

# Fundamental Algorithm Techniques

## Problem Set #1

Due: October 4, 2025

**Problem 1** (Seems smart?). Consider a sparse vector of size  $n$  with only 0's and a single 1:

$$v = [0, 0, \dots, 1, \dots, 0]$$

- (a) Write pseudo recursive code that perform **just** the binary divide and conquer method without creating a copy of the vector and runs until vector fully decomposed into sizes 1.
- function  $\text{divide}(v, \dots, 2)$
  - function  $\text{divide}(v, \dots, m)$  for any  $m \ll n$
- (b) Analyse complexity of above divide and conquer:  $m=2$ ,  $m=3, \dots$  tertiary division, which  $T(n)$  and which  $\mathcal{O}$ ?
- (c) Next, once the division has reach sizes of 1, we collect the unique 1 and its position.
- Evaluate the cost  $f(n)$
  - What is the recurrence relation  $T(n)$
  - is complexity now  $\mathcal{O}(\log(n))$  or  $\mathcal{O}(n)$  (use master Theorem)
- (d) Compare with simpler approach: run over all indices...

**Problem 2** (School Multiplication was easy back then!). Similar to Paesant multiplication, but in basis 10. it is just the first multiplication you learned at school.

represent  $x, y$  in  $\mathbb{N}^+$  with the vectors/arrays  $X$  and  $Y$ , such that  $x = \sum_{i=0}^{n_x} X[i] \cdot 10^i$ ,  $y = \sum_{j=0}^{n_y} Y[j] \cdot 10^j$ .

1. write a multiplication pseudocode using above vector
2. tweak code such that results can be larger than the limit of your standard integer
3. what is the time complexity of your code?
4. Explain how above multiplication can be described with the divide and conquer recursion to find  $T(n) \approx 4T(\frac{n}{2})$  or Karatsuba algorithm:  $T(n) \approx 3T(\frac{n}{2})$ , finding with Master Theorem resp.  $\mathcal{O}(n^2)$  and  $\mathcal{O}(n^{1.585})$  **hard**

$$\text{Hint 1: } x \cdot y = (x_1 \cdot 10^{n/2} + x_0) (y_1 \cdot 10^{n/2} + y_0)$$

$$\text{Hint 2: } x \cdot y = z_2 \cdot 10^n + z_1 \cdot 10^{n/2} + z_0$$

5.  $\sum_{i=1}^n i$  or  $n \cdot n - 1 \cdot \dots \cdot 2 \cdot 1$  can be computed by one application of school multiplication:  $n! = \frac{1}{2} \text{mult}(v, w)$  with which  $v, w$ ? **hard** but simple math...