TABLE 10.2
Quemo Chemical Company Information

PROJECT	NET PRESENT VALUE	YEAR 1	YEAR 2
Catalytic converter	\$25,000	\$8,000	\$7,000
Software	\$18,000	\$6,000	\$4,000
Warehouse expansion	\$32,000	\$12,000	\$8,000
Available funds		\$20,000	\$16,000

Capital Budgeting Example

A common capital budgeting decision involves selecting from a set of possible projects when budget limitations make it impossible to select all of these. A separate 0–1 variable can be defined for each project. We will see this in the following example.

Quemo Chemical Company is considering three possible improvement projects for its plant: a new catalytic converter, a new software program for controlling operations, and an expansion of the warehouse used for storage. Capital requirements and budget limitations in the next 2 years prevent the firm from undertaking all of these at this time. The net present value (the future value discounted back to the present time) of each project, the capital requirements for each project, and the available funds for the next 2 years are given in Table 10.2.

To formulate this as an integer programming problem, we identify the objective function and the constraints as follows:

$$\begin{aligned}&\text{Maximize net present value of projects undertaken}\\ &\text{subject to}\qquad \text{Total funds used in year 1} \le \$20{,}000\\ &\qquad\qquad\qquad\ \ \text{Total funds used in year 2} \le \$16{,}000\end{aligned}$$

We define the decision variables as

$$X_1 = \begin{cases} 1 \text{ if catalytic converter project is funded} \\ 0 \text{ otherwise} \end{cases}$$

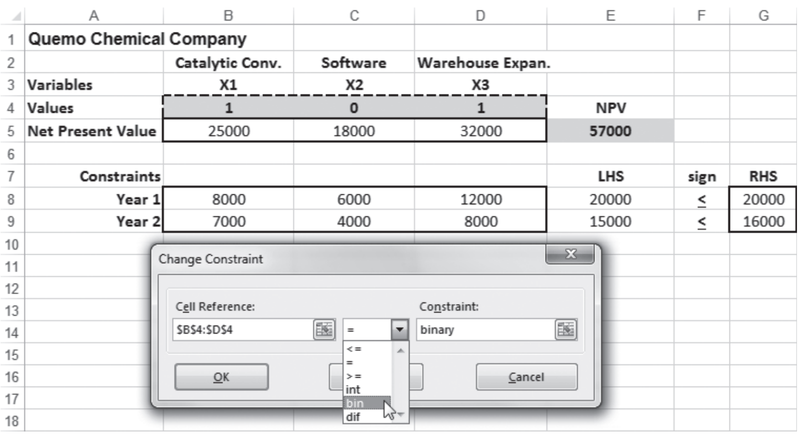
$$X_2 = \begin{cases} 1 \text{ if software project is funded} \\ 0 \text{ otherwise} \end{cases}$$

$$X_3 = \begin{cases} 1 \text{ if warehouse expansion project is funded} \\ 0 \text{ otherwise} \end{cases}$$

The mathematical statement of the integer programming problem becomes

$$\begin{aligned}\text{Maximize NPV} &= 25{,}000X_1 + 18{,}000X_2 + 32{,}000X_3\\ \text{subject to}\qquad & 8{,}000X_1 + 6{,}000X_2 + 12{,}000X_3 \le 20{,}000\\ & 7{,}000X_1 + 4{,}000X_2 + 8{,}000X_3 \le 16{,}000\\ & X_1, X_2, X_3 = 0 \text{ or } 1\end{aligned}$$

PROGRAM 10.5
Excel 2016 Solver Solution for Quemo Chemical Problem



Solver Parameter Inputs and Selections

Set Objective: E5

By Changing cells: B4:D4

To: Max

Subject to the Constraints:

E8:E9 <= G8:G9

B4:D4 = binary

Solving Method: Simplex LP

☑ **Make Variables Non-Negative**

Key Formulas

	E
5	=SUMPRODUCT($B$4:$D$4,B5:D5)

Copy E5 to E8:E9

Program 10.5 provides the Solver solution in Excel 2016. You specify the variables to be binary (0–1) by selecting *bin* from the Change Constraint window. The optimal solution is $X_1 = 1$, $X_2 = 0, X_3 = 1$, with an objective function value of 57,000. This means that Quemo should fund the catalytic converter project and the warehouse expansion project but not the new software project. The net present value of these investments will be $57,000.

Limiting the Number of Alternatives Selected

One common use of 0–1 variables involves limiting the number of projects or items that are selected from a group. Suppose that in the Quemo Chemical Company example, the company is required to select no more than two of the three projects *regardless* of the funds available. This could be modeled by adding the following constraint to the problem:

$$X_1 + X_2 + X_3 \leq 2$$

If we wished to force the selection of *exactly* two of the three projects for funding, the following constraint should be used:

$$X_1 + X_2 + X_3 = 2$$

This forces exactly two of the variables to have values of 1, whereas the other variable must have a value of 0.

Dependent Selections

At times, the selection of one project depends in some way on the selection of another project. This situation can be modeled with the use of 0–1 variables. Now suppose in the Quemo Chemical problem that the new catalytic converter can be purchased only if the software is also purchased. The following constraint would force this to occur:

$$X_1 \leq X_2$$

or, equivalently,

$$X_1 - X_2 \leq 0$$

Thus, if the software is not purchased, the value of X_2 is 0, and the value of X_1 must also be 0 because of this constraint. However, if the software is purchased $(X_2 = 1)$, then it is possible that the catalytic converter could also be purchased $(X_1 = 1)$, although this is not required.

If we wished for the catalytic converter and the software projects to either both be selected or both not be selected, we should use the following constraint:

$$X_1 = X_2$$

or, equivalently,

$$X_1 - X_2 = 0$$

Thus, if either of these variables is equal to 0, the other must also be 0. If either of these is equal to 1, the other must also be 1.