COMP0147: Discrete Mathematics Notes

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Contents

Introduction

Notes are based on lecture notes by Professor Max Kanovich [Kan19] and discrete mathematics books [Ros12].

Chapter 1

Logic and Proofs

1.1 Propositional Logic

1.1.1 Propositions

Definition 1.1.1.1. A proposition is statement which is either *true* or *false* but not both.

Propositions can be denoted via uppercase letters, $P, Q, R, S, \ldots$.

Example 1.1.1.2. Let $P =$ "Computer Science is life".

Definition 1.1.1.3. The negation of a proposition P can be denoted as $\neg P$ or $\bar{P}$.

P	$\neg P$
0	1
1	0

Table 1.1: Truth Table of $\neg P$

Example 1.1.1.4. "Computer Science is not life" can be denoted as $\neg P$ or $\overline{P}$.

Definition 1.1.1.5. The logical AND or conjunction of P and Q can be denoted as $P \wedge Q$.

P	Q	$P \wedge Q$
0	0	0
0	1	0
1	0	0
1	1	1

Table 1.2: Truth Table of $P \wedge Q$

Definition 1.1.1.6. The logical OR or disjunction of P and Q can be denoted as $P \vee Q$.

P	Q	$P \vee Q$
0	0	0
0	1	1
1	0	1
1	1	1

Table 1.3: Truth Table of $P \vee Q$

Definition 1.1.1.7. The logical XOR or exclusive or of P and Q can be denoted as $P \oplus Q$.

P	Q	$P \oplus Q$
0	0	0
0	1	1
1	0	1
1	1	0

Table 1.4: Truth Table of $P \oplus Q$

Definition 1.1.1.8. The implication between P and Q can be denoted as $P \to Q$.

Note that in $P \to Q$, P is known as the premise and Q is the conclusion.

P	Q	$P \to Q$
0	0	1
0	1	1
1	0	0
1	1	1

Table 1.5: Truth Table of $P \oplus Q$

Definition 1.1.1.9. Given implication $P \to Q$, its converse is then $Q \to P$.

Definition 1.1.1.10. Given implication $P \to Q$, its contrapositive is then $\neg Q \to \neg P$.

Note that the contrapositive $\neg Q \to \neg P$ is *equivalent* to the original implication $P \to Q$.

Definition 1.1.1.11. The biconditional or bi-implication between P and Q can be denoted as $P \leftrightarrow Q$.

1.1.2 Composition of Propositions

Example 1.1.2.1. Given a compound proposition $(P \wedge Q) \to (\neg Q \vee P)$

P	Q	$P \leftrightarrow Q$
0	0	1
0	1	0
1	0	0
1	1	1

Table 1.6: Truth Table of $P \leftrightarrow Q$

P	Q	$(P \wedge Q)$	$\neg Q$	$(\neg Q \vee P)$	$(P \wedge Q) \to (\neg Q \vee P)$

Table 1.7: Truth Table of $P \leftrightarrow Q$

Bibliography

[Kan19] Max Kanovich. Discrete mathematics lecture notes. http://www.cs.ucl.ac.uk/people/M.Kanovich.html/, 2019. Accessed: January 29, 2019.

[Ros12] Kenneth H. Rosen. *Discrete Mathematics and its Applications*. The McGraw-Hill Companies, Inc, seventh edition, 2012.