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Robot task

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Trajectory Planning — 規劃路徑軌跡，包含位置及速度等資訊

↓ $q_d, \dot{q}_d$ 軌跡交給控制器做監控

Feedback / Feedforward control

feed forward ：預先預測動態進行補償

feedback ：動態執行後，發現誤差，進行補償修正

↓ Torque (F=ma) 力或力矩

Dynamic Equations — 計算機器人的運動方程式

↓ $\ddot{q}_d$ 加速度或角加速度

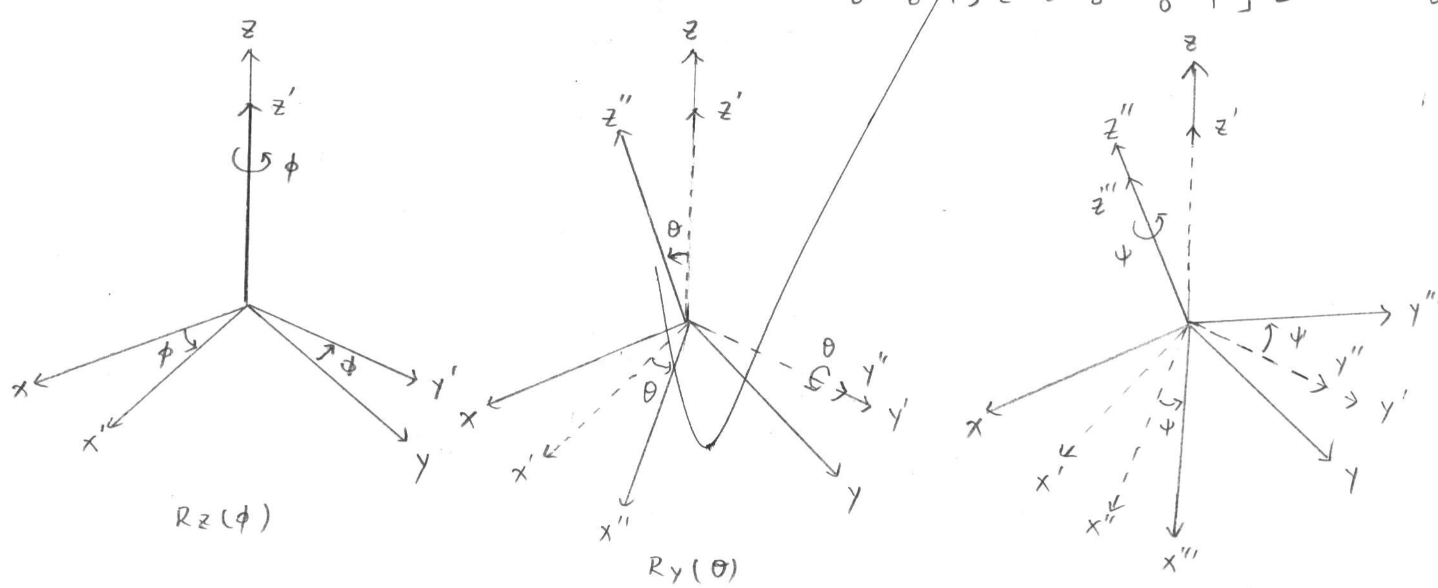
Motion Integration — 整合所有運動並輸出並將速度與位置送回至控制器，看規劃結果與當前結果差多少，去進行補償

Main challenges：
動力學是最難去模擬的地方，因為很難對物理性質做出一個精確的描述。

2.

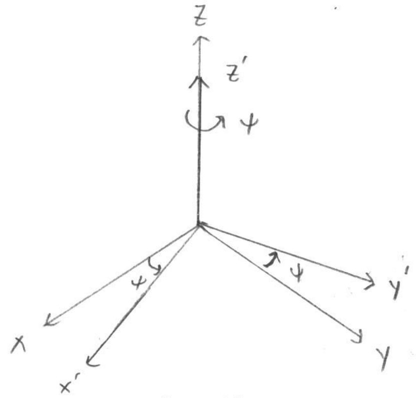
$$Euler(\phi, \theta, \psi) = R_z(\phi) R_y(\theta) R_z(\psi)$$

① post - multiplication ： $R_z(\phi) R_y(\theta) R_z(\psi) = \begin{bmatrix} \cos\phi & -\sin\phi & 0 & 0 \\ \sin\phi & \cos\phi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\psi & -\sin\psi & 0 & 0 \\ \sin\psi & \cos\psi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

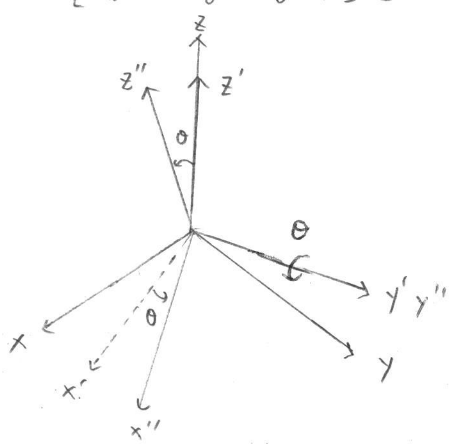


$R_z(\phi)$ $R_y(\theta)$ $R_z(\psi)$

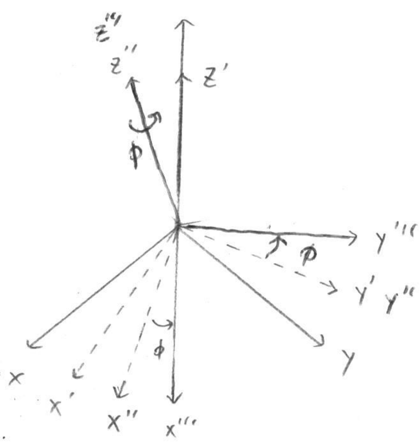
pre - multiplication: $R_z(\psi)\, R_y(\theta)\, R_z(\phi) = \begin{bmatrix} \cos\psi & -\sin\psi & 0 & 0 \\ \sin\psi & \cos\psi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\phi & -\sin\phi & 0 & 0 \\ \sin\phi & \cos\phi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$



$R_z(\psi)$



$R_y(\theta)$



$R_z(\phi)$

$$\underbrace{\begin{bmatrix} \cos\phi & -\sin\phi & 0 & 0 \\ \sin\phi & \cos\phi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\psi & -\sin\psi & 0 & 0 \\ \sin\psi & \cos\psi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}^{\text{post - multiplication}} = \begin{bmatrix} \cos\phi\cos\theta & -\sin\phi & \cos\phi\sin\theta & 0 \\ \sin\phi\cos\theta & \cos\phi & \sin\phi\sin\theta & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\psi & -\sin\psi & 0 & 0 \\ \sin\psi & \cos\psi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\phi\cos\theta\cos\psi - \sin\theta\sin\psi & -\cos\phi\cos\theta\sin\psi - \sin\phi\cos\theta \checkmark\ -0 & \cos\phi\sin\theta & 0 \\ \sin\phi\cos\theta\cos\psi + \cos\phi\sin\psi & -\sin\phi\cos\theta\sin\psi + \cos\phi\cos\psi & \sin\phi\sin\theta & 0 \\ -\sin\theta\cos\theta \checkmark\ -0 & \sin\theta\sin\psi & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \underbrace{\begin{bmatrix} \cos\psi & -\sin\psi & 0 & 0 \\ \sin\psi & \cos\psi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\phi & -\sin\phi & 0 & 0 \\ \sin\phi & \cos\phi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{\text{pre - multiplication}}$$

?

$$R_k(\theta) = Trans(P_x, P_y, P_z) \cdot C R_z(\theta) C^{-1} \cdot Trans(-P_x, -P_y, -P_z)$$

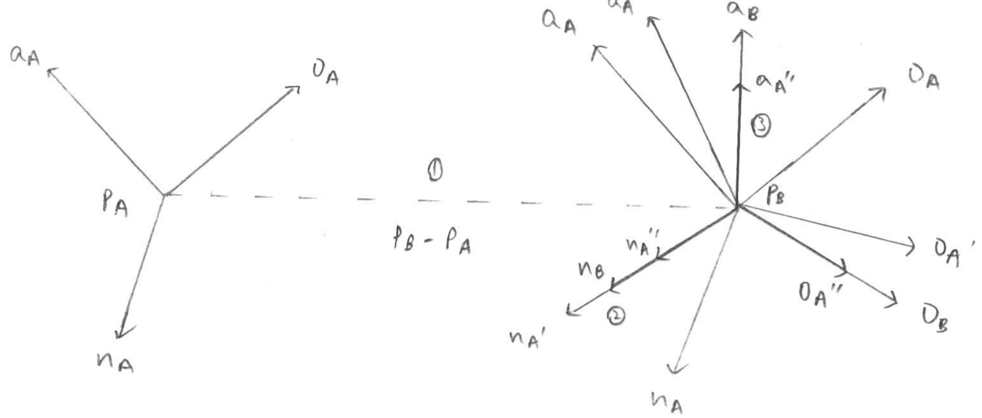
$$= \begin{bmatrix} 1 & 0 & 0 & P_x \\ 0 & 1 & 0 & P_y \\ 0 & 0 & 1 & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} n_x & O_x & a_x & 0 \\ n_y & O_y & a_y & 0 \\ n_z & O_z & a_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} n_x & n_y & n_z & 0 \\ O_x & O_y & O_z & 0 \\ a_x & a_y & a_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -P_x \\ 0 & 1 & 0 & -P_y \\ 0 & 0 & 1 & -P_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} k_x^2 \operatorname{vers}\theta + \cos\theta & k_x k_y \operatorname{vers}\theta - k_z \sin\theta & k_x k_z \operatorname{vers}\theta + k_y \sin\theta & -P_x n_x' - P_y O_x' - P_z a_x' + P_x \\ k_x k_y \operatorname{vers}\theta + k_z \sin\theta & k_y^2 \operatorname{vers}\theta + \cos\theta & k_y k_z \operatorname{vers}\theta - k_x \sin\theta & -P_x n_y' - P_y O_y' - P_z a_y' + P_y \\ k_y k_z \operatorname{vers}\theta - k_y \sin\theta & k_y k_z \operatorname{vers}\theta + k_x \sin\theta & k_z^2 \operatorname{vers}\theta + \cos\theta & -P_x n_z' - P_y O_z' - P_z a_z' + P_z \\ 0 & 0 & 0 & \end{bmatrix}$$

$$= \begin{bmatrix} n_x' & O_x' & a_x' & P_x - (P_x n_x' + P_y O_x' + P_z a_x') \\ n_y' & O_y' & a_y' & P_y - (P_x n_y' + P_y O_y' + P_z a_y') \\ n_z' & O_z' & a_z' & P_z - (P_x n_z' + P_y O_z' + P_z a_z') \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & P_x \\ 0 & 1 & 0 & P_y \\ 0 & 0 & 1 & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} n_x' & O_x' & a_x' & 0 \\ n_y' & O_y' & a_y' & 0 \\ n_z' & O_z' & a_z' & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -P_x \\ 0 & 1 & 0 & -P_y \\ 0 & 0 & 1 & -P_z \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

4.



① translate P_A to P_B, make P_A and P_B coincide $\rightarrow$ $Trans(P_{Bx}-P_{Ax}, P_{By}-P_{Ay}, P_{Bz}-P_{Az})$

② rotate about $\vec{n_A}\times\vec{n_B}$ axis for angle θ, make the n_A axis of frame A and n_B axis of frame B coincide. $\longrightarrow$ $R_{\vec{n_A}\times\vec{n_B}}(\theta)$

跟 A 和 B 垂直的轴

③ rotate frame A about n_B axis for angle ϕ $\rightarrow$ $R_{n_B}(\phi)$

$\Rightarrow$ $Trans(P_{Bx}-P_{Ax},\ P_{By}-P_{Ay},\ P_{Bz}-P_{Az})\cdot R_{\vec{n_A}\times\vec{n_B}}(\theta)\cdot R_{n_B}(\phi)$

$$= \begin{bmatrix} 1 & 0 & 0 & P_{Bx}-P_{Ax} \\ 0 & 1 & 0 & P_{By}-P_{Ay} \\ 0 & 0 & 1 & P_{Bz}-P_{Az} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} K_x K_x \operatorname{vers}\theta + \cos\theta & K_x K_y \operatorname{vers}\theta - K_z \sin\theta & K_x K_z \operatorname{vers}\theta + K_y \sin\theta & 0 \\ K_x K_y \operatorname{vers}\theta + K_z \sin\theta & K_y^2 \operatorname{vers}\theta + \cos\theta & K_y K_z \operatorname{vers}\theta - K_x \sin\theta & 0 \\ K_x K_z \operatorname{vers}\theta - K_y \sin\theta & K_y K_z \operatorname{vers}\theta + K_x \sin\theta & K_z^2 \operatorname{vers}\theta + \cos\theta & 0 \\ \underset{\overset{\|}{n}}{0} & \underset{\overset{\|}{o}}{0} & \underset{\overset{\|}{a}}{0} & \underset{p}{\|} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi & 0 \\ 0 & \sin\phi & \cos\phi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} n_x & o_x\cos\phi + a_x\sin\phi & -o_x\sin\phi + a_x\cos\phi & P_{Bx}-P_{Ax} \\ n_y & o_y\cos\phi + a_y\sin\phi & -o_y\sin\phi + a_y\cos\phi & P_{By}-P_{Ay} \\ n_z & o_z\cos\phi + a_z\sin\phi & -o_z\sin\phi + a_z\cos\phi & P_{Bz}-P_{Az} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$ 约