Linear Programming

STATISTICS 186

Part 2

Last week?

- Linear Programming! (You guessed it)
- Steps
- Going to tackle another example

Introduction

- Interested in optimizing objective functions
- Can involve maximizing OR minimizing
- From business point of view
  - **<u>Maximum</u>** production
  - **<u>Minimum</u>** cost
- Next example will deal with minimizing cost

Example 2

- 2 Brands of Turkey Feed (Product A, Product B)?
- Brand 1
  - 150g ingredient A
  - 120g ingredient B
  - 15g ingredient C
- Brand 2
  - 300g ingredient A
  - 90g ingredient B
  - 0g ingredient C
- Minimum monthly intakes per turkey (notice its already in correct amounts per unit)
  - 2700 g ingredient A
  - 1440g ingredient B
  - 45g ingredient C

Example 2

- Brand 1 costs R2 per kg
- Brand 2 costs R3 per kg
- Need **lowest** cost diet

Step 1 – State objective function

- Want to minimize cost
- What is cost of feeds?

$$K = 2V_1 + 3V_2$$

- Want the above minimized

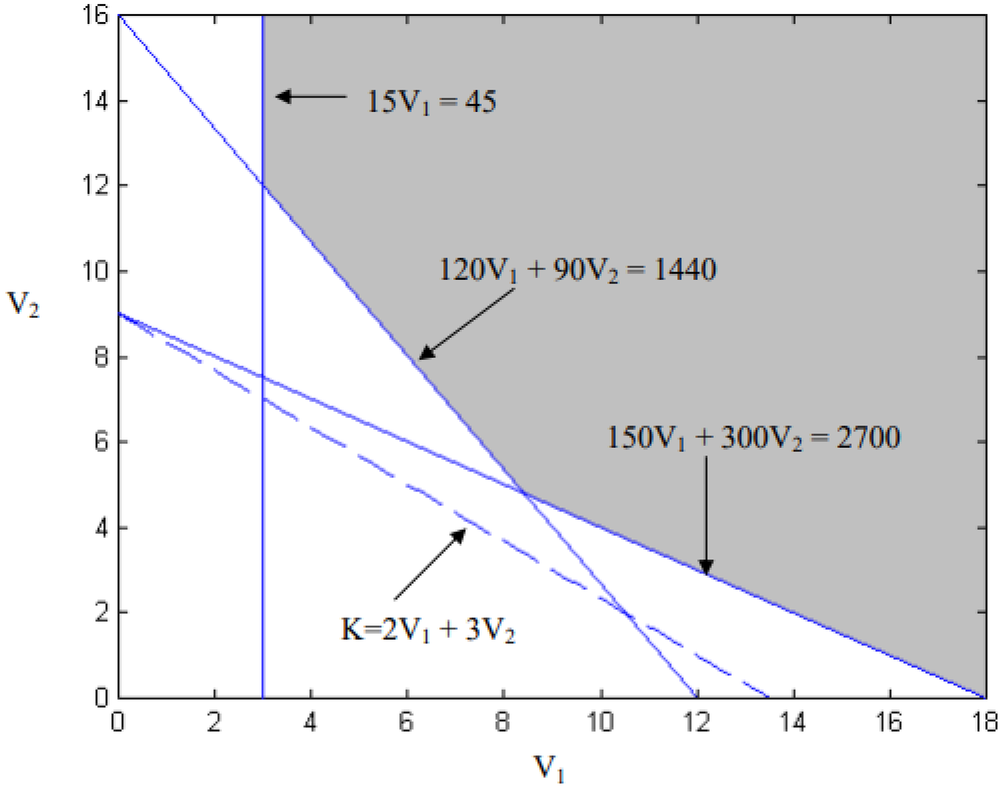
Step 2 – Determine Constraints

- Remember there was a limit to monthly intake of ingredients
- Constraint!

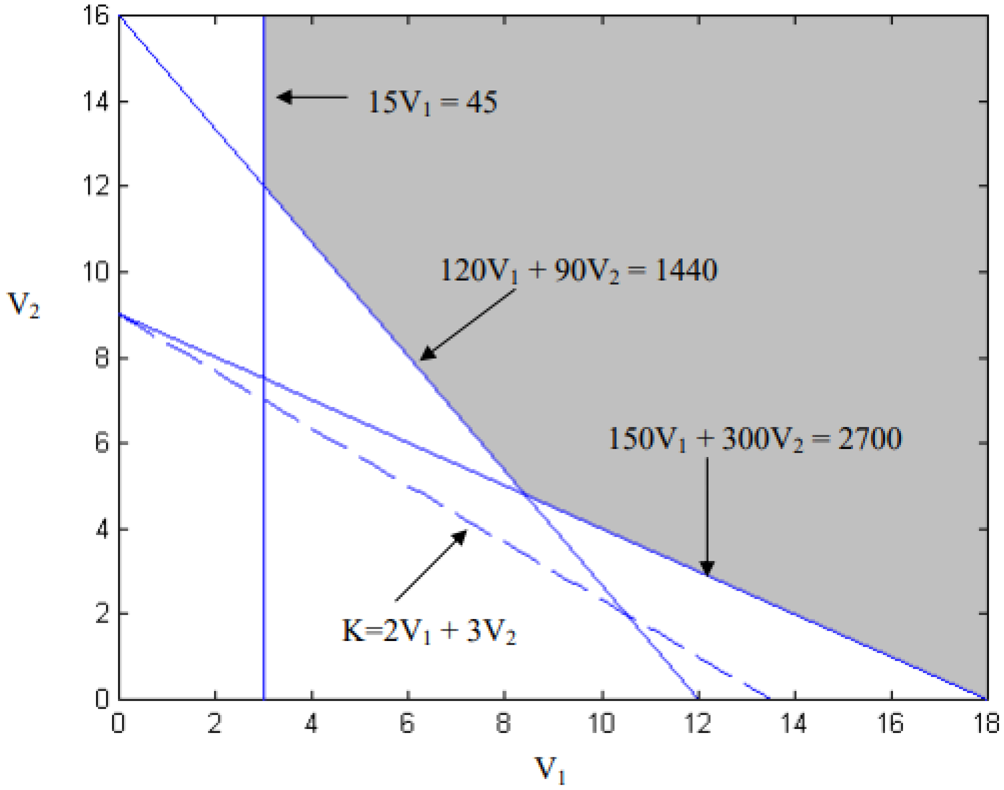
$$150V_1 + 300V_2 \geq 2700\ (A)$$
$$120V_1 + 90V_2 \geq 1440\ (B)$$
$$15V_1 \geq 45\ (C)$$

- Work out intercepts and draw lines
  - Take note of the greater than equal sign to shade above region

Step 3 – Represent constraints graphically



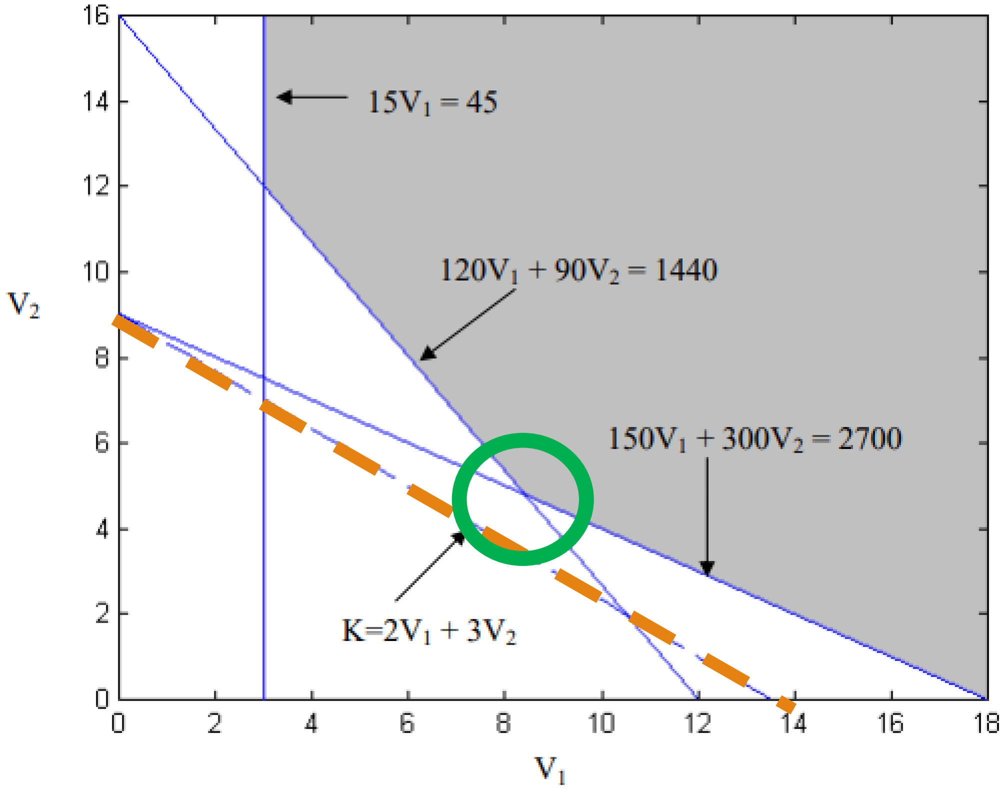
Step 4 – Represent Objective Function Graphically



- Start by choosing random value for K
- Choose one in such a way that the y-intercept is in the middle of the axis (Try 9) and set $V_1 = 0$
  - Results in $K = 27$
- Hence objective function equation

$$2V_1 + 3V_2 = 27$$

Step 4 – Represent Objective Function Graphically



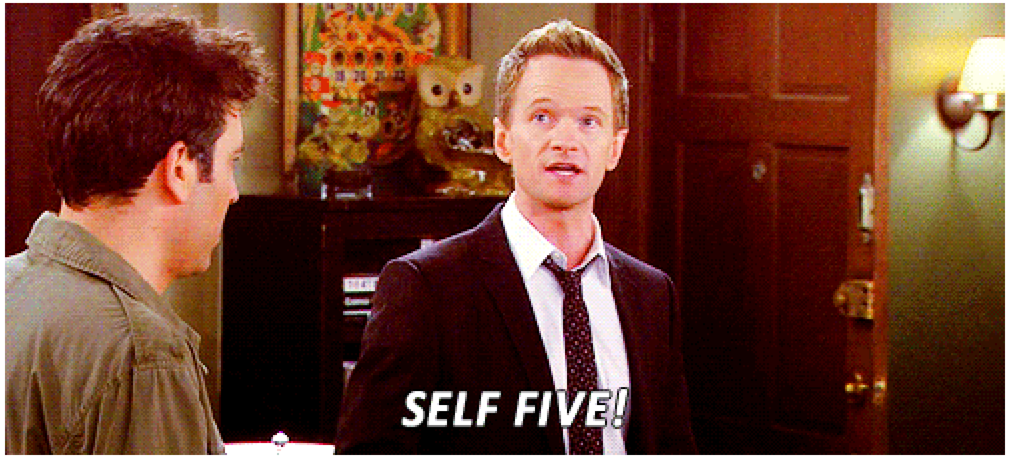
- Now want to minimize cost
- So move intercept until a line touches the feasible area from the bottom
- See it is point where lines
- $150V_1 + 300V_2 = 2700$ and
- $120V_1 + 90V_2 = 1440$ intersect

Step 5 - Calculate optimal solution

- Solve simultaneous equations obtain
  - $V_1 = 8.4$
  - $V_2 = 4.8$
- Amount of feed and not number of units produced so decimals are accepted
- Cheapest mix

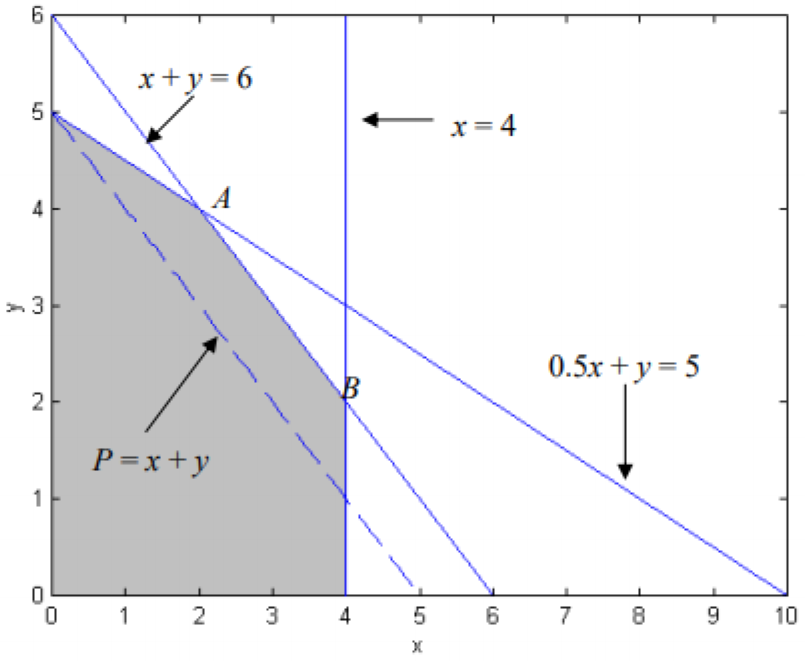
$$K = 2(8.4) + 3(4.8) = R31.20$$

We did it!

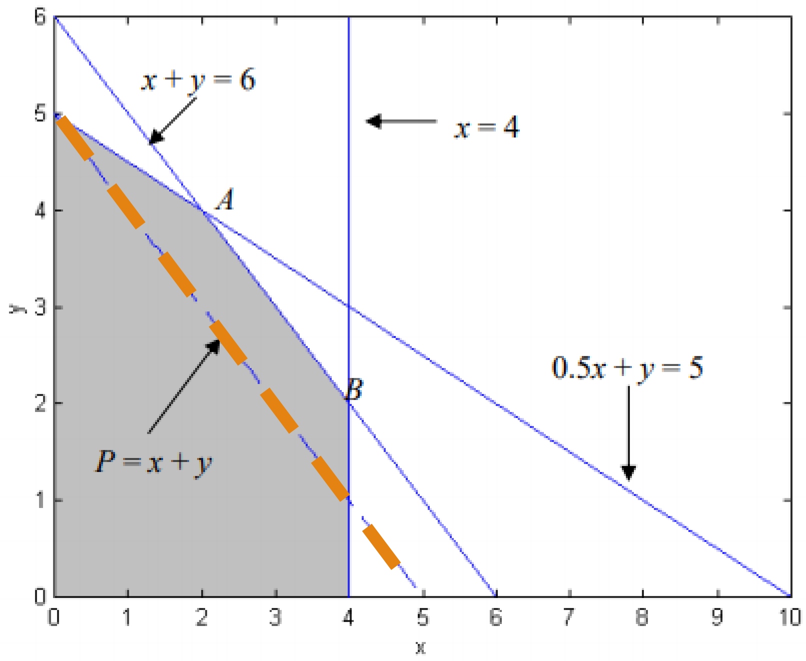


Another Example

- This example shows that optimal solution does not need to be unique



Step 5 – Graphically represent objective function



- If we had to draw objective function
- Will see that point(s) that it touches while still being in the feasible region forms a line
- Hence any point on that AB line will be an optimal solution
- Special case where objective function is parallel to one of the constraint lines

Other Methods

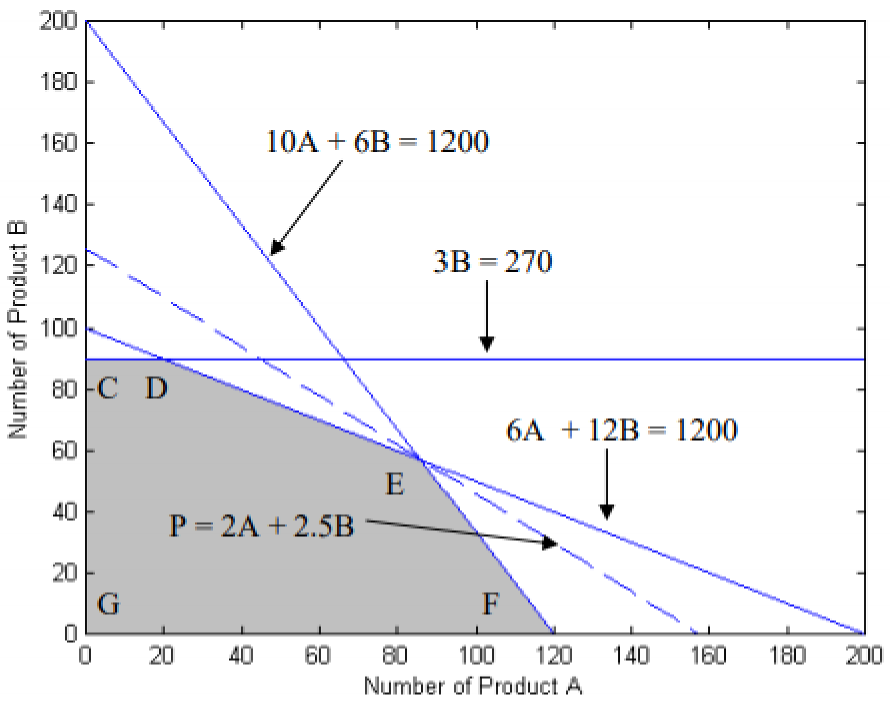
- So far only looked at geometrical method
- Also another method called Extreme point method
  - Must take note that in test or exam either method can be asked so know both!

Extreme Point Method

- Uses the fact that optimal solution will always be at extreme point
- Does not require Step 4 (Plotting objective function) but does require more calculations
- Let’s look at example 1

Extreme Point – Example 1

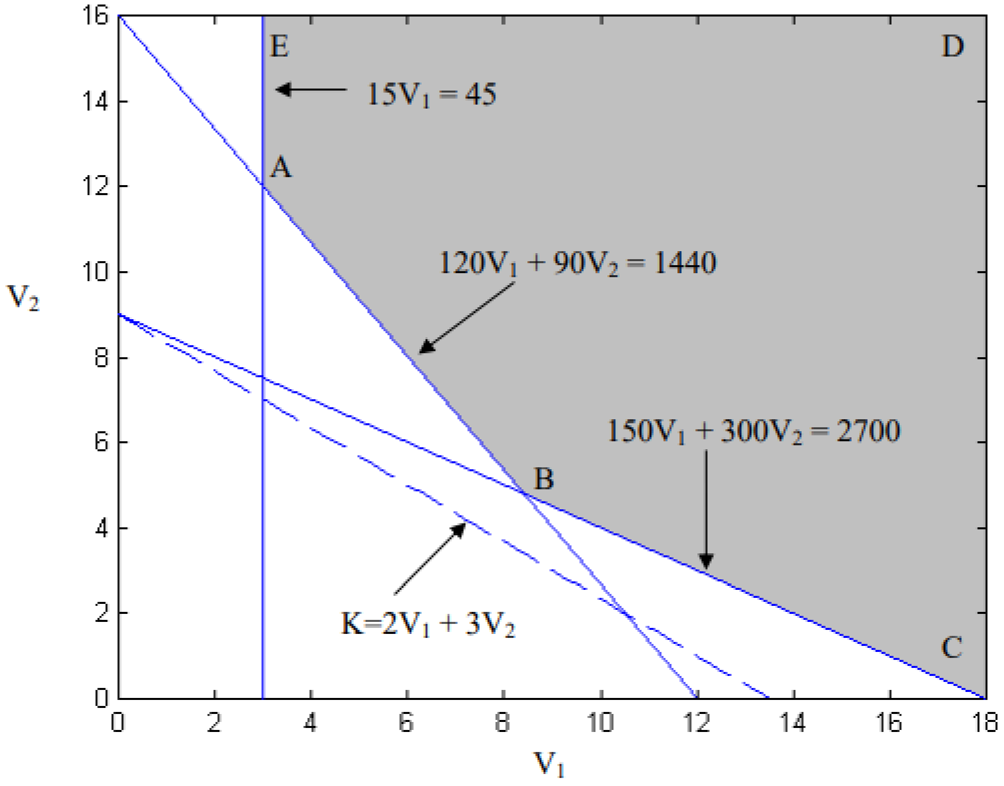
Denoting the extreme points by C through G:



- Extreme points can be viewed as the "corners" of the feasible region
- Calculate P for all points

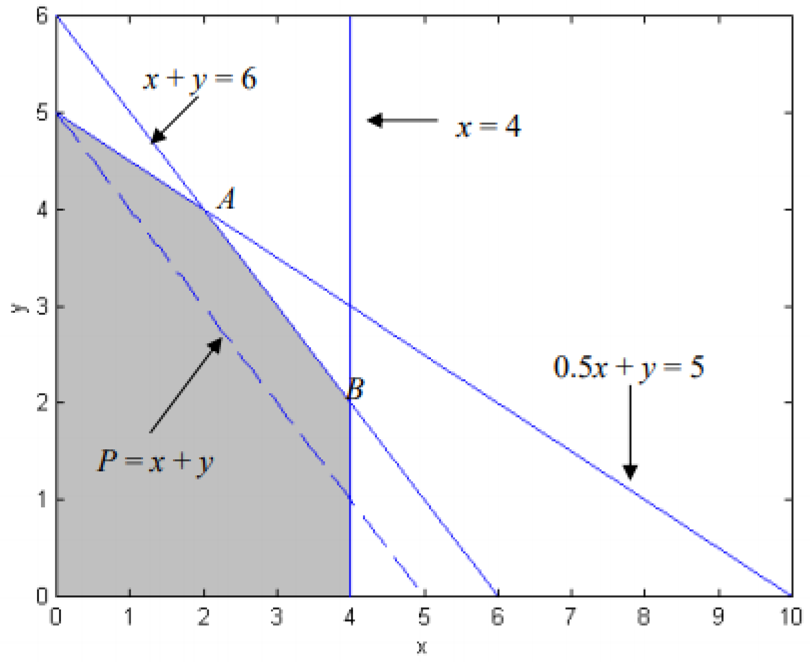
Extreme point	Coordinates	Profit
C	(0; 90)	225
D	(20; 90)	265
E	(85; 57)	312.5
F	(120; 0)	240
G	(0; 0)	0

Extreme Point – Example 2



- Remember in this scenario wanted to minimize cost
- Note that C,D,E are not extreme points since they are not “corners” only the border of the graph
- Once we calculate K at points A, B and C – see that B minimizes cost

Extreme Point – Example 3



- Are also other extreme points but not labelled
- Since not unique solution extreme point A or B will give you optimal solution

Summary

- Did 2 more examples
  - Noticed that optimization applies to both maximizing and minimizing objective functions
  - Saw that there are cases where no unique solution exists
- Used another method – Extreme Point Method
- Did very well today!