

Sorting Algorithms and Master Theorem Summary 1. Bubble Sort Not a divide-and-conquer algorithm.

Complexity:  $\Theta(n^2)$  time,  $O(1)$  space. 2. Merge Sort Recurrence:  $T(n) = 2T(n/2) + \Theta(n)$

Master Theorem ( $a=2$ ,  $b=2$ ,  $f(n)=\Theta(n)$ )  $\Rightarrow$  Case 2  $\Rightarrow \Theta(n \log n)$ .

Space:  $O(n)$ . 3. Quick Sort (Random Pivot) Average:  $T(n) = 2T(n/2) + \Theta(n) \Rightarrow \Theta(n \log n)$

Worst:  $\Theta(n^2)$

Space:  $O(\log n)$ . 4. Quick Sort (Median-of-Three Pivot) Better pivot choice, fewer unbalanced cases.

Average:  $\Theta(n \log n)$ , Worst:  $\Theta(n^2)$ , Space:  $O(\log n)$ . 5. Heap Sort Build heap  $O(n)$  +  $n \times \text{extract}$   $O(\log n) = O(n \log n)$

Space:  $O(1)$ . Summary Table

Algorithm	Best	Average	Worst	Space
Bubble Sort	$O(n)$	$O(n^2)$	$O(n^2)$	$O(1)$
Quick Sort (random)	$O(n \log n)$	$O(n \log n)$	$O(n^2)$	$O(\log n)$
Quick Sort (median-3)	$O(n \log n)$	$O(n \log n)$	$O(n \log n)$	$O(\log n)$
Merge Sort	$O(n \log n)$	$O(n \log n)$	$O(n \log n)$	$O(n)$
Heap Sort	$O(n \log n)$	$O(n \log n)$	$O(n \log n)$	$O(1)$

**Conclusion:** According to the Master Theorem, Merge Sort and Quick Sort have  $\Theta(n \log n)$  expected complexity, while Bubble Sort is much slower.