o Example 2

↳ minimisation rather than maximisation

- Brand 1 : * 150g A
  * 120g B
  * 15g C

  } per Kg

- Brand 2 : * 300g A
  * 90g B
  * 0g C

- Min monthly intake : * 2,7 kg A
  * 1,44 kg B
  * 0,045 kg C

- Brand 1 costs £2 } per kg
  Brand 2 costs £3

- AIM: lowest-cost diet
  ↳ meet minimum monthly intake requirements

Solution:

- Let V_1 = amount of Brand 1 (kg)
  V_2 = amount of Brand 2 (kg)

- State the objective function

$$K = 2 \cdot V_1 + 3 \cdot V_2$$

- Specify constraints

A: $150 \cdot V_1 + 300 V_2 \geq 2700$

B: $120 V_1 + 90 V_2 \geq 1440$

C: $15 V_1 \geq 45$

note:
Implicit constrai[nts]
$V_1, V_2 \geq 0$
∴ positive quadrant

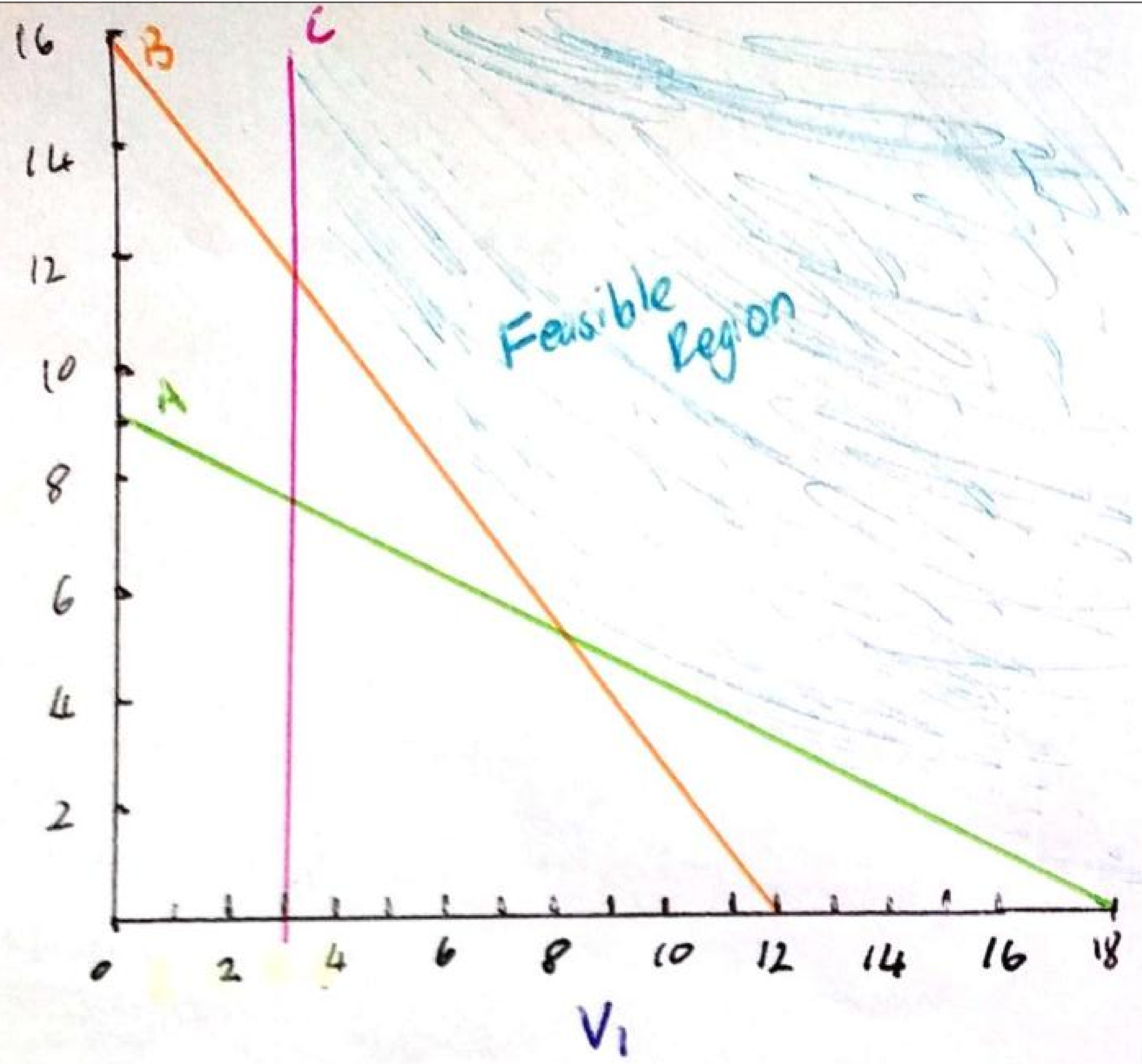
- Represent constraints graphically
  ↳ i.e find intercepts

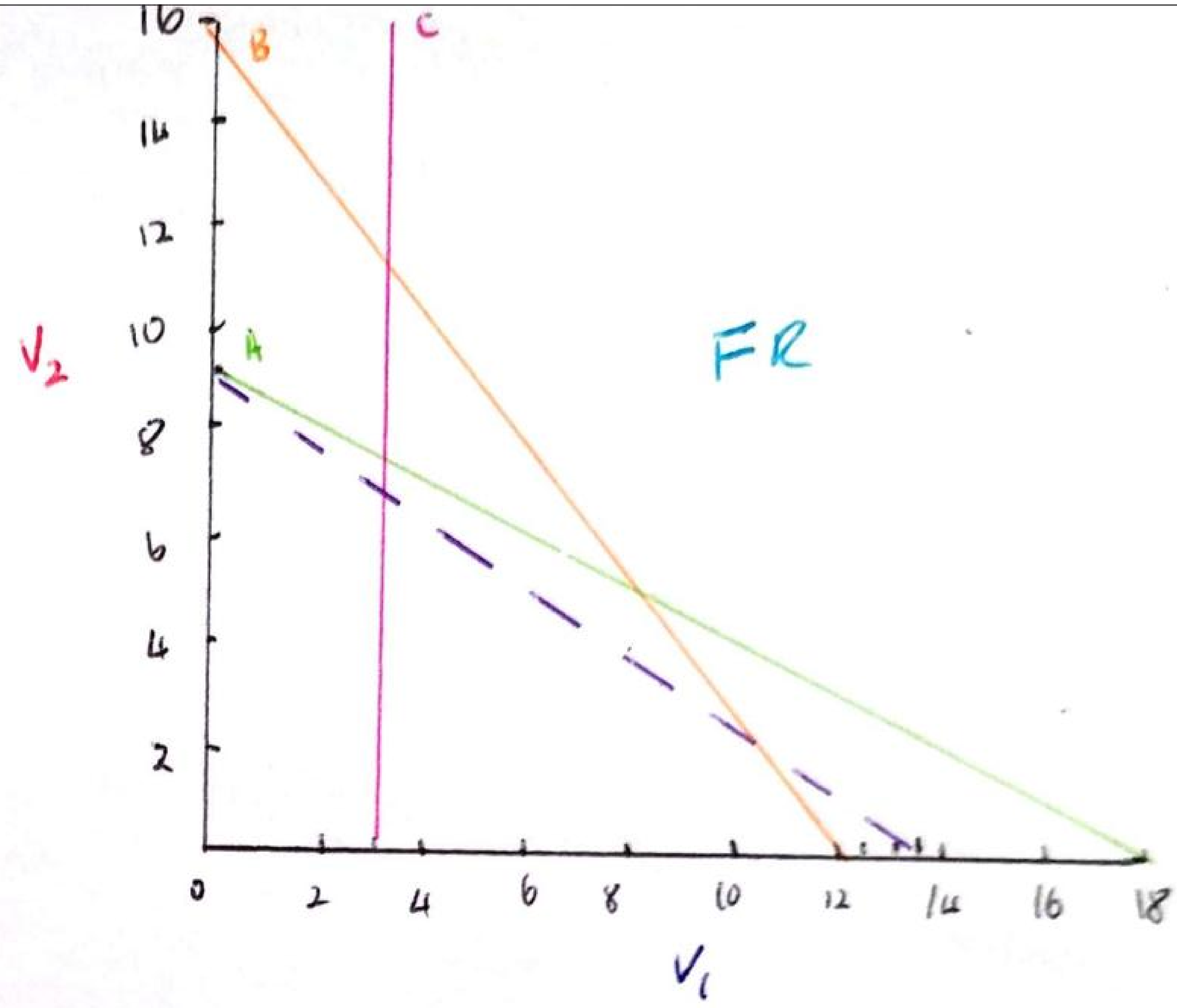
A: * $V_1 = 0 \Rightarrow V_2 = 9$
* $V_2 = 0 \Rightarrow V_1 = 18$

B: * $V_1 = 0 \Rightarrow V_2 = 16$
* $V_2 = 0 \Rightarrow V_1 = 12$

C: * $V_1 = 3$

note:
Sign of inequality "$\geq$" means above the line





- Represent objective function graphically

$$K = 2V_1 + 3V_2$$

↳ rewrite equation

$$V_2 = -\frac{2}{3}V_1 + \frac{K}{3}$$

↳ Set intercept to middle of y-axis

$\therefore$ Let $\frac{K}{3} = 9$

$\Rightarrow K = 27$

$\Rightarrow 27 = 2V_1 + 3V_2$

- Calculate optimal solution

↳ cheapest mix where lines

A $\quad 150V_1 + 300V_2 = 2700$

B $\quad 120V_1 + 90V_2 = 1440$

cross

↳ 2 methods

Method 1: make 1 of the variables look the same $\quad A \div \frac{10}{3}$

$$45V_1 + 90V_2 = 810$$
$$120V_1 + 90V_2 = 1440$$
$$-75V_1 = -630$$

$\Leftrightarrow V_1 = 8.4$ ~~...~~

2RC L3

$\Rightarrow 120(8{,}4) + 90V_2 = 1440$

$\Rightarrow V_2 = 4{,}8$

Method 2: $V_1 = 18 - 2V_2$ A

↳ sub into B

$120(18 - 2V_2) + 90V_2 = 1440$

$-150V_2 = \cancel{\quad} -720$

$V_2 = 4{,}8$

↳ plug into A or B

↳ A $150V_1 + 300(4{,}8) = 2700$

$\Rightarrow V_1 = 8{,}4$

- Finally

↳ cheapest mix : • 8,4 kg Brand 1
• 4,8 kg Brand 2

$K = 2(8{,}4) + 3(4{,}8)$

$= R\,31{,}20$

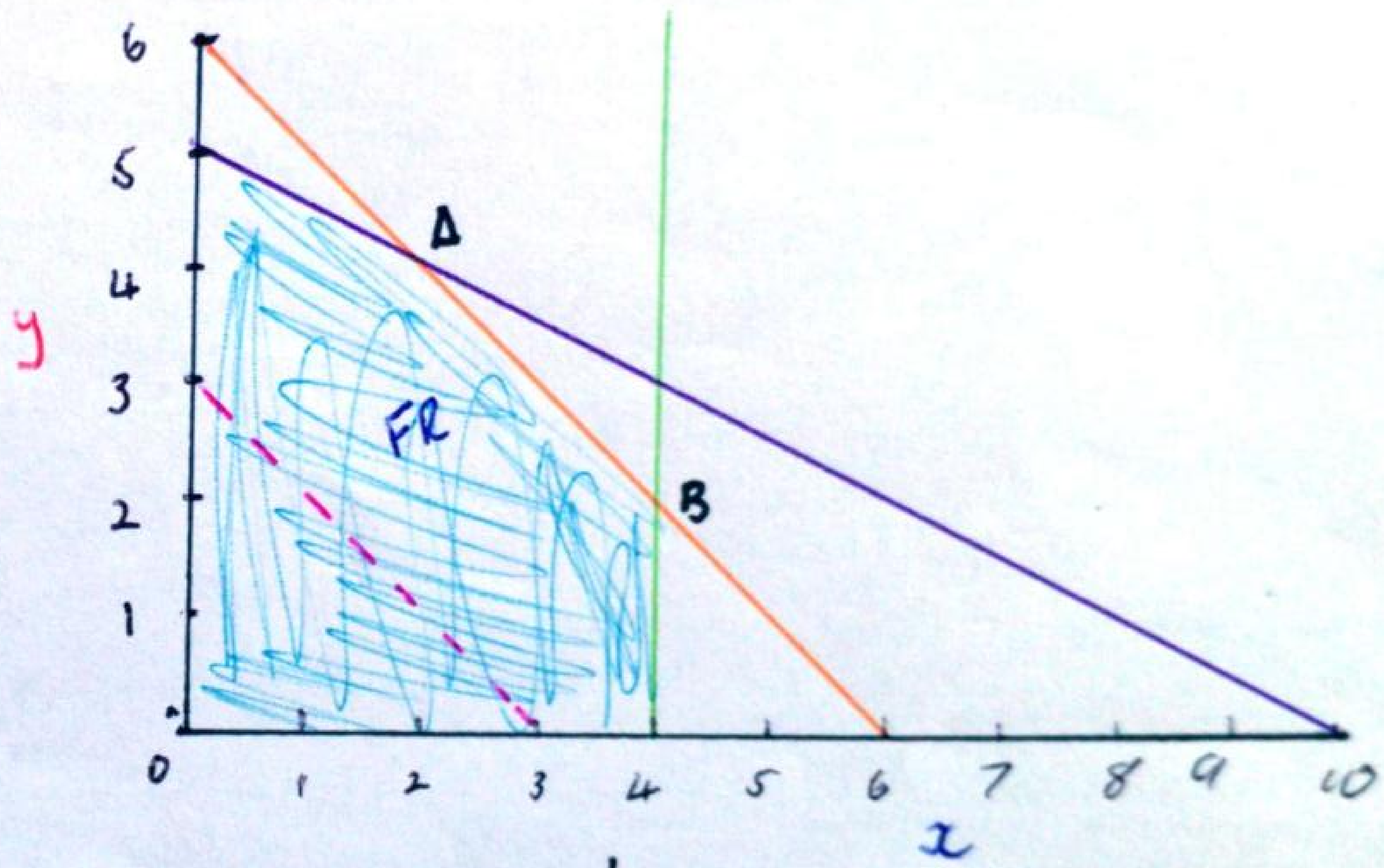
- Example 3

↳ optimal solution need not be unique

↳ can have more than 1 → ∞ solutions

Problem: * Maximise $x + y = P$

* constraints
(1) $x \leq 4$
(2) $x + y \leq 6$
(3) $0{,}5x + y \leq 5$
(4) $x, y \geq 0$



→ optimal solution is when $x + y = 6$

↳ same as equation of 2nd constraint

∴ Any point on line AB is optimal

↳ note: can only happen when one constraint line ∥ objective function

3EC L3

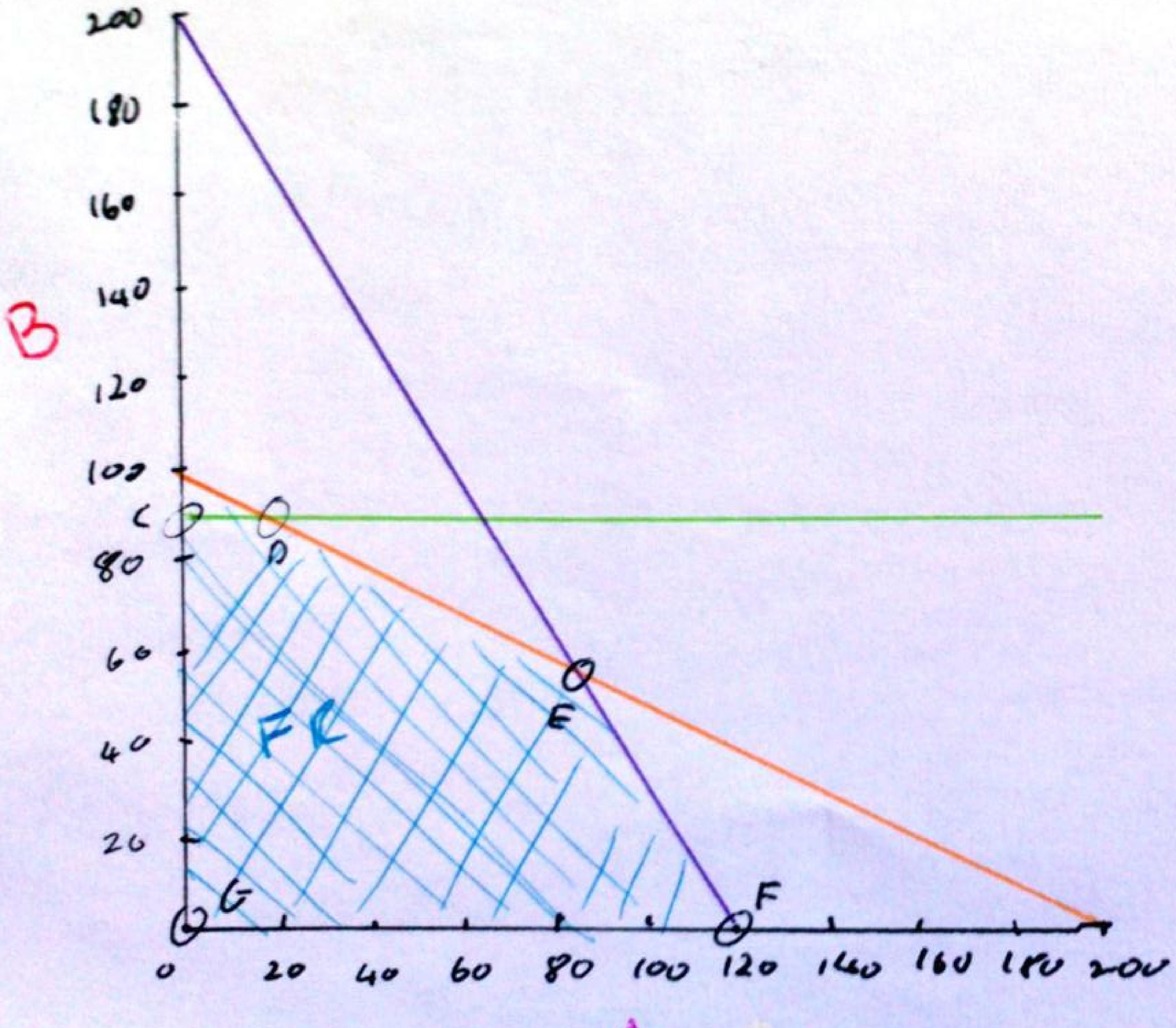
◊ Extreme Point Method

↳ optimal solution is <u>always</u> at one of the extreme points of the feasible region

↳ no plotting of objective function

↳ involves more calculations

○ Example 1



↳ find coordinates of all extreme points

↳ calculate profit at each of those points

↳ objective function:

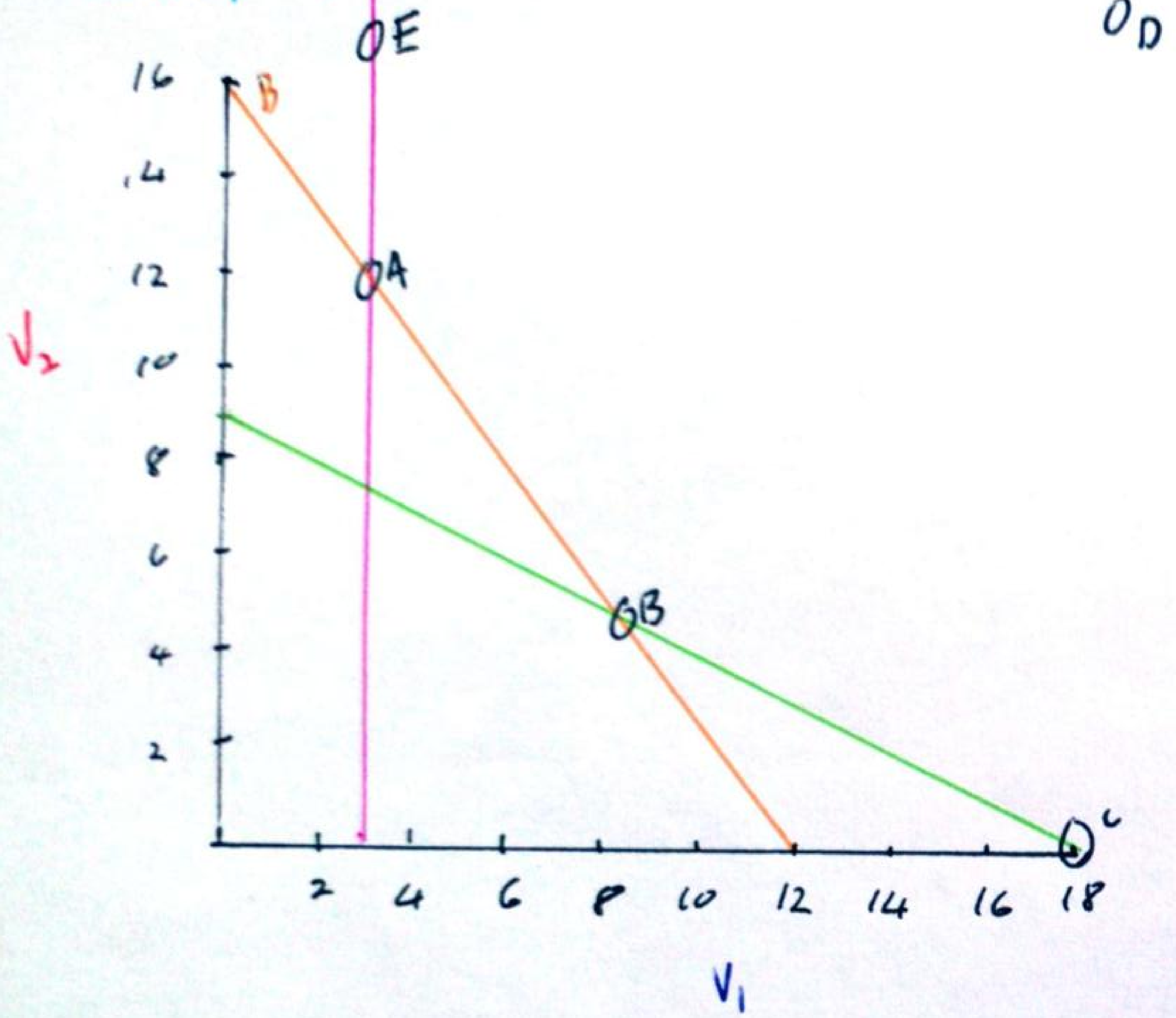
$$P = 2A + 2{,}5B$$

Extreme point	coordinate	Profit
C	(0,90)	225
D*	(20,90)	265
E*	(85,57)	312,5
F	(120,0)	240
G	(0,0)	0

↳ point E yields optimal profit

* need to solve simultaneous equations to obtain coordinates

o Example 2



↳ note: only A, B & C are extreme points

↳ D and E are <u>not</u> extreme points.

o Example 3

↳ A, B and 3 other points are extreme points

↳ optimal solution at A and at B

↳ all points between A & B are also optimal solutions.