DEPARTEMENT STATISTIEK EN AKTUARIËLE WETENSKAP

STATISTIEK 186

TUTORIAAL 1 OPLOSSING

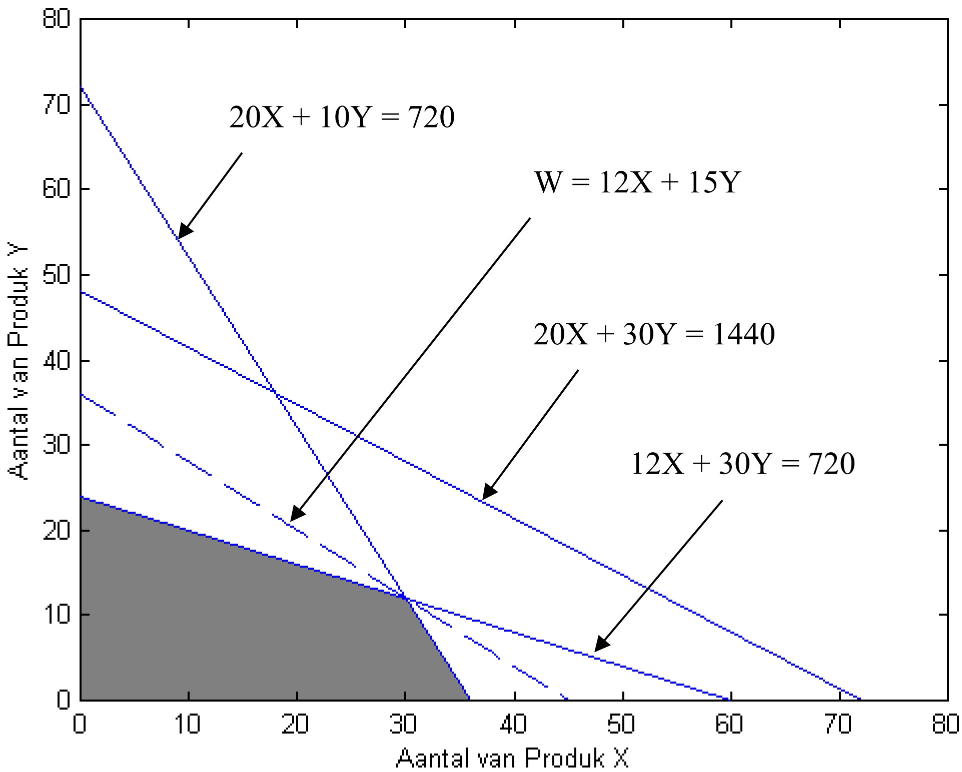
Probleem 1 / *Problem 1*

(a) Doelwitfunksie / *Objective function*: $W = 12X + 15Y$

Beperkings / *Constraints*:

$20X + 30Y \leq 1440$	(Ontwerp / *Design*)
$12X + 30Y \leq 720$	(Vervaardiging / *Manufacture*)
$20X + 10Y \leq 720$	(Verpakking / *Packaging*)
$X \geq 0, Y \geq 0$	

(b)



(c) Uit die grafiek is dit duidelik dat die optimale oplossing by die snypunt van die volgende twee lyne lê / *It is clear from the graph that the optimal solution lies at the intersection of the following two lines*:

$$20X + 10Y = 720$$
$$12X + 30Y = 720$$

Die oplossing van hierdie twee vergelykings in twee onbekendes is: $X = 30$ en $Y = 12$. Die optimale hoeveelhede om te produseer is dus 30 eenhede van produk X en 12 eenhede van produk Y. Die optimale wins is $W = 12(30) + 15(12) = \text{R}540$. / *The solution of these two equations in two unknowns is:* $X = 30$ *and* $Y = 12$. *The optimal quantities to produce are therefore 30 units of product X and 12 units of product Y. The optimal profit is* $W = 12(30) + 15(12) = \text{R}540$.

(d) Omdat die Ontwerp afdeling se beperking nie krities is nie (m.a.w. dit speel nie 'n rol by die bepaling van die toelaatbare gebied nie) is die skaduprys van hierdie afdeling 0. Die skaduppryse van die ander twee afdelings kan verkry word deur die volgende twee vergelykings op te los: / *Since the constraint of the Design department is not critical (it does not play a role in determining the feasible region) the shadow price of the Design department is 0. The shadow prices of the other two departments are found by solving the following two equations*:

$$12a + 20b = 12$$
$$30a + 10b = 15$$

Hier skryf ons "a" vir die Vervaardiging afdeling en "b" vir die Verpakking afdeling. Die oplossing van hierdie twee vergelykings gee vir ons die twee skaduppryse: / *In these equations we write "a" for the shadow price of the Manufacturing department and "b" for the shadow price of the Packaging department. The solution of these two equations gives us the required shadow prices*:

Skaduprys vir Vervaardiging / *Shadow price of Manufacturing* = *a* = R0.375.
Vir elke addisionele minuut vervaardigingstyd neem die optimale wins toe met R0.375. / *For every additional minute of manufacturing time, optimal profit will increase by* R0.375.
Skaduprys vir Verpakking / *Shadow price of Packaging* $= b =$ R0.375.
Vir elke addisionele minuut verpakkingstyd neem die optimale wins toe met R0.375. / *For every additional minute of packaging time, optimal profit will increase by* R0.375.

(e) Gestel ons maak 'n wins van a rand vir elke eenheid van X. Die doelwitfunksie word $W = aX + 15Y$, dit wil sê $Y = -\frac{a}{15}X + \frac{W}{15}$. Die twee kritiese beperkings is: / *Suppose we make a profit of* a *rand for each unit of X. The objective function becomes* $W = aX + 15Y$, *in other words* $Y = -\frac{a}{15}X + \frac{W}{15}$. *The two critical constraints are*:

$$Y = -2X + 72$$
$$Y = -\frac{2}{5}X + 24.$$

Ons bepaal nou X se bydrae, aangedui deur a hierbo, sodat / *We now determine the contribution of X, denoted by a above, from*

$$-\frac{a}{15} = -2 \quad \text{en / } \textit{and} \quad -\frac{a}{15} = -\frac{2}{5}.$$

Dit gee $a = 30$ en $a = 6$. Dus, die optimale oplossing sal verander indien X se bydrae onder R6 daal of bo R30 styg. / *This gives* $a = 30$ *and* $a = 6$. *Hence, the optimal solution will change if the contribution of X drops below R6 or rises above R30.*

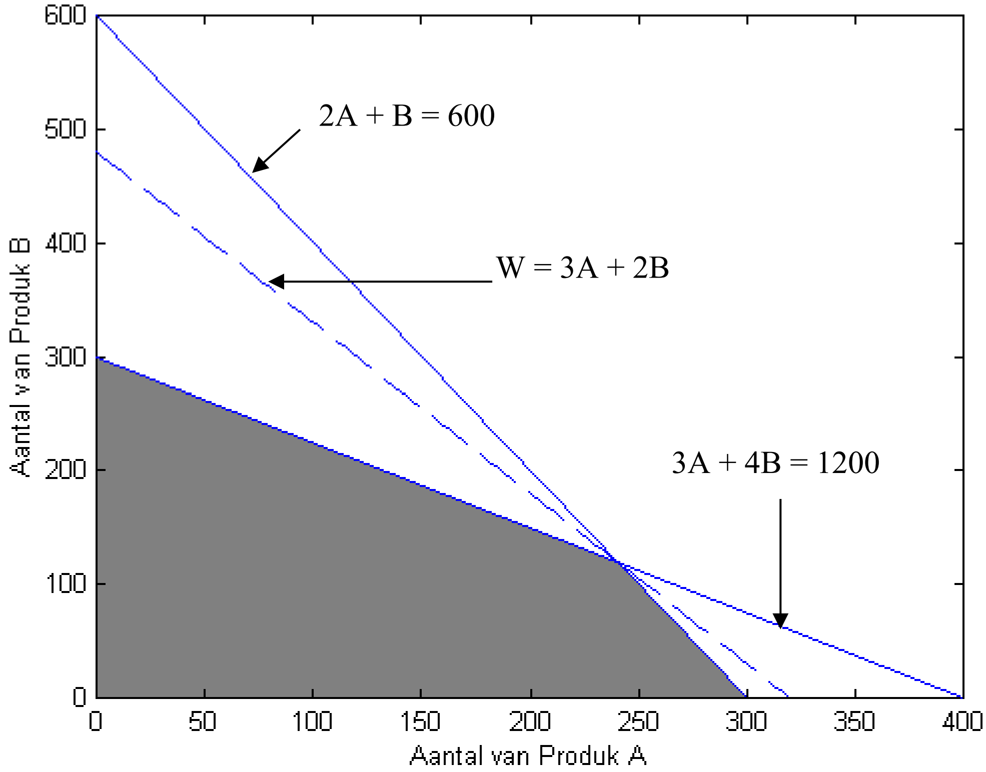
Probleem 2 / *Problem 2*

(a) Doelwitfunksie / *Objective function*: $W = 3A + 2B$

Beperkings / *Constraints*:

$3A + 4B \le 1200$ (Grondstof / *Raw material*)
$2A + B \le 600$ (Masjien-tyd / *Machine time*)
$A \ge 0,\ B \ge 0$

(b)



(c) Uit die grafiek is dit duidelik dat die optimale oplossing by die snypunt van die volgende twee lyne lê: / *It is clear from the graph that the optimal solution lies at the intersection of the following two lines*:

$$3A + 4B = 1200$$
$$2A + B = 600.$$

Die oplossing van hierdie twee vergelykings in twee onbekendes is: $A = 240$ en $B = 120$. Die optimale hoeveelhede om te produseer is dus 240 eenhede van produk A en 120 eenhede van produk B. Die optimale wins is $W = 3(240) + 2(120) = \text{R}960$. / *The solution of these two equations in two unknowns is: $A = 240$ and $B = 120$. The optimal quantities to produce are therefore 240 units of product A and 120 units of product B. The optimal profit is* $W = 3(240) + 2(120) = \text{R}960$.

(d) Ons verkry die skaduprys van grondstof deur die volgende stelsel van twee vergelykings in twee onbekendes op te los en dan die nuwe maksimum wins te bereken: / *We obtain the shadow price of raw material by solving the following system of two equations in two unknowns and calculating the new maximum profit*:

$$\begin{aligned} 3A + 4B &= 1201 \\ 2A + B &= 600 \end{aligned}$$

Die oplossing is $A = \frac{1199}{5}$, $B = \frac{602}{5}$ en die nuwe maksimum wins werk uit op R960.20. Die skaduprys van grondstof is gevolglik / *The solution is* $A = \frac{1199}{5}$, $B = \frac{602}{5}$ *and the new maximum profit turns out to be R*960.20. *The shadow price of raw material is consequently given by*

R960.20 – R960.00 = R0.20.

Dit beteken dat die optimale wins met 20 sent toeneem vir elke 1 kg toename in die beskikbare grondstof. / *We conclude that our optimal profit increases by* 20 *cents for each extra kg of raw material that we have available.*

(e) Gestel ons maak 'n wins van a rand vir elke eenheid van A. Die doelwitfunksie word $W = aA + 2B$, dit wil sê $B = -\frac{a}{2}A + \frac{W}{2}$. Die twee kritiese beperkings is: / *Suppose we make a profit of a rand for each unit of A. The objective function becomes* $W = aA + 2B$ *, in other words* $Y = -\frac{a}{2}A + \frac{W}{2}$. *The two critical constraints are*:

$$\begin{aligned} 3A + 4B &= 1200 \\ 2A + B &= 600. \end{aligned}$$

Ons bepaal nou A se bydrae, aangedui deur a hierbo, sodat / *We now determine the contribution of A, denoted by a above, from*

$$-\frac{a}{2} = -\frac{3}{4} \text{ en / } \textit{and } -\frac{a}{2} = -2.$$

Dit gee $a = 1.5$ en $a = 4$. Dus, die optimale oplossing sal verander indien A se bydrae onder R1.50 daal of bo R4 styg. / *This gives $a = 1.5$ and $a = 4$. Hence, the optimal solution will change if the contribution of A drops below R*1.50 *or rises above R*4.

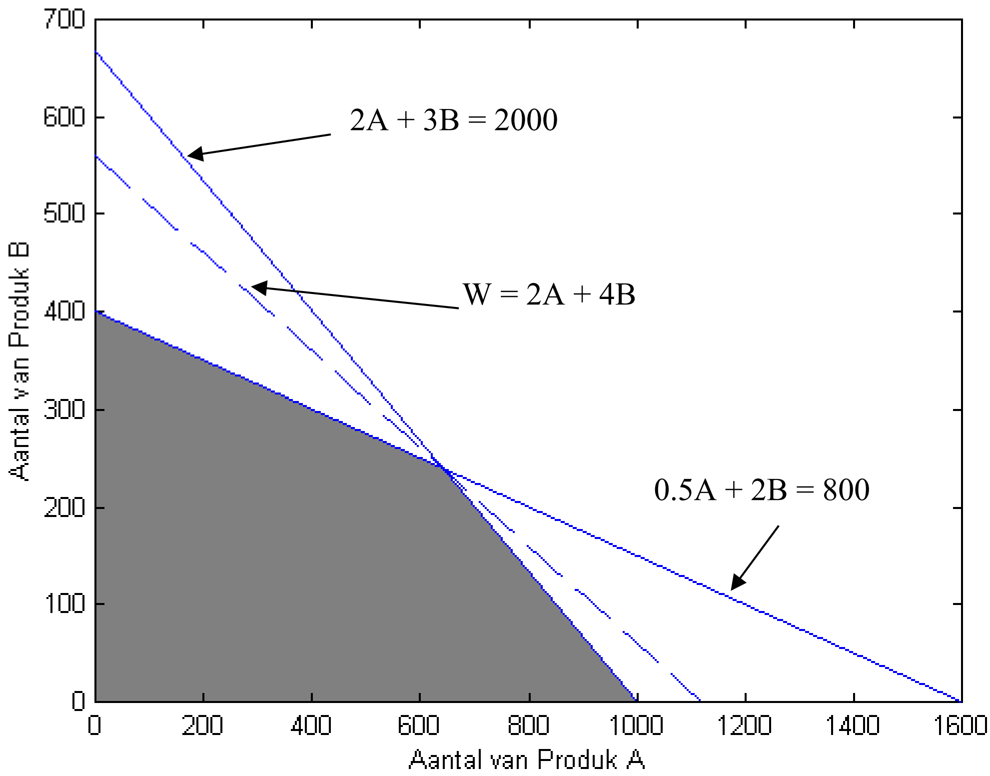
Probleem 3 / *Problem 3*

(a) Doelwitfunksie / *Objective function*: $W = 2A + 4B$

Beperkings / *Constraints*:

$2A + 3B \leq 2000$ (Grondstof / *Raw material*)
$0.5A + 2B \leq 800$ (Masjientyd / *Machine time*)
$A \geq 0, B \geq 0$

(b)



(c) Uit die grafiek is dit duidelik dat die optimale oplossing by die snypunt van die volgende twee lyne lê: / *It is clear from the graph that the optimal solution lies at the intersection of the following two lines*:

$$2A + 3B = 2000$$
$$0.5A + 2B = 800$$

Die oplossing van hierdie twee vergelykings in twee onbekendes is: $A = 640$ en $B = 240$. Die optimale hoeveelhede om te produseer is dus 640 eenhede van produk A en 240 eenhede van produk B. Die optimale inkomste is $W = 2(640) + 4(240) =$ R2 240. / *The solution of these two equations in two unknowns is: $A = 640$ and $B = 240$. The optimal quantities to produce are therefore 640 units of product A and 240 units of product B. The optimal income is $W = 2(640) + 4(240) =$ R2 240.*

(d) Die skadupryse van die twee beperkende faktore word verkry deur die volgende twee vergelykings op te los / *The shadow prices of the two limiting factors are obtained by solving the following two equations*:

$$2g + 0.5m = 2$$
$$3g + 2m = 4$$

Hier skryf ons "g" vir die skaduprys van grondstof en "m" vir die skaduprys van masjien-tyd. Die oplossing van hierdie twee vergelykings gee vir ons die twee skadupryse: / *In these equations we write "g" for the shadow price of raw material and "m" for the shadow price of machine time. The solution of these two equations gives us the required shadow prices*:

Skaduprys vir grondstof / *Shadow price for raw material* $= g =$ R0.80.
Vir elke addisionele kilogram grondstof neem die optimale wins toe met R0.80. / *For every additional kilogram of raw material, optimal profit will increase by* R0.80.
Skaduprys vir masjientyd / *Shadow price for machine time* $= m =$ R0.80.
Vir elke addisionele uur masjientyd neem die optimale wins toe met R0.80. / *For every additional hour of machine time, optimal profit will increase by* R0.80.

(e) Gestel ons verdien 'n inkomste van x rand vir elke eenheid van A. Die objekfunksie word $W = xA + 4B$, dit wil sê $B = -\frac{x}{4}A + \frac{W}{4}$. Die twee kritiese beperkings is: / *Suppose we earn an income of x rand for each unit of A. The objective function becomes $W = xA + 4B$, in other words $B = -\frac{x}{4}A + \frac{W}{4}$. The two critical constraints are*

$$B = -\frac{2}{3}A + \frac{2000}{3}$$

$$B = -\frac{1}{4}A + 400.$$

Ons bepaal nou A se bydrae, aangedui deur x hierbo, sodat / *We now determine the contribution of A, denoted by x above, from*

$$-\frac{x}{4} = -\frac{2}{3} \text{ en / } \textit{and } -\frac{x}{4} = -\frac{1}{4}.$$

Dit gee $x = 2.66$ en $x = 1$. Dus, die optimale oplossing sal verander indien A se bydrae onder R1 daal of bo R2.66 styg. Let op dat 2.666… afgerond moet word omdat 2.67 groter as 2.666… is. / *This gives $x = 2.66$ and $x = 1$. Hence, the optimal solution will change if the contribution of A drops below R*1 *or rises above R*2.66*. Remark that we must round down 2.666… to 2.66 since 2.67 is larger than 2.666….*

(f) Gestel ons vervaardig a eenhede van produk A en koop dus $1500-a$ eenhede, en ons vervaardig b eenhede van produk B en koop dus $1200-b$ eenhede. Die nuwe optimeringsprobleem is: maksimeer $W=0.75a+b$ onderhewig aan dieselfde beperkings as tevore, naamlik: / *Suppose we manufacture a units of product A and we therefore buy $1500-a$ units, and we manufacture b units of product B and therefore buy $1200-b$ units. The new optimisation problem is: maximise $W=0.75a+b$ subject to the same constraints as before, namely:*

$2a+3b \leq 2000$	(Grondstof / *Raw material*)
$0.5a+2b \leq 800$	(Masjien-tyd / *Machine time*)

Die oplossing is nou by die punt $(a=1000,\ b=0)$. Dit beteken dat ons 1000 eenhede van produk A moet vervaardig en 500 moet inkoop. Verder moet ons geen eenhede van produk B vervaardig nie, maar die volle daaglikse kwota van 1200 eenhede inkoop. Die nuwe optimale inkomste is: / *The solution now lies at the point $(a=1000,\ b=0)$. This means that our optimal policy is to manufacture 1000 units of product A and to buy 500 units of this product. We should also buy 1200 units of product B, i.e. we should not manufacture any of this type of product. The new optimal income is:*

$$W=2(1000)+1.25(500)+3(1200)=\text{R6 225}.$$

Probleem 4 / *Problem 4*

(a) Laat / *Let*

X = die aantal eenhede van Pluto / *the number of units of Pluto*
Y = die aantal eenhede van Atlantic / *the number of units of Atlantic.*

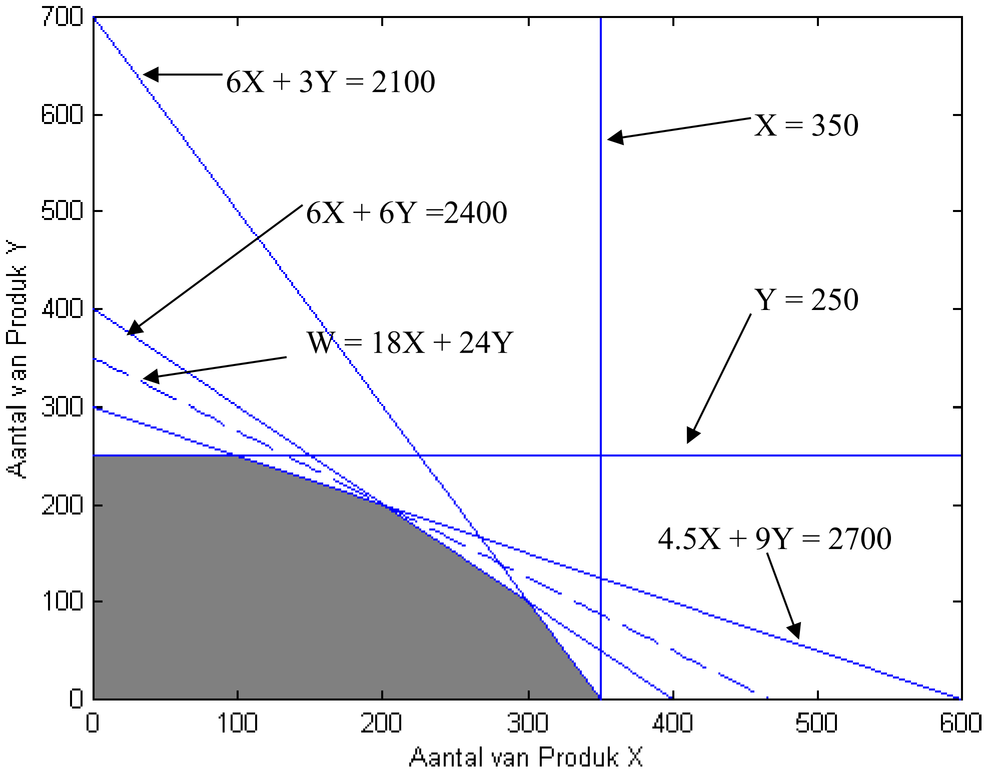
Dan is / *Then we have:*

Doelwitfunksie / *Objective function*: $W=18X+24Y$

Beperkings / *Constraints*:

$6X+3Y \leq 2100$	(Grondstof A / *Raw material A*)
$6X+6Y \leq 2400$	(Grondstof B / *Raw material B*)
$4.5X+9Y \leq 2700$	(Grondstof C / *Raw material C*)
$X \leq 350$, $Y \leq 250$	(Aanvraag / *Demand*)
$X \geq 0$, $Y \geq 0$	

(b)



(c) Uit die grafiek is dit duidelik dat die optimale oplossing by die snypunt van die volgende twee lyne lê: / *It is clear from the graph that the optimal solution lies at the intersection of the following two lines*:

$$6X + 6Y = 2400$$
$$4.5X + 9Y = 2700.$$

Die oplossing van hierdie twee vergelykings in twee onbekendes is: $X = 200$ en $Y = 200$. Die optimale hoeveelhede om te produseer is dus 200 eenhede van produk Pluto en 200 eenhede van produk Atlantic. Die optimale wins is $W = 18(200) + 24(200) =$ R8 400. / *The solution of these two equations in two unknowns is:* $X = 200$ *and* $Y = 200$. *The optimal quantities to produce are therefore 200 units of product Pluto and 200 units of product Atlantic. The optimal profit is* $W = 18(200) + 24(200) =$ R8400.

(d) Los op / *Solve*:

$$6X + 6Y = 2401$$
$$4.5X + 9Y = 2700.$$

Dit gee / *This gives*: $X = \frac{1803}{9}$, $Y = \frac{3597}{18}$. Die nuwe optimale wins is dus R8402 / *The new optimal profit is therefore R8402*.

Dus die skaduprys vir grondstof B = R2. / *Therefore the shadow price for raw material B* = R2.

Vir elke addisionele kilogram van Grondstof B neem die optimale wins toe met R2. / *For every additional kilogram of Raw Material B, optimal profit will increase by* R2.

(e) Gestel ons verdien ’n inkomste van x rand vir elke eenheid van Pluto. Die doelwitfunksie word $W = xX + 24Y$, dit wil sê $Y = -\frac{x}{24}X + \frac{W}{24}$. Die twee kritiese beperkings is: / *Suppose we earn an income of x rand for each unit of Pluto. The objective function becomes $W = xX + 24Y$, in other words $Y = -\frac{x}{24}X + \frac{W}{24}$. The two critical constraints are:*

$$Y = -X + \frac{2400}{6}$$

$$Y = -\frac{1}{2}X + \frac{2700}{9}.$$

Ons bepaal nou Pluto se bydrae, aangedui deur x hierbo, sodat / *We now determine the contribution of Pluto, denoted by x above, from*

$$-\frac{x}{24} = -1 \text{ en / } \textit{and } -\frac{x}{24} = -\frac{1}{2}.$$

Dit gee $x = 24$ en $x = 12$. Dus, die optimale oplossing sal verander indien Pluto se bydrae onder R12 daal of bo R24 styg. / *This gives $x = 24$ and $x = 12$. Hence, the optimal solution will change if the contribution of Pluto drops below R12 or rises above R24.*