COMP0147 Discrete Mathematics for Computer Scientists Notes

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March 27, 2019

Notes adapted from lecture notes by Max Kanovich and Robin Hirsch [1].

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1 Foundations

Contents

1.1 Set Theory and Functions

1.1.1 Set Notations

- Set definition: $A = \{a, b, c\}$
- Set membership (element-of): $a \in A$
- Set builder notation: $\{x \mid x \in \mathbb{R} \wedge x^2 = x\}$
- Empty set: $\emptyset$

1.1.2 Properties

- No structure
- No order
- No copies

For example, a, b, c are references to actual objects in

$$\{a, b, c\} \Leftrightarrow \{c, a, b\} \Leftrightarrow \{a, b, c, b\}$$

1.1.3 Set Equality

Definition 1.1.1 (Set Equality)**.** Set $A = B$ iff:

1. $A \subseteq B \implies \forall\, x(x \in A \rightarrow x \in B)$
2. $B \subseteq A \implies \forall\, y(y \in B \rightarrow y \in A)$

Remark. $A = B \Leftrightarrow A \subseteq B \wedge B \subseteq A$

1.1.4 Set Operations

- *Union*: $A \cup B \equiv \{x \mid x \in A \lor x \in B\}$
- *Intersection*: $A \cap B \equiv \{x \mid x \in A \land x \in B\}$
- *Relative Complement*: $A \setminus B \equiv \{x \mid x \in A \land x \notin B\}$
- *Absolute Complement*: $A^c \equiv U \setminus A \equiv \{x \mid x \in U \land x \notin A\}$
- *Symmetric Difference*: $A\Delta B \equiv (A \setminus B) \cup (B \setminus A) \equiv (A \cup B) \setminus (A \cap B)$
- *Cartesian Product*: $A \times B \equiv \{(x, y) \mid x \in A \land y \in B\}$

1.1.5 Boolean Algebra

Definition 1.1.2 (De Morgan's Laws).

$$\neg(p \lor q) \equiv \neg p \land \neg q \tag{1.1}$$
$$\neg(p \land q) \equiv \neg p \lor \neg q \tag{1.2}$$

Definition 1.1.3 (Idempotent Laws).

$$p \lor p \equiv p \tag{1.3}$$
$$p \land p \equiv p \tag{1.4}$$

Definition 1.1.4 (Commutative Laws).

$$p \lor q \equiv q \lor p \tag{1.5}$$
$$p \land q \equiv q \land p \tag{1.6}$$

Definition 1.1.5 (Associative Laws).

$$p \lor (q \lor r) \equiv (p \lor q) \lor r \tag{1.7}$$
$$p \land (q \land r) \equiv (p \land q) \land r \tag{1.8}$$

Definition 1.1.6 (Distributive Laws).

$$p \land (q \lor r) \equiv (p \land q) \lor (p \land r) \tag{1.9}$$
$$p \lor (q \land r) \equiv (p \lor q) \land (p \lor r) \tag{1.10}$$

Definition 1.1.7 (Identity Laws).

$$p \lor \mathrm{F} \equiv p \tag{1.11}$$
$$p \lor \mathrm{T} \equiv \mathrm{T} \tag{1.12}$$
$$p \land \mathrm{T} \equiv p \tag{1.13}$$
$$p \land \mathrm{F} \equiv \mathrm{F} \tag{1.14}$$

Definition 1.1.8 (Absorption Laws)**.**

$$p \lor (p \land q) \equiv p \tag{1.15}$$
$$p \land (p \lor q) \equiv p \tag{1.16}$$

Definition 1.1.9 (Implication and Negation Laws)**.**

- *Identity*: $p \to q \equiv \neg p \lor q$
- *Counter-example*: $\neg(p \to q) \equiv p \land \neg q$

Bibliography

[1] Max Kanovich and Robin Hirsch.
"Lecture Notes on Discrete Mathematics for Computer Scientists".
URL: http://www.cs.ucl.ac.uk/1819/a4u/t2/comp0147_discrete_mathematics_for_computer_scientists/.