CS 225 Spring 2019 :: TA Lecture Notes
4/3 Disjoint Sets Implementation

By Wenjie

Disjoint Sets ADT

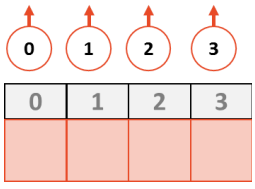
- Maintain a collection $S = \{s_0, s_1, \ldots s_k\}$ *(a set of disjoint sets).*
- Each set has an element as its representatives
- API:
  - `void makeSet(const T & t);` *(make set with one element)*
  - `void union(const T & k1, const T & k2);` *(set1 + set2)*
  - `T & find(const T & k);` *(find the representative element)*

Implementation 1

- The array indices are the keys of the elements. Any type of elements could be converted to int through a hash function
- Find(k): O(1)
- Union(k1, k2):
  - Naive implementation: going through entire array to update representations
  - O(n)

Implementation 2

- The array indices are the keys of elements
- The value of the array at index **i** would be
  - **-1**: if **i** is an representative element
  - **The index of the parent of i**: if we haven't found the rep. element.
- We call these **UpTrees**



Example of Implementation 2

- Initial state:

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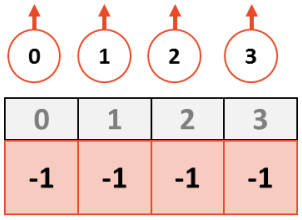
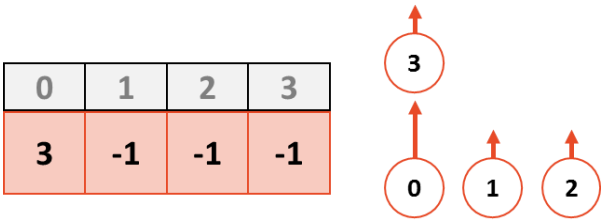
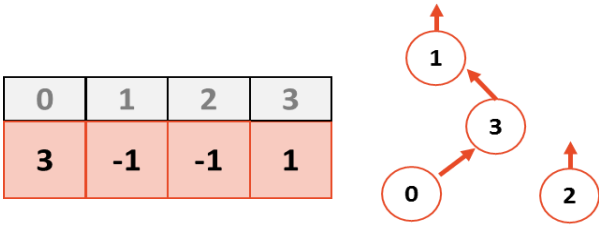


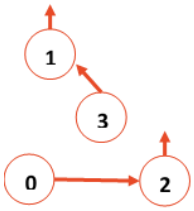
- **Union (3,0)** - we are going to point 0 to 3 and update the value for index 0



- **Union(1, 3)** – 3 will point to 1 and we will update the value for index 3:



- **Union(2,0):**
    - **BAD PRACTICE:** if we just follow the previous step, point 0 to 2, we get



      which is **is not good**
    - Instead, we need to union those roots
    - Union(find(2), find(0)) = Union(2, 1)

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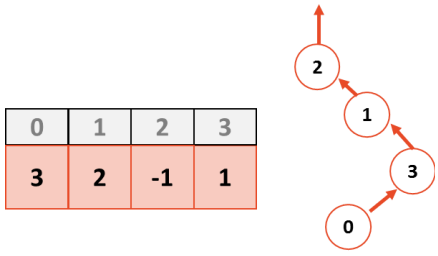
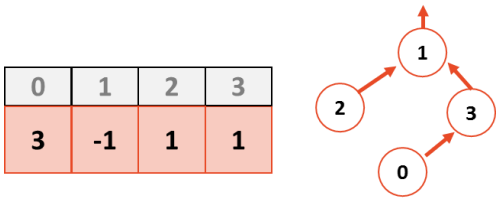


- Notice that, this is **NOT** the unique UpTree created by this set
    - We can also do Union(1, 2) and we would get:



    - This gives us a better tree since the height is smaller
    - We resolve this issue later - if we want the shortest UpTree

Disjoint Set Find

```
1  int DisjointSets::find() {
2    if ( s[i] < 0 ) { return i; }
3    else { return _find( s[i] ); }
4  }
```

- Algorithm
    - If we have **-1**: we have the root
    - If not, we recursively call find() on the parent node
- Running time
    - $O(h) <= O(n)$.
    - worst case could be

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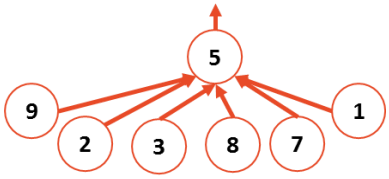
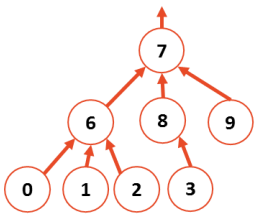
- The ideal UpTree: every element is the direct child of the root!



  - O(1) time!

Disjoint Set Union



0	1	2	3	4	5	6	7	8	9	10	11
6	6	6	8	-1	10	7	-1	7	7	4	5

- We want 7 a child of 4 (or vice versa), since that makes the total height smaller
- **Union by height** (*Keep the height of the tree as small as possible*):
  - Make root of a taller tree the parent of the root of the shorter tree (add the shorter tree to the taller tree);
  - For this approach, we need to keep track of heights:

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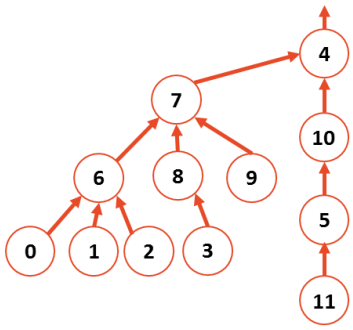
- In every **root** node, we store the **negative value of the height of the tree - 1** (to make sure -0 doesn't happen)
- **root := - h - 1**

0	1	2	3	4	5	6	7	8	9	10	11
6	**6**	**6**	**8**	**-4**	**10**	**7**	**-3**	**7**	**7**	**4**	**5**



- After union by height we have

0	1	2	3	4	5	6	7	8	9	10	11
6	**6**	**6**	**8**	**-4**	**10**	**7**	**4**	**7**	**7**	**4**	**5**

- **Union by Size** (*Minimize the number of nodes that increase in height*):
  - **root := -n**
    - where n is the size of the tree

0	1	2	3	4	5	6	7	8	9	10	11
6	**6**	**6**	**8**	**-4**	**10**	**7**	**-8**	**7**	**7**	**4**	**5**

- The results after union:

0	1	2	3	4	5	6	7	8	9	10	11
6	**6**	**6**	**8**	**7**	**10**	**7**	**-12**	**7**	**7**	**4**	**5**

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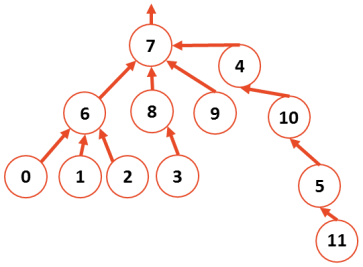

    - Union by size keeps the average case average, but worsen the worst case (node 11 gets height increased by one)
- Both guarantee the height of the tree to be **O(log n)**