機器人學 Project1 成果報告

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一、 介面說明

- 開發平台: Matlab

- 如何執行:

1. 開啟 robotics_project1_forward_311605012.m 及 robotics_project1_backward_311605012.m
2. 執行程式

二、 程式架構說明

- robotics_project1_forward_311605012.m

I. 程式運行流程

1. 程式執行後，在 command window 依序輸入 6 個 joint variable($\theta_1, \theta_2, d_3, \theta_4, \theta_5, \theta_6$)。
2. 每個 joint 皆有其限制的輸入範圍，當超出範圍時，會出現紅字提醒使用者發生錯誤，並要求重新輸入數值。
3. 輸入完成後，即會分別計算出(n, o , a, p)以及 Cartesian point($x, y, z, \phi, \theta, \psi$)，下圖為輸出結果。

```
>> robotics_project1_forward_311605012
----------------------------INPUT----------------------------

Please enter the joint variable of thetal between -160 ~ 160 :
165
θ1 is out of range ,please type it again.
Please enter the joint variable of thetal between -160 ~ 160 :
20
Please enter the joint variable of thetal between -125 ~ 125 :
-130
θ2 is out of range ,please type it again.
Please enter the joint variable of thetal between -125 ~ 125 :
20
Please enter the joint variable of thetal between -30 ~ 30 :
50
θ3 is out of range ,please type it again.
Please enter the joint variable of thetal between -30 ~ 30 :
20
Please enter the joint variable of thetal between -140 ~ 140 :
150
θ4 is out of range ,please type it again.
Please enter the joint variable of thetal between -140 ~ 140 :
20
Please enter the joint variable of thetal between -100 ~ 100 :
160
θ5 is out of range ,please type it again.
Please enter the joint variable of thetal between -100 ~ 100 :
20
Please enter the joint variable of thetal between -260 ~ 260 :
-300
θ6 is out of range ,please type it again.
Please enter the joint variable of thetal between -260 ~ 260 :
20

----------------------------OUTPUT----------------------------

(n, o, a, p):
   0.312899788315632  -0.777300157424024   0.545800501777546   4.247497683164255
   0.777300157424024   0.539800721952620   0.323140288184588   8.330096026320385
  -0.545800501777546   0.323140288184588   0.773099066363012  18.793852415718170
                   0                   0                   0   1.000000000000000

Cartesian point (x, y, z, ϕ, θ, Ψ):
4.247498 8.330096 18.793852 30.627584 39.366996 30.627584
```

II. 核心程式碼說明

1. 依據 kinematic table 定義已知的參數
2. 提供輸入介面供使用者輸入 joint variable
3. 每次輸入時，檢查是否超出輸入範圍，超出範圍，即跳出錯誤，並要求使用者再次輸入
4. 計算 A1~A6 的矩陣，並計算 A1 至 A6 矩陣運算後的結果 T
5. 依據 T 矩陣結果，計算出 Cartesian point(x,y,z,ϕ,θ,ψ)，並輸出結果

- robotics_project1_backward_311605012.m

I. 程式運行流程

1. 程式執行後，在 command window 輸入 Cartesian point (n,o,a,p)(需加入左右括號[])
2. 輸入完成後,會輸出 8 種 joint variable($\theta_1, \theta_2, d_3, \theta_4, \theta_5, \theta_6$)的解
3. 每一組解會判斷 joint variable 是否位在限制的工作範圍內，若超出範圍，會告知使用者該 joint variable 已超出範圍，下圖為輸出結果

```
>> robotics_project1_backward_311605012
----------------------------INPUT----------------------------

Please enter the Cartesian point:
[    0.312899788315632   -0.777300157424024    0.545800501777546    4.247497683164255
     0.777300157424024    0.539800721952620    0.323140288184588    8.330096026320385
    -0.545800501777546    0.323140288184588    0.773099066363012   18.793852415718170
                     0                    0                    0    1.000000000000000]

--------------------------OUTPUT--------------------------

corresponding variables ans1 (θ1, θ2, θ3, θ4, θ5, θ6):
20.000000000000007  20.000000000000000  20.000000000000004  19.999999999999975  20.000000000000011  20.000000000000021
corresponding variables ans2 (θ1, θ2, θ3, θ4, θ5, θ6):
θ4 is out of range!
20.000000000000007  20.000000000000000  20.000000000000004  -160.000000000000028  -20.000000000000011  -159.999999999999972
corresponding variables ans3 (θ1, θ2, θ3, θ4, θ5, θ6):
θ2 is out of range!
θ4 is out of range!
θ5 is out of range!
20.000000000000007  -160.000000000000000  -20.000000000000004  160.000000000000028  160.000000000000000  -159.999999999999972
 corresponding variables ans4 (θ1, θ2, θ3, θ4, θ5, θ6):
θ2 is out of range!
θ5 is out of range!
20.000000000000007  -160.000000000000000  -20.000000000000004  -19.999999999999972  -160.000000000000000  20.000000000000025
corresponding variables ans5 (θ1, θ2, θ3, θ4, θ5, θ6):
-74.033869384222157  -19.999999999999993  20.000000000000000  79.515775878897287  38.612519362654176  62.647839145120216
corresponding variables ans6 (θ1, θ2, θ3, θ4, θ5, θ6):
-74.033869384222157  -19.999999999999993  20.000000000000000  -100.484224121102727  -38.612519362654176  -117.352160854879784
corresponding variables ans7 (θ1, θ2, θ3, θ4, θ5, θ6):
θ2 is out of range!
θ5 is out of range!
-74.033869384222157  160.000000000000028  -20.000000000000000  100.484224121102699  141.387480637345817  -117.352160854879841
corresponding variables ans8 (θ1, θ2, θ3, θ4, θ5, θ6):
θ2 is out of range!
θ5 is out of range!
-74.033869384222157  160.000000000000028  -20.000000000000000  -79.515775878897301  -141.387480637345817  62.647839145120187
```

II. 核心程式碼說明

1. 提供輸入介面供使用者輸入 Cartesian point (n,o,a,p)
2. 依據 kinematic table 定義已知的參數
3. 利用代數法分別解出$\theta_1, \theta_2, d_3, \theta_4, \theta_5, \theta_6$各自對應的計算式
4. 判斷 joint variable 是否位在限制的工作範圍內，如超出範圍，顯示該 joint variable 已超出範圍
5. 輸出 8 組結果

三、 數學運算說明

- **robotics_project1_forward_311605012.m**

六軸機器人各軸之間座標系統的轉換，frames n-1 與 n 的關係:

(1)以Z_{n-1}為中心，旋轉 θn

(2)沿Z_{n-1}移動距離d_n

(3)依循X_n平移距離a_n

(4)以X_n為中心旋轉α_n

以上的平移與旋轉過程皆能透過矩陣A_n表示

$$A_n = \begin{bmatrix} cos\theta_n & -sin\theta_n cos\alpha_n & sin\theta_n sin\alpha_n & a_n cos\theta_n \\ sin\theta_n & cos\theta_n cos\alpha_n & -cos\theta_n sin\alpha_n & a_n sin\theta_n \\ 0 & sin\alpha_n & cos\alpha_n & d_n \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

六軸旋轉與平移後的結果為

$$T_6 = A_1 \times A_2 \times A_3 \times A_4 \times A_5 \times A_6 = \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = [n, o, a, p]$$

$$x = p_x, y = p_y, z = p_z,$$

$$\phi = \tan^{-1}\left(\frac{a_y}{a_x}\right), \theta = \tan^{-1}\left(\frac{a_x \cos\phi + a_y \sin\phi}{a_z}\right),$$

$$\psi = \tan^{-1}\left(\frac{-n_x sin\phi + n_y cos\phi}{-o_x sin\phi + o_y cos\phi}\right)$$

- robotics_project1_backward_311605012.m
  - 通過下列算式推導，發現在$P_z = 0$時，會有 singular point 的情形

1.

$$A_1^{-1} \cdot T_6 = A_2 A_3 A_4 A_5 A_6$$

$$\begin{bmatrix} C_1 & S_1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ -S_1 & C_1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \vdots & \vdots & \vdots & d_3 S_2 \\ \vdots & \vdots & \vdots & -d_3 C2 \\ \vdots & \vdots & & \vdots \; d_2 \\ \vdots & \vdots & & \vdots \; 1 \end{bmatrix}$$

$$-S_1 p_x + C_1 p_y = d_2, let\ p_x = pcos\phi, p_y = psin\phi,$$

$$p = \sqrt{p_x^2 + p_x^2}, \phi = Atan2(p_x, p_y)$$

$$\boldsymbol{\theta_1 = Atan2(p_y, p_x) - Atan2(d_2, \pm\sqrt{p_x^2 + p_y^2 - d_2^2})}$$

2.

$$\begin{cases} C_1 p_x + S_1 p_y = d_3 S_2 \\ -p_z = -d_3 C_2 \end{cases} \rightarrow 上下取平方相加$$

$$d_3^2(s_2^2 + c_2^2) = \left(C_1 p_x + S_1 p_y\right)^2 + p_z^2$$

$$\boldsymbol{d_3 = \pm\sqrt{\left(C_1 p_x + S_1 p_y\right)^2 + p_z^2}}$$

3.

$$A_3^{-1} A_2^{-1} A_1^{-1} T_6 = A_4 A_5 A_6$$

$$\begin{bmatrix} C_1C_2 & S_1C_2 & -S_2 & 0 \\ -S_1 & C_1 & 0 & 0 \\ C_1C_2 & S_1S_2 & C_2 & 0 \\ C_1d_3S_2 - S_1d_2 & C_1d_2 - S_2S_1d_3 & C_2d_3 & 1 \end{bmatrix} \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} =$$

$$\begin{bmatrix} C_4C_5C_6 - S_4S_6 & -C_4C_5S_6 - S_4C_6 & C_4S_5 & 0 \\ S_4C_5C_6 + C_4S_6 & -S_4C_5S_6 + C_4C_6 & S_4S_5 & 0 \\ -S_5C_6 & S_5S_6 & C_5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{cases} C_1\left(C_1 p_x + S_1 p_y\right) = S_2 p_z \\ S_2\left(C_1 p_x + S_1 p_y\right) = d_3 - C_2 p_z \end{cases} \rightarrow \textit{上下取平方相加}$$

$$\left(C_1 p_x + S_1 p_y\right)^2 = d_3^2 - 2 d_3 C_2 p_z + p_z^2$$

$$C_2 = \frac{-\left(C_1 p_x + S_1 p_y\right)^2 + d_3^2 + p_z^2}{2 d_3 p_z}$$

$$S_2 = \frac{C_2(C_1 p_x + S_1 p_y)}{p_z}$$

$$\boldsymbol{\theta_2 = Atan2(S_2, C_2)}$$

4.

$$\begin{cases} C_1C_2a_x + S_1C_2a_y - S_2a_z = C_4S_5 - (1) \\ -S_1a_x + C_1a_y = S_4S_5 - (2) \\ C_1S_2a_x + S_1S_2a_y + C_2a_z = C_5 - (3) \end{cases}, \sqrt{(1)^2 + (2)^2}$$

$$\sqrt{\left(C_1C_2a_x + S_1C_2a_y - s_2a_z\right)^2 + \left(-S_1a_x + C_1a_y\right)^2} = \pm S_5$$

$$\boldsymbol{\theta_5 = Atan2(S_5, C_5)}$$

5.

$$C_4 = \frac{C_1C_2a_x + S_1C_2a_y - S_2a_z}{S_5}, S_4 = \frac{-S_1a_x + C_1a_y}{S_5}.$$

$$\boldsymbol{\theta_4 = Atan(S_4, C_4)}$$

6.

$$\begin{cases} \dfrac{C_1S_2o_x + S_1S_2o_y + C_2o_z}{S_5} = S_6 \\ \dfrac{C_1S_2n_x + S_1S_2n_y + C_2n_z}{-S_5} = C_6 \end{cases}$$

$$\boldsymbol{\theta_6 = Atan2(S_6, C_6)}$$

四、 加分題(討論兩種逆向運動學(代數法，幾何法)的優缺點)

- 正向運動學

  優點：計算簡單

  缺點：不容易到達明確位置

- 逆向運動學

  優點：結果對執行工作較有幫助

  缺點：計算複雜