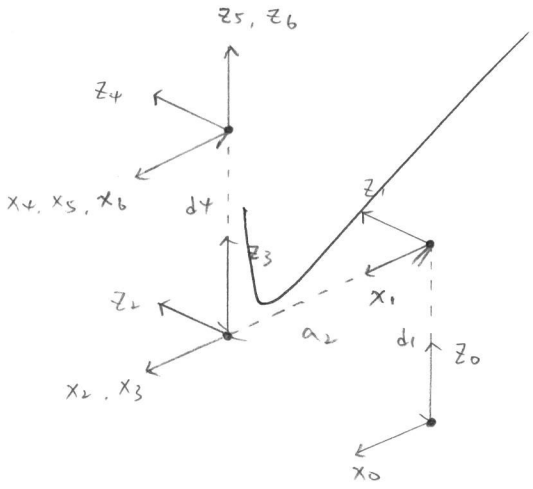
Robotics : HW2 311605012 揚曜瑋

1.

(a)



(b)

$$A_n = Rot(z, \theta_n) \cdot Trans(0, 0, d_n) \cdot Trans(a_n, 0, 0) \cdot Rot(x, \alpha_n)$$

$$= \begin{bmatrix} \cos\theta_n & -\sin\theta_n\cos\alpha_n & \sin\theta_n\sin\alpha_n & a_n\cos\theta_n \\ \sin\theta_n & \cos\theta_n\cos\alpha_n & -\cos\theta_n\sin\alpha_n & a_n\sin\theta_n \\ 0 & \sin\alpha_n & \cos\alpha_n & d_n \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_1 = \begin{bmatrix} C\theta_1 & 0 & S\theta_1 & 0 \\ S\theta_1 & 0 & -C\theta_1 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_2 = \begin{bmatrix} C\theta_2 & -S\theta_2 & 0 & a_2C\theta_2 \\ S\theta_2 & C\theta_2 & 0 & a_2S\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_3 = \begin{bmatrix} C\theta_3 & 0 & -S\theta_3 & 0 \\ S\theta_3 & 0 & C\theta_3 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_4 = \begin{bmatrix} C\theta_4 & 0 & S\theta_4 & 0 \\ S\theta_4 & 0 & -C\theta_4 & 0 \\ 0 & 1 & 0 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_5 = \begin{bmatrix} C\theta_5 & 0 & -S\theta_5 & 0 \\ S\theta_5 & 0 & C\theta_5 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_6 = \begin{bmatrix} C\theta_6 & -S\theta_6 & 0 & 0 \\ S\theta_6 & C\theta_6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_6 = A_1 \times A_2 \times A_3 \times A_4 \times A_5 \times A_6 = \begin{bmatrix} n_x & O_x & a_x & P_x \\ n_y & O_y & a_y & P_y \\ n_z & O_z & a_z & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_6 = \begin{bmatrix} C\theta_1 & 0 & S\theta_1 & 0 \\ S\theta_1 & 0 & -C\theta_1 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} C\theta_2 & -S\theta_2 & 0 & a_2C\theta_2 \\ S\theta_2 & C\theta_2 & 0 & a_2S\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} C\theta_3 & 0 & -S\theta_3 & 0 \\ S\theta_3 & 0 & C\theta_3 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} C\theta_4 & 0 & S\theta_4 & 0 \\ S\theta_4 & 0 & -C\theta_4 & 0 \\ 0 & 1 & 0 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\cdot \begin{bmatrix} C\theta_5 & 0 & -S\theta_5 & 0 \\ S\theta_5 & 0 & C\theta_5 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} C\theta_6 & -S\theta_6 & 0 & 0 \\ S\theta_6 & C\theta_6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$n_x = c_1 (c_{23} (c_4 c_5 c_6 - s_4 s_6) - s_{23} s_5 c_6) - s_1 (s_4 c_5 c_6 + c_4 s_6)$$

$$n_y = s_1 (c_{23} (c_4 c_5 c_6 - s_4 s_6) - s_{23} s_5 c_6) + c_1 (s_4 c_5 c_6 + c_4 s_6)$$

$$n_z = s_{23} (c_4 c_5 c_6 - s_4 s_6) + c_{23} s_5 c_6$$

$$o_x = - c_1 (c_{23} (c_4 c_5 s_6 + s_4 c_6) - s_{23} s_5 s_6) + s_1 (s_4 c_5 s_6 - c_4 c_6)$$

$$o_y = - s_1 (c_{23} (c_4 c_5 s_6 + s_4 c_6) - s_{23} s_5 s_6) - c_1 (s_4 c_5 s_6 - c_4 c_6)$$

$$o_z = - s_{23} (c_4 c_5 s_6 + s_4 c_6) - c_{23} s_5 s_6$$

$$a_x = - c_1 (c_{23} c_4 s_5 + s_{23} c_5) + s_1 s_4 s_5$$

$$a_y = - s_1 (c_{23} c_4 s_5 + s_{23} c_5) - c_1 s_4 s_5$$

$$a_z = - s_{23} c_4 s_5 + c_{23} c_5$$

$$p_x = c_1 (a_2 c_2 - d_4 s_{23})$$

$$p_y = s_1 (a_2 c_2 - d_4 s_{23})$$

$$p_z = d_1 + a_2 s_2 + d_4 c_{23}$$

(c)

① algebraic

use p_x, p_y if $a_2 c_2 - d_4 s_{23} \neq 0 \Rightarrow \frac{p_y}{p_x} = \frac{s_1}{c_1} = \tan\theta_1$, $\theta_1 = \tan^{-1}\left(\frac{p_y}{p_x}\right)$

$$A_1^{-1} T_6 = {}^1T_6 = A_2 A_3 A_4 A_5 A_6$$

$$\begin{bmatrix} c_1 & s_1 & 0 & 0 \\ 0 & 0 & 1 & -d_1 \\ s_1 & -c_1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = \left[\begin{array}{ccc:c} & & & a_2 c_2 - d_4 s_{23} \\ & & & a_2 s_2 + d_4 c_{23} \\ & & & 0 \\ \hdashline 0 & 0 & 0 & 1 \end{array}\right] \rightarrow \begin{aligned} c_1 p_x + s_1 p_y &= a_2 c_2 - d_4 s_{23} \quad ① \\ p_z - d_1 &= a_2 s_2 + d_4 c_{23} \quad ② \\ s_1 p_x - c_1 p_y &= 0 \quad ③ \end{aligned}$$

$$①^2 + ②^2 + ③^2 = p_x^2 + (p_z - d_1)^2 + p_y^2 - a_2^2 + d_4^2 - 2 a_2 d_4 (c_2 s_{23} - s_2 c_{23}) = a_2^2 + d_4^2 - 2 a_2 d_4 s_3$$

$$\sin\theta_3 = \frac{p_x^2 + p_y^2 + (p_z - d_1)^2 - a_2^2 - d_4^2}{-2 a_2 d_4}$$

$$\theta_3 = \sin^{-1}\left(\frac{p_x^2 + p_y^2 + (p_z - d_1)^2 - a_2^2 - d_4^2}{-2 a_2 d_4}\right) \quad *$$

$$A_2^{-1} A_1^{-1} T_6 = T_2^{-1} T_6 = A_3 A_4 A_5 A_6$$

$$\begin{bmatrix} c_1 c_2 & c_1 c_2 & s_2 & -a_2 - d_1 s_2 \\ -c_1 s_2 & -s_1 s_2 & c_2 & -d_1 c_2 \\ s_1 & -c_1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} n & o & a & p \\ n & o & a & p \\ n & o & a & p \\ 0 & 0 & 0 & 1 \end{bmatrix} = \left[\begin{array}{ccc:c} & & & -d_4 s_3 \\ & & & d_4 c_3 \\ & & & 0 \\ \hdashline 0 & 0 & 0 & 1 \end{array}\right] \rightarrow \begin{aligned} & c_2 (c_1 p_x + c_1 p_y) + c_2 p_z - a_2 \\ & \quad - d_1 s_2 = -d_4 s_3 \quad ① \\ & -s_2 (c_1 p_x + s_1 p_y) + c_2 p_z - d_1 c_2 \\ & \quad = d_4 c_3 \quad ② \end{aligned}$$

$$①^2 + ②^2 = (c_1 p_x + s_1 p_y)^2 + (p_z - d_1)^2 = d_4^2 + a_2^2 - 2 a_2 d_4 s_3$$

$$T_3^{-1} T_6 = {}^3T_6 = A_4 A_5 A_6$$

$$\begin{bmatrix} C_1C_{23} & S_1C_{23} & S_{23} & -d_1S_{23} - a_2C_3 \\ -S_1 & C_1 & 0 & 0 \\ -C_1S_{23} & -S_1S_{23} & C_{23} & -d_1C_{23} + a_2S_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} n & o & a & p \\ n & o & a & p \\ n & o & a & p \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} & & & 0 \\ & & & 0 \\ & & & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$C_{23}(C_1P_x + S_1P_y) + S_{23}(P_z - d_1) - a_2C_3 = 0$$

$$-S_1P_x + C_1P_y = 0 \quad \rightarrow \quad C_{23}(C_1P_x + S_1P_y) + S_{23}(P_z - d_1) = a_2C_3 \quad -①$$

$$-S_{23}(C_1P_x + S_1P_y) + C_{23}(P_z - d_1) + a_2S_3 = d_4 \qquad -S_{23}(C_1P_x + S_1P_y) + C_{23}(P_z - d_1) = d_4 - a_2S_3 \quad -②$$

$$①^2 + ②^2 = \sqrt{P_x^2 + P_y^2 + (P_z - d_1)^2} = \sqrt{(a_2C_3)^2 + (d_4 - a_2S_3)^2} = t$$

$$\sin\phi = \frac{C_1P_x + S_1P_y}{t}, \quad \cos\phi = \frac{P_z - d_1}{t}, \quad \phi = \tan^{-1}\left(\frac{C_1P_x + S_1P_y}{P_z - d_1}\right)$$

$$C_{23}S\phi + C_{23}C\phi = \sin(\theta_2 + \theta_3 + \phi) = \frac{a_2C_3}{t}$$

$$-S_{23}S\phi + C_{23}C\phi = \cos(\theta_2 + \theta_3 + \phi) = \frac{d_4 - a_2C_3}{t} \qquad \tan(\theta_2 + \theta_3 + \phi) = \frac{a_2C_3}{d_4 - a_2C_3}$$

$$\theta_2 + \theta_3 + \phi = \tan^{-1}\left(\frac{a_2C_3}{d_4 - a_2C_3}\right)$$

$$\theta_2 = \tan^{-1}\left(\frac{a_2C_3}{d_4 - a_2C_3}\right) - \tan^{-1}\left(\frac{C_1P_x + S_1P_y}{P_z - d_1}\right) - \sin^{-1}\left(\frac{P_x^2 + P_y^2 + (P_z - d_1)^2 - a_2^2 - d_4^2}{-2a_2d_4}\right)$$

$${}^3T_6 = \begin{bmatrix} C_4C_5C_6 - S_4S_6 & -C_4C_5S_6 - S_4C_6 & -C_4S_5 & 0 \\ S_4C_5C_6 + C_4S_6 & -S_4C_5S_6 + C_4C_6 & -S_4S_5 & 0 \\ S_5C_6 & -S_5S_6 & C_5 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \rightarrow \begin{array}{l} C_{23}(C_1a_x + S_1a_y) + S_{23}a_z = -C_4S_5 \\ -S_1a_x + C_1a_y = -S_4S_5 \end{array}$$

if $\theta_5 \neq 0$

$$\tan\theta_4 = \frac{-S_4S_5}{-C_4S_5} = \frac{-S_1a_x + C_1a_y}{C_{23}(C_1a_x + S_1a_y) + S_{23}a_z} \qquad \theta_4 = \tan^{-1}\left(\frac{-S_1a_x + C_1a_y}{C_{23}(C_1a_x + S_1a_y) + S_{23}a_z}\right)$$

$$T_4^{-1} T_6 = {}^4T_6 = A_5 A_6$$

$$\begin{bmatrix} C_1C_{23}C_4 - S_1S_4 & S_1C_{23}C_4 + C_1S_4 & S_{23}C_4 & \cdots \\ -C_1S_{23} & -S_1S_{23} & C_{23} & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \end{bmatrix} \begin{bmatrix} n & o & a & p \\ n & o & a & p \\ n & o & a & p \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} C_5C_6 & -C_5S_6 & -S_5 & 0 \\ S_5C_6 & -S_5S_6 & C_5 & 0 \\ -S_6 & -C_6 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$(C_1C_{23}C_4 - S_1S_4)a_x + (S_1C_{23}C_4 + C_1S_4)a_y + S_{23}C_4a_z = -S_5$$

$$-C_1S_{23}a_x - S_1S_{23}a_y + C_{23}a_z = C_5$$

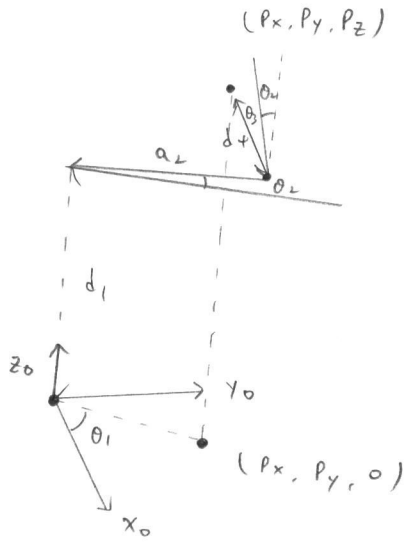
$$\tan\theta_5 = \frac{-S_5}{-C_5},\quad \theta_5 = \tan^{-1}\left(\frac{-S_5}{-C_5}\right) = \tan^{-1}\left(\frac{(C_1C_{23}C_4 - S_1S_4)a_x + (S_1C_{23}C_4 + C_1S_4)a_y + S_{23}C_4 a_z}{S_{23}a_x + S_1 S_{23} a_y - C_{23}a_z}\right)$$

$$3_{76}: \; -C_1 S_{23} n_x - S_1 S_{23} n_y + C_{23} n_z = S_5 C_6$$

$$-C_1 S_{23} o_x - S_1 S_{23} o_y + C_{23} o_z = -S_5 S_6$$

$$\tan\theta_6 = \frac{-S_5 S_6}{-S_5 C_6},\quad \theta_6 = \tan^{-1}\left(\frac{-C_1 S_{23} o_x - S_1 S_{23} o_y + C_{23} o_z}{C_1 S_{23} n_x + S_1 S_{23} n_y - C_{23} n_z}\right) \#$$

① geometric :



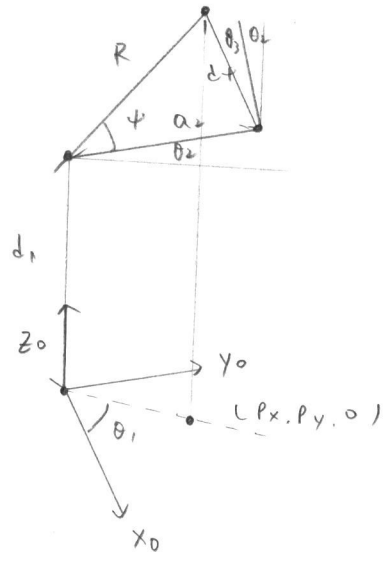
if $P_x \neq 0$, $P_y \neq 0$, then $\tan\theta_1 = \frac{P_y}{P_x}$, $\theta_1 = \tan^{-1}\left(\frac{P_y}{P_x}\right)$

$$P_z - d_1 = a_2 S_2 + d_4 C_{23}$$

$$\sqrt{P_x^2 + P_y^2} = a_2 C_2 - d_4 S_{23}$$

$$P_x^2 + P_y^2 + (P_z - d_1)^2 = a_2^2 + d_4^2 - 2a_2 d_4 S_3$$

$$\theta_3 = \sin^{-1}\left(\frac{P_x^2 + P_y^2 + (P_z - d_1)^2 - a_2^2 - d_4^2}{-2a_2 d_4}\right)$$



$$\tan\phi = \frac{P_z - d_1}{\sqrt{P_x^2 + P_y^2}} \qquad \phi = \tan^{-1}\left(\frac{P_z - d_1}{\sqrt{P_x^2 + P_y^2}}\right)$$

$$S_1 P_x = C_1 P_y \rightarrow S_1 P_x - C_1 P_y = 0$$

$$\rightarrow P_x^2 + P_y^2 = (C_1^2 + S_1^2) P_x^2 + (C_1^2 + S_1^2) P_y^2 + 2C_1 S_1 P_x P_y - 2C_1 S_1 P_x P_y$$

$$= (C_1 P_x + S_1 P_y)^2 + \underbrace{(S_1 P_x - C_1 P_y)^2}_{\to 0}$$

$$\phi = \tan^{-1}\left(\frac{P_z - d_1}{C_1 P_x + S_1 P_y}\right) \qquad \psi = \cos^{-1}\left(\frac{a_2^2 + R^2 - d_4^2}{2a_2 R}\right)$$

$$R = \sqrt{P_x^2 + P_y^2 + (P_z - d_1)^2}$$

$$\theta_2 = \phi - \psi = \tan^{-1}\left(\frac{P_z - d_1}{C_1 P_x + S_1 P_y}\right) - \cos^{-1}\left(\frac{a_2^2 + P_x^2 + P_y^2 + (P_z - d_1)^2 - d_4^2}{2a_2\sqrt{P_x^2 + P_y^2 + (P_z - d_1)^2}}\right)$$

2.

Advantage：① 只需計算四個參數，較為容易

② 簡單的 4×4 matric 就能計算

Disadvantage：① 不能自由設定坐標

② 校正不同，有誤差不好調整

③ 沒有統一答案

④ 有些答案會因參數不夠無法解出來

⑤ Tool, Base frame 不能隨意放置

3.

① 結構誤差，溫度，摩擦力，風阻，磨損...

② 影響每個手臂的實際位置，重複性

③ Inaccuracy 一定會影響 accuracy，但不一定影響 repeatability

accuracy 能通過校正加強，repeatability 無法校正

④ 如果指令是到 x，但每一次都精準到 y

則要使它到 y，就必要給它到 x 的指令

⑤ 找觀察性好且抗 noise 的點

要修正 D-H formulation，D-H 模型中只有 4 個參數，校正需要使用所有參數校正，較為精準

⑥ 選數個代表性的點即可，通常是固定範圍就找一個點代表，可減少量測數量，但找越多點，accuracy 會比較好