o\ a\ p] \rightarrow \frac{\partial T}{\partial \theta_5} = \left[{}^{T}d_{5x}\ {}^{T}d_{5y}\ {}^{T}d_{5z}\ {}^{T}\delta_{5x}\ {}^{T}\delta_{5y}\ {}^{T}\delta_{5z}\right]^T$

$\frac{\partial T}{\partial \theta_5} = [0, 0, 0, S_6, C_6, 0]^T$

⑥ let ${}^{5}T_6 = [n\ o\ a\ p] \rightarrow \frac{\partial T}{\partial \theta_6} = \left[{}^{T}d_{6x}\ {}^{T}d_{6y}\ {}^{T}d_{6z}\ {}^{T}\delta_{6x}\ {}^{T}\delta_{6y}\ {}^{T}\delta_{6z}\right]^T$

$\frac{\partial T}{\partial \theta_6} = [0, 0, 0, 0, 0, 1]^T$.

$\Rightarrow J = [①\ ②\ ③\ ④\ ⑤\ ⑥]$ #

(2) Inverse Jacobian:

① $-S_1 P_x + C_1 P_y = d_3 \rightarrow$ 對 θ_1 微分 $\rightarrow -C_1 P_x - \frac{S_1 dP_x}{d\theta_1} - S_1 P_y + \frac{C_1 dP_y}{d\theta_1} = 0 \rightarrow d\theta_1 = \frac{-S_1 dP_x + C_1 dP_y}{C_1 P_x + S_1 P_y}$

if $C_1 P_x + S_1 P_y = 0 \Rightarrow$ singular point #

② $a_3 S_3 + d_4 S_3 = \frac{P_x^2 + P_y^2 + P_z^2 - a_2^2 - a_3^2 - d_3^2 - d_4^2}{2a_2} \rightarrow$ 對 θ_3 微分 $\rightarrow d\theta_3 = \frac{P_x dP_x + P_y dP_y + P_z dP_z}{-a_2(a_3 S_3 - d_4 C_3)}$

if $a_3 C_3 + d_4 C_3 = 0 \Rightarrow$ singular point #

③ $C_2(C_1 P_x + S_1 P_y) - S_2 P_z = d_4 S_3 + a_3 C_3 + a_2 \rightarrow$ 對 θ_2 微分 $\rightarrow$ $\boxed{\begin{aligned} & d\theta_2 (S_2 C_1 P_x + S_1 S_2 P_y + C_2 P_z) \\ &= C_1 C_2 dP_x + S_1 C_2 dP_y - S_2 dP_z \end{aligned}}$

詳見最後一頁

其中 θ_1 包含 $P_x, P_y \rightarrow$ 包含 θ_2
因此 C_1, S_1 也要對 θ_2 微分

④ $\tan\theta_4 = \frac{-S_1 a_x + C_1 a_y}{C_1 C_{23} a_x + S_1 C_{23} a_y - S_{23} a_z}$

對 θ_4 微分 $\rightarrow \frac{1}{\cos^2\theta_4} = \frac{\left(-S_1 \frac{da_x}{d\theta_4} + C_1 \frac{da_y}{d\theta_4}\right)(C_1 C_{23} a_x + S_1 C_{23} a_y - S_{23} a_z) - (-S_1 a_x + C_1 a_y)\left(C_1 C_{23} \frac{da_x}{d\theta_4} + S_1 C_{23} \frac{da_y}{d\theta_4} - S_{23} \frac{da_z}{d\theta_4}\right)}{(C_1 C_{23} a_x + S_1 C_{23} a_y - S_{23} a_z)^2}$

$\rightarrow d\theta_4 = \frac{[(-S_1 da_x + C_1 da_y)(C_1 C_{23} a_x + S_1 C_{23} a_y - S_{23} a_z) - (-S_1 a_x + C_1 a_y)(C_1 C_{23} da_x + S_1 C_{23} da_y - S_{23} da_z)] C_4^2}{(C_1 C_{23} a_x + S_1 C_{23} a_y - S_{23} a_z)^2}$

↳ if $C_1 C_{23} a_x + S_1 C_{23} a_y - S_{23} a_z = 0$
$\Rightarrow$ singular point.

⑤ $-S_1 a_x + C_1 a_y = S_4 S_5$ → 對 θ_5 微分 → $\boxed{S_1 \dfrac{da_x}{d\theta_5} + C_1 \dfrac{da_y}{d\theta_5} = S_4 C_5}$

詳見最後一頁

其中 θ_4 含有 a_x, a_y, a_z → 包含 θ_5
因此 S_4 也要對 θ_5 微分

⑥ $$\tan\theta_6 = \frac{C_1 S_{23} O_x + S_1 S_{23} O_y + C_{23} O_z}{-(C_1 S_{23} n_x + S_1 S_{23} n_y + C_{23} n_z)}, \quad \text{suppose } \tan\theta_6 = \frac{A}{-B}$$

→ 對 θ_6 微分 → $$\frac{1}{\cos^2\theta_6} = \frac{-\left(C_1 S_{23}\frac{dO_x}{d\theta_6} + S_1 S_{23}\frac{dO_y}{d\theta_6} + C_{23}\frac{dO_z}{d\theta_6}\right)B + \left(C_1 S_{23}\frac{dn_x}{d\theta_6} + S_1 S_{23}\frac{dn_y}{d\theta_6} + C_{23}\frac{dn_z}{d\theta_6}\right)A}{B^2}$$

$$\rightarrow d\theta_6 = \frac{\left[-(C_1 S_{23} dO_x + S_1 S_{23} dO_y + C_{23} dO_z)B + (C_1 S_{23} dn_x + S_1 S_{23} dn_y + C_{23} dn_z)A\right] C_6^2}{B^2}$$

(b) from the method in "An Efficient Solution of ... Robots", 已知 J_w

$$J_w = \begin{bmatrix} J_{11} & J_{12} & J_{13} & 0 & 0 & 0 \\ J_{21} & 0 & 0 & 0 & 0 & 0 \\ J_{31} & J_{32} & J_{33} & 0 & 0 & 0 \\ J_{41} & 0 & 0 & 0 & J_{45} & J_{46} \\ 0 & 1 & 1 & 0 & J_{55} & J_{56} \\ J_{61} & 0 & 0 & 1 & 0 & J_{66} \end{bmatrix}$$

$d\bar{x} = J_w d\bar{q}$, $d\bar{x} = [dx \; dy \; dz \; \delta x \; \delta y \; \delta z]^T$

$w_i = \bar{\delta}_i + \varepsilon \bar{d}_i$, $\bar{\delta}_i = [\delta x_i \; \delta y_i \; \delta z_i]^T$, $\bar{d}_i = [dx_i \; dy_i \; dz_i]^T$

$w_{\delta T} = \bar{\delta} + \varepsilon \bar{d} = w_1 dq_1 + w_2 dq_2 + \cdots w_6 dq_6$

$$N_i = \det\begin{pmatrix} w_1 & w_2 & \cdots & w_i \\ w_1\times w_1 & w_1 \times w_2 & \cdots & w_1 \times w_i \\ \vdots & \vdots & & \vdots \\ w_{i-1}\times w_1 & w_{i-1}\times w_2 & & w_i \times w_i \end{pmatrix}$$

$w_i \times w_j = \bar{\delta}_i \times \bar{\delta}_j + \bar{d}_i \times \bar{d}_j$

$N_i \times w_j = 0$, for $j > i$

$$dq_i = \frac{\left(w_{\delta T} - \sum_{j=i+1}^{N} dq_j w_j\right) \times N_i}{w_i \times N_i}, \quad dq_N = \frac{w_{\delta T} \times N_N}{w_N \times N_N}$$

$$dq_1 = \frac{w_{\delta T}^{w} \times N_1^{w}}{w_1^{w} \times N_1^{w}}, \quad dq_i = \frac{\left(w_{\delta T}^{w} - \sum_{j=1}^{i-1} dq_j w_j^{w}\right) \times N_i^{w}}{w_i^{w} \times N_i^{w}}, \quad i = 2, 3$$

$$dq_i = \frac{\left[\left(w_{\delta T}^{w} - \sum_{j=i+1}^{6} dq_j w_j^{w}\right) - \sum_{j=1}^{3} dq_j w_j^{w}\right] N_i^{w}}{w_i^{w} \times N_i^{w}}, \quad i = 4, 5, 6$$

$J_{11} = -J_{61} d_3$

$J_{21} = a_2 C_2 + a_3 J_{61} - d_4 J_{41}$

$J_{31} = J_{41} d_3$

$J_{41} = -S_{23}$

$J_{61} = C_{23}$

$J_{12} = a_2 S_3 + d_4$

$J_{32} = -a_2 C_3 - a_3$

$J_{13} = d_4$

$J_{33} = -a_3$

$J_{45} = -S_4$

$J_{55} = C_4$

$J_{46} = C_4 S_5$

$J_{56} = S_4 S_5$

$J_{66} = C_5$

$$w_1^w = J_{41}\bar{i}^w + J_{61}\bar{k}^w + \varepsilon\left(J_{11}\bar{i}^w + J_{21}\bar{j}^w + J_{31}\bar{k}^w\right)$$

$$w_2^w = \bar{j}^w + \varepsilon\left(J_{12}\bar{i}^w + J_{32}\bar{k}^w\right)$$

$$w_3^w = \bar{j}^w + \varepsilon\left(J_{13}\bar{i}^w + J_{33}\bar{k}^w\right)$$

$$w_4^w = \bar{k}^w$$

$$w_5^w = J_{45}\bar{i}^w + J_{55}\bar{j}^w$$

$$w_6^w = J_{46}\bar{i}^w + J_{56}\bar{j}^w + J_{66}\bar{k}^w$$

$$N_1^w = \varepsilon\bar{j}^w$$

$$N_2^w = \varepsilon\left(J_{33}\bar{i}^w - J_{13}\bar{k}^w\right)$$

$$N_3^w = \varepsilon\left(J_{13}\bar{i}^w + J_{33}\bar{k}^w\right)$$

$$N_6^w = J_{55}\bar{i}^w - J_{45}\bar{j}^w$$

$$N_5^w = J_{45}\bar{i}^w + J_{55}\bar{j}^w$$

$$N_4^w = \bar{k}^w$$

$$dq_1 = \frac{dyw}{J_{21}}$$

$$dq_2 = \frac{AJ_{33} - BJ_{13}}{J_{12}J_{33} - J_{13}J_{32}}$$

$$dq_3 = \frac{(A - J_{12}dq_2)J_{13} + (B - J_{32}dq_2)J_{33}}{J_{13}^2 + J_{33}^2}$$

$$dq_6 = \frac{CJ_{55} - DJ_{55}}{J_{46}J_{55} - J_{45}J_{56}}$$

$$dq_5 = \left(C - J_{46}dq_6\right)J_{45} + \left(D - J_{56}dq_6\right)J_{55}$$

$$dq_4 = \delta zw - J_{61}dq_1 - J_{66}dq_6$$

$A = dxw - J_{11}dq_1$, $B = dzw - J_{31}dq_1$, $C = \delta xw - J_{41}dq_1$, $D = \delta yw - dq_2 - dq_3$

$$J_{21} = a_2C_2 + a_3J_{61} - d_4J_{41} = 0$$

$$J_{12}J_{33} - J_{13}J_{32} = 0.$$

$$J_{46}J_{55} - J_{45}J_{56} = S_5 = 0.$$

$$d\bar{q} = J_w^{-1}\, d\bar{x}_w, \quad d\bar{x}_w = \begin{bmatrix} dx_w & dy_w & dz_w & \delta x_w & \delta y_w & \delta z_w \end{bmatrix}^T$$

$$J_w^{-1} = \begin{bmatrix} 0 & G_{12} & 0 & 0 & 0 & 0 \\ G_{21} & G_{22} & G_{23} & 0 & 0 & 0 \\ G_{31} & G_{32} & G_{33} & 0 & 0 & 0 \\ G_{41} & G_{42} & G_{43} & G_{44} & G_{45} & 1 \\ G_{51} & G_{52} & G_{53} & G_{54} & G_{55} & 0 \\ G_{61} & G_{62} & G_{63} & G_{64} & G_{65} & 0 \end{bmatrix}$$

$$G_{12} = \frac{1}{J_{12}}, \quad G_{21} = \frac{J_{33}}{L}, \quad G_{22} = \frac{G_{12}M}{L}, \quad G_{23} = \frac{-J_{13}}{L}$$

$$G_{31} = \frac{(J_{13} - G_{21}P)}{N}, \quad G_{32} = \frac{-(G_{12}Q + G_{22}P)}{N}$$

$$G_{33} = \frac{(J_{33} - G_{23}P)}{N}, \quad G_{61} = \frac{J_{45}(G_{21} + G_{31})}{R}$$

$$G_{62} = \frac{[J_{45}(G_{22} + G_{32}) - J_{41}J_{55}G_{12}]}{R}, \quad G_{63} = \frac{(G_{23} + G_{33})J_{45}}{R}$$

$$G_{64} = \frac{J_{55}}{R}, \quad G_{65} = \frac{-J_{45}}{R}, \quad G_{51} = -J_{55}(G_{21} + G_{31}) - G_{61}S$$

$$G_{52} = -J_{41}J_{45}G_{12} - J_{55}(G_{22} + G_{32}) - G_{62}S, \quad G_{53} = -J_{55}(G_{23} + G_{33}) - G_{63}S$$

$$G_{54} = J_{45} - G_{64}S, \quad G_{55} = J_{55} - G_{65}S, \quad G_{41} = -J_{66}G_{61}$$

$$G_{42} = -J_{61}G_{12} - J_{66}G_{62}, \quad G_{43} = -J_{66}G_{63}, \quad G_{44} = -J_{66}G_{64}$$

$$G_{45} = -J_{66}G_{65}$$

$$L = J_{12}J_{33} - J_{13}J_{32}, \quad M = J_{13}J_{31} - J_{11}J_{33}, \quad N = J_{13}^2 + J_{33}^2$$

$$P = J_{12}J_{13} + J_{32}J_{33}, \quad Q = J_{11}J_{13} + J_{31}J_{33}, \quad R = J_{46}J_{55} - J_{45}J_{56}$$

$$S = J_{45}J_{46} + J_{55}J_{56}$$

singular 條件：

$$J_{21} = a_2 \cos\theta_2 + a_3 J_{61} - d_4 J_{41} = 0$$

$$J_{12}J_{33} - J_{13}J_{32} = a_2 d_4 \cos\theta_3 - a_2 a_3 \sin\theta_3 = 0$$

$$J_{46}J_{55} - J_{45}J_{56} = \sin\theta_5 = 0$$

① 從 ^2T 得 $C_2(C_1P_x + S_1P_y) - S_2P_z = d_4S_3 + a_3C_3 + a_2$

→ 對 θ_2 微分 →

$$-S_2C_1P_x + \frac{C_2dC_1P_x}{d\theta_2} + \frac{C_1C_2dP_x}{d\theta_2} - S_2S_1P_y + \frac{C_2dS_1P_y}{d\theta_2} + \frac{C_2S_1dP_y}{d\theta_2} - C_2P_z - \frac{S_2dP_z}{d\theta_2} = \frac{d_4dS_3}{d\theta_2} + \frac{a_3dC_3}{d\theta_2}$$

$$\Rightarrow (S_2C_1P_x + S_2S_1P_y + C_2P_z)\,d\theta_2 = C_2dC_1P_x + C_1C_2dP_x + C_2dS_1P_y + C_2S_1dP_y - S_2dP_z - d_4dS_3 - a_3dC_3$$

$$\Rightarrow d\theta_2 = \frac{C_2dC_1P_x + C_1C_2dP_x + C_2dS_1P_y + C_2S_1dP_y - S_2dP_z - d_4dS_3 - a_3dC_3}{S_2C_1P_x + S_2S_1P_y + C_2P_z}$$ #

if $S_2C_1P_x + S_2S_1P_y + C_2P_z = 0 \Rightarrow$ sigular point #

⑤ 從 ^3T 得 $-S_1a_x + C_1a_y = S_4S_5$

→ 對 θ_5 微分 →

$$-S_1\frac{da_x}{d\theta_5} + C_1\frac{da_y}{d\theta_5} = \frac{(dS_4)S_5}{d\theta_5} + S_4C_5$$

$$\Rightarrow (S_4C_5)\,d\theta_5 = -S_1da_x + C_1da_y - (dS_4)S_5$$

$$d\theta_5 = \frac{-S_1da_x + C_1da_y - (dS_4)S_5}{S_4C_5}$$ #

if $S_4C_5 = 0 \Rightarrow$ singular point #

〈補充 Inverse Jacobian 的 ② 與 ⑤〉