

DEPARTMENT OF STATISTICS AND ACTUARIAL SCIENCE

STATISTICS 186

TUTORIAL 1

1. A company produces two products, X and Y. Each item goes through three departments, namely Design, Manufacture and Packaging. The following table provides the number of items of the two products that can be handled per hour in each of the three departments.

	Product X	Product Y
Design	3	2
Manufacture	5	2
Packaging	3	6

The departments only have a limited amount of time available each day. The following table summarises this information.

Department	Total number of minutes per day
Design	1 440
Manufacture	720
Packaging	720

The company makes a profit of R12 for each unit of X that it manufactures, and a profit of R15 for each unit of Y.

- (a) Write down the objective function and the constraints.
- (b) Give a graphical representation of this LP problem.
- (c) Solve the LP problem and calculate the maximum profit.
- (d) Calculate the shadow prices of the three departments of the company.
- (e) Do a sensitivity analysis for product X.

2. A company manufactures two products, A and B. The marginal income for product A is R3 per unit, while product B has a marginal income of R2 per unit. It requires 3 kg of raw material and 2 hours machine time to manufacture one unit of product A. Similarly, it requires 4 kg of raw material and 1 hour of machine time to manufacture one unit of product B. Daily availability of raw material and machine time are limited to 1200 kg and 600 hours respectively.

- (a) Write down the objective function and the constraints for this LP problem.
- (b) Give a graphical representation of the LP problem.
- (c) Solve the LP problem and calculate the maximum profit.
- (d) Calculate the shadow price of raw material.
- (e) Do a sensitivity analysis for product A.

3. A company produces two products, namely A and B. Each item that is produced requires a certain amount of raw material and a certain amount of machine time. The following table summarises the requirements with respect to raw material and machine time for the two products.

	Product A	Product B
Raw material	2 kg	3 kg
Machine time	$\frac{1}{2}$ hour	2 hours

The company has 2000 kg raw material and 800 hours machine time available per day. Product A has a marginal income of R2 per unit and product B has a marginal income of R4 per unit.

- (a) Write down the objective function and the constraints for this LP problem.
- (b) Give a graphical representation of this LP problem.
- (c) Solve the LP problem and calculate the maximum income.
- (d) Calculate the shadow prices of the two constraining factors.
- (e) Do a sensitivity analysis for product A.
- (f) Suppose product A can be bought in at a price that yields a marginal income of R1.25 per unit, and that product B can be bought in at a price that yields a marginal income of R3 per unit. The constraints do not change, and the daily demand for the two products is 1500 units for A and 1200 units for B. Determine the new optimal quantities to be produced of the two products and calculate the new optimal income.

4. Company ABC manufactures chemicals for swimming pools. The products are called Pluto and Atlantic, and three types of raw material are required in their manufacture namely A, B, and C. Limited amounts of the raw materials are available: 2100 kg of A, 2400 kg of B, and 2700 kg of C per week. The following table summarises the requirements in terms of raw materials for one unit of each of the two products.

	Product Pluto	Product Atlantic
Raw material A	6 kg	3 kg
Raw material B	6 kg	6 kg
Raw material C	4.5 kg	9 kg

The weekly demand for Pluto is 350 units and 250 units for Atlantic. The company makes a profit of R18 per unit of Pluto and R24 per unit of Atlantic.

- Write down the objective function and the constraints for this LP problem.
- Give a graphical representation of the LP problem.
- Solve the LP problem and calculate the maximum weekly profit of the company.
- Calculate the shadow price of raw material B.
- Do a sensitivity analysis for product Pluto.