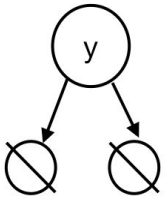
CS 225 Spring 2019 :: TA Lecture Notes
2/18 Tree Traversal

By Wenjie

- **Number of null pointers in a Binary Tree**
  - **Theorem:** A binary tree with **n** data items has **n+1** null pointers.
  - *Proof.* Let **NULL**(n) be the number of null pointers in a tree with **n** nodes.
    - Target: **NULL**(n) = n+1, for **n** >= 0
    - Proof by induction:
    - Base case:
      - when n=0, **NULL**(0) = 1, Just a empty tree with one null pointer: root = nullptr

        
      - when n=1, **NULL**(1) = 2

      - when n=2, **NULL**(2) = 3

    - **Induction step:**
      - Inductive hypothesis: **NULL**(j) = j+1, for any j<n
      - Target: for tree **T** with **n** nodes:
        NULL(n) = n+1
      - Look at the root:
        #nodes in left subtree + #nodes in right subtree = n-1
      - Suppose left subtree has **q** nodes, right subtree has **n-q-1** nodes
        - Then, **q<n**, and **n-q-1<n**
        - Then, since the root has no null pointers,
          NULL(n) = **NULL**(q) + **NULL**(n-q-1) = q+1 + n-q-1+1 = n+1
  - **Observation**: the overhead of the tree data is proportional to the number of nodes. Since NULLs don't take up a lot of memory, this is very good.

CS 225 Spring 2019 :: TA Lecture Notes 2/18 Tree Traversal

By Wenjie

- **Traversal**
  - To traverse a tree means we process every element exactly once in a tree
    - **Pre-order:** process the data first, then left child, then the right child
    - **In-order:** left child, process the data, right child
    - **Post-order:** left child, right child, process the data the last

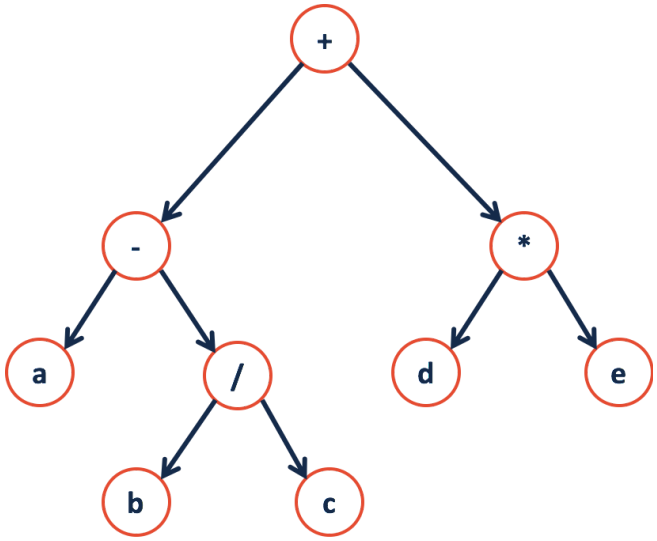
BinaryTree.cpp

```
1  void BinaryTree<T>::preOrder(TreeNode * cur) {
2    if (cur != NULL) {
3      yell(cur->data);      // yell is some imaginary function
4      preOrder(cur->left);
5      preOrder(cur->right);
6    }
7  }
8
9  void BinaryTree<T>::inOrder(TreeNode * cur) {
10   if (cur != NULL) {
11     inOrder(cur->left);
12     yell(cur->data);
13     inOrder(cur->right);
14   }
15 }
16
17 void BinaryTree<T>::postOrder(TreeNode * cur) {
18   if (cur != NULL) {
19     postOrder(cur->left);
20     postOrder(cur->right);
21     yell(cur->data);
22   }
23 }
```

CS 225 Spring 2019 :: TA Lecture Notes
2/18 Tree Traversal

By Wenjie

- **In-order print out of the tree**



  - Recursion all the way to the left, print out **a**, then **-**
  - Then to **b / c**; then **+, d, *, e;**
  - It will be **a - b / c + d * e!**
  - Tree is used a lot in parsing - in this example, a language with **binary operators** is parsed into a **binary tree**
  - Therefore, if we use different methods of traversals we will have different outputs and meanings.

- **Level Order Traversal**:
  - We use it to traverse the tree by every level
  - We will use iterative approach, by using a queue to keep track of each node we encounter on each level

```
1. Enqueue the root
2. While queue is not empty
      a. Dequeue e
      b. Yell e->data
      c. Enqueue(e->left),
         Enqueue(e->right)
```