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Queueing Theory Part 2 / Toustaanteorie Deel 2

LSCM344 - July 2022
Lecturer - HW Freiboth

Slides adapted from those originally compiled
for the module by Dr Neil Jacobs (2019)

Kendall-Lee notasie / Kendall-Lee notation

$M/M/1/GD/\infty/\infty$-tou
ⓐ/ⓑ/ⓒ/ⓓ/ⓔ/ⓕ -tou

- ⓐ Aard van die tussenaankomstye
- ⓑ Aard van die dienstye
- ⓒ Aantal dienspunte
- ⓓ Tou-reëls
- ⓔ Aantal klïente toelaatbaar in die stelsel
- ⓕ Aantal kliënte in die bediende populasie

$M/M/1/GD/\infty/\infty$-queue
ⓐ/ⓑ/ⓒ/ⓓ/ⓔ/ⓕ -queue

- ⓐ Nature of the interarrival times
- ⓑ Nature of the service times
- ⓒ Number of service points
- ⓓ Queuing rules
- ⓔ Number of customers allowable in the system
- ⓕ Number of customers in the calling population

Kendall-Lee notation

Posisies en betekenis van geassosieerde simbole

ⓐ M - tussenaankomstye volg die eksponensiële verdeling.

ⓑ
- M - dienstye volg die eksponensiële verdeling
- D - dienstye is deterministies

ⓒ
- "1" - 1 bediener of dienspunt in die stelsel.
- "m" - m-aantal bedieners in die stelsel.

ⓓ GD - "Eerste in eerste uit"-tou-dissipline.

ⓔ
- "N" - Ruimte vir N-aantal kliënte in die stelsel
- "∞" - Geen ruimte beperkings oftewel ruimte vir 'n oneindige aantal (∞) kliënte

ⓔ
- "N" - Grootte van die besoekende populasie (wat bekend moet wees) is N-aantal kliënte
- "∞" - Oneindige aantal kliënte

Kendall-Lee notation

Positions and meaning of associated symbols

ⓐ M - interrival times are exponential distributed

ⓑ
- M - service times are exponential distributed
- D - service times are deterministic

ⓒ
- "1" - 1 server or service point in the system
- "m" - m-number of service points in the system

ⓓ GD - First in first out queue discipline

ⓔ
- "N" - Space for N-number of customers in the system
- "∞" - No space limitations or space for an infinite (∞) number of customers

ⓔ
- "N" - Size of the calling population (must be known) is N number of customers
- "∞" - Unlimited number of customers.

Weer Blitz / Blitz again

Mnr Blitz weet daar kom gemiddeld 50 aflewerings voertuie/uur aan om pakette af te haal en sy laaipunt kan 54 aflewerings- voertuie/uur help. Nou wil hy die volgende gemiddeldes weet aangaande aantal voertuie :

1. Voertuie wat wag om bedien te word.
2. Voertuie by die laaipunt.
3. Totaal in die perseel.

Mr Blitz knows that 50 delivery vehicles arrive/hour to pick up parcels and his loading point can handle 54 delivery vehicles/hour. Now he wants to know the following regarding the number of vehicles :

1. Vehicles waiting to be served.
2. Vehicles at the loading point.
3. Total on the premises.

Weer Blitz / Blitz again

Hy wil ook die volgende weet aangaande die gemiddelde tyd wat voertuie in die perseel spandeer

4. Tyd wat 'n voertuig wag om bedien te word.
5. Tyd wat 'n voertuig by die laaipunt staan.
6. Tyd wat 'n voertuig in die perseel spandeer.

He also wants to know the following about the average time vehicles spends at his premises

4. Time a vehicle waits to be serviced.
5. Time a vehicle stands at the loading point.
6. Time a vehicle spends inside the premises.

Weer Blitz / Blitz again

Hy stel ook in die volgende belang

7. Benutting van sy laaipunt.
8. Waarskynlikheid dat die laaipunt ledig is.
9. Waarskynlikheid dat daar meer as 8 afleweringsvoertuie in sy perseel is.

Onder sekere toestande kan toustaanteorie Mnr Blitz se vrae volledig beantwoord.

He is also interested in the following

7. Utilization of his loading point.
8. Probability that the loading point is idle.
9. Probability that there are more than 8 vehicles on his premises.

Under certain conditions queueing theory can answer Mr Blitz's questions completely.

$M/M/1/GD/\infty/\infty$- toustaanstelsel aannames / queueing system assumptions

1. Kliënte in die tou volg die algemene tou-dissipline (GD) wat 'n eerste in eerste bedien logika is.
2. Kliënte draai nie om voordat hulle in die tou inval nie.
3. Kliënte bly in die tou tot hulle diens ontvang, daar is met ander woorde geen uitval uit die tou nie.
4. Tussenaankomstye is onderling onafhanklik en volg die identiese eksponensiële verdeling met aankomstempo (λ) bekend.

1. Clients in the queue follow the general queueing discipline (GD) which is a first come first serve logic.
2. Clients do not turn around before joining the queue, thus no balking takes place
3. Clients remain in the queue until they receive service, thus no reneging takes place.
4. Interarrival times are mutually independent and follow the identical exponential distribution with arrival rate (λ) known.

$M/M/1/GD/\infty/\infty$- toustaanstelsel / queueing system

Aannames vervolg

5. Dienstye is onderling onafhanklik en volg die identiese eksponensiële verdeling met dienstempo (μ) bekend.
6. Die besoekende populasie is oneindig groot.
7. Daar is geen ruimtebeperking in die stelsel nie.
8. Die aankomstempo is kleiner as die dienstempo, met ander woorde $\lambda < \mu \equiv \frac{\lambda}{\mu} < 1$.

Assumptions continued

5. Service times are mutually independent and follow the identical exponential distribution with service rate μ known.
6. The calling population is infinitely large.
7. There is no space constraint in the system
8. The arrival rate is less than the service rate, in other words $\lambda < \mu \equiv \frac{\lambda}{\mu} < 1$.

Blitz se antwoorde / Blitzes answers

Mnr Blitz weet daar kom gemiddeld 50 aflewerings voertuie/uur aan om pakette af te haal en sy laaipunt kan 54 aflewerings- voertuie/uur help. Nou wil hy die volgende gemiddeldes weet aangaande aantal voertuie :

1. Voertuie wat wag om bedien te word. L_q
2. Voertuie by die laaipunt. L_s
3. Totaal in die perseel. $L = L_s + L_q$.

Mr Blitz knows that 50 delivery vehicles arrive/hour to pick up parcels and his loading point can handle 54 delivery vehicles/hour. Now he wants to know the following regarding the number of vehicles :

1. Vehicles waiting to be served. L_q
2. Vehicles at the loading point. L_s
3. Total on the premises. $L = L_s + L_q$.

Weer Blitz / Blitz again

Hy wil ook die volgende weet aangaande die gemiddelde tyd wat voertuie in die perseel spandeer

4. Tyd wat 'n voertuig wag om bedien te word. W_q
5. Tyd wat 'n voertuig by die laaipunt staan. W_s
6. Tyd wat 'n voertuig in die perseel spandeer. $W = W_q + W_s$

He also wants to know the following about the average time vehicles spends at his premises

4. Time a vehicle waits to be serviced. W_q
5. Time a vehicle stands at the loading point. W_s
6. Time a vehicle spends inside the premises. $W = W_q + W_s$

Weer Blitz / Blitz again

Hy stel ook in die volgende belang

7. Benutting van sy laaipunt. ρ
8. Waarskynlikheid dat die laaipunt ledig is. P_0
9. Waarskynlikheid dat daar meer as 8 afleweringsvoertuie in sy perseel is. $P_{n>8}$

Onder sekere toestande kan toustaanteorie Mnr Blitz se vrae volledig beantwoord.

He is also interested in the following

7. Utilization of his loading point. ρ
8. Probability that the loading point is idle. P_0
9. Probability that there are more than 8 vehicles on his premises. $P_{n>8}$

Under certain conditions queueing theory can answer Mr Blitz's questions completely.

Formulas

$$L_q = \frac{\lambda^2}{\mu(\mu - \lambda)} \tag{10}$$

$$L_s = \frac{\lambda}{\mu} \tag{11}$$

$$L = \frac{\lambda}{\mu - \lambda} \tag{12}$$

$$W_q = \frac{\lambda}{\mu(\mu - \lambda)} \tag{13}$$

$$W_s = \frac{1}{\mu} \tag{14}$$

$$W = \frac{1}{\mu - \lambda} \tag{15}$$

Formulas

$$\rho = \frac{\lambda}{\mu} \tag{16}$$

$$P_0 = 1 - \frac{\lambda}{\mu} \tag{17}$$

$$P_{n>k} = \left(\frac{\lambda}{\mu}\right)^{k+1} \tag{18}$$