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Queueing Theory Part 3 / Toustaanteorie Deel 3

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Lecturer - HW Freiboth

Slides adapted from those originally compiled
for the module by Dr Neil Jacobs (2019)

Little se vloeivergelykings / Little's flow equations

$$L = \lambda W, \tag{19}$$
$$L_q = \lambda W_q, \tag{20}$$
$$L_s = \lambda W_s, \tag{21}$$

waaruit volg dat,

from which it follows that,

$$W = \frac{L}{\lambda}, \tag{22}$$
$$W_q = \frac{L_q}{\lambda}, \tag{23}$$
$$W_s = \frac{L_s}{\lambda}. \tag{24}$$

$M/M/m$-toustaanstelsels / $M/M/m$-queuing systems

$M/M/m$ beteken volgens konvensie dieselfde as $M/M/m/GD/\infty/\infty$

$M/M/m$ means the same as $M/M/m/GD/\infty/\infty$

$M/M/m$-toustaanstelsels / $M/M/m$-queuing systems

Bondige uitleg van die Kendall-Lee notasie:

Tusssenaankomstye en dienstye is eksponensieel verdeel, daar is m-aantal bedieners in parallel wat kliënte uit 'n enkele tou bedien, kliënte in die tou volg die algemene toudissipline, daar is oneindig groot ruimte in die stelsel vir kliënte en die kliënte populasie is oneindig groot.

Brief explanation of the Kendall-lee notation:

Interarrival times and service times are exponentially distributed, there are m number of servers in parallel serving clients from a single queue, clients in the queue follow the general queuing discipline, there is infinite large space in the system for clients and the client population is infinitely large.

Voorbeelde van $M/M/m$-toustaanstelsels / Examples of $M/M/m$-queuing systems

- Multi-teller banksaal
- Enkeltou supermark kassiere
- Inbelsentrum
- Tolhek
- Doeane
- Voertuigwassery
- Voertuig-diensfasiliteit
- Kaaie
- Houer-berging en uithaal

- Multi-teller banking hall
- Single queue supermarket cashiers
- Call centre
- Tollgate
- Customs
- Vehicle washer
- Vehicle service facility
- Quays
- Container storage and retrieval

$M/M/m$-toustaanstelsel / $M/M/m$-queuing system

Moz Port is 'n ystererts uitvoer-hawe operateur. Ystererts kom aan in treinvragte teen 'n tempo van 9 massa treine per dag. Dit neem Moz Port gemiddeld 9.5 uur om 'n trein af te laai by enige van hulle 4 aflaai-plekke, In terme van die kontrak tussen Moz Port en trein-eienaars, ontvang trein einenaars 'n boete van R5 000 vir elke dag wat 'n trein by Moz Port deurbring. Moz Port het die geleentheid om hulle fasiliteite uit te brei na 5 aflaai-plekke teen 'n ekstra jaarlikse koste van R15 mil, wat alle vaste en veranderlike koste insluit. Behoort Moz Port uit te brei?

Moz Port is a iron-ore export-port operator. Iron-ore arrives by trainload at a rate of 9 bulk trains per day. It takes Moz Port on average 9.5 hours to unload a train at any one of their 4 off-loading facilities. In terms of the contract between Moz Port and train owners, train owners receive a penalty of R5 000 for every day that a train spends at Moz Port. Moz Port has the opportunity to expand their facility to 5 off-loading facilities at a extra yearly cost of 15 million R, that includes all fixed and variable cost. Should Moz Port expand?

Bestuur wil eers weet / Owner first wants to know

Aangaande gemiddeldes van die aantal treine:

1. Treine wat wag om bedien te word.
2. Treine in die aflaai-plekke.
3. Totaal by Moz Port.

Regarding averages of the number of trains :

1. Trains waiting to be served.
2. Trains in the off-loading facilities.
3. Total at Moz Port.

Aangaande die gemiddelde tyd wat Treine in MP spandeer

4. Tyd wat 'n trein wag om bedien te word.
5. Tyd wat 'n trein in die aflaai-plek deurbring.
6. Tyd wat 'n trein in die hawe deurbring.

Regarding the average time trains spend at MP

4. Time a train waits to receive service.
5. Time a train stands in the off-loading facility.
6. Time a train spends at the port.

Bestuur wil weet / Management wants to know

Hulle stel ook in die volgende belang

7. Benutting van die fasiliteit.
8. Waarskynlikheid dat die fasiliteit ledig is.

Onder sekere toestande kan toustaanteorie die bestuur se vrae volledig beantwoord.

They are also interested in the following

7. Utilization of the facility.
8. Probability that the facility is idle.

Under certain conditions queueing theory can answer management's questions completely.

Antwoorde vir die bestuur/ Answers to management

Aangaande gemiddeldes van die aantal treine:

1. Treine wat wag om bedien te word. L_q
2. Treine in die aflaai-plekke. L_s
3. Totaal by Moz Port. L

Aangaande die gemiddelde tyd wat Treine in MP spandeer

4. Tyd wat 'n trein wag om bedien te word. W_q
5. Tyd wat 'n trein in die aflaai-plek deurbring. W_s
6. Tyd wat 'n trein in die hawe deurbring. W

Regarding averages of the number of trains :

1. Trains waiting to be served. L_q
2. Trains in the off-loading facilities. L_s
3. Total at Moz Port. L

Regarding the average time trains spend at MP

4. Time a train waits to receive service. W_q
5. Time a train stands in the off-loading facility. W_s
6. Time a train spends at the port. W

Antwoorde vir die bestuur/ Answers to management

Hulle stel ook in die volgende belang

7. Benutting van die fasiliteite. ρ
8. Waarskynlikheid dat die fasiliteit ledig is. P_0

Onder watter toestande kan toustaanteorie die bestuur se vrae volledig beantwoord?

They are also interested in the following

7. Utilization of his facility.ρ
8. Probability that the facility is idle. P_0

Under which conditions can queueing theory answer management's questions completely?

$M/M/m/GD/\infty/\infty$- toustaanstelsel aannames / queueing system assumptions

1. Kliënte in die tou volg die algemene tou-dissipline (GD) wat 'n eerste in eerste bedien logika is.
2. Kliënte draai nie om voordat hulle in die tou inval nie.
3. Kliënte bly in die tou tot hulle diens ontvang, daar is met ' woorde geen uitval uit die tou nie.
4. Tussenaankomstye is onderling onafhanklik en volg die identiese eksponensiële verdeling met aankomstempo λ bekend.

1. Clients in the queue follow the general queueing discipline (GD) which is a first come first serve logic.
2. Clients do not turn around before joing the queue, thus no balking takes place
3. Clients remain in the queue until they receive service, thus no reneging takes place.
4. Interarrival times are mutually independent and follow the identical exponential distribution with arrival rate λ known.

$M/M/m/GD/\infty/\infty$- toustaanstelsel / queueing system

Aannames vervolg

5. Dienstye is onderling onafhanklik en volg die identiese eksponensiële verdeling met dienstempo μ bekend.
6. Die besoekende populasie is oneindig groot.
7. Daar is geen ruimtebeperking in die stelsel nie.
8. Die aankomstempo is kleiner as die gesamentlike dienstempo van die m bedieners, met ander woorde $\lambda < m\mu \equiv \frac{\lambda}{m\mu} < 1$

Assumptions continued

5. Service times are mutually independent and follow the identical exponential distribution with service rate μ known.
6. The calling population is infinitely large.
7. There is no space constraint in the system
8. The arrival rate is less than the combined service rate of the m servers, in other words $\lambda < m\mu \equiv \frac{\lambda}{m\mu} < 1$

Formules / Formulas

$$P_0 = \frac{1}{\left[\sum_{n=0}^{n=m-1} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n\right] + \frac{1}{m!}\left(\frac{\lambda}{\mu}\right)^m \frac{m\mu}{m\mu-\lambda}} \tag{25}$$

$$L = \frac{\lambda\mu\left(\frac{\lambda}{\mu}\right)^m}{(m-1)!\,(m\mu-\lambda)^2} P_0 + \frac{\lambda}{\mu} \tag{26}$$

$$W = \frac{\mu\left(\frac{\lambda}{\mu}\right)^m}{(m-1)!\,(m\mu-\lambda)^2} P_0 + \frac{1}{\mu} = \frac{L}{\lambda} \tag{27}$$

$$L_q = L - \frac{\lambda}{\mu} \tag{28}$$

$$W_q = W - \frac{1}{\mu} = \frac{L_q}{\lambda} \tag{29}$$

$$\rho = \frac{\lambda}{m\mu} \tag{30}$$