The differential relationship of linear velocity of the links of a robot (revolute joint) 100

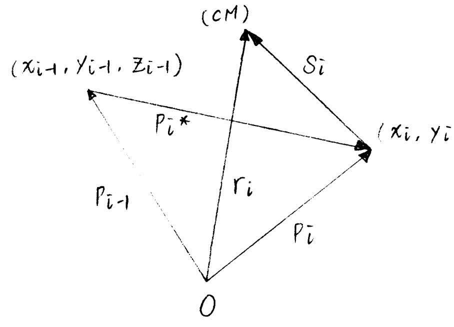
$$V_{i+1} = W_{i+1} \times P_{i+1}^* + V_i \rightarrow \dot{V}_{i+1} = \dot{W}_{i+1} \times P_{i+1}^* + W_{i+1} \times (W_{i+1} \times P_{i+1}^*) + \dot{V}_{i+1}$$

Newton - Euler equation

$$F_i = \frac{d}{dt}(m_i V_{ci}) = m_i \dot{V}_{ci}$$, V_{ci} is the velocity of center of mass (CM)

Euler Equation

$$N_i = \frac{d}{dt}(J_i W_i) = J_i \dot{W}_i + W_i \times (J_i W_i)$$

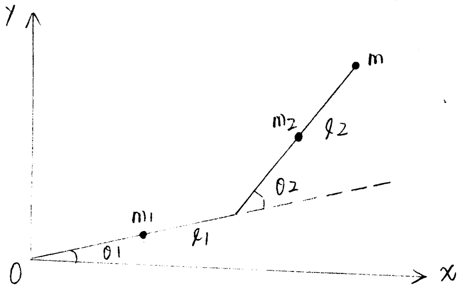


$$f_i = F_i + f_{i+1}$$

$$n_i = N_i + n_{i+1} + P_i^* \times f_{i+1} + (P_i^* + S_i) \times F_i$$

$$\Gamma_i = n_i^t \cdot \vec{Z}_{i-1} \quad , \quad R$$

$$\Gamma_i = f_i^t \cdot \vec{Z}_{i-1} \quad , \quad P$$



for the two-link robot manipulator example in the class with CM shift to the middle of each link and with a load m held at the top of the second link

$$W_{1z} = \dot{\theta}_1 \qquad \dot{W}_{1z} = \ddot{\theta}_1$$

$$\dot{V}_1 = \dot{W}_1 \times P_1^* + W_1 \times (W_1 \times P_1^*) + \dot{V}_0 \quad , \quad \text{where } \dot{V}_0 = \begin{bmatrix} 0 \\ g \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0 \\ \ddot{\theta}_1 \end{bmatrix} \begin{bmatrix} l_1 C_1 \\ l_1 S_1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix} \times \left(\begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix} \begin{bmatrix} l_1 C_1 \\ l_1 S_1 \\ 0 \end{bmatrix} \right) + \begin{bmatrix} 0 \\ g \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} -l_1 S_1 \ddot{\theta}_1 - l_1 C_1 \dot{\theta}_1^2 \\ l_1 C_1 \ddot{\theta}_1 - l_1 S_1 \dot{\theta}_1^2 + g \\ 0 \end{bmatrix}$$

Note that :

$C_1 = \cos \theta_1$

$S_1 = \sin \theta_1$

$C_{12} = \cos(\theta_1 + \theta_2)$

$S_{12} = \sin(\theta_1 + \theta_2)$

$$V_{ci} = W_i \times S_i + V_i$$

$$\dot{V}_{ci} = \dot{W}_i \times S_i + W_i \times (W_i \times S_i) + \dot{V}_i$$

$$\dot{V}_{c1} = \dot{W}_1 \times S_1 + W_1 \times (W_1 \times S_1) + \dot{V}_1 \quad , \quad \text{where } S_1 = -\frac{1}{2} P_1^*$$

$$= \begin{bmatrix} -\frac{1}{2} l_1 S_1 \ddot{\theta}_1 - \frac{1}{2} l_1 C_1 \dot{\theta}_1^2 \\ \frac{1}{2} l_1 C_1 \ddot{\theta}_1 - \frac{1}{2} l_1 S_1 \dot{\theta}_1^2 + g \\ 0 \end{bmatrix}$$

$$F_1 = m_1 \dot{V}_{c1} \qquad J_1 = \begin{bmatrix} \frac{1}{4} m_1 l_1^2 & 0 & 0 \\ 0 & \frac{1}{4} m_1 l_1^2 & 0 \\ 0 & 0 & \frac{1}{4} m_1 l_1^2 \end{bmatrix} , \qquad N_1 = J_1 \dot{W}_1 + W_1 \times (J_1 W_1)$$