◇ shadow prices

↳ scarce resource
  ↳ limits production process

↳ shadow price
  ↳ marginal contribution to profit
  ↳ increase in profit per additional unit of scarce resource

Method 1

↳ direct application of definition

○ Example 1

Scarce resources

→ Machine : $6A + 12B \leq 1200$

→ Finishing : $10A + 6B \leq 1200$

☐ Step 1: state critical constraints ↗

Question: What is the shadow price of machine time?

☐ Step 2: obtain original optimal profit

↳ note: use un rounded answer

$A = 85,714$

$B = 57,143$

↳ origind contribution $= 2(85,714) + 2,5(57,143)$

$= 314,2859$

☐ Step 3: obtain new optimal profit by adding one additional unit of resource

→ Machine : $6A + 12B = 1201$ (↓)

→ Finishing : $10A + 6B = 1200$ (*)

↳ solve : $1 \cdot (↓) - 2 \cdot (*)$

$-14A = -1199$

$\Rightarrow A = 85,6429$

↳ Sub into (↓):

$6(85,6429) + 12B = 1201$

$\Rightarrow B = 57,2619$

New optimal profit:

$P = 2(85,6429) + 2,5(57,2619)$

$= 314,4406$

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□ Step 4: Subtract old and new & determine shadow price

(new) (old)
↳ 314,14406 − 314,2859 = 0,1547

∴ Shadow price for machine time = R0,1547 per minute

↳ Exercise: follow procedure but for finishing time

↳ SP for FT = R0,1071 per min.

Interpretation:

↳ every additional minute of machine time, optimal profit increases by R 0,1547

↳ every additional minute of finishing time, optimal profit increases by R 0,1071

Method 2

↳ preferable method

□ Step 1: Represent critical constraints in a table.

→ MT : 6A + 12B ≤ 1200

→ FT : 10A + 6B ≤ 1200

	A	B	limit
MT	6	12	1200
FT	10	6	1200
Contribution	2	2,5	

↳ note: equations appear horizontally

□ Step2: Obtain shadow price by representing equations vertically

↳ Let M = shadow price of machine time
F = shadow price of finishing time

↳ (1) 6M + 10F = 2

(2) 12M + 6F = 2,5

↳ solve for M and F

↳ 2(1) - (2)

$$12M + 20F = 4$$
$$\ominus \quad 12M + 6F = 2{,}5$$
$$14F = 1{,}5$$

$$\Rightarrow F = 0{,}1071$$

↳ sub into (1)

$$\Rightarrow 6M + 10(0{,}1071) = 2$$

$$\Rightarrow M = 0{,}1548$$

$\therefore$ $M = R0{,}1548$ per min

$F = R0{,}1071$ per min

<u>why does this work?</u>

↳ consider equation (1)

↳ now consider the units

$$6\ \frac{\text{minutes}}{\text{unit}} \cdot M + 10\ \frac{\text{minutes}}{\text{unit}} F = 2\ \frac{\text{Rand}}{\text{unit}}$$

$\therefore$ Units of M & F must be $\frac{\text{Rand}}{\text{minute}}$ $\frac{\text{Rand}}{\text{minutes}}$

$$\frac{\text{Rand}}{\text{Minute}} \cdot \frac{\text{Minute}}{\text{unit}} = \frac{\text{Rnd}}{\text{unit}}$$

◇ Application of shadow prices:
Special offer

↳ obtain minimum quoted price on a special offer

○ Example 1

↳ special order of another product C

↳ require 100 units

↳ amount of resources:

raw material : 90 kg
Machine time : 200 minutes
Finishing time : 140 minutes

} per 100 units

↳ resource <u>costs</u>:

raw material : R10/kg
machine time : R5/minute
Finishing time : R10/minute

↳ Aim: obtain <u>lowest</u> selling price that should be quoted before we accept the offer.

note: Shadow price vs Resource costs

△ profit
or $\frac{\text{Profit}}{\text{unit}}$
↳ cost are already implicit

Labour, electricity, repairs, etc.
↳ does not involve calculation of profits

↳ cost price of product C

Raw material : 90 x 10 = R900
Machine time : 200 x 5 = R1000
Finishing time : 140 x 10 = R1400
Total cost R3300

↳ Resources spent on C means less resources spent on A and B
∴ lose out on profit of A and B
↳ where shadow prices come in
↳ use shadow prices to calculate the loss in contributions of A and B
↳ Shadow price : $\frac{\text{contribution}}{\text{unit of scarce resource}}$

Raw material
↳ used in producing B only
↳ 57 units of B currently produced
↳ each unit uses 3kg raw material
* Raw material available = 270kg
* Raw material for B = 3x57 = 171kg
∴ unused raw material = 99kg
↳ only need 90kg for C
∴ rm does not cause loss in contribution.

Machine time & Finishing time

↳ scarce resources

↳ Shadow prices:

M = R 0,1548 per minute

F = R 0,1071 per min.

↳ Calculate loss in contribution:

Machine time: 200 × R 0,1548

= R 30,96

Finishing time: 140 × R 0,1071

= R 14,994

Lowest selling price

* Manufacturing cost	R 3300,00
* loss in contribution	
MT	R 30,96
FT	R 14,99
Total cost	R 3345,95

Alternative Method:

↳ redo linear programming

↳ Subtract resource requirements of C

i.e new constraints:

$$6A + 12B \leq (1200 - 200) = 1000$$

$$10A + 6B \leq (1200 - 140) = 1060$$

$$3B \leq 270 - 90 = 180$$

↳ obtain optimal profit (new)

↳ Subtract: old profit − new profit

= loss in contribution = R 345,95

↳ add loss in contribution to manufacturing costs to obtain lowest selling price

Remarks

↳ do not round

↳ Situation when Alt method has to be used:

* when opportunity affects non-scarce resource

* Eg. Suppose C used 150 kg raw Material

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