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Simulation Part 2 / Simulasie Deel 2

LSCM344 - August 2022
Lecturer - HW Freiboth

Slides adapted from those originally compiled for the module by Dr Neil Jacobs (2019)

Kansgetal generering / Generating random numbers

Diskrete verdeling voorbeeld / Discrete distribution example

Gebruik Tabel 13.4 op bladsy 485 van Render en Stair, om 'n reeks van 20 waardes te simuleer van die aantal drywers wat op 'n dag afwesig is van die werk. Die aantal afwesig volg die diskrete verdeling in die onderstaande tabel.

Use Table 13.4 on page 485 of Render and Stair to simulate a series of 20 values of the number of drivers who are absent from work on a day. The number absent follows the discrete distribution in the table below.

Aantal afwesig Number absent	Waarskynlikheid Probability
0	0.5
1	0.25
2	0.1
3	0.1
4	0.05

Kansgetal generators / Random number generators

Simulasie verg kansgetalle wat $U[0,1]$-verdeel is.

Tabelle soos wat in die vorige voorbeeld gebruik is, is van beperkte nut wanneer groot hoeveelhede kansgetalle gegenereer moet word. Daarom bestaan daar 'n behoefte aan 'n metode wat kansgetalle kan genereer wat

1. rekenaarmatig vinnig is,
2. min rekenaar geheue verg,
3. identies nadoenbaar is,
4. genoegsaam verspreid is en
5. 'n lang siklus het.

Simulation requires random numbers which are $U[0,1]$ distributed.

Tables as used in the previous example is of limited use when large numbers of random numbers are to be generated. Therefore a need exist for a method which can generate random numbers that are

1. computationally fast,
2. requires little computer memory,
3. is sufficiently spread out,
4. is identically replicable and
5. has a long cycle.

Lineêr kongruensiële kansgetal generator / Linear congruential random number generator

Die Lineêr Kongruensiële Kansgetal Generator-metode, of LKG, is 'n metode wat, **as dit korrek toegepas word**, aan die vereistes voldoen.

Linear Congruential Random Number Generator-method, or LCG, is a method which, **if it is applied correctly**, fulfills the requirements.

Elke kansgetal R_i, $i \leq n$, in die kansgetal reeks van n kansgetalle, word bereken met die formule:

Every random number R_i, $i \leq n$, in the random number series of n random numbers , are calculated with the formula:

$$R_i = \frac{x_i}{m} \tag{5}$$

waar / where

$$\tag{6}$$

$$x_i = ax_{i-1} + c \mod m. \tag{7}$$

Lineêr kongruensiële kansgetal generator / Linear congruential random number generator

Komponente van

Components of

$$x_i = ax_{i-1} + c \mod m. \tag{8}$$

word soos volg genoem

- a is die konstante vermenigvuldiger,
- c is die inkrement,
- m is die modulus,
- x_0 is die saadgetal.

are called the following

- a is the constant multiplier,
- c is the increment,
- m is the modulus,
- x_0 is seed number.

Goeie parameters vir die LKG / Good parameters for the LCG

$$a = 7^5 \tag{9}$$

$$m = 2^{31} - 1 \tag{10}$$

dus / thus

$$\tag{11}$$

$$x_i = 7^5 x_{i-1} + c \mod (2^{31} - 1). \tag{12}$$

werk goed, met x_0 en c deur die modelleerder gespesifiseer.

works well, with x_0 and c specified by the modeller.

Kontinue waarskynlikheidsverdelings van belang / Continuous probability distributions of importance

Daar is drie kontinue waarskynlikheidsverdelings van belang in hierdie kursus. Hoewel die drie baie gebruik word in die praktyk is daar baie meer waarskynlikheidsverdelings om van te kies in modellering. Die drie is

- die uniforme verdeling,
- die eksponensiële verdeling en
- die normaal verdeling.

There are three continuous probability distributions of importance in this course. Although these three are used a lot in practice there are many more probability distributions to choose from in practice. The three are

- the uniform distribution,
- the exponential distribution and
- the normal distribution.

Uniforme verdeling / Uniform distribution

$$t \sim \mathrm{Uniform}\,(a, b)$$

Waarskynlikheidsdigtheidsfunksie: Probability density function:

$$f(t) = \begin{cases} 0 & t < a, t > b \\ \frac{1}{b-a} & a \leq t \leq b \end{cases}$$

Kumulatiewe waarskynlikheidsdigtheidfunksie: Cumulative probability density function:

$$F(t) = \begin{cases} 0 & t < a \\ \frac{x-a}{b-a} & a \leq t \leq b \\ 1 & t > b \end{cases}$$

Inverse transform: Inverse transform:

$$t = a + (b - a)R_i$$

Eksponensiële verdeling / Exponential distribution

$$t \sim \mathrm{Expon}\left(\lambda\right)$$

Waarskynlikheidsdigtheidsfunksie: Probability density function:

$$f(t) = \begin{cases} 0 & t < 0 \\ \lambda \mathrm{e}^{-\lambda x} & t \geq 0, \lambda > 0 \end{cases}$$

Kumulatiewe waarskynlikheidsdigtheidfunksie: Cumulative probability density function:

$$F(t) = \begin{cases} 0 & t < 0 \\ 1 - \mathrm{e}^{-\lambda t} & t \geq 0, \lambda > 0 \end{cases}$$

Inverse transform: Inverse transform:

$$t_i = -\frac{1}{\lambda} \ln\left(1 - R_i\right)$$

Normaal verdeling / Normal distribution

$$t \sim \mathrm{Normal}[\mu, \sigma]$$

Waarskynlikheidsdigtheidsfunksie: Probability density function:

$$f(t) = \mathrm{e}^{\frac{-(t-\mu)^2}{2\sigma^2}}$$

Gebruik die Excel funksie "=norm.inv(R_i, μ, σ)" om kansveranderlike uitkomste te verkry.

Use the Excel function "=norm.inv(R_i, μ, σ)" to find random variable outcomes.