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Determination of the linear expansion coefficient of solids

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1 Introduction

This experiment was conducted to determine the coefficient of linear thermal expansion (α) for several solid materials, namely brass, copper, and steel. The method involved measuring the initial length (L_0) of each metallic rod at a known initial temperature. Water was then passed through the rods to raise their temperature to a final, uniform temperature ($T_f \approx 55°\text{C}$). The resulting elongation (ΔL) of each rod was precisely measured using a dial gauge.

We need to find the difference in temperature and length which we can do in this way:

$$\Delta T \ = T_1 - T_0 \tag{1}$$

$$\Delta L \ = L_1 - L_0 \tag{2}$$

To calculate the linear expansion coefficient, we need to transform the following formula:

$$\Delta L = \alpha \, L_0 \, \Delta T \tag{3}$$

After transformation we have:

$$\alpha = \frac{\Delta L}{L_0 \Delta T} \tag{4}$$

Where α is the linear expansion coefficient, L_0 is the original length at T_0

2 Measurements

We need to find the length of bars in room temperature (approximately 20(°C)), using special lens and measure. We must find x_1 and x_2 to determinate original length.

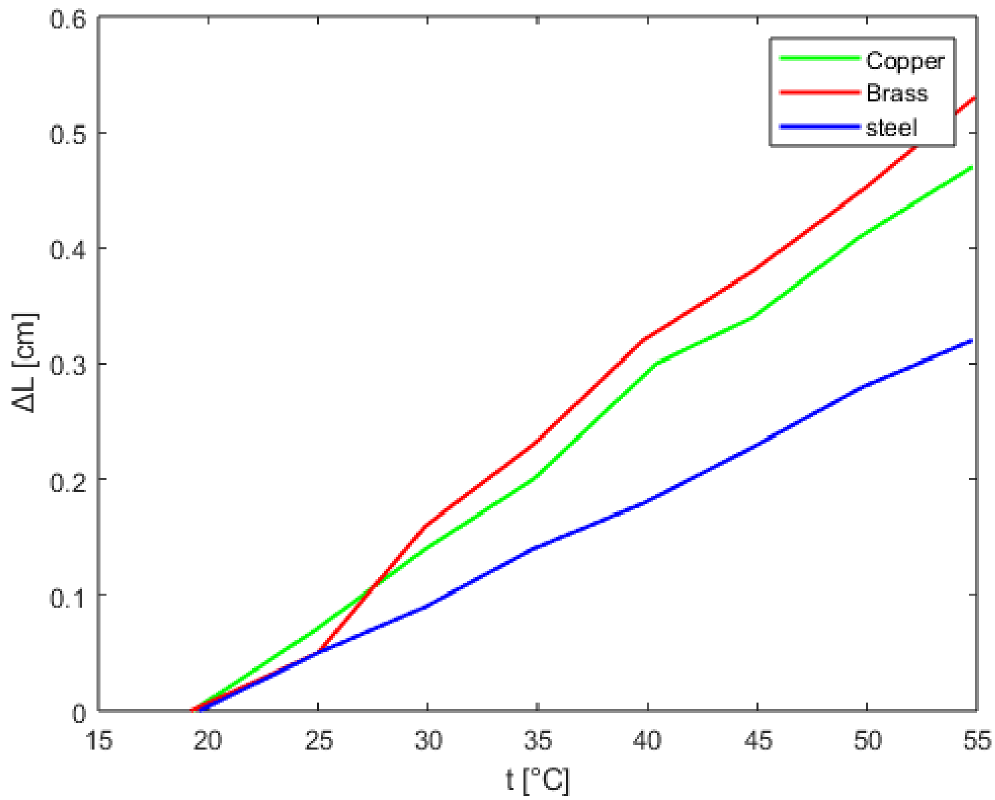
Table 1: Measurement data for Steel, Brass, and Copper

Material	x_1 [mm]	x_2 [mm]	$L = x_2 - x_1$ [mm]
Steel	0.25	773.6	773.35
Brass	0.25	772.4	772.15
Copper	0.25	773.3	773.05

The experiment was conducted measuring the elongation of three different materials cooper, brass and steel. The rods were heated up through a thermal heating system, the elongation was measured through micrometer sensor and the temperature was observed using a digital thermometer. Measurement were taken in approximately 5°C intervals.

Table 2: Temperature and elongation data for Copper (C), Brass (B), and Steel (S)

Copper (C)		Brass (B)		Steel (S)	
T (°C)	ΔL (mm)	T (°C)	ΔL (mm)	T (°C)	ΔL (mm)
19.3	0.00	19.2	0.00	19.6	0.00
24.9	0.07	25.0	0.09	25.1	0.05
29.9	0.14	29.9	0.16	29.9	0.09
34.8	0.20	34.8	0.23	34.9	0.14
40.4	0.30	39.8	0.32	39.9	0.18
44.8	0.34	44.8	0.38	45.0	0.23
49.7	0.41	49.8	0.45	49.8	0.28
54.8	0.47	54.9	0.53	54.8	0.32


Copper
Brass
steel
ΔL [cm]
t [°C]
0
0.1
0.2
0.3
0.4
0.5
0.6
15
20
25
30
35
40
45
50
55

3 Calculations

To determine α, a linear regression of the measured data has to be performed to be able to treat Δl as a linear function of T:

$$\Delta l = aT + B \tag{4}$$

The gradient obtained through the linear regression and the initial length measured l_0 the linear expansion coefficient can be determined by calculating:

$$\alpha = \frac{a}{l_0} \tag{5}$$

The gradients are:

$$\alpha_{copper} = \frac{1.3466 * 10^{-2} mm/^{\circ}C}{773.05 mm} = 1.742 * 10^{-5} K^{-1} \tag{5}$$

$$\alpha_{brass} = \frac{1.4770 * 10^{-2} mm/^{\circ}C}{772.15 mm} = 1.9128 * 10^{-5} K^{-1} \tag{6}$$

$$\alpha_{steel} = \frac{9.1616 * 10^{-3} mm/^{\circ}C}{773.35 mm} = 1.1847 * 10^{-5} K^{-1} \tag{7}$$

4 Measurement Error

The combined standard uncertainty, $u(\alpha)$, is determined using Gaussian error propagation. This method combines the relative uncertainties from the slope of the regression (a) and the initial length (L_0). The uncertainty for the initial length, $u(L_0)$, is ± 0.05 mm (half the 0.1 mm resolution of the measuring scale).

The formula for the propagated uncertainty is:

$$u(\alpha) = \alpha \sqrt{\left(\frac{u(a)}{a}\right)^2 + \left(\frac{u(L_0)}{L_0}\right)^2} \tag{8}$$

To complete these calculations, the standard error of the slope, $u(a)$, must be obtained from the linear regression statistics for each material.

Uncertainty for Steel

$$\begin{aligned}
u(\alpha)_{\text{Steel}} &= \alpha_{\text{Steel}} \times \sqrt{\left(\frac{u(a)_{\text{steel}}}{a_{\text{steel}}}\right)^2 + \left(\frac{u(L_0)}{L_0}\right)^2} \\
&= (1.1847 \times 10^{-5}) \times \sqrt{\left(\frac{0.000157}{9.1616 \times 10^{-3}}\right)^2 + \left(\frac{0.05}{773.35}\right)^2} \\
&= (1.1847 \times 10^{-5}) \times 0.01714 \\
&= 2.03 \times 10^{-7} \text{ K}^{-1}
\end{aligned}$$

$$\alpha_{\textbf{Steel}} = (1.1847 \pm 0.20) \times 10^{-5} \ \mathbf{K}^{-1}$$

Uncertainty for Brass

$$\begin{aligned}
u(\alpha)_{\text{Brass}} &= \alpha_{\text{Brass}} \times \sqrt{\left(\frac{u(a)_{\text{brass}}}{a_{\text{brass}}}\right)^2 + \left(\frac{u(L_0)}{L_0}\right)^2} \\
&= (1.9128 \times 10^{-5}) \times \sqrt{\left(\frac{0.000183}{1.4770 \times 10^{-2}}\right)^2 + \left(\frac{0.05}{772.15}\right)^2} \\
&= (1.9128 \times 10^{-5}) \times 0.01239 \\
&= 2.37 \times 10^{-7} \text{ K}^{-1}
\end{aligned}$$

$$\alpha_{\textbf{Brass}} = (1.9128 \pm 0.24) \times 10^{-5} \ \mathbf{K}^{-1}$$

Uncertainty for Copper

$$
\begin{aligned}
u(\alpha)_{\text{Copper}} &= \alpha_{\text{Copper}} \times \sqrt{\left(\frac{u(a)_{\text{copper}}}{a_{\text{copper}}}\right)^2 + \left(\frac{u(L_0)}{L_0}\right)^2} \\
&= (1.742 \times 10^{-5}) \times \sqrt{\left(\frac{0.000134}{1.3466 \times 10^{-2}}\right)^2 + \left(\frac{0.05}{773.05}\right)^2} \\
&= (1.742 \times 10^{-5}) \times 0.00995 \\
&= 1.73 \times 10^{-7}\ \text{K}^{-1}
\end{aligned}
$$

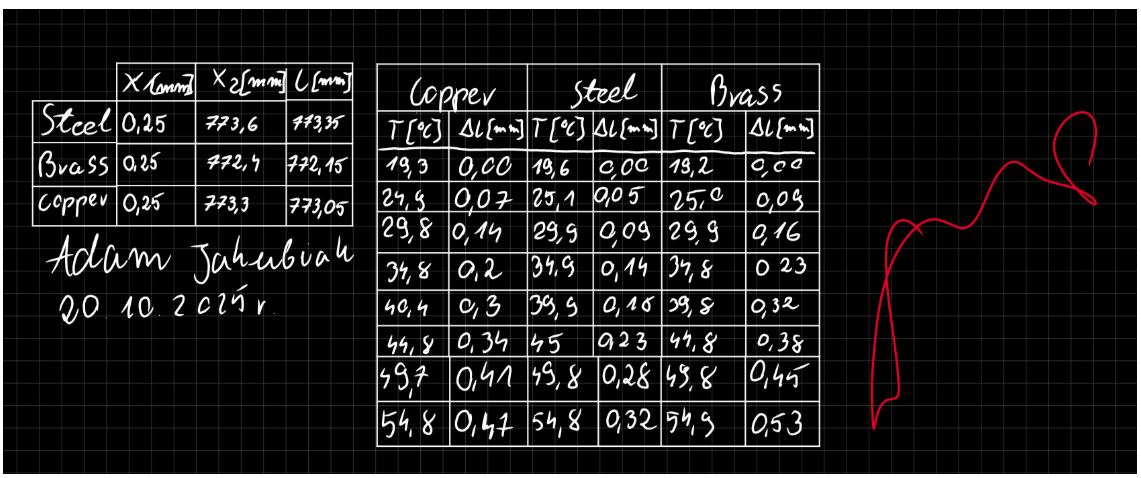
$\alpha_{\textbf{Copper}} = (1.742 \pm 1.7) \times 10^{-5}\ \mathbf{K}^{-1}$

Several other sources of uncertainty also impact the final values. Instrumental errors are present in each measurement: the dial gauge introduces uncertainty in ΔL (e.g., ± 0.005 mm) and the digital thermometer has its own accuracy limits for T (e.g., $\pm 0.1°$C). Systemic errors might also be present, such as incomplete thermal equilibrium or non-uniform heating along the rod. As shown in the calculations, the statistical uncertainty from the linear regression (the $u(a)$ term) is the dominant source of error, as the uncertainty from the initial length L_0 is negligible.

5 Conclusion

In this experiment, the thermal expansion of three solid materials was investigated by measuring the elongation of their rods as the temperature increased. The results are in strong agreement with the theory, confirming that over a small temperature range, the change in length is linearly proportional to the change in temperature.The calculated expansion coefficients for all three materials align with known values from the literature.

The final calculated expansion coefficients, including their measurement uncertainty, are:
$\alpha_{\text{Steel}} = (11.85 \pm 0.20) \times 10^{-6}\ \text{K}^{-1}$
$\alpha_{\text{Brass}} = (19.13 \pm 0.24) \times 10^{-6}\ \text{K}^{-1}$
$\alpha_{\text{Copper}} = (17.42 \pm 0.17) \times 10^{-6}\ \text{K}^{-1}$
These calculated expansion coefficients for all three materials align with known values from the literature within their uncertainty bounds.



	X1 [mm]	X2 [mm]	L [mm]
Steel	0,25	773,6	773,35
Brass	0,25	772,4	772,15
Copper	0,25	773,3	773,05

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Copper		Steel		Brass	
T [°C]	ΔL [mm]	T [°C]	ΔL [mm]	T [°C]	ΔL [mm]
19,3	0,00	19,6	0,00	19,2	0,00
24,9	0,07	25,1	0,05	25,0	0,09
29,8	0,14	29,9	0,09	29,9	0,16
34,8	0,2	34,9	0,14	34,8	0 23
40,4	0,3	39,9	0,16	39,8	0,32
44,8	0,34	45	0,23	44,8	0,38
49,7	0,41	49,8	0,28	49,8	0,45
54,8	0,47	54,8	0,32	54,9	0,53