In this section, we present an inventory problem with two decision variables and two probabilistic components. The owner of a hardware store would like to establish the *order quantity* and *reorder point* for a particular product that has probabilistic (uncertain) daily demand and reorder lead time. He wants to make a series of simulation runs, trying out various order quantities and reorder points, to minimize his total inventory cost for the item. Inventory costs in this case include ordering, holding, and stockout costs.

## Simkin's Hardware Store

Mark Simkin, owner and general manager of Simkin Hardware, wants to find a good, low-cost inventory policy for one particular product: the Ace model electric drill. Due to the complexity of this situation, he has decided to use simulation to help with this. The first step in the simulation process seen in Figure 13.1 is to define the problem. Simkin specifies this to be finding a good inventory policy for the Ace electric drill.

In the second step of this process, Simkin identifies two types of variables: the controllable and uncontrollable inputs. The controllable inputs (or decision variables) are the order quantity and the reorder point. Simkin must specify the values that he wishes to consider. The other important variables are the uncontrollable inputs: the fluctuating daily demand and the variable lead time. Monte Carlo simulation is used to simulate the values for both of these.

Daily demand for the Ace model drill is relatively low but subject to some variability. Over the past 300 days, Simkin has observed the sales shown in column 2 of Table 13.6. He converts this historical frequency data into a probability distribution for the variable daily demand (column 3). A cumulative probability distribution is formed in column 4. Finally, Simkin establishes an interval of random numbers to represent each possible daily demand (column 5).

When Simkin places an order to replenish his inventory of Ace electric drills, there is a delivery lag of 1 to 3 days. This means that lead time can also be considered a probabilistic variable. The number of days it took to receive the past 50 orders is presented in Table 13.7. In a fashion similar to that for the demand variable, Simkin establishes a probability distribution for the lead time variable (column 3 of Table 13.7), computes the cumulative distribution (column 4), and assigns random number intervals for each possible time (column 5).

The third step in the simulation process is to develop the simulation model. A **flow diagram**, or **flowchart**, is helpful in the logical coding procedures for programming this simulation process (see Figure 13.3).

In flowcharts, special symbols are used to represent different parts of a simulation. The rectangular boxes represent actions that must be taken. The diamond-shaped figures represent branching points where the next step depends on the answer to the question in the diamond. The beginning and ending points of the simulation are represented as ovals or rounded rectangles.

The fourth step of this simulation is to specify the values of the variables that we wish to test.

***A delivery lag is the lead time in receiving an order—the time between when the order was placed and when it was received.***

The first inventory policy that Simkin Hardware wants to simulate is an order quantity of 10 with a reorder point of 5. That is, every time the on-hand inventory level at the end of the day is 5 or less, Simkin will call his supplier and place an order for 10 more drills. If the lead time is 1 day, by the way, the order will not arrive the next morning but at the beginning of the following working day.

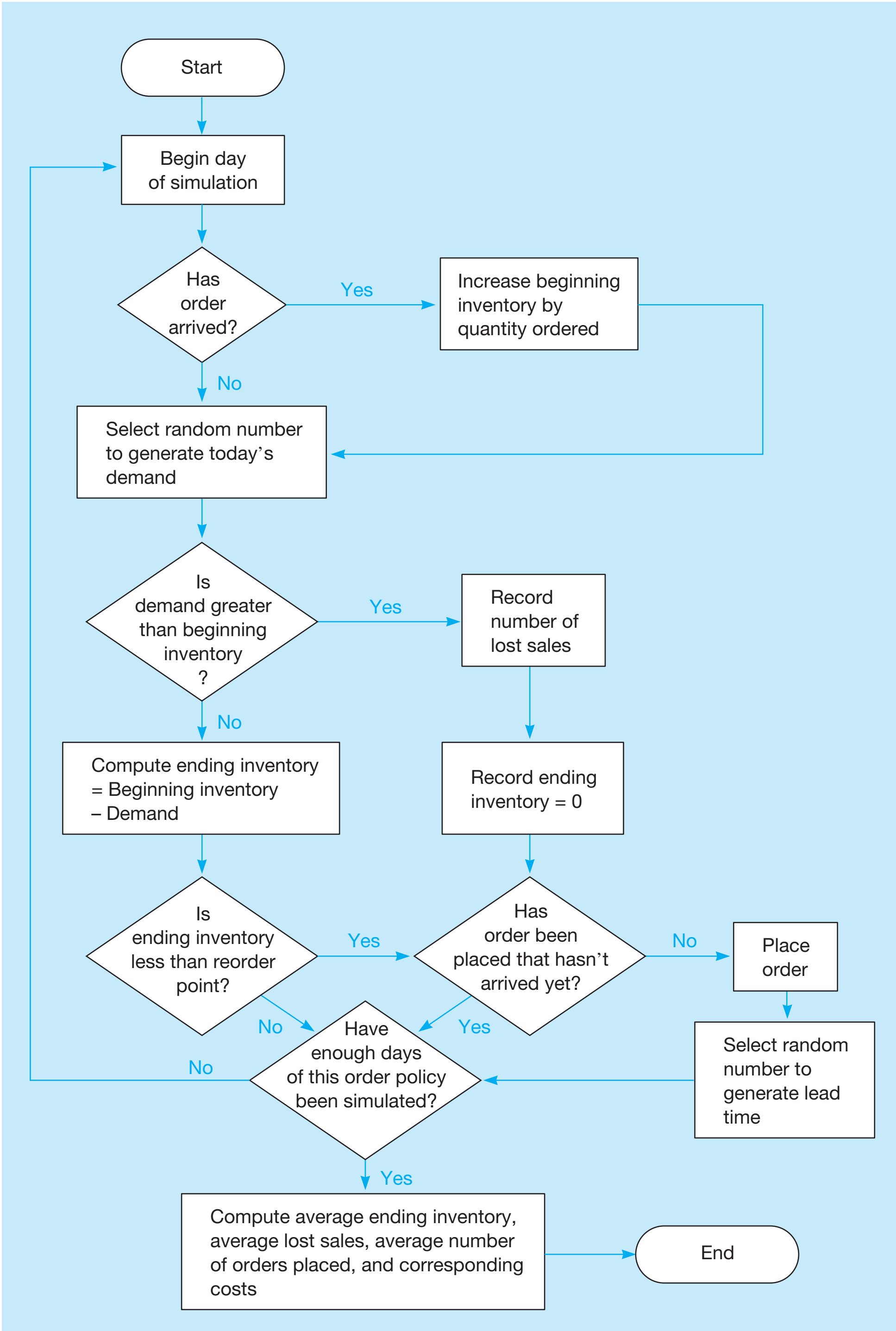
**TABLE 13.6**
Probabilities and Random Number Intervals for Daily Ace Drill Demand

| (1) DEMAND FOR ACE DRILL | (2) FREQUENCY (DAYS) | (3) PROBABILITY | (4) CUMULATIVE PROBABILITY | (5) INTERVAL OF RANDOM NUMBERS |
|---|---|---|---|---|
| 0 | 15 | 0.05 | 0.05 | 01 to 05 |
| 1 | 30 | 0.10 | 0.15 | 06 to 15 |
| 2 | 60 | 0.20 | 0.35 | 16 to 35 |
| 3 | 120 | 0.40 | 0.75 | 36 to 75 |
| 4 | 45 | 0.15 | 0.90 | 76 to 90 |
| 5 | 30 | 0.10 | 1.00 | 91 to 00 |
| | 300 | 1.00 | | |

**TABLE 13.7**
Probabilities and Random Number Intervals for Reorder Lead Time

| (1) LEAD TIME (DAYS) | (2) FREQUENCY (ORDERS) | (3) PROBABILITY | (4) CUMULATIVE PROBABILITY | (5) RANDOM NUMBER INTERVAL |
|---|---|---|---|---|
| 1 | 10 | 0.20 | 0.20 | 01 to 20 |
| 2 | 25 | 0.50 | 0.70 | 21 to 70 |
| 3 | 15 | 0.30 | 1.00 | 71 to 00 |
| | 50 | 1.00 | | |

**FIGURE 13.3**
Flow Diagram for Simkin's Inventory Example



The fifth step of the simulation process is to actually conduct the simulation, and the Monte Carlo method is used for this. The entire process is simulated for a 10-day period in Table 13.8. We can assume that the beginning inventory is 10 units on day 1. (Actually, it makes little difference in a long simulation what the initial inventory level is. Since we would tend in real life to simulate hundreds or thousands of days, the beginning values would tend to be averaged out.) Random numbers for Simkin's inventory problem are selected from the second column of Table 13.4.

Table 13.8 is filled in by proceeding one day (or line) at a time, working from left to right. It is a four-step process:

*Here is how we simulated the Simkin Hardware example.*

1. Begin each simulated day by checking whether any ordered inventory has just arrived (column 2). If it has, increase the current inventory (in column 3) by the quantity ordered (10 units, in this case).
2. Generate a daily demand from the demand probability distribution in Table 13.6 by selecting a random number. This random number is recorded in column 4. The demand simulated is recorded in column 5.
3. Compute the ending inventory every day and record it in column 6. Ending inventory equals beginning inventory minus demand. If on-hand inventory is insufficient to meet the day's demand, satisfy as much as possible and note the number of lost sales (in column 7).
4. Determine whether the day's ending inventory has reached the reorder point (5 units). If it has and if there are no outstanding orders, place an order (column 8). Lead time for a new order is simulated by first choosing a random number from Table 13.4 and recording it in column 9. (We can continue down the same string of the random number table that we were using to generate numbers for the demand variable.) Finally, we convert this random number into a lead time by using the distribution set in Table 13.7.

**TABLE 13.8** Simkin Hardware's First Inventory Simulation

| ORDER QUANTITY = 10 UNITS | | | | REORDER POINT = 5 UNITS | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| (1) DAY | (2) UNITS RECEIVED | (3) BEGINNING INVENTORY | (4) RANDOM NUMBER | (5) DEMAND | (6) ENDING INVENTORY | (7) LOST SALES | (8) ORDER | (9) RANDOM NUMBER | (10) LEAD TIME |
| 1 | … | 10 | 06 | 1 | 9 | 0 | No | | |
| 2 | 0 | 9 | 63 | 3 | 6 | 0 | No | | |
| 3 | 0 | 6 | 57 | 3 | 3[a] | 0 | Yes | 02[b] | 1 |
| 4 | 0 | 3 | 94[c] | 5 | 0 | 2 | No[d] | | |
| 5 | 10[e] | 10 | 52 | 3 | 7 | 0 | No | | |
| 6 | 0 | 7 | 69 | 3 | 4 | 0 | Yes | 33 | 2 |
| 7 | 0 | 4 | 32 | 2 | 2 | 0 | No | | |
| 8 | 0 | 2 | 30 | 2 | 0 | 0 | No | | |
| 9 | 10[f] | 10 | 48 | 3 | 7 | 0 | No | | |
| 10 | 0 | 7 | 88 | 4 | 3 | 0 | Yes | 14 | 1 |
| | | | | | Total 41 | 2 | | | |

[a]This is the first time inventory dropped below the reorder point of 5 drills. Because no prior order was outstanding, an order is placed.

[b]The random number 02 is generated to represent the first lead time. It was drawn from column 2 of Table 13.4 as the next number in the list being used. A separate column could have been used to draw lead time random numbers from if we had wanted to do so, but in this example we did not do so.

[c]Again, notice that the random number 02 was used for lead time (see footnote b), so the next number in the column is 94.

[d]No order is placed on day 4 because there is one outstanding from the previous day that has not yet arrived.

[e]The lead time for the first order placed is 1 day, but as noted in the text, an order does not arrive the next morning but at the beginning of the following working day. Thus, the first order arrives at the start of day 5.

[f]This is the arrival of the order placed at the close of business on day 6. Fortunately for Simkin, no lost sales occurred during the 2-day lead time until the order arrived.

## Analyzing Simkin's Inventory Costs

Now that the simulation results have been generated, Simkin is ready to proceed to step 6 of this process—examining the results. Since the objective is to find a low-cost solution, Simkin must determine, given these results, what the costs would be. In doing this, Simkin finds some interesting results. The average daily ending inventory is

$$\text{Average ending inventory} = \frac{41 \text{ total units}}{10 \text{ days}} = 4.1 \text{ units per day}$$

We also note the average lost sales and number of orders placed per day:

$$\text{Average lost sales} = \frac{2 \text{ sales lost}}{10 \text{ days}} = 0.2 \text{ unit per day}$$

$$\text{Average number of orders placed} = \frac{3 \text{ orders}}{10 \text{ days}} = 0.3 \text{ order per day}$$

These data are useful in studying the inventory costs of the policy being simulated.

Simkin Hardware is open for business 200 days per year. Simkin estimates that the cost of placing each order for Ace drills is \$10. The cost of holding a drill in stock is \$6 per drill per year, which can also be viewed as 3 cents per drill per day (over a 200-day year). Finally, Simkin estimates that the cost of each shortage, or lost sale, is \$8. What is Simkin's total daily inventory cost for the ordering policy of order quantity $Q = 10$ and reorder point $\text{ROP} = 5$?

Let us examine the three cost components:

$$\begin{aligned}
\text{Daily order cost} &= (\text{Cost of placing one order}) \\
&\quad \times (\text{Number of orders placed per day}) \\
&= \$10 \text{ per order} \times 0.3 \text{ order per day} = \$3 \\
\text{Daily holding cost} &= (\text{Cost of holding one unit for one day}) \\
&\quad \times (\text{Average ending inventory}) \\
&= \$0.03 \text{ per unit per day} \times 4.1 \text{ units per day} \\
&= \$0.12 \\
\text{Daily stockout cost} &= (\text{Cost per lost sale}) \\
&\quad \times (\text{Average number of lost sales per day}) \\
&= \$8 \text{ per lost sale} \times 0.2 \text{ lost sale per day} \\
&= \$1.60 \\
\text{Total daily inventory cost} &= \text{Daily order cost} + \text{Daily holding cost} \\
&\quad + \text{Daily stockout cost} = \$4.72
\end{aligned}$$

Thus, the total daily inventory cost for this simulation is \$4.72. Annualizing this daily figure to a 200-day working year suggests that this inventory policy's cost is approximately \$944.

***It is important to remember that the simulation should be conducted for many, many days before it is legitimate to draw any solid conclusions.***

Once again we want to emphasize something very important. This simulation should be extended many more days before we draw any conclusions as to the cost of the inventory policy being tested. If a hand simulation is being conducted, 100 days would provide a better representation. If a computer is doing the calculations, 1,000 days would be helpful in reaching accurate cost estimates.

Let's say that Simkin *does* complete a 1,000-day simulation of the policy that order quantity = 10 drills, reorder point = 5 drills. Does this complete his analysis? The answer is *no*—this is just the beginning! We should now verify that the model is correct and validate that the model truly represents the situation on which it is based. As indicated in Figure 13.1, once the results of the model are examined, we may want to go back and modify the model that we have developed. If we are satisfied that the model performed as we expected, then we can specify other values of the variables. Simkin must now compare *this* potential strategy to other possibilities. For example, what about $Q = 10$, $\text{ROP} = 4$; or $Q = 12$, $\text{ROP} = 6$; or $Q = 14$, $\text{ROP} = 5$? Perhaps every combination of values of $Q$ from 6 to 20 drills and ROP from 3 to 10 drills should be simulated. After simulating all reasonable combinations of order quantities and reorder points, Simkin would go to step 7 of the simulation process and probably select the pair that yields the lowest total inventory cost.