## Welcome to CS 186, Section 6!

TA: Bryan Munar

OH: Mondays 11-12pm and Thursdays 2:30-3:30pm (651 Soda)

**DISC:** Tuesdays 11-12am (136 Barrows) and Wednesdays 10-11am (130 Wheeler)



#### Announcements and Such

- Project/HW 3 due on Monday!
- How was the midterm? And how'd you do? Come to OH if you wanna discuss anything
- Any inconsistencies with score? Ask for a regrade (when scores are released!)

#### Discussion 6: Relational Algebra, Entity-Relationship Diagrams, and Functional Dependencies

#### Overview:

- 1. Relational Algebra
- 2. Worksheet exercises
- 3. Entity-Relationship Diagrams
- 4. Worksheet Exercises
- 5. Functional Dependencies
- 6. Worksheet Exercises

(A majority of the slides are from Michelle and lecture!)

## Relational Algebra



## Relational Algebra

- Input and output: Relation instances (tables)
- Has set semantics
  - No duplicate tuples in a relation
- Useful for representing semantics of execution plans in a DBMS (more later!)

## Relational Algebra

Operation	Symbol	Explanation
Selection	σ	Selects rows
Projection	π	Selects columns
Union	U	Tuples in r1 or r2
Intersection	Λ	Tuples in r1 and r2
Cross-product	×	Combines two relations
Join	$\bowtie$	Conditional cross- product
Difference	_	Tuples in r2 not in r1

#### Selection

Select rows

• Example:  $\sigma_{gpa>3.5}(R)$ 

name	sid	gpa
Bob	1	3.7
Sue	3	2.9
Ron	2	1.2
Al	4	4.0
Sally	5	3.6

#### Selection

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## Projection

Select columns

• Example:  $\pi_{\text{name, sid}}(R)$ 

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#### Union

Set union between two relations

• Example:  $\sigma_{sid<3}(R) \cup \sigma_{sid\%2=0}(R)$ 

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Set intersection between two relations

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• Example:  $\sigma_{sid<3}(R) \cap \sigma_{sid\%2=0}(R)$ 

name	sid	gpa
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#### Cross Product

Takes all rows from A and combines with all rows in B

• Example:  $\pi_{\text{name}}(R) \times \pi_{\text{gpa}}(R)$ 

name	sid	gpa
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Sue
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gpa
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2.9
1.2

#### Cross Product

Takes all rows from A and combines with all rows in B

• Example:  $\pi_{name}(R) \times \pi_{gpa}(R)$ 

X

name
Bob
Sue
Ron

gpa
3.7
2.9
1.2

name	gpa
Bob	3.7
Bob	2.9
Bob	1.2
Sue	3.7
Sue	2.9
Sue	1.2
Ron	3.7
Ron	2.9
Ron	1.2

### Join

Joins A and B based on some column

• Example:  $\pi_{\text{name,sid}}(R) \bowtie \pi_{\text{name,gpa}}(R)$ 

name	sid
Bob	1
Sue	3
Ron	2



name	gpa
Bob	3.7
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Takes rows in A that are not in B

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## Do first page of worksheet!

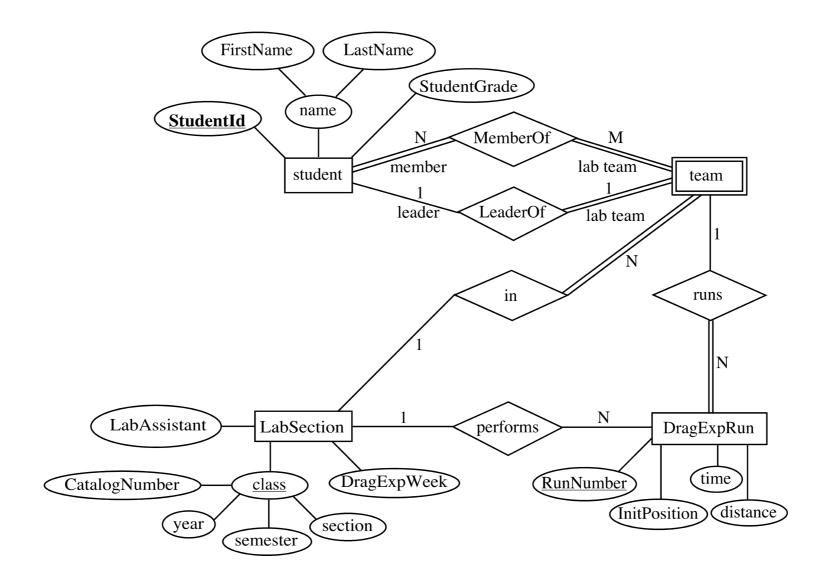


# Entity Relationship Diagrams



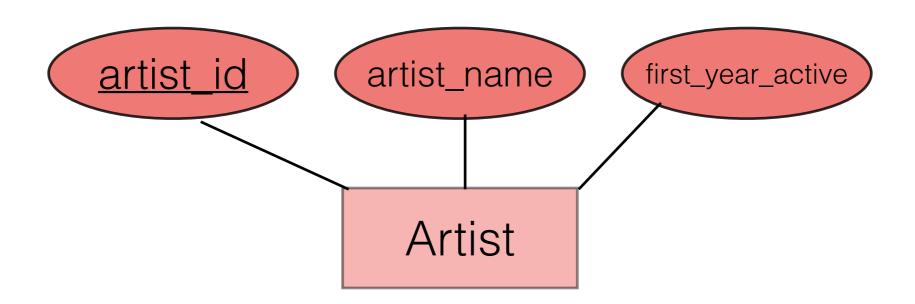
#### Motivation

Visualize data schema



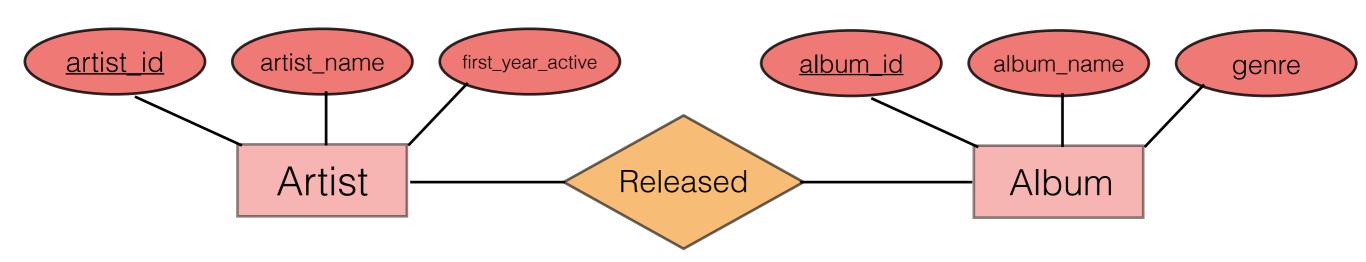
#### Entities

- Entity: "thing"
- Attribute: Property of the entity
  - Primary key underlined



### Relationships

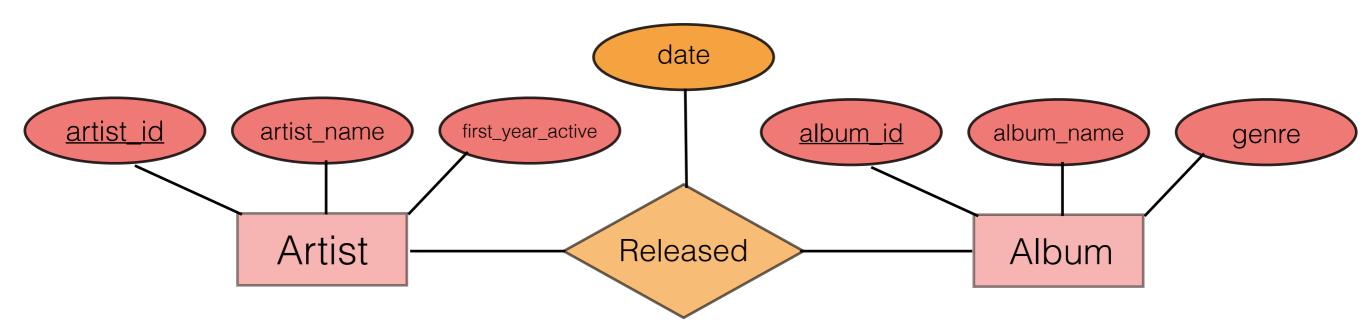
How two entities interact



Artist 4 released album 2.

## Relationships

- How two entities interact
  - Interactions can have attributes



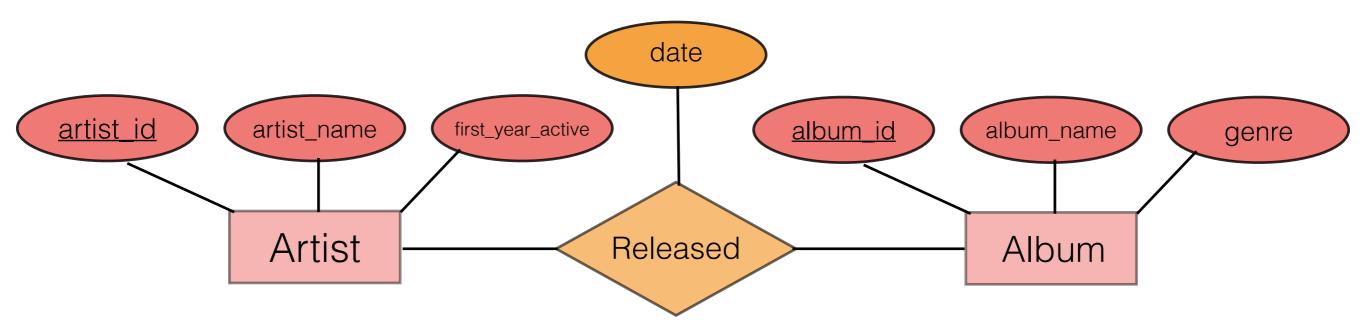
Artist 4 released album 2 on February 27, 2015.

#### Constraints

- Make relationship lines meaningful
  - Participation constraint (Partial/Total)
    - Total participation: participates at least once
  - Key/Non-key constraint
    - Key: Participates at most once

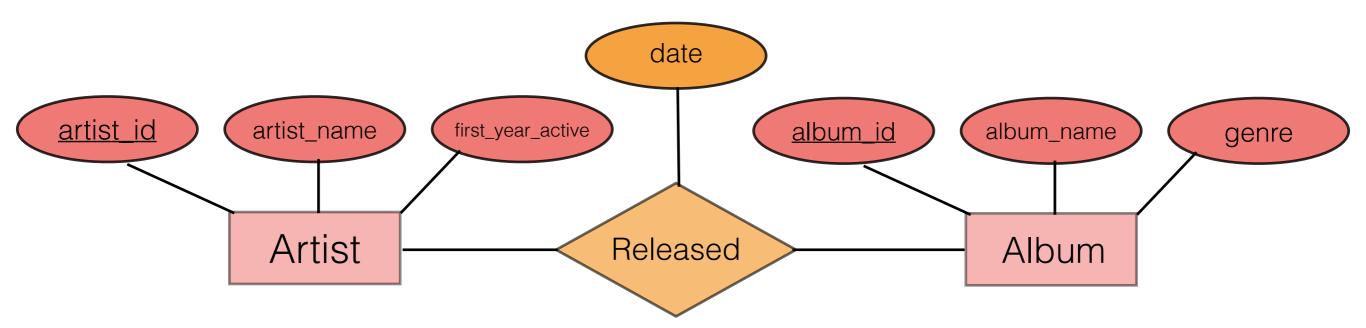
	Partial Participation	Total Participation
Non-Key	0 or More ———	1 or More ———
Key	0 or 1 ———	Exactly 1 ——

#### Constraints



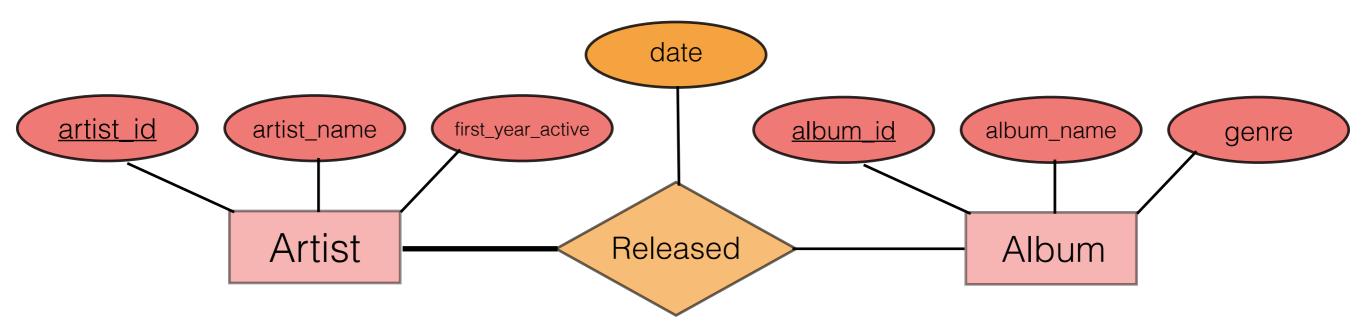
Non-Key constraint with partial participation

#### Constraints

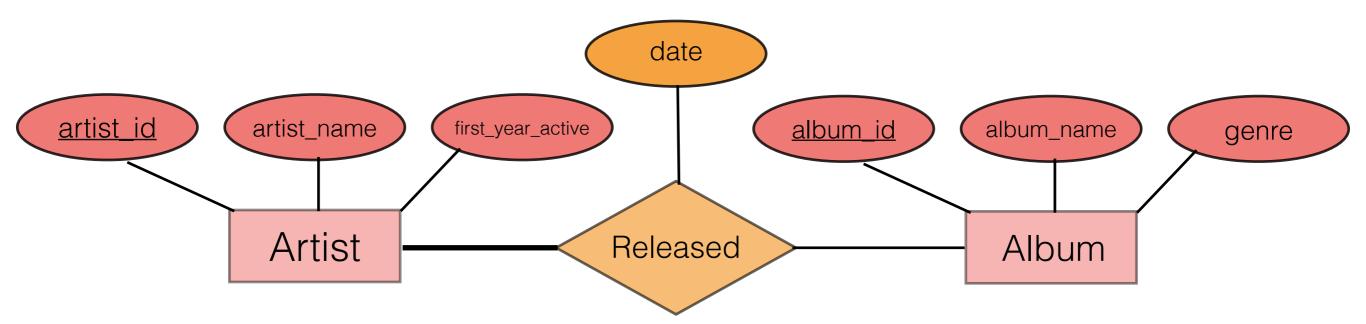


Non-Key constraint with partial participation

An artist can release an album zero or more times.

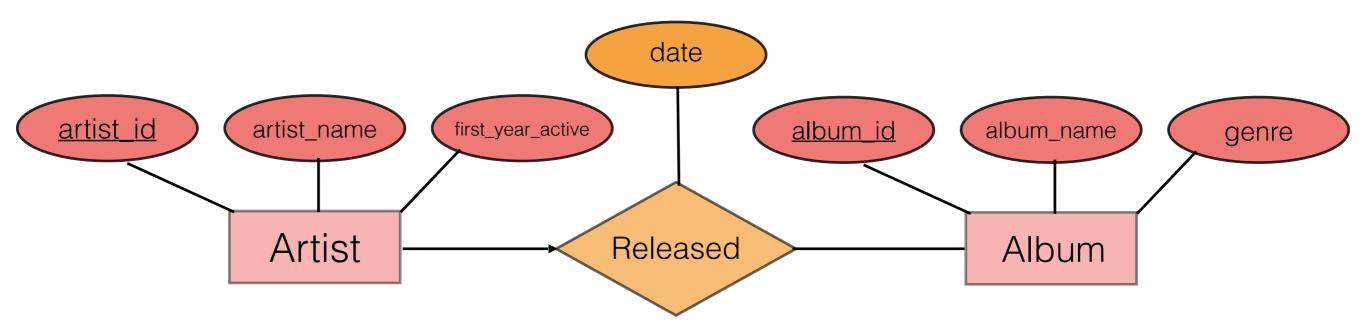


Non-Key constraint with total participation

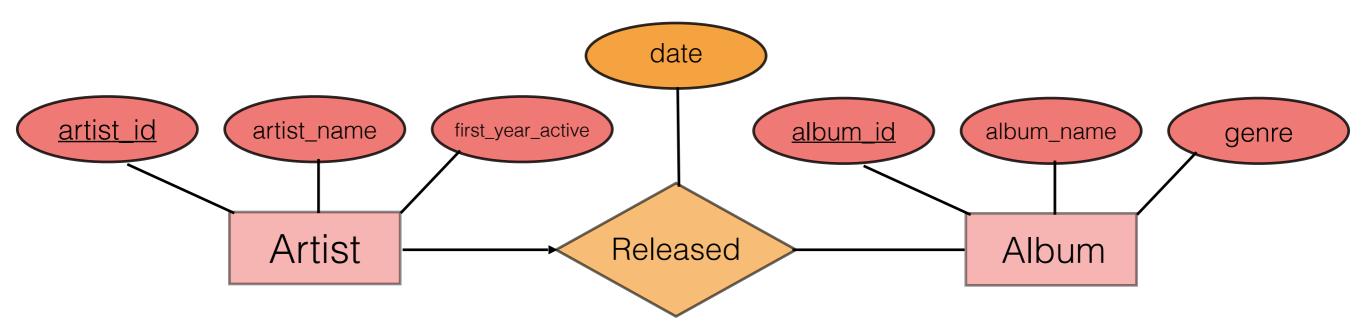


Non-Key constraint with total participation

An artist can release an album one or more times.

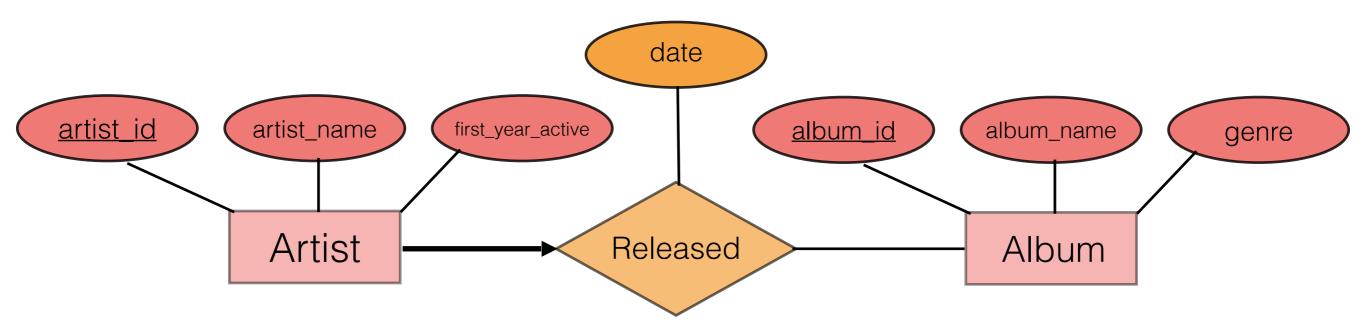


Key constraint with partial participation

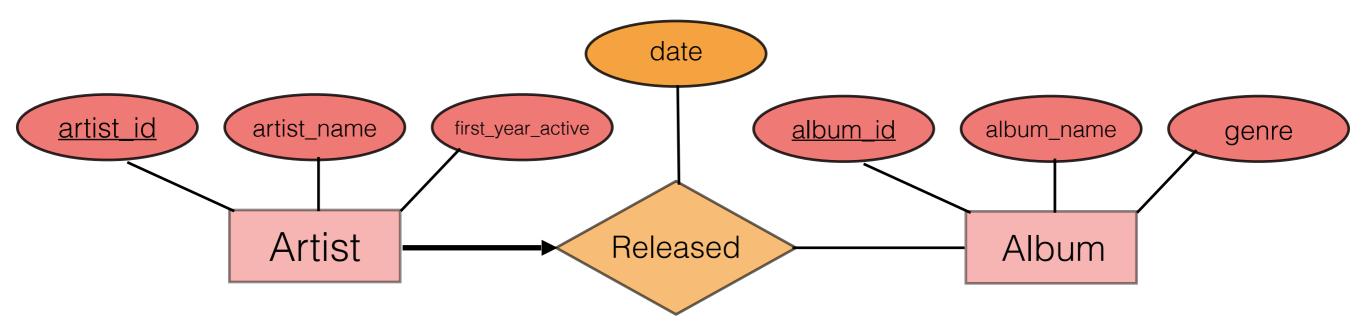


Key constraint with partial participation

An artist can release an album zero or one times.



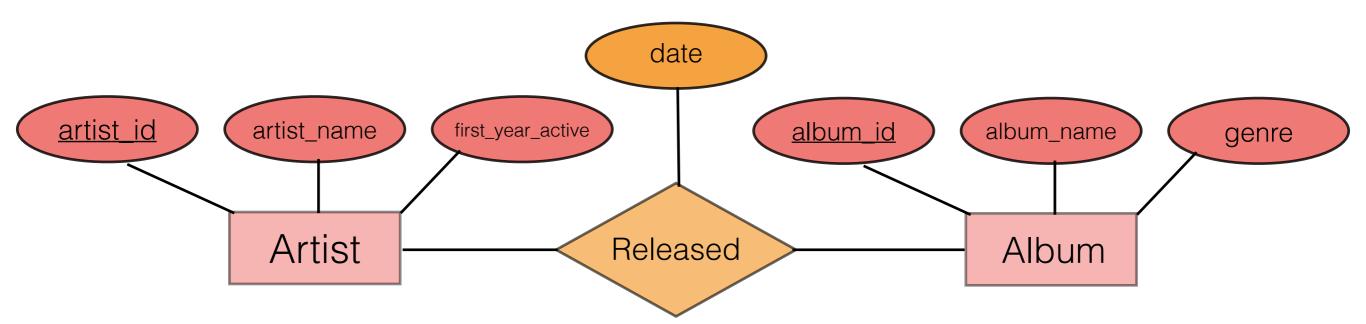
Key constraint with total participation



Key constraint with total participation

An artist can release exactly one album.

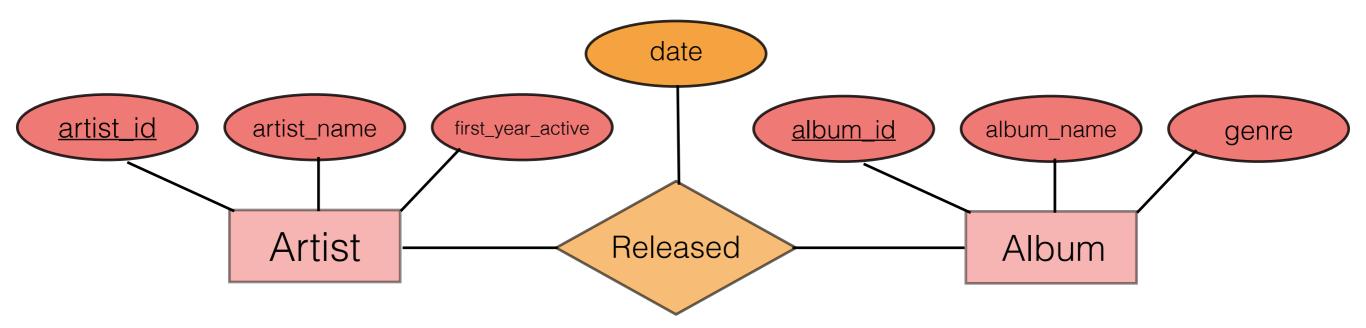
#### We want...



Non-Key constraint with partial participation

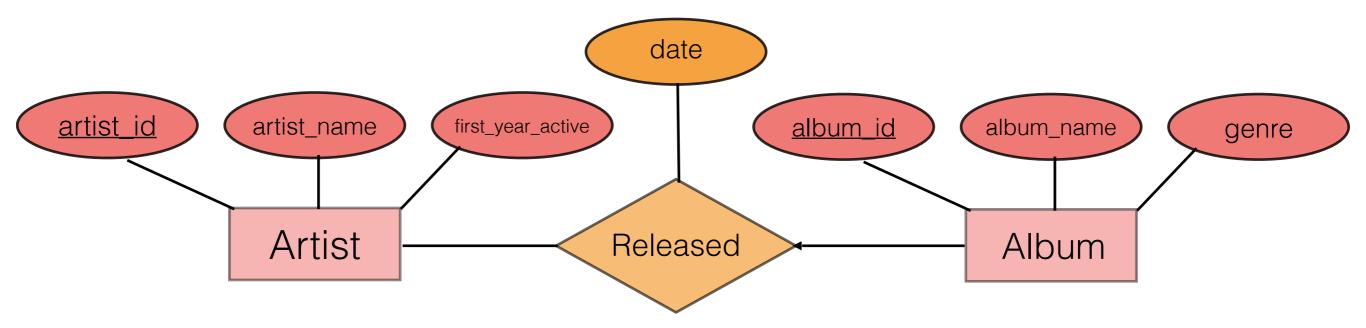
An artist can release an album zero or more times.

# What constraint do we want from Album?

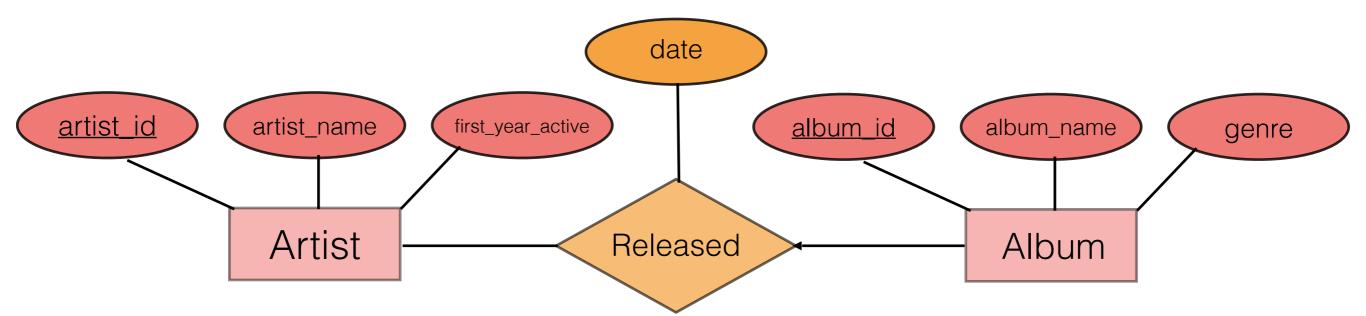


- A. An album can be released 0 or more times. ———
- B. An album can be released 1 or more times.
- C. An album can be released 0 or 1 times. ———
- D. An album is released exactly once. ———

# What constraint do we want from Album?

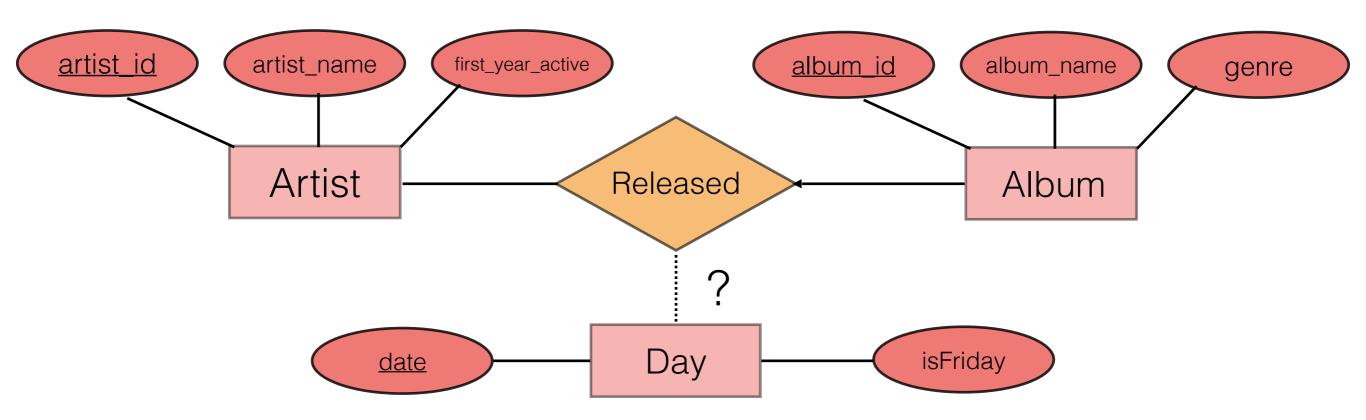


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- B. An album can be released 1 or more times. ———
- C. An album can be released 0 or 1 times. ———
- D. An album is released exactly once. ———



An artist may release zero or more albums. An album may be released or unreleased.

# Ternary Relations

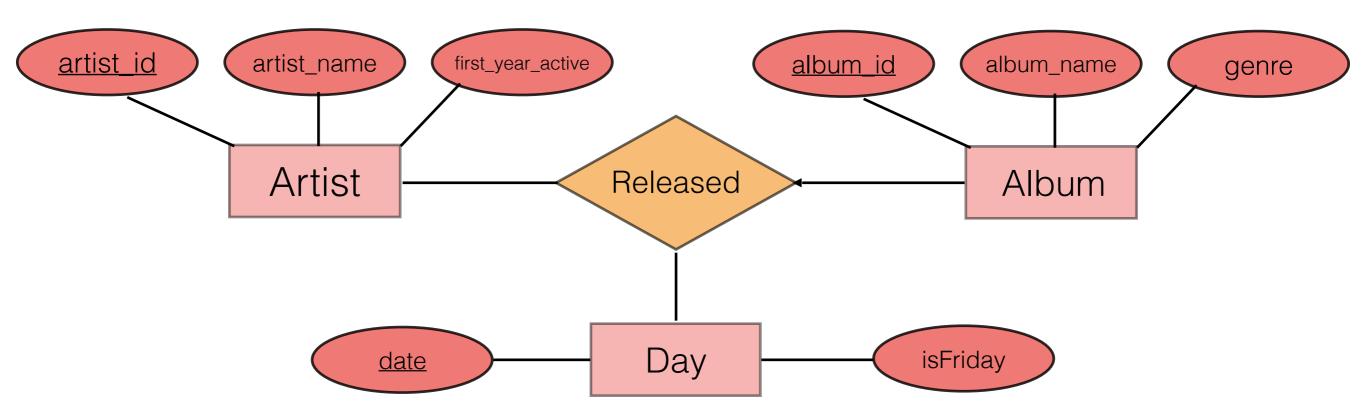


An artist may release zero or more albums.

An album may be released or unreleased.

Releasing an album can occur ??? times a day.

# Ternary Relations



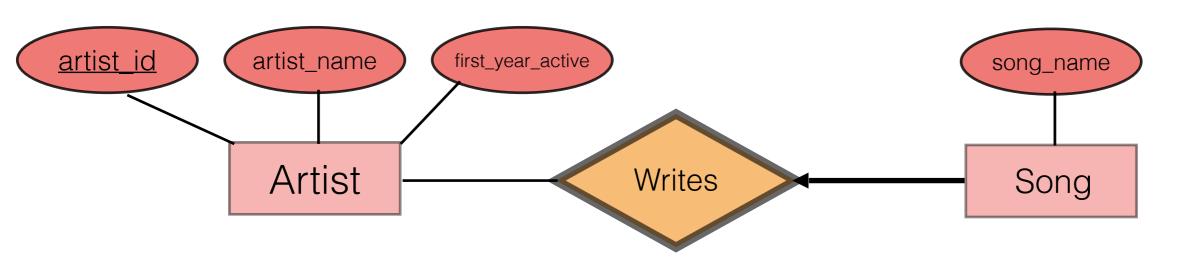
An artist may release zero or more albums.

An album may be released or unreleased.

Releasing an album can occur 0 or more times a day.

#### Weak Entities

 Weak entity can only be identified only when considering primary key of another (owner) entity.



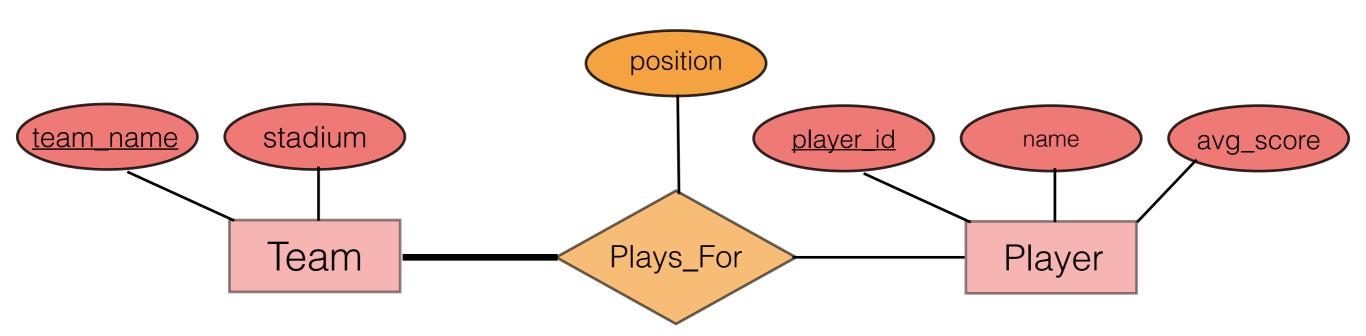
- Song's key is actually (Artist.artist\_id, Song.song\_name)
- Can there be two songs with the same name?
  - How about by the same artist?
- Can a song exist without an artist?

# Do second page of worksheet!



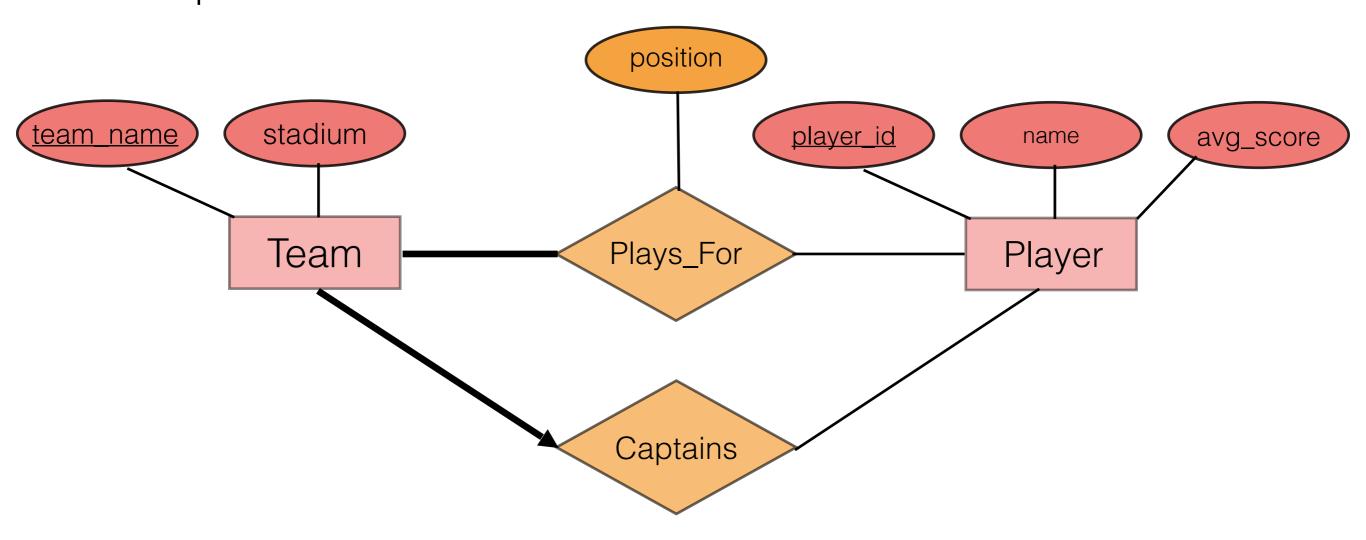
 Assume that a player can play in more than one team (Yes, our league has different rules!) and that a team needs at least one player.

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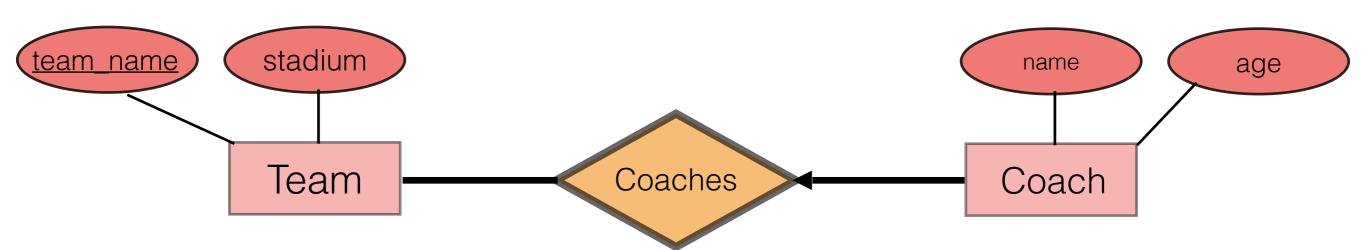
 Now let's say we want to also track who is the captain of every team. How will the ER diagram change from the previous case? Note: Every team needs exactly one captain!

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 Are there are any weak-entity relationships in either of our ER diagrams?

- Are there are any weak-entity relationships in either of our ER diagrams?
- No. A weak entity can be identified uniquely only by considering the primary key of another (owner) entity.
- Example of possible weak entity: Coaches



A team can have many coaches, but each coach exactly coaches one team.



- X → Y reads "X determines Y"
- Used to detect redundancies and refine schema

id	rating	wage
1	3	20
2	2	10
3	3	20
4	2	10

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rating → wage

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id	rating	wage
1	3	20
2	2	10
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id → rating, wage

Key → All attributes of relation

id	rating	wage
1	3	20
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id → rating, wage

# Armstrong's Axioms

- Reflexivity: if  $X \supseteq Y$ , then  $X \rightarrow Y$ 
  - Examples: A → A, AB → A
- Augmentation: if X → Y, then XZ → YZ for any Z
- Transitivity: if  $X \to Y$  and  $Y \to Z$ , then  $X \to Z$
- Useful rules derived from Armstrong's Axioms:
  - Union: if  $X \to Y$  and  $X \to Z$ , then  $X \to YZ$
  - Decomposition: if  $X \rightarrow YZ$ , then  $X \rightarrow Y$  and  $X \rightarrow Z$

# Armstrong's Axioms

If XA → YA, can you infer X → Y?

# Armstrong's Axioms

- If XA → YA, can you infer X → Y?
  - Trivial example: A → YA

#### Closures

- Functional dependency closure: F+
  - Set of all FDs implied by F, including trivial dependencies
  - Example:  $F = \{A \rightarrow B, B \rightarrow C\}$ 
    - $F + = \{A \rightarrow B, B \rightarrow C, A \rightarrow C, A \rightarrow A, A \rightarrow AB, ...\}$

# Closures

- Attribute closure: X+
  - Given just X, what can we determine?
  - Example:  $F = \{A \rightarrow B, B \rightarrow C\}$ 
    - A+=ABC

- A methodical algorithm, given a set of FDs F:
  - Initialize X+ := X
  - Repeat until no change:
    - If U → V is in F such that U is in X+, add V to X+

- R = ABCDE
- $F = \{B \rightarrow CD, D \rightarrow E, B \rightarrow A, E \rightarrow C, AD \rightarrow B\}$
- B+=?

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- B + = B

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- R = ABCDE
- $F = \{B \rightarrow CD, D \rightarrow E, B \rightarrow A, E \rightarrow C, AD \rightarrow B\}$
- B+ = ABCDE B is a key of R!

## Boyce-Codd Normal Form (BCNF)

- Motivation: Schema design is hard, want a way to ensure a reasonable design
- BCNF ≈ "reasonable schema"

## Boyce-Codd Normal Form (BCNF)

- Definition: Relation R with FDs F is in BCNF if for all X → A in F+:
  - X → A is reflexive (a trivial FD) OR
  - X is a superkey for R
    - Superkey: Key that does not need to be minimal

- If X → A violates BCNF, decompose R into R A and XA
  - Repeat as necessary

- R = ABCEG
- $F = \{AB \rightarrow C, AC \rightarrow B, BC \rightarrow G, E \rightarrow G\}$

- If X → A violates BCNF, decompose R into R A and XA
  - Repeat as necessary

- R = ABCEG
- $F = \{AB \rightarrow C, AC \rightarrow B, BC \rightarrow G, E \rightarrow G\}$
- ABEG, ABC

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- ABE, EG, ABC

- Lossless join: Can we reconstruct R?
  - Decomposing R into X and Y is lossless iff:
    - $X \cap Y \rightarrow X$ , or
    - $\bullet$   $X \cap Y \rightarrow Y$

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  - Decomposing R into X and Y is lossless iff:
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- Example:
  - ABC decomposed to AB, BC
  - FDs:  $A \rightarrow B$ ,  $C \rightarrow B$
  - This is lossy! AB ∩ BC = B → B

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- Example:
  - ABC decomposed to AC, BC
  - FDs:  $A \rightarrow B$ ,  $C \rightarrow A$
  - This is lossless! AC ∩ BC = C → AC

- Example: ABE, EG, ABC
  - $F = \{AB \rightarrow C, AC \rightarrow B, BC \rightarrow G, E \rightarrow G\}$

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- Example: ABE, EG, ABC
  - $F = \{AB \rightarrow C, AC \rightarrow B, BC \rightarrow G, E \rightarrow G\}$
  - This dependency was not preserved!
  - We can fix this by adding BCG, but this may break BCNF.

- G for a set of FDs F
  - Closure of G = closure of F
  - Right hand side of each FD is G is a single attribute
- Implies lossless join and dependency preserving decomposition

Example: A → B, ABCD → E, EF → GH, ACDF → EG

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- A → B

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# Do last third page of worksheet!



Find the set of functional dependencies:

Flights(Flight\_no, Date, fRom, To, Plane\_id),

ForeignKey(Plane\_id)

Planes(Plane\_id, tYpe)

Seat(Seat\_no, Plane\_id, Legroom), ForeignKey(Plane\_id)

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Flights(Flight\_no, Date, fRom, To, Plane\_id),

ForeignKey(Plane\_id)

Planes(Plane\_id, tYpe)

Seat(Seat\_no, Plane\_id, Legroom), ForeignKey(Plane\_id)

- FD → RTP
- P → Y
- SP → L

Now consider the attribute set R = ABCDE and the FD set  $F = \{AB \rightarrow C, A \rightarrow D, D \rightarrow E, AC \rightarrow B\}$ . Compute the closure for the following attributes.

- A:
- AB:
- B:
- D

Now consider the attribute set R = ABCDE and the FD set  $F = \{AB \rightarrow C, A \rightarrow D, D \rightarrow E, AC \rightarrow B\}$ . Compute the closure for the following attributes.

- A: ADE
- AB: ABCDE
- B: B
- D: DE

ABEFG, ABCD

ABEFG, ABCD

- ABEFG, ABCD
- BEFG, ABCD, AG

- ABEFG, ABCD
- BEFG, ABCD, AG
- BEG, FG, ABCD, AG

- ABEFG, ABCD
- BEFG, ABCD, AG
- BEG, FG, ABCD, AG

Does BEG, FG, ABCD, AG preserve dependencies?  $F = \{AB \rightarrow CD, C \rightarrow EF, G \rightarrow A, G \rightarrow F, CE \rightarrow F\}.$ 

Does BEG, FG, ABCD, AG preserve dependencies?  $F = \{AB \rightarrow CD, C \rightarrow EF, G \rightarrow A, G \rightarrow F, CE \rightarrow F\}.$ 

No, C → EF and CE → F are not preserved.

Give a minimal cover for:  $F = \{AB \rightarrow CD, C \rightarrow EF, G \rightarrow A, G \rightarrow F, CE \rightarrow F\}.$  Give a minimal cover for:

$$F = \{AB \rightarrow CD, C \rightarrow EF, G \rightarrow A, G \rightarrow F, CE \rightarrow F\}.$$

- AB → C
- AB → D

Give a minimal cover for:

$$F = \{AB \rightarrow CD, C \rightarrow EF, G \rightarrow A, G \rightarrow F, CE \rightarrow F\}.$$

- AB → C
- AB → D
- C → F
- C → E

$$F = \{AB \rightarrow CD, C \rightarrow EF, G \rightarrow A, G \rightarrow F, CE \rightarrow F\}.$$

- AB → C
- AB → D
- C → F
- C → E
- $G \rightarrow A$

$$F = \{AB \rightarrow CD, C \rightarrow EF, G \rightarrow A, G \rightarrow F, CE \rightarrow F\}.$$

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More lossless practice: F = {AB -> CDE, BE -> X, A -> E} Is ABC, BCDEX a lossless decomposition?

## More lossless practice:

 $F = \{AB -> CDE, BE -> X, A -> E\}$ 

Is ABC, BCDEX a lossless decomposition?

 No, it is lossy. ABC ∩ BCDEX = BC, which is not a superkey of ABC nor BCDEX. R: ABCDE

Given FD ={AE  $\rightarrow$  BC, AC  $\rightarrow$  D, CD  $\rightarrow$  BE, D  $\rightarrow$  E}

Give three candidate keys.

R: ABCDE

Given FD ={AE → BC, AC → D, CD → BE, D → E}

Give three candidate keys.

 AE, AC and AD are candidate keys, as each of their attribute closures include all attributes and no subset of them is a super key by itself. R: ABCDE

Given FD ={AE  $\rightarrow$  BC, AC  $\rightarrow$  D, CD  $\rightarrow$  BE, D  $\rightarrow$  E}

Is R already in BCNF?

R: ABCDE

Given FD ={AE  $\rightarrow$  BC, AC  $\rightarrow$  D, CD  $\rightarrow$  BE, D  $\rightarrow$  E}

Is R already in BCNF?

 No, because both CD → BE and D → E violate BCNF. R: ABCD Given FD ={A  $\rightarrow$  B, B  $\rightarrow$  D, C  $\rightarrow$  D} Decomposed to AB, CD, AC. Is this lossless? R: ABCD Given FD ={A  $\rightarrow$  B, B  $\rightarrow$  D, C  $\rightarrow$  D} Decomposed to AB, CD, AC. Is this lossless?

 Yes, a lossless decomposition would be: ABC CD which is lossless because C is a key for CD and then a further decomposition of ABC into AB and AC which is lossless because A is a key for AB.