◊ Sensitivity Analysis

↳ How sensitive is our optimal solution to changes in the objective function?

↳ By how much does the coefficients in our objective function need to change by so that we obtain a different optimal solution?

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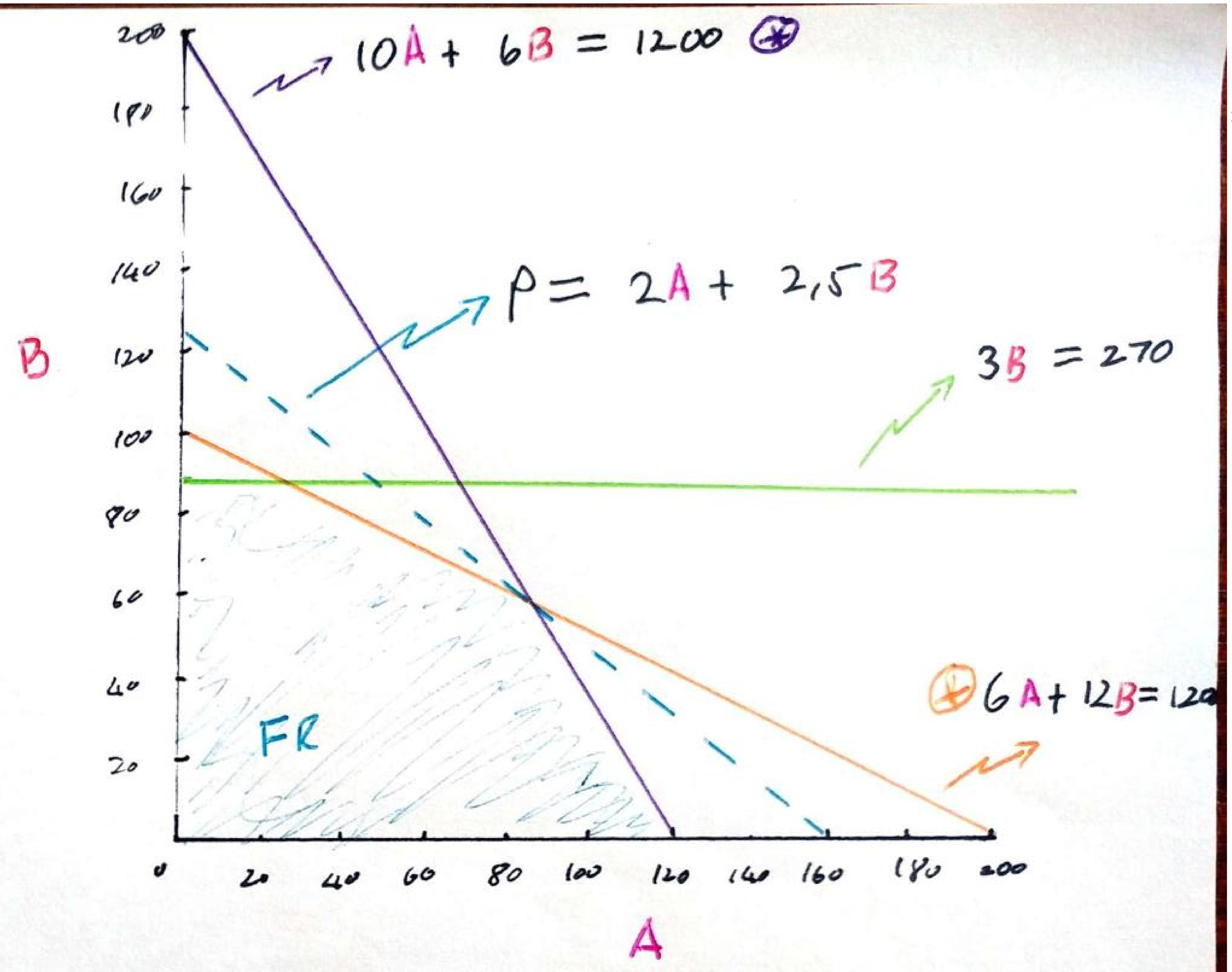
o Example 1

Question: "By how much would the contribution of product B have to change for there to be a change in the optimal solution?

Optimal solution: Product mix of A and B that maximises profit



↳ get different optimal solution if drastic change in objective function occurs

↳ slope becomes greater than ⊛

↳ slope becomes less than ⊛

↳ use sensitivity analysis to avoid redoing LP when changes in contribution occur

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Procedure:

- specify contribution of B as a variable
- set slope of objective function equal to slopes of both limiting constraints
- solve for contribution of B
  - from both equations

☐ Step 1: Determine the slope of the objective function

* Let b = contribution per unit of B
* $P = 2A + bB$
  - get in another form: equation for B

$$B = -\frac{2}{b}A + \frac{P}{b}$$

∴ Slope of objective function: $-\frac{2}{b}$

☐ Step 2: Determine the slopes of 2 critical constraints

MT: $6A + 12B = 1200$

$\Rightarrow B = -\frac{1}{2}A + 100$

∴ slope: $-\frac{1}{2}$ ⊛

FT: $10A + 6B = 1200$

$\Rightarrow B = -\frac{10}{6}A + 200$

∴ slope: $-\frac{10}{6}$ ⊛

☐ Step 3: Solve for contribution from both equations

MT: $-\frac{2}{b} = -\frac{1}{2} \Rightarrow b = \cancel{2}\,4$

FT: $-\frac{2}{b} = \frac{-10}{6} \Rightarrow b = \cancel{4}\,1{,}2$

☐ Step 4: State result in words

* optimal solution will change if the contribution per unit of product B rises above R4 or drops below R1,20

Exercise: Perform procedure for product A

2RC L6

Remarks:

* Check your answer
  ↳ original contribution should fall between obtained limits

i.e B : R1,20 ≤ R2,50 ≤ R4,00

* Reason for doing this
  ↳ avoids redoing LP when small changes occur

  ↳ eg. Suppose RM becomes more expensive

  ↳ contribution of B decreases from R2,50 to R1,50

  ↳ From sensitivity analysis we <u>know</u> that our optimal solution will <u>not</u> change

  i.e. still produce 57 units of B

→ note however : optimal profit will change.

$P = 2(85) + 1{,}5(57)$

$= R255{,}50$

↳ still the maximum profit in this case

3BC L6