◇ Functions in one variable:

- $y = -3x^2 + 6x^1 + 4$ ⊛

↳ Value of x that maximises y?

- $\frac{dy}{dx} = -6x + 6$

- Set $-6x + 6 = 0$

↳ solve: $-6x = -6$

$\Rightarrow x = 1$

- Sub $x = 1$ into ⊛

↳ $y = -3(1)^2 + 6(1) + 4$

$= 7$

- - - - - - - - - - - - - - - - - -

◇ Linear Functions in more than one variable

- $y = -3x_1 + 6x_2 + 4x_3$ ∇

↳ Differences:

- Several indep. variables
- ⊛ is quadratic not linear
- ∇ linear in variables
- ⊛ can be visualised graphically
- ∇ cannot be visualised graphically
- ∇ cannot be optimised using differentiation

eg. $\frac{dy}{dx_1} = -3 \rightarrow$ no x_1

$\therefore$ cannot solve for x_1

- How does one optimise ∇ then?

↳ Technique : Linear Programming

↳ Aim : Finding values for

x_1, x_2 and x_3

that optimise y.

↳ Subject to contraints

1RC L2

◇ Methods Available :

- Geometrical method
  - ↳ graphical representation
  - ↳ restrict variable to 2
  - ↳ 2 variables ≡ 2 axis
- Extreme point method
  - ↳ only mention briefly
  - ↳ viable alternative
- Simplex method
  - ↳ not covered.

◇ Optimisation

o Example 1:

→ 2 Departments
Machining & Finishing

→ 2 products A, B } requires processing in each dep.

→ Capacity Machining, Finishing } 1200 min. each

→ Machining : 10 units of A or 5 units of B } each hour

→ Finishing : 6 units of A or 10 units of B } each hour

→ Each unit of B requires 3kg raw material
↳ max. 270 kg raw material per day

→ Contribution per unit of product :
A : R2
B : R2,50

→ Aim : Specify the product mix which maximises daily contribution

Step 1: State the objective function

- $P = 2A + 2{,}5B$

Step 2: Determine the constraints

- 3 constraints
  - → machine time ①
  - → finishing time ②
  - → raw material ③

① * 1200 m.t. available
* Amount of time per unit of A?

↳ 10 units per 60 min
⇒ 60 minutes per 10 units
⇒ 6 minutes per unit

* Amount of time per unit of B?

↳ 12 min. per unit

∴ M.T. constraint:

$$6A + 12B \leq 1200$$

② * Similarly

↳ $10A + 6B \leq 1200$

③ $3B \leq 270$

- constraints in summary:

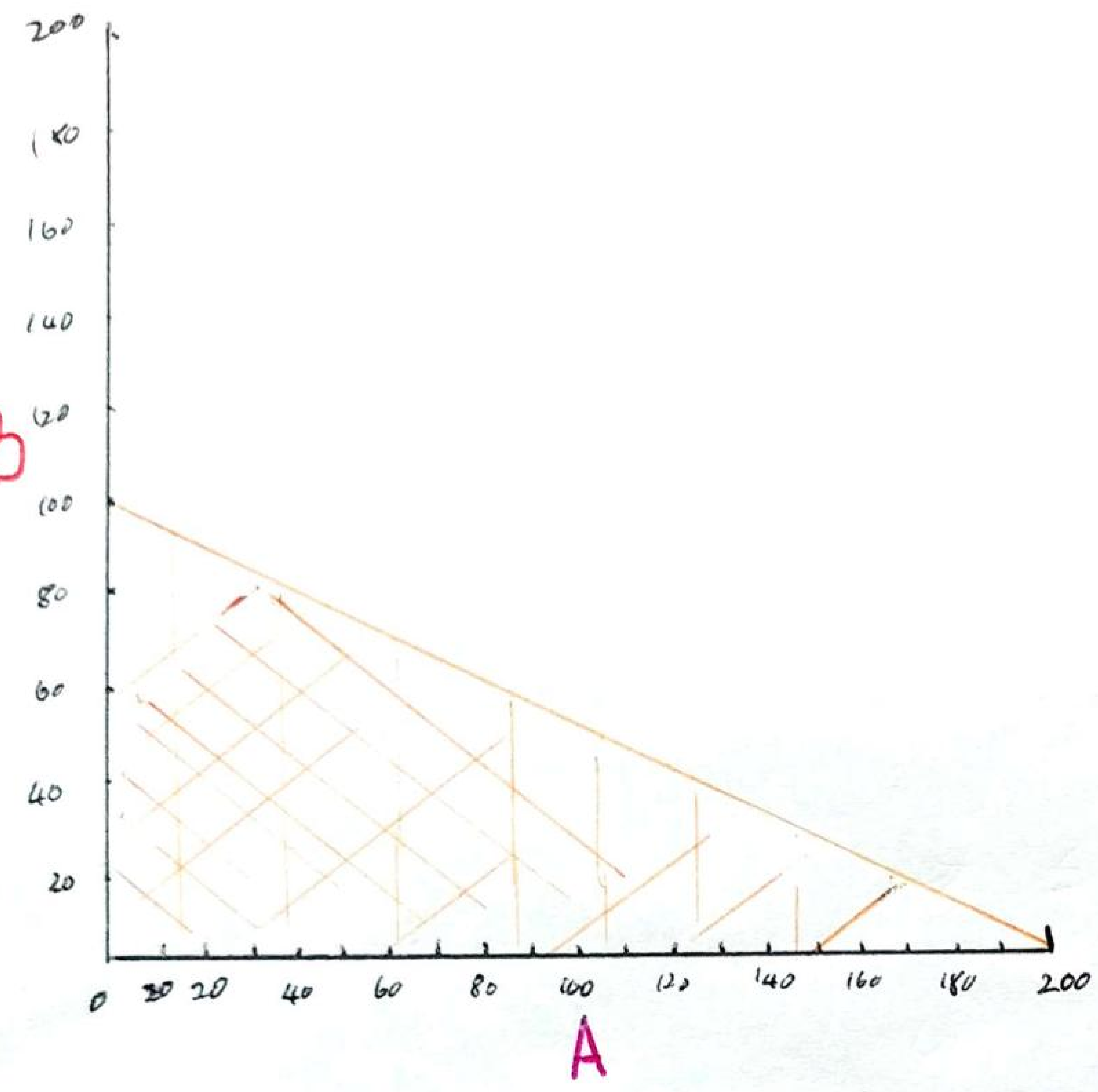
① $6A + 12B \leq 1200$
② $10A + 6B \leq 1200$
③ $3B \leq 270$

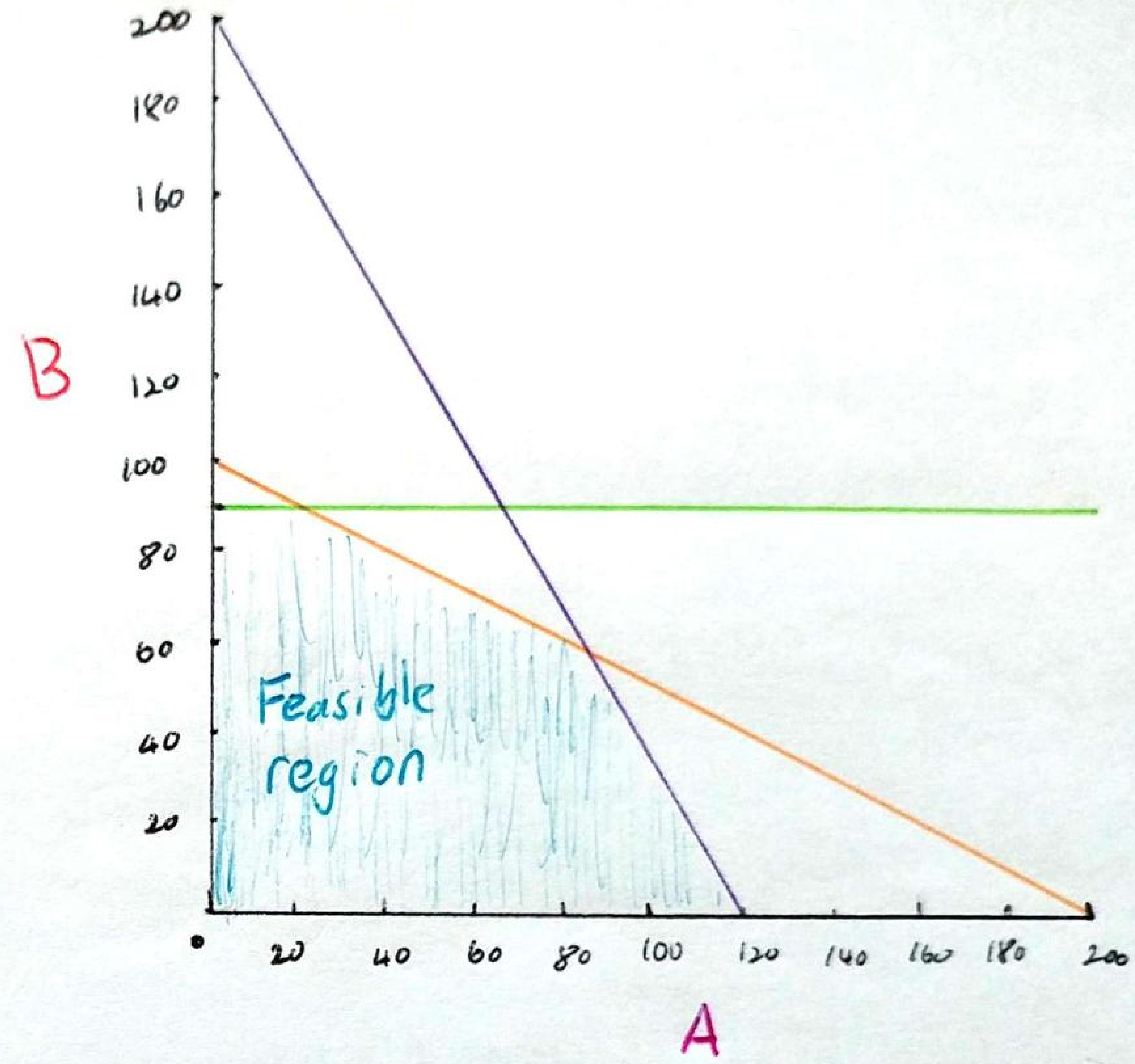
- note: since cannot produce negative number of units

↳ Implicit constraint:
$A \geq 0$
$B \geq 0$

Step 3: Represent constraints graphically

- since $A, B \geq 0$ → consider only positive quadrant
- plot constraint ①:
  - ↳ rewrite equation: $B \leq -0{,}5A + 100$
  - ↳ "$\leq$" indicates feasible region is below the line
  - ↳ easy way to plot: look for x & y intercepts





→ now graph other constraints

(2) $10A + 6B \leq 1200$

$\Rightarrow B \leq -\frac{5}{3}A + 200$

↳ B intercept : 200 } draw line
A intercept : 120

(3) $3B \leq 270$

4RC L2

Step 4: Represent the objective function graphically

- $P = 2A + 2,5B$

$\Leftrightarrow B = -0,8A + \frac{P}{2,5}$

- Aim: To maximise profit

$\therefore$ Want profit line to have as large an intercept as possible

↳ Start with arbitrary P (choose so that it is middle of y-axis)

Let $\frac{P}{2,5} = 100 \Rightarrow P = 250$

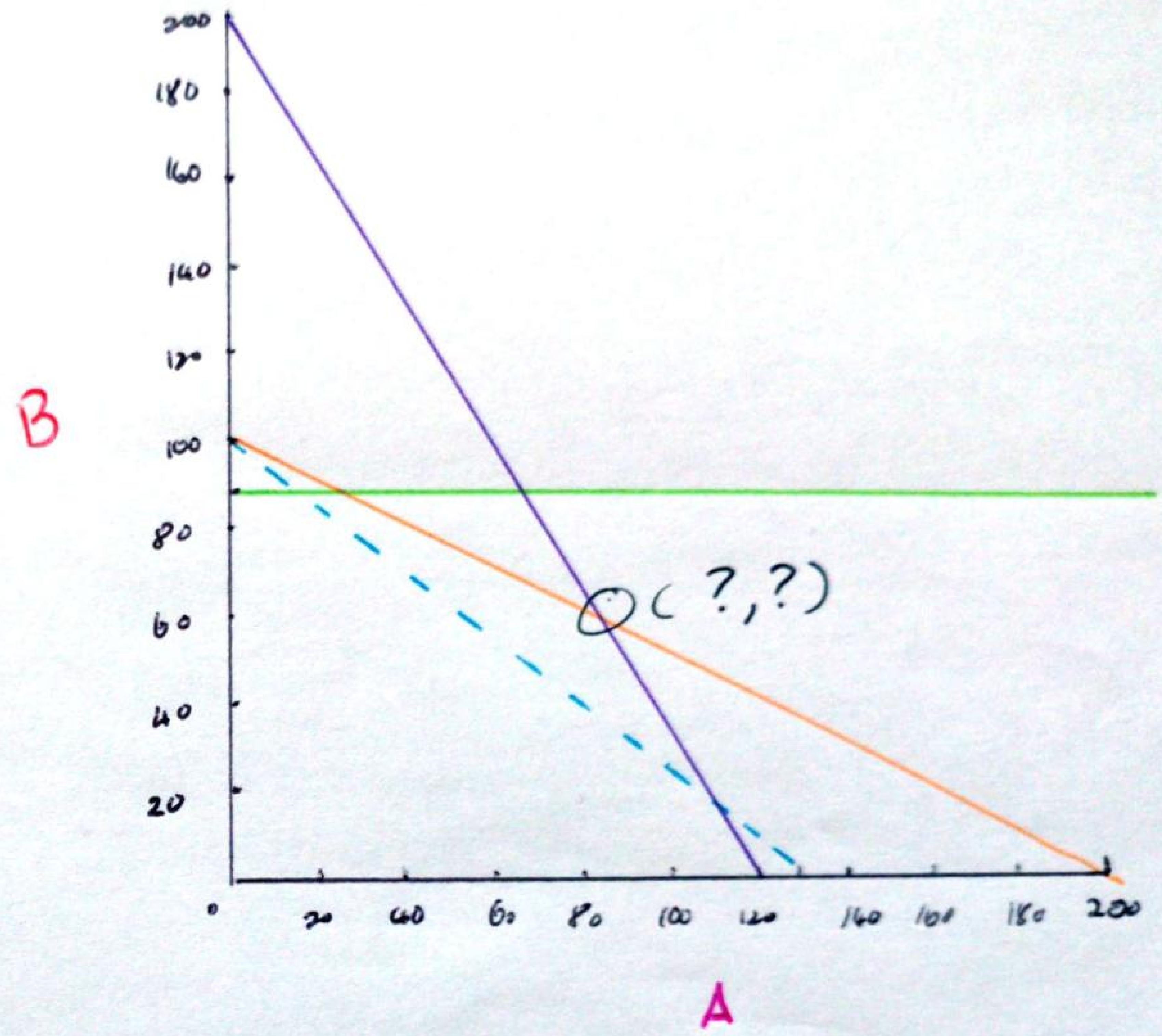
↳ $\therefore$ graph $B = -0,8A + 100$

↳ Since different values of P only affect intercept

$\Rightarrow$ move profit line parallel to the one we got

↳ Until intercept is as large as possible

↳ at least 1 point is in feasible region



$\Rightarrow$ Line obtained crosses point where ① and ② cross!

☐ Step 5: Calculate the optimal solution and resulting optimal profit

- Solve the equations:

(1) $6A + 12B = 1200$

and

(2) $10A + 6B = 1200$

$\Rightarrow A = 85{,}71$

$\Rightarrow B = 57{,}14$

- note: * numbers need rounding
  * fraction of units not allowed

- Final solution

↳ produce 85 units of A
  & 57 units of B

↳ optimal profit

$P = 2\cdot(85) + 2{,}5\cdot(57)$

$= £312{,}50$