

Midterm Robotics
December 1, 2022 (Open Book)

1.(36%) For a robot manipulator with the following kinematic table, (a) please draw its structure with the coordinate frames (6%), and also find its T_3 (6%). (b) Please solve its corresponding joint solutions from T_3 using both algebraic and geometric approaches (20%). (c) With the presence of joint range, please describe what the workspace will be like (4%).

Joint	d	a	α	θ
1(r)	d_1	0	90°	θ_1
2(r)	0	0	90°	θ_2
3(p)	d_3	0	0°	0°

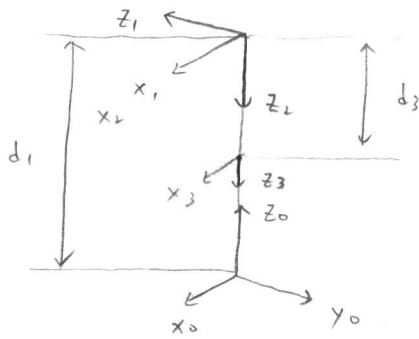
2.(20%) (a) Explain why the singular points are present for a multi-DOF robot manipulator, and what are their mathematical and physical meaning (8%). (b) What is the definition of the wrist-partitioned type of robot manipulator (3%)? What is its benefit in solving the inverse kinematic solutions (3%)? Is the Mitsubishi type of robot manipulator for the course project wrist-partitioned, and how about human arm (3%)? (c) Describe the singular point for the robot manipulator in Problem 1 (3%).

3.(18%) (a) What are the main factors needed to be considered in designing a feasible path for a robot manipulator (4%)? (b) Describe how to plan a feasible path in the presence of static obstacle(s) for (i) a mobile robot and (ii) a multi-DOF robot manipulator (10%). (c) If the workspaces of two multi-DOF robot manipulators are overlapping, what will be your design for them to work together when considering both safety and efficiency (4%)?

4.(18%) (a) Please give the general dynamic equation for a multi-DOF robot manipulator, describe the four main components, and also evaluate their importance on control (8%). For this dynamic equation to be realistic, what should be done (4%)? (b) Explain what the learning approach is? Why it is not effective in dealing with inverse kinematic and inverse Jacobian problems for a multi-DOF robot manipulator (4%)? (c) This question is about human-robot cooperation. If you want to have a robot to work with a human, what will be your main considerations for system design (4%)? Please name at least two items.

5.(12%) Please just use a few words to describe what "Control" is. What is the benefit of "Feedforward Control"? How to achieve "Feedforward Control"? Why the industry usually adopts "Feedback Control" instead of more advanced "Feedforward Control" for controller design? For all the questions, please just describe the concept.

1. (a)



$$T_3 = A_1 A_2 A_3$$

$$= \begin{bmatrix} c_1 & 0 & s_1 & 0 \\ s_1 & 0 & -c_1 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_2 & 0 & s_2 & 0 \\ s_2 & 0 & -c_2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= A_1 \begin{bmatrix} c_2 & 0 & s_2 & s_2 d_3 \\ s_2 & 0 & -c_2 & -c_2 d_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} c_1 c_2 & s_1 & c_1 s_2 & c_1 s_2 d_3 \\ s_1 c_2 & -c_1 & s_1 s_2 & s_1 s_2 d_3 \\ s_2 & 0 & -c_2 & -c_2 d_3 + d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(b)

Algebraic

$$\begin{bmatrix} c_1 c_2 & s_1 & c_1 s_2 & c_1 s_2 d_3 \\ s_1 c_2 & -c_1 & s_1 s_2 & s_1 s_2 d_3 \\ s_2 & 0 & -c_2 & -c_2 d_3 + d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{cases} p_x = c_1 s_2 d_3 \\ p_y = s_1 s_2 d_3 \\ p_z = -c_2 d_3 + d_1 \end{cases} \quad \text{if } s_2 d_3 \neq 0, \quad \theta_{11} = \tan^{-1} \left(\frac{p_y}{p_x} \right) = \tan^{-1} \left(\frac{s_1}{c_1} \right)$$

$$\theta_{12} = \tan^{-1} \left(\frac{p_y}{p_x} \right) + 180^\circ$$

$$A^{-1} T_3 = \begin{bmatrix} c_1 & s_1 & 0 & 0 \\ 0 & 0 & 1 & -d_1 \\ s_1 & -c_1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} c_2 & 0 & s_2 & s_2 d_3 \\ s_2 & 0 & -c_2 & -c_2 d_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{cases} c_1 p_x + s_1 p_y = s_2 d_3 \\ p_z - d_1 = -c_2 d_3 \end{cases} \Rightarrow \theta_{21} = \tan^{-1} \left(\frac{c_1 p_x + s_1 p_y}{d_1 - p_z} \right)$$

$$\theta_{22} = \tan^{-1} \left(\frac{-c_1 p_x - s_1 p_y}{d_1 - p_z} \right)$$

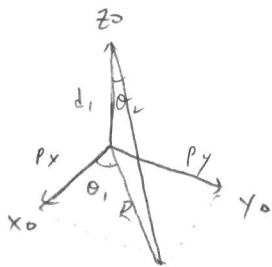
$$A_2^{-1} A_1^{-1} T_3 = \begin{bmatrix} c_2 & s_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ s_2 & -c_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_1 & s_1 & 0 & 0 \\ 0 & 0 & 1 & -d_1 \\ s_1 & -c_1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} n & o & a & p \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} c_2 c_1 & c_2 s_1 & s_2 & -d_1 s_2 \\ s_1 & -c_1 & 0 & 0 \\ c_1 s_2 & s_1 s_2 & -c_2 & d_1 c_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$c_1 s_2 p_x + s_1 s_2 p_y - c_2 p_z + d_1 c_2 = d_3$$

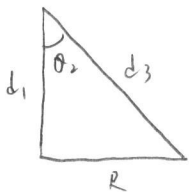
$$\Rightarrow s_2 (c_1 p_x + s_1 p_y) - c_2 (p_z - d_1) = d_3$$

Geometric



$$\theta_{11} = \tan^{-1} \left(\frac{\sin \theta_{11}}{\cos \theta_{11}} \right) = \tan^{-1} \left(\frac{p_y}{p_x} \right)$$

$$\theta_{12} = \tan^{-1} \left(\frac{\sin \theta_{12}}{\cos \theta_{12}} \right) = \tan^{-1} \left(\frac{p_y}{p_x} \right) + 180^\circ$$



$$R = p_x c_1 + p_y s_1$$

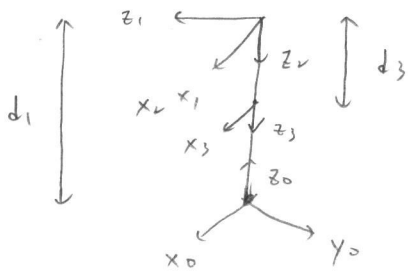
$$p_x = p_x c_1 + p_y s_1 - d_3 s_2$$

$$p_y = d_1 - p_z = d_3 c_2$$

$$d_3 > 0 \Rightarrow \theta_2 = \tan^{-1} \left(\frac{p_x c_1 + p_y s_1}{d_1 - p_z} \right)$$

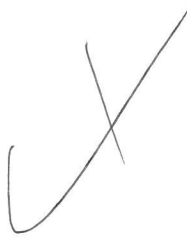
$$d_3 = \sqrt{(p_x c_1 + p_y s_1)^2 + (d_1 - p_z)^2}$$

(c)



3(p) 可伸縮 d_3 的距離

1(r) 擁有 d_1 的移動距離



———>
背面為
第 2 題

2. (a) why present

在移動上可能會有某些角度或位置抵達不了, 即需要 singular point 來表示 — 2

physical meaning:

從幾何結構上, 機械手臂的某一個末端點可能因為長度變數為 0, 或移動變數的 joint 投影回原點, 使其在某個運動方向喪失自由度

mathematical meaning:

從解 differential kinematic 的角度出發, 當透過坐標間轉換求解 Jacobian 時, 如果其行列式值 $\det J = 0$, 意味著無法做出 inverse Jacobian 的運算, 則此點為奇異點

(b) Definition:

工業機器人常使用的設計方式, 將所有軸分兩種

Primary joints: for the positional workspace (多為前三軸)

Minor joints: for the orientational workspace (多為剩下後幾軸)

⇒ 前三軸決定末端點位置, 後幾軸決定末端點朝向角

Benefits:

求解 inverse kinematic solutions 一定有 closed form solutions, 在不考慮 tool 長度情況下, 可先用前三軸找到 wrist 的位置, 將前三軸矩陣利用 inverse 移開, 用後三軸決定方向, 方便計算

Mitsubishi type:

是, 人的手臂是更為精密的 wrist-partitioned.

~~Human?~~

(c)

singularity 意味在路徑移動上失去其部分自由度

當無法計算出 inverse Jacobian, 即存在 singular point

→

8

(a)

Easy to specify

smoothness

2

configuration

singularity

task, kinematics, dynamics, obstacle

(b)

(i) 定位空間中障礙物位置與範圍

在不碰撞到物體情況下, 連線出一條路徑
在移動時不斷偵測與修正, 具體方法?

(ii)

定位空間中障礙物位置範圍

在不碰撞物體情況下, 連線出一條路徑

∩

(c)

做好完善的路徑規劃與同步控制, 使手臂間

能順暢合作, 不會互相碰撞, 同時建立 Master 與

slave, 在互相重疊時, master priority 較高

ok!

4. (a)

$$\tau = H(\theta) \ddot{\theta} + C(\theta, \dot{\theta}) \dot{\theta} + G(\theta) + K^T(\theta) f_{ext}$$

4 速度, 加速度, 角加速度, 位置 \rightarrow 確認移動軌跡與方向, 方便控制

$$L(\theta, \dot{\theta}) = k(\theta, \dot{\theta}) - p(\theta)$$

(6) 學習控制是透過疊代學習, 不斷調整自身控制參數, 達到最佳化控制的方式

不一定!

3.

(1) 計算複雜, 且無法確認是否收斂與穩定.

say more.

(1) 碰撞到人的判斷機制

2 路徑上有人時修正路徑或停止的方法

for safety, other?

5. 機器可動路策次

8 ① 確保依循人類所制定的規則(定值, 路徑...) 運行,
即為控制

② 能預先就做好補償, 不需事後才不斷修正

③ feed forward 需對系統有足夠了解, 且應用上成本較高,
故多使用 feed backward. OK!