Linear Programming

STATISTICS 186

Part 5

Last week?

- Looked at changes in the objective function
- Did an application where could buy same products elsewhere
  - Involved calculating new objective function

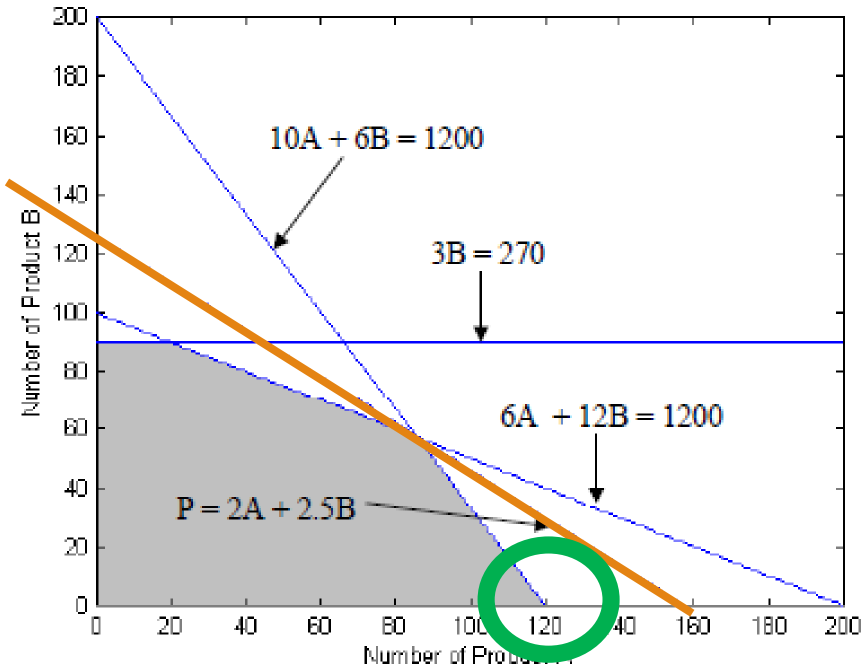
Today?

- Performing sensitivity analysis
- Trying to see
  - By how much would the contribution of product B **have to change** for there to be a change in the optimal solution

Sensitivity Analysis

- In a business needs some idea of how sensitive your original decision is to changes in the estimates of the contributions
- Must be informed of your business decisions

Revisit Example 1



- Question is how much would the contribution of B have to change for there to be a change in the optimal solution?
- Different optimal solution will occur if we change slope of objective function
  - In this case want slope to increase in absolute value to beyond that of
  - $10A + 6B = 1200$

Procedure of Sensitivity Analysis

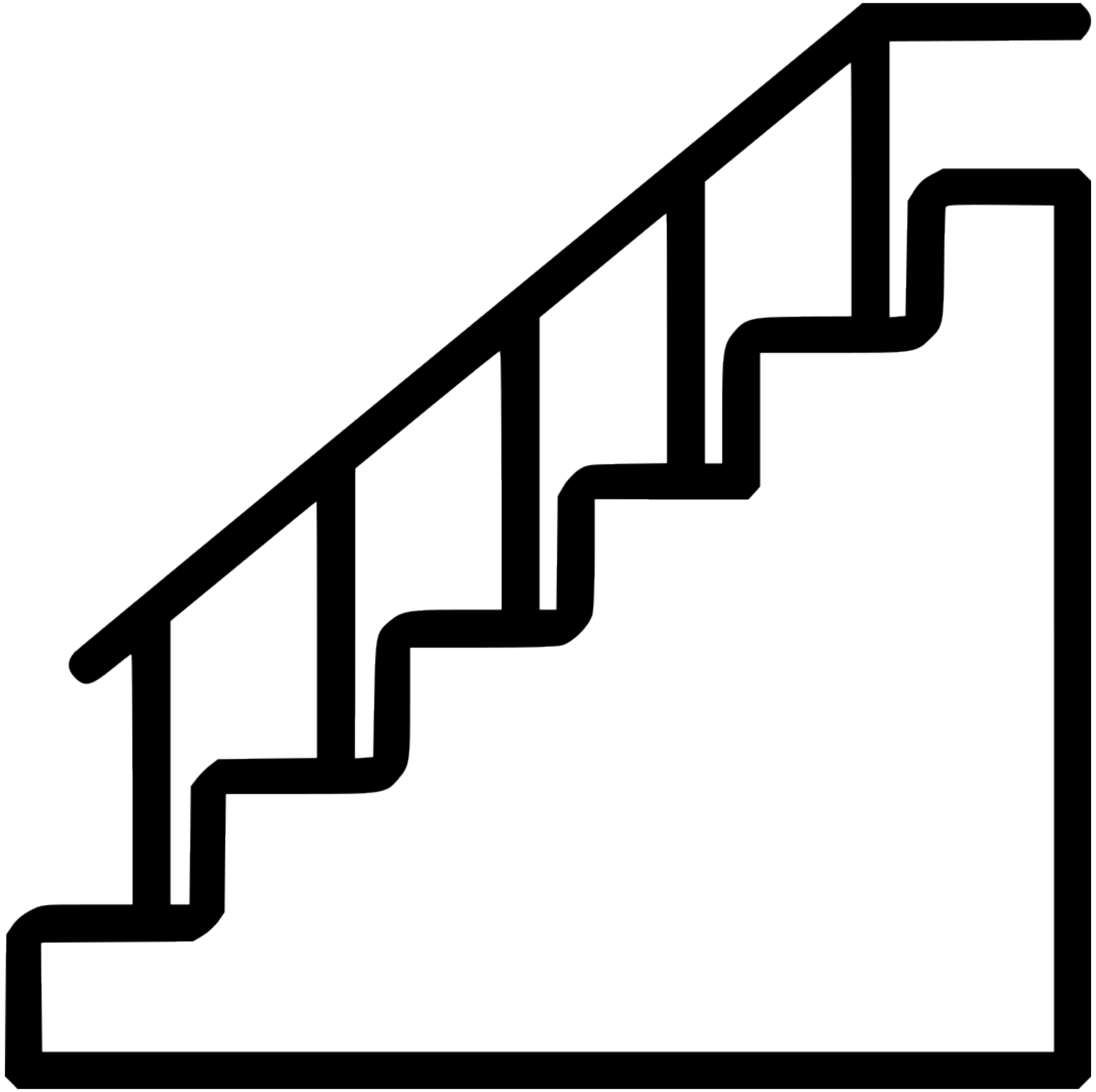
- Specifying contribution of B as a variable and not a constant
  - Hence

$$P = 2.5A + 2B$$

  - Changes to

$$P = 2.5A + bB$$

- Setting the slope of the objective function equal to the slopes of both limiting constraints and solving for the contribution of B from both equations



Steps

1. Determine slope of the objective function
2. Determine the slopes of the two critical constraints
3. Solve for the contribution from both equations
4. State the result in words

Step 1 – Determine the slope of the objective function

- Let b = contribution per unit of product B

$$P = 2.5A + bB$$

- Hence,

$$B = \frac{-2}{b}A + \frac{P}{b} \qquad "y = mx + c"$$

- Slope of objective function $\dfrac{-2}{b}$

Step 2 – Determine the slopes of the two critical constraints

- Finishing time: $10A + 6B = 1200 \Rightarrow B = -\frac{10}{6}A + 200$
- Machine time: $6A + 12B = 1200 \Rightarrow B = -\frac{1}{2}A + 100$

- Slope of finishing time = -10/6
- Slope of machine time = -1/2

- Set slope of constraints and objective function equal and solve for b

Step 3 – Solve for contribution from both equations

- Finishing time: $-\frac{2}{b} = -\frac{10}{6} \Rightarrow b = 1.2$
- Machine time: $-\frac{2}{b} = -\frac{1}{2} \Rightarrow b = 4$

Step 4 – State results in words

- Optimal solution will change if contribution per unit of B rises above R4 or drops below R1.2
- Can do same process for product A – where you set slope = a

Remarks

- Able to check you answer!
  - Original contribution must always fall between limits (things you worked out)
  - Hence for product A: R1.25 < R2.00 < R4.16
  - Product B: R1.20 < R2.50 < R4.00

Reason for sensitivity analysis

- If small changes in contribution occurs can see if new optimal solution is needed

  - Product A: R1.25 < R2.00 < R4.16
  - Product B: R1.20 < R2.50 < R4.00

- If contribution for A increased to R3.00
  - No need to redo exercise to determine optimal solution since it will stay the same (still between the limits)

Where to use LP?

Application of LP

Inventory managers

- Lower cost of products as well as transport

Portfolio Managers

- Minimise risk
- How to allocate assets

Yield Managers

- Schedule crews to flights of an airline

Manufacturing and logistics managers use LP

Telecommunications managers use LP for

- Call routing
- Network Design
- Internet Traffic

Summary

- Sensitivity analysis allows you to determine how much a contribution for a product must change before a new optimal solution must be calculated
- Go through the steps!