Fundamental Algorithm Techniques

Problem Set #3

Due: October 25, 2025

Problem 1 (Fibonacci Super Fast!). *1. compute Fibonacci with the relation:*

$$\begin{bmatrix} F_{n+1} \\ F_n \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^n \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

2. This can also be expressed as:

$$\begin{bmatrix} F_{n+1} \\ F_n \end{bmatrix} = \left(\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \right)^{n/2} \begin{bmatrix} 1 \\ 0 \end{bmatrix},$$

use Master Theorem to discuss the complexity of this decomposition and show, explain why time complexity is then $\log_2(n)$*.*

Problem 2 (0/1 Knapsack Algorithm!). *1. Why is Knapsack not greedy Algo., why dynamical programming?*

2. Solve the Knapsack Algorithm for the course example.

3. Can you get space complexity to $\mathcal{O}(W)$*?*

Problem 3 (Neuro Computing!). *1. Generate 100 random binary vectors of length N, like* $[1, 0, 0, 1, \cdots, 0, 1]$

2. Define a similarity functions of form:

$$\mathrm{sim}(\mathbf{x}, \mathbf{y}) = \frac{\mathbf{x} \cdot \mathbf{y}}{\|\mathbf{x}\|_1 \, \|\mathbf{y}\|_1} = \frac{\sum_{i=1}^n x_i y_i}{\left(\sum_{i=1}^n x_i\right)\left(\sum_{i=1}^n y_i\right)}$$

$$\mathrm{Jacc}(\mathbf{x}, \mathbf{y}) = \frac{\sum_{i=1}^n x_i y_i}{\sum_{i=1}^n \max(x_i, y_i)} = \frac{|\mathbf{x} \cap \mathbf{y}|}{|\mathbf{x} \cup \mathbf{y}|}$$

and observe that the similarities distribute like a Gaussian.

3. What happens when repeating the experiance for larger N's? Why?

4. consider next a huge sparse binary vector of length $N = 2000$ *with* $w = 5$ *the number of random ones. How many possible such vectors are possible?*

5. Can you think about a notion of capacity for these vectors?