# CS 225 Spring 2019 :: TA Lecture Notes
# 3/6 KD Tree

By Wenjie

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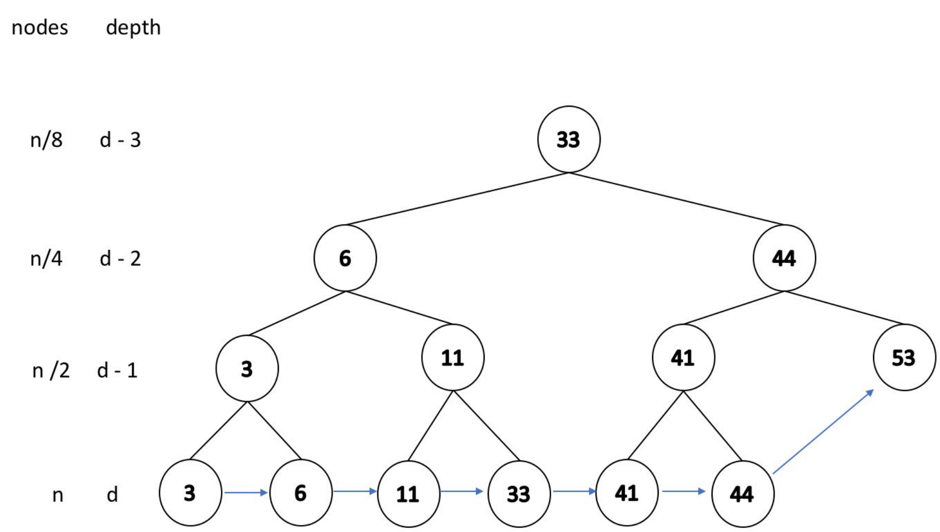
- **Range based search in a kD-Tree**
  - Given a set of points P = $\{p_n,...,p_n\}$, find the points in a range [a, b]
  - Build the kD-tree: use an AVL tree, but put all data in the leaves
  - Search is very similar with BST:
    - If dataToFind <= node.data, go left
    - If dataToFind > node.data, go right
  - If the exact data was not found (eg. find 42), we get **one past the data**, an upper bound
  - We want to find all numbers between [11,41] efficiently
    - Need to jump from 11 to 41 efficiently
    - So we make a linked list on data nodes




  - This is like running a **binary search** on a linked list
  - Runtime for finding all integers in a range [a, b]
    - find lower bound **a** in the tree: **O(lg n)**
    - find upper bound **b** in the tree: **O(lg n)**
    - traverse the linked list to find the elements in the middle: **O(k)**, where **k** is the number of elements in this range
    - So total is **<u>O(lg(n) + k)</u>**
  - Analysis
    - runtime depends on the situation: **k** or **lg(n)** might dominate the term
    - No better than running a binary search on an array, but no worse

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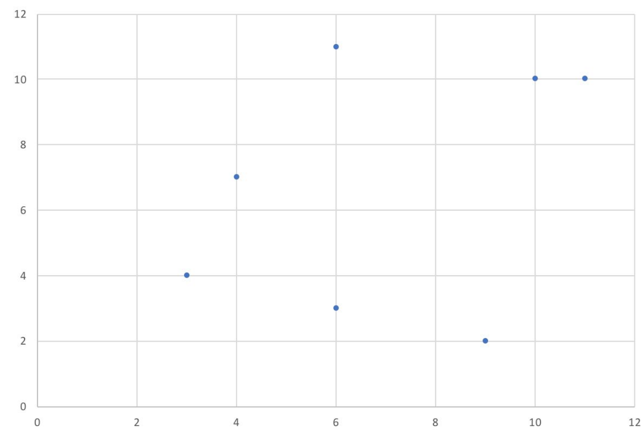
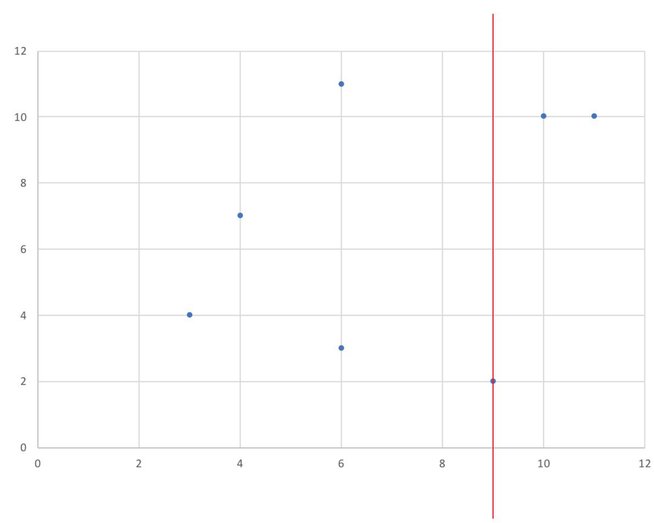
- **Range search in 2-dimensions**
  - Given P = {$p_0$, $p_1$, ..., $p_n$}, where each $p_i$ is a point in 2-D space
    - find all the points in a rectangle {($x_1$, $y_1$), ($x_2$, $y_2$)}
    - find the nearest point to (x, y)
  - Very widely asked questions in the real world. Eg: in image recognition
  - We still want to do a binary search, but an simple array will not suffice. We build a kD-tree.




    - Build a root node: find a proper **x** value to divide the space into 2 parts




    - Continue partitioning: find a proper **y** value for each subspace, and further divide each subspace into 2

# CS 225 Spring 2019 :: TA Lecture Notes
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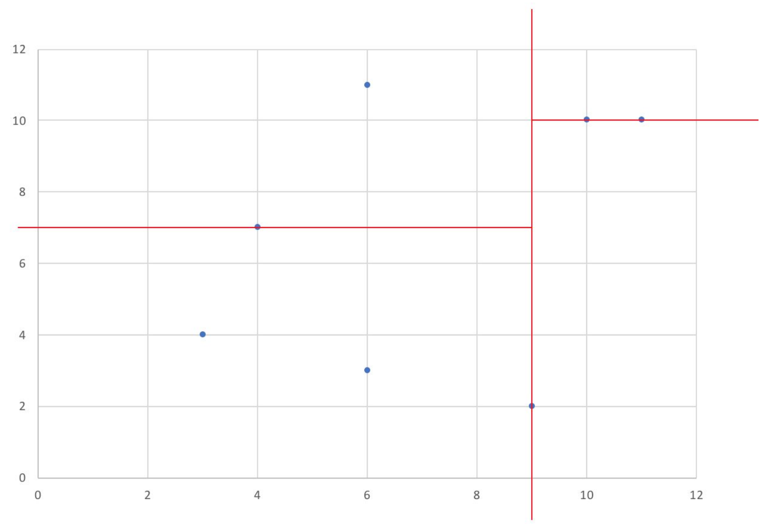
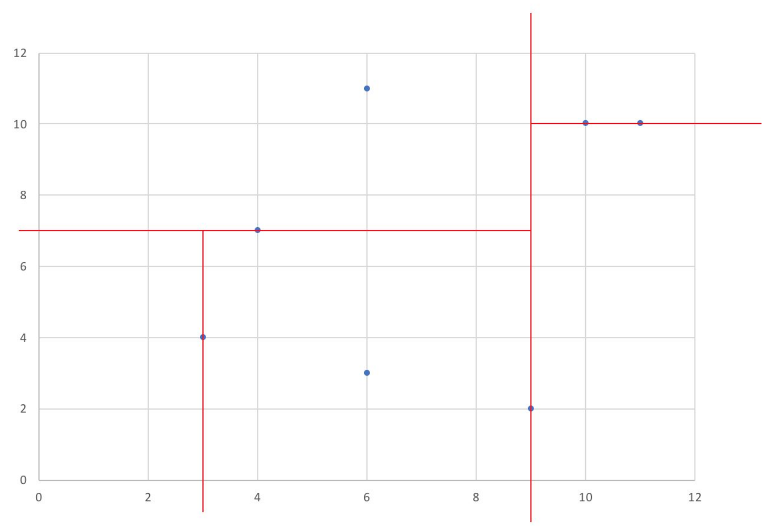



- Continue partitioning, and iterating through dimensions. So next we will partition each part depending on the **x** value (for 3-D spaces, we divide on **x, y, z** dimension)




- Stop until every part only has one point. That point will be the leaf data.

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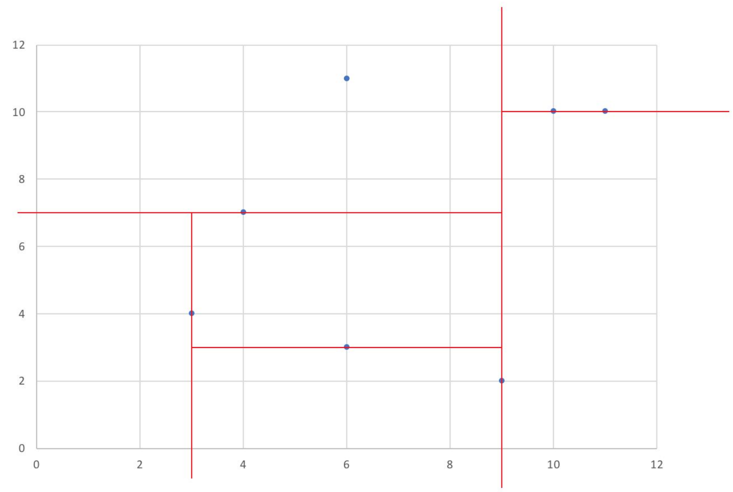
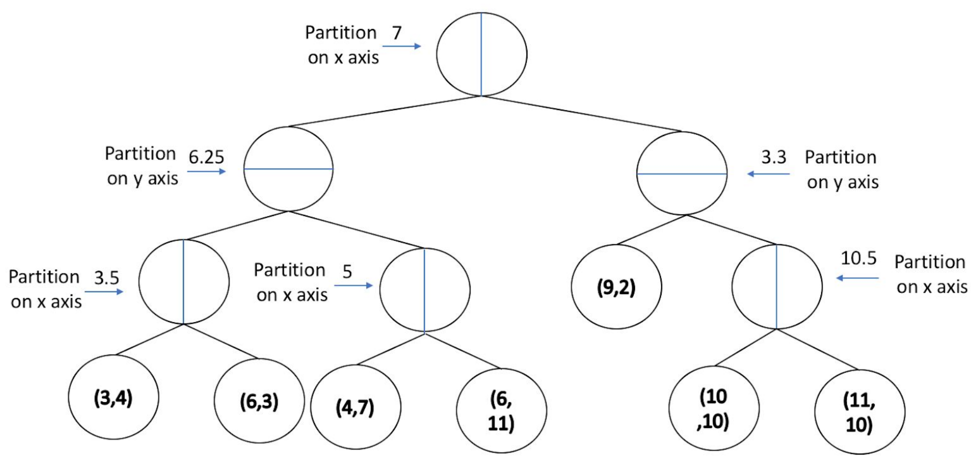



- The actual tree will be like this




- Search: similar to a binary search
  - Compare on the **x** value, then the **y** value, then the **x** value....
  - Alternating until we find the point we want
  - Each comparison eliminates half of the points in the search

- **BTrees**
  - Motivation
    - We cannot always keep data in the memory
    - Where do we put data?

        - Disk
        - “Cloud” ¯_(ツ)_/¯
        - Quantum Protons

    - Runtime
        - **3GHz** CPU performs **3 million** operations in **1ms**
        - Hard Drives
            - Bleeding-edge Disks can be very fast, ~32MB/ms
            - But most large disks are very slow, ~30 MB/s
        - The Cloud is slow
            - good ping time is 20-40 ms
            - takes ~40ms to going through an edge in a tree: very slow
            - want to lower the height even more!

- **BTree (of order m)**
    - Please note that BTree is not a binary tree at all.
    - Goal: to minimize the number of reads.
    - In practice, every node will store exactly (1 network packet/1 disk block/...)
    - BTree node:
        - Every node is a sorted array
        - Contains up to m - 1 keys
        - For example: Btree of m = 9

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| **-3** | **8** | **23** | **25** | **31** | **42** | **43** | **55** |

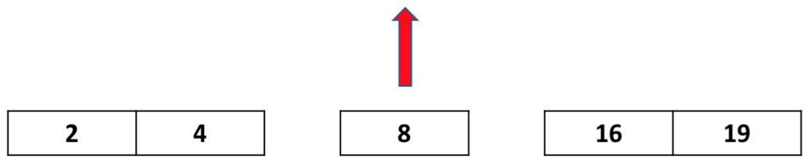
m = 9

    - **Insertion (m=5)**
        - We add to the tree until we reach the maximum number of keys in a node.

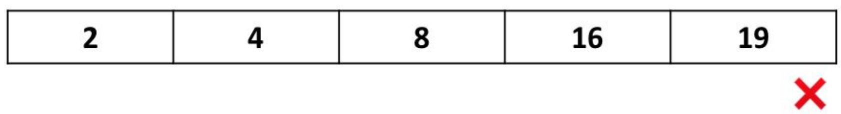
| | | | |
|---|---|---|---|
| **2** | **4** | **8** | **16** |

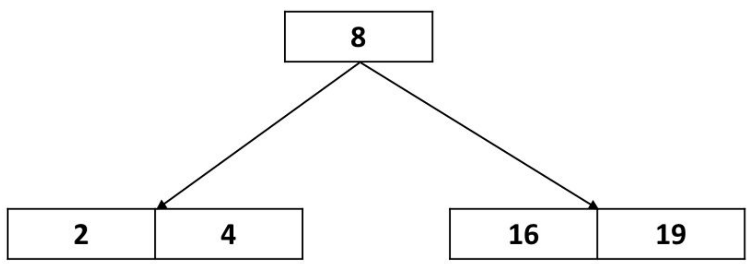
        - If we insert for example 19, we will exceed the allowed number of keys in this node. This is a overfilled array

- we split the data and throw up the middle element.




- Now we have a BTree with order m=5




- **Recursive call of split**
  - m = 3