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Determination of gravitational acceleration using a reversible and mathematical pendulum

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The primary objective of this experiment is to determine the local gravitational acceleration, g, by analyzing the periodic motion of two distinct types of pendulum: a reversible physical pendulum and a simple mathematical pendulum. The underlying principle for both methods is the relationship between a pendulum's period of oscillation (T), its characteristic length, and the gravitational field it swings in.

$$T = 2\pi\sqrt{\frac{l}{g}} \tag{1}$$

and then after transforming the formula to find g we have:

$$g = \frac{4\pi^2 L_r}{T^2} \tag{2}$$

Mathematical pendulum

Set the length of string and deflect the ball for 5° and then press the button to measure time of 10 cycles and repeat that 3 times to get an average (AVG) value, after that we must change a position and repeat the process.

Table 1: Measurements of mathematical pendulum

L [cm]	**I [s]**	**II [s]**	**III [s]**	**AVG [s]**
25	10,475	9,996	9,998	10,156
35	12,223	11,902	12,459	12,195
45	13,336	13,336	13,329	13,333

After the measurements we are using the formula for calculating g.

Table 2: Calculations of g

L [cm]	**AVG [s]**	**g**$[m/s^2]$
25	10,156	9,57
35	12,195	9,29
45	13,333	9,99

We also need to take into account the measurement error using the Student-Fisher t-distribution. The average value is:

$$g_{avg} = \frac{g_1 + g_2 + g_3}{3} \approx 9.617 \ \frac{\mathrm{m}}{\mathrm{s}^2} \tag{3}$$

Next, we find the sample standard deviation (σ), using the $(N-1)$ denominator for a small sample:

$$\sigma = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(g_i - g_{avg})^2} \approx 0.352 \ \frac{\mathrm{m}}{\mathrm{s}^2} \tag{4}$$

The standard error of the mean ($\sigma_{\bar{g}}$) is:

$$\sigma_{\bar{g}} = \frac{\sigma}{\sqrt{N}} \approx 0.203 \ \frac{\mathrm{m}}{\mathrm{s}^2} \tag{5}$$

For $N = 3$ measurements, we have $\nu = N - 1 = 2$ degrees of freedom. For a **68%** confidence level (two-sided), the Student-Fisher coefficient is

$$t \approx 1.312. \tag{6}$$

The error (Δg) at 68% confidence is therefore:

$$\Delta g = t \times \sigma_{\bar{g}} \approx 1.312 \times 0.203 \approx 0.27 \ \frac{\mathrm{m}}{\mathrm{s}^2} \tag{7}$$

So our final result for the mathematical pendulum is:

$$g = (9.62 \pm 0.27) \ \frac{\mathrm{m}}{\mathrm{s}^2} \tag{8}$$

Reversible pendulum

Set the adjustable weight to x (ex. 100 cm) position and deflect the pendulum for 5° then press the button to measure time of 10 cycles and repeat that 3 times to get an average value, after that we must change a position and repeat the process. After measuring all wanted lengths turn the pendulum around and repeat the process paying attention to the length which need to be the same.

Table 3: Measurements of first pivot of reversible pendulum

L [cm]	**I [s]**	**II [s]**	**III [s]**	**AVG [s]**	**Period T [s]**
32	18,745	18,747	18,753	18,748	1.8748
42	18,385	18,383	18,377	18,382	1.8382
52	18,199	18,195	18,190	18,195	1.8195
62	18,139	18,138	18,149	18,142	1.8142
72	18,225	18,223	18,213	18,220	1.8220
82	18,395	18,407	18,399	18,400	1.8400
92	18,647	18,647	18,659	18,651	1.8651

Table 4: Measurements of second pivot of reversible pendulum

L [cm]	**I** [s]	**II** [s]	**III** [s]	**AVG** [s]	**Period T** [s]
32	18,893	18,887	18,890	18,891	1.8891
42	18,242	18,245	18,246	18,244	1.8244
52	17,698	17,688	17,687	17,691	1.7691
62	17,297	17,274	17,272	17,280	1.7280
72	17,215	17,208	17,217	17,213	1.7213
82	17,710	17,710	17,717	17,721	1.7721
92	19,390	19,389	19,395	19,396	1.9396

After measuring all the values, we divide the average time for 10 cycles (AVG 10T) by 10 to find the period (T) for each position, as shown in Tables 3 and 4. These periods are then plotted against the length L. The intersection of the two resulting curves gives the points where the period is the same for both pivots.

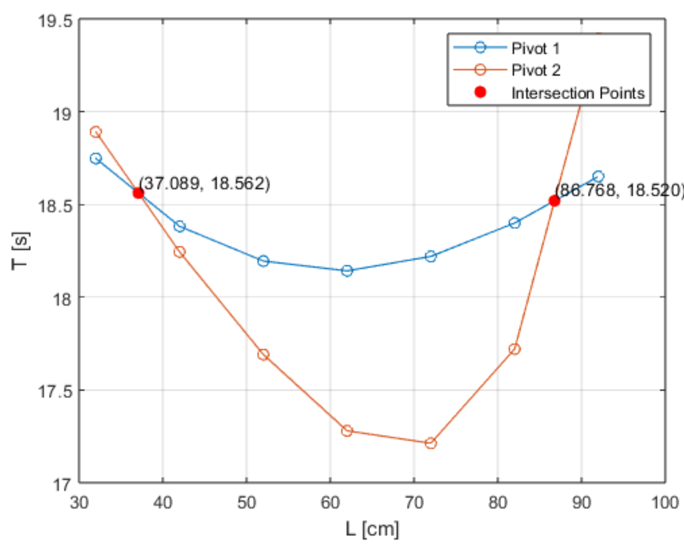

Figure 1: Plot of Period T vs. Length L for both pivots.

We take the intersection periods from the corrected Figure 1 to be: $T_1 = 1.8562$ [s] $T_2 = 1.8620$ [s]

The average period (T) is:

$$T = \frac{T_1 + T_2}{2} = \frac{1.8562 + 1.8620}{2} = 1.8591 \text{ [s]} \tag{9}$$

We use the reduced length $L_r = 0.870 \pm 0.001$ [m]. We can now calculate g using formula (2):

$$g = \frac{4\pi^2 L_r}{T^2} = \frac{4\pi^2(0.870 \text{ m})}{(1.8591 \text{ s})^2} \approx 9.939\frac{m}{s^2} \tag{10}$$

To find the error Δg, we use the logarithmic differential method (error propagation). The relative error is:

$$\frac{\Delta g}{g} = \sqrt{\left(\frac{\Delta L_r}{L_r}\right)^2 + \left(2\frac{\Delta T}{T}\right)^2} \tag{11}$$

We need the error in T, ΔT. We can estimate this from the two intersection values: $\Delta T = \frac{|T_2 - T_1|}{2} = \frac{|1.8620-1.8562|}{2} = 0.0029$ [s].

Plugging in the values:

$$\frac{\Delta g}{g} = \sqrt{\left(\frac{0.001}{0.870}\right)^2 + \left(2\frac{0.0029}{1.8591}\right)^2} \approx \sqrt{(0.00115)^2 + (0.00312)^2} \approx 0.00333 \tag{12}$$

The absolute error is:

$$\Delta g = g \times 0.00333 = 9.939 \times 0.00333 \approx 0.033\frac{m}{s^2} \tag{13}$$

So our final result for the reversible pendulum, after rounding, is:

$$g = (9.94 \pm 0.03)\frac{m}{s^2} \tag{14}$$

Conclusion

Table 5: Final Results for Gravitational Acceleration (g)

Method	**Value for g** $[m/s^2]$
Mathematical Pendulum	$g_{mat} = 9.62 \pm 0.27$
Reversible Pendulum	$g_{rev} = 9.94 \pm 0.03$
Literature Value	$g_{lit} = 9.8126 \pm 0.0002$

The difference is most likely due to measurement inaccuracies in the reduced length or timing errors. Overall, the experiment confirms the theoretical relationship between period and length of a pendulum, with acceptable deviations caused by experimental limitations.