COMP0147 Discrete Mathematics for Computer Scientists Notes

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Notes adapted from lecture notes by Max Kanovich and Robin Hirsch [1].

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1 Foundations

Contents

1.1 Set Theory and Functions

1.1.1 Set Notations

- Set definition: $A = \{a, b, c\}$
- Set membership (element-of): $a \in A$
- Set builder notation: $\{x \mid x \in \mathbb{R} \wedge x^2 = x\}$
- Empty set: $\emptyset$

1.1.2 Properties

- No structure
- No order
- No copies

For example, a, b, c are references to actual objects in

$$\{a, b, c\} \Leftrightarrow \{c, a, b\} \Leftrightarrow \{a, b, c, b\}$$

1.1.3 Set Equality

Definition 1.1.1 (Set Equality). Set $A = B$ iff:

1. $A \subseteq B \implies \forall\, x(x \in A \to x \in B)$
2. $B \subseteq A \implies \forall\, y(y \in B \to y \in A)$

Remark. $A = B \Leftrightarrow A \subseteq B \wedge B \subseteq A$

1.1.4 Set Operations

- *Union*: $A \cup B \equiv \{x \mid x \in A \vee x \in B\}$
- *Intersection*: $A \cap B \equiv \{x \mid x \in A \wedge x \in B\}$
- *Relative Complement*: $A \setminus B \equiv \{x \mid x \in A \wedge x \notin B\}$
- *Absolute Complement*: $A^c \equiv U \setminus A \equiv \{x \mid x \in U \wedge x \notin A\}$
- *Symmetric Difference*: $A \Delta B \equiv (A \setminus B) \cup (B \setminus A) \equiv (A \cup B) \setminus (A \cap B)$
- *Cartesian Product*: $A \times B \equiv \{(x, y) \mid x \in A \wedge y \in B\}$

1.1.5 Boolean Algebra

Definition 1.1.2 (De Morgan's Laws).

$$\neg(p \vee q) \equiv \neg p \wedge \neg q \tag{1.1}$$
$$\neg(p \wedge q) \equiv \neg p \vee \neg q \tag{1.2}$$

Definition 1.1.3 (Idempotent Laws).

$$p \vee p \equiv p \tag{1.3}$$
$$p \wedge p \equiv p \tag{1.4}$$

Definition 1.1.4 (Commutative Laws).

$$p \vee q \equiv q \vee p \tag{1.5}$$
$$p \wedge q \equiv q \wedge p \tag{1.6}$$

Definition 1.1.5 (Associative Laws).

$$p \vee (q \vee r) \equiv (p \vee q) \vee r \tag{1.7}$$
$$p \wedge (q \wedge r) \equiv (p \wedge q) \wedge r \tag{1.8}$$

Definition 1.1.6 (Distributive Laws).

$$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r) \tag{1.9}$$
$$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r) \tag{1.10}$$

Definition 1.1.7 (Identity Laws).

$$p \vee \mathrm{F} \equiv p \tag{1.11}$$
$$p \vee \mathrm{T} \equiv \mathrm{T} \tag{1.12}$$
$$p \wedge \mathrm{T} \equiv p \tag{1.13}$$
$$p \wedge \mathrm{F} \equiv \mathrm{F} \tag{1.14}$$

Definition 1.1.8 (Absorption Laws).

$$p \lor (p \land q) \equiv p \tag{1.15}$$
$$p \land (p \lor q) \equiv p \tag{1.16}$$

Definition 1.1.9 (Implication and Negation Laws).

- *Identity*: $p \to q \equiv \neg p \lor q$
- *Counter-example*: $\neg(p \to q) \equiv p \land \neg q$
- *Equivalences*: $p \to q \to r \equiv (p \land q) \to r \equiv q\ to(p \to r)$
- *Absorbance*:
  $p \to \mathrm{T} \equiv T$
  $p \to \mathrm{F} \equiv \neg p$
  $\mathrm{T} \to p \equiv p$
  $\mathrm{F} \to p \equiv T$
- *Contrapositive*: $p \to q \equiv \neg q \to \neg p$
- *Law of Excluded Middle*:
  $p \lor \neg p \equiv \mathrm{T}$
  $p \land \neg p \equiv \mathrm{F}$
- *Double Negation*: $\neg\neg p \equiv p$
- *Reduction to Absurdity*: $\neg p \to \mathrm{F} \equiv p$

Bibliography

[1] Max Kanovich and Robin Hirsch.
"Lecture Notes on Discrete Mathematics for Computer Scientists".
URL: http://www.cs.ucl.ac.uk/1819/a4u/t2/comp0147_discrete_mathematics_for_computer_scientists/.