Linear Programming

STATISTICS 186

Part 4

Last week?

- Linear Programming!
- Touched on Shadow Prices
  - Amount of profit per additional unit of resource available
  - Two methods to calculate it
    - Definition
    - Constructing table
- Use shadow prices to help us finding lowest selling price if we were given a special offer on another product

Today?

- What happens when there are changes to the objective function
- In context
  - What happens if we can buy units from another producer to complete an order
  - What will be the optimal number of units to produce?

Example 1

- Assume demand
  - Product A: 200 units
  - Product B: 150 units
- Products can now be bought in at a price that will yield profit
  - Product A: R1.50 per unit
  - Product B: R1.00 per unit

$$P = 1.5A + 1B$$

- Constraints do **not** change

- Question: What are the number of items that needs to be produced and the number which has to be bought in of both products such that contribution will be maximised

Step 1: Set up Differential Contribution Table

- First **row** current objective function
- Second **row** outside contribution function
- Last **row** will be the difference of the columns
- First **column** will be the first product
- Second **column** will be the second product

$$P = 2.5A + 2B$$

$$P = 1.5A + 1B$$

Step 1 – Set up Differential Contribution Table

$$P = 2.5A + 2B$$

$$P = 1.5A + 1B$$

	Product A	Product B
Current contribution	R2.00	R2.50
Outside contribution	R1.50	R1.00
Differential contribution	R0.50	R1.50

New objective function: $P = 0.5A + 1.5B$

Step 2 – Obtain optimum solution given new objective function

- New objective function

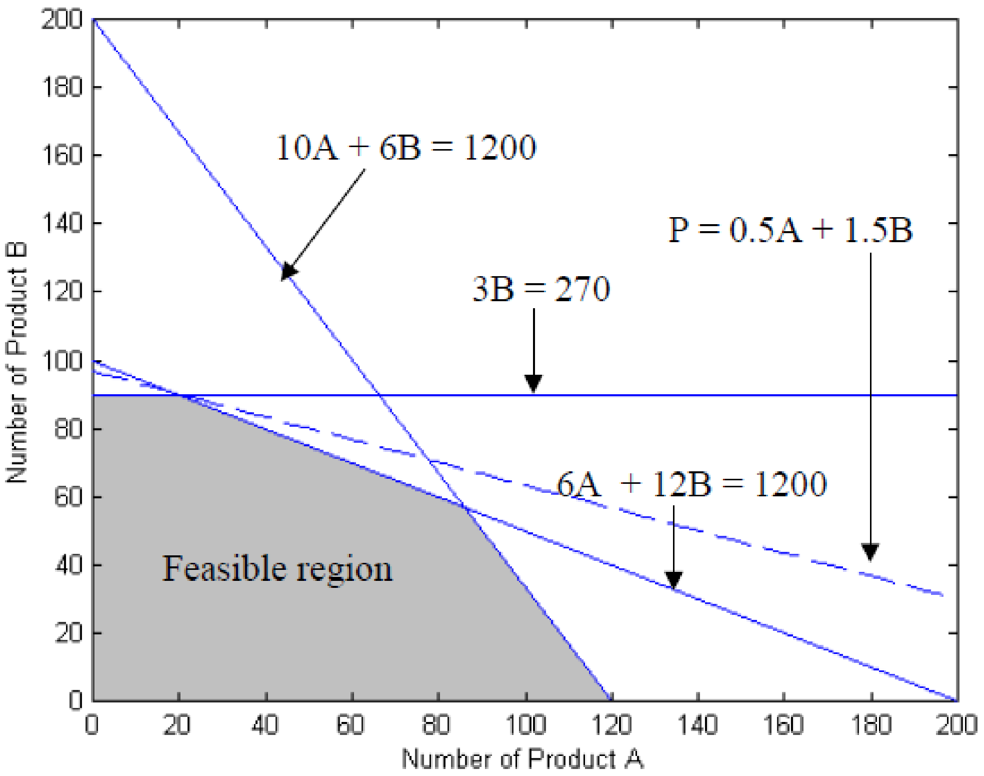
$$P = 0.5A + 1.5B$$

- Constraints remain the same

$$6A + 12B \leq 1200$$
$$10A + 6B \leq 1200$$
$$3B \leq 270$$

- Graphically represent constraints and objective function (Lecture 1 &2)

Step 2 – Obtain optimum solution given new objective function



- Note that new optimum solution is different than before
- Now intersection between lines
- $6A + 12B = 1200$ and $3B = 270$
- Set up simultaneous equations and solve
- Obtain
  - B=90
  - A=20

Amount that needs to be produced internally – rest must be bought in

Step 3 – Set up table of contribution

- First two **rows** will be related to internal production
- Last two **rows** will be related to external purchase
- First **column** will be amount of units bought/produced
- Second **column** will be amount of contribution
- Third **column** will be total

Step 3 – Set up table of contribution

Obtained as solution from new objective function

Profit per unit produced

Product	Internal Production	Contribution	Total
A	20	2.00	40
B	90	2.50	225
	External purchase		
A	180 = (200 -20)	1.50	270
B	60 = (150 – 90)	1.00	60
			R595

Demand – Internal Production

Profit per unit bought

Optional Exercise

- If original optimal solution was produced with the rest of the demand bought what will the total consumption be?
- Total consumption in last slide R595
- Set up table

Optional Exercise

Product	Internal Production	Contribution	Total
A	85	2.00	170
B	57	2.50	142.50
	External purchase		
A	115 = (200 -85)	1.50	172.50
B	93 = (150 – 57)	1.00	93
			R578

R578 < R595