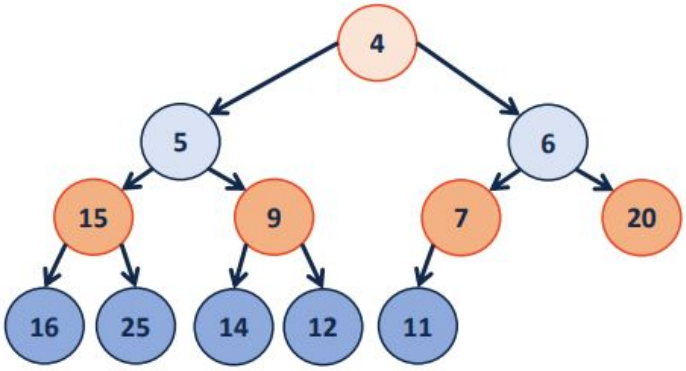
CS 225 Spring 2019 :: TA Lecture Notes 3/29 Heaps II

By Wenjie

The minHeap opeartions

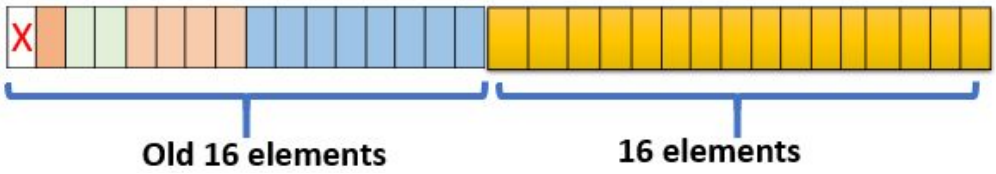
- ADT:
  - `insert()`
  - `remove()`
  - `isEmpty()`



- The visualization is a tree, but the actual implementation will be an array (or vector)
  - Root is at index 1
  - For the node on index i, its
    - Left child is at index $2 * i$
    - Right child is at index $2 * i + 1$
    - Parent is at index $\lfloor i/2 \rfloor$

Insertion

- Check if we still have the array capacity
  - If not, we double the size of the array



  - This is just adding a new layer to the tree

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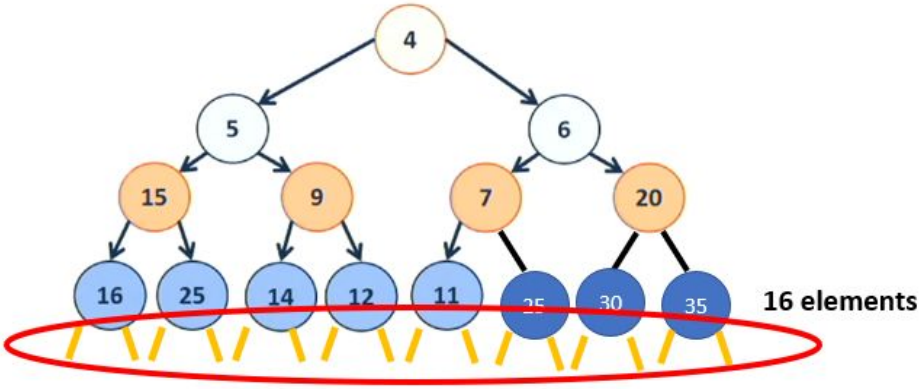
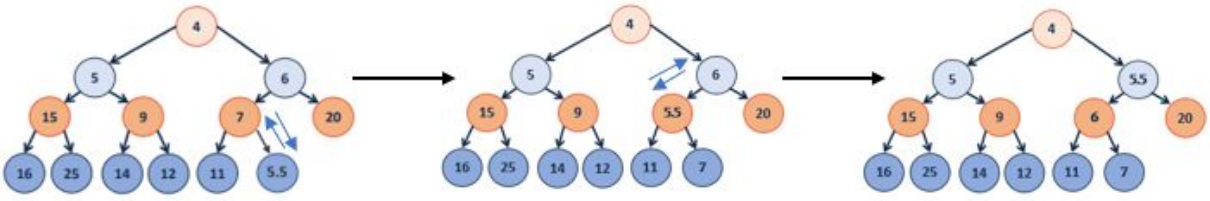


- Insert the element at the end of the array, and make sure the resulted heap is still a heap (applying heapify-up if needed)

```
1  template <class T>
2  void Heap<T>::_insert(const T & key) {
3    // Check to ensure there's space to insert an element
4    // ...if not, grow the array
5    if ( size_ == capacity_ ) { _growArray(); }
6
7    // Insert the new element at the end of the array
8    item_[++size] = key;
9
10   // Restore the heap property
11   _heapifyUp(size);
12 }
```

Heapify-Up



- Starting from the inserted node, and also assuming the heap is valid everywhere above that inserted node
- If the current element is not the root, and smaller than its parent:
    - Swap the current element with its parent node

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- Continue to applying heapifyUp on the parent node

```
1  template <class T>
2  void Heap<T>::_heapifyUp(unsigned index) {
3    if ( index > 1 ) {
4      if ( item_[index] < item_[ parent(index) ] ) {
5        std::swap( item_[index], item_[ parent(index) ] );
6        _heapifyUp(parent(index));
7      }
8    }
9  }
```

- Runtime of Insertion operation
  - `growArray()` takes O(1) amortized
  - `insertion` takes O(1)
  - `heapify-up` takes O(h) = O(lg n) since the tree is complete
  - Total runtime: O(lg n)

Remove

- Swap the root with the last element
- Then remove the last element
- Heapify-Down to ensure the heap property is perserved.

```
1  template <class T>
2  void Heap<T>::_removeMin() {
3    // Swap with the last value
4    T minValue = item_[1];
5    item_[1] = item_[size_];
6    size--;
7
8    // Restore the heap property
9    heapifyDown();
10
11   // Return the minimum value
12   return minValue;
13 }
```

Heapify-Down

- Starting from the root node with the assumption that both substrees are valid heaps
- If current is not leaf, find the minChild among the two children

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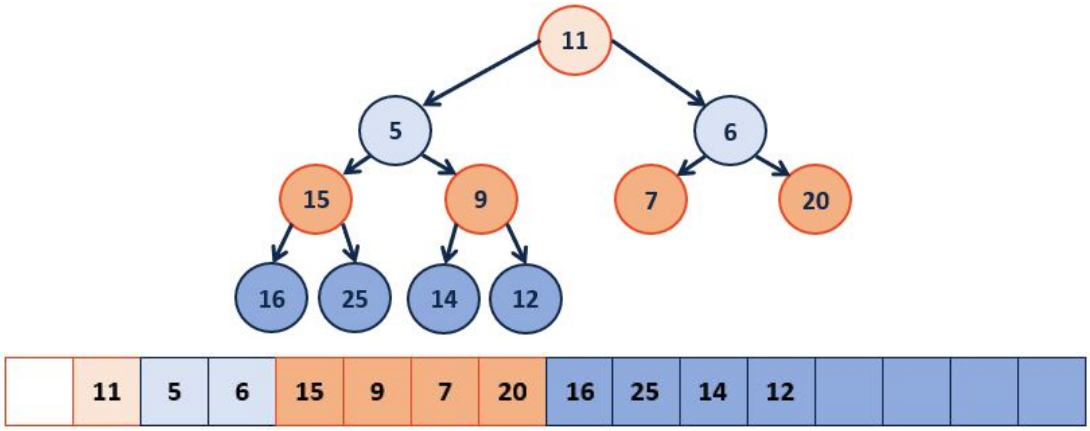
- Swap the value of minChild and subRoot if needed
- Continue on the minChild node if swap happened

```
1  template <class T>
2  void Heap<T>::_heapifyDown(int index) {
3    if ( !_isLeaf(index) ) {
4      int minChildIndex = _minChild(index);
5      if ( item_[index] > item_[minChildIndex] ) {
6        std::swap( item_[index], item_[minChildIndex] );
7        _heapifyDown(minChildIndex);
8      }
9    }
10 }
```

- Runtime of `Remove()`
  - swap takes O(1)
  - heapify-down takes O(h) = O(lg n) since the tree is complete
  - **Total runtime: O(log n)**

- **Example of HeapifyDown**



HeapifyDown(1):
minChildIndex = 2;
if(11 > 5) 😊 (true)
swap elements;
heapifyDown(2);

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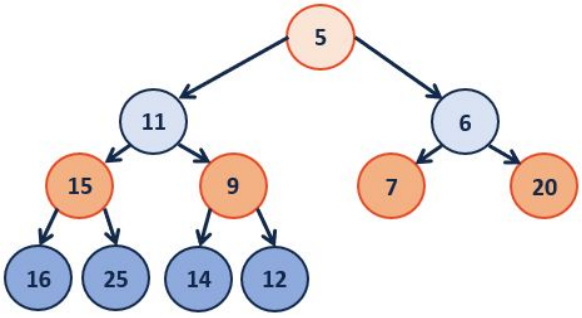
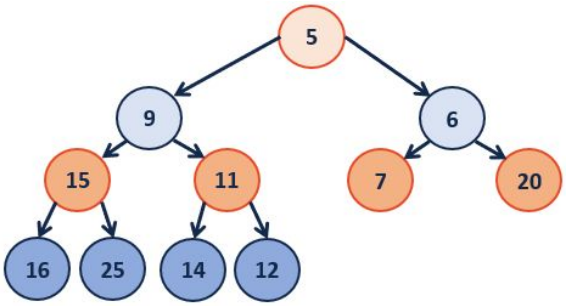


HeapifyDown(2):
    minChildIndex = 5;
    if(11 > 9) 😊
        swap elements;
        heapifyDown(5);



HeapifyDown(5):
    minChildIndex = 11;
    if(11 > 12) ☹

Done: Heap property restored!

Everything so far can be done using an AVL tree under the same runtime
But the below function, buildHeap, gives Heaps the edge over AVLs

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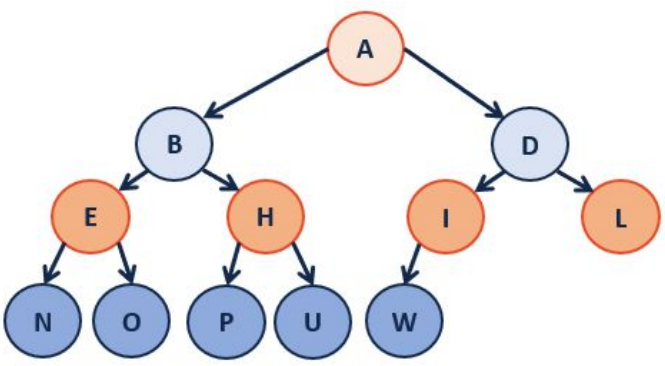
BuildHeap

- We want to build a heap using a given array:
- **Method 1: sorting**

  - A sorted array is always a heap



  - Runtime: $O(n \log n)$
- **Method 2: heapify-up**
  - Call Heapify-Up on every element from the root

```
1  template <class T>
2  void Heap<T>::buildHeap() {
3    for (unsigned i = 2; i <= size_; i++) {
4      heapifyUp(i);
5    }
6  }
```

  - Takes $O(\log n)$ for every element, so $O(n \log n)$ in total

- **Method 3: heapify-down**
  - Call Heapify-Down on every element from the end of the array

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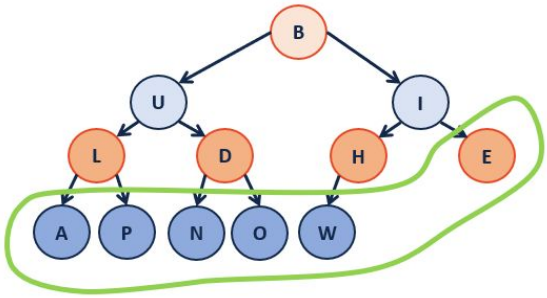
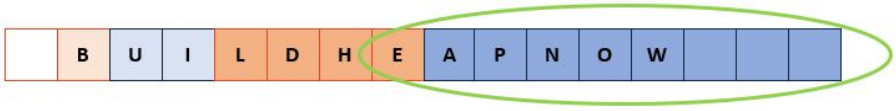
- Notice that the last level already has the heap property!
- We can start from the second-last level
- In the case below, “H” is the first element that is not a heap






```
1  template <class T>
2  void Heap<T>::buildHeap() {
3    for (unsigned i = parent(size); i > 0; i--) {
4      heapifyDown(i);
5    }
6  }
```

- Since heapify-down runs in O(h) time:
  - heapify on “H” takes 1 unit of work
  - heapify on “I” takes 2 units of work
  - heapify on “B” takes 3 units of work
  - In total, we have 1+1+1+2+2 + 3 = 10 units of work, this is just linear to the number of elements

- Then we have the runtime: $O(n)$
  - Proof in next lecture