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Queueing Theory Part 4 / Toustaanteorie Deel 4

LSCM344 - August 2022
Lecturer - HW Freiboth

Slides adapted from those originally compiled
for the module by Dr Neil Jacobs (2019)

$M/M/1$ met eindige bron / $M/M/1$ with finite source

"$M/M/1$ met eindige bron" is eintlik $M/M/1/GD/N/N$

"$M/M/1$ with finite source" is actually $M/M/1/GD/N/N$

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"$M/M/1$ met eindige bron" is eintlik $M/M/1/GD/N/N$

Tusssenaankomstye en dienstye is eksponensieel verdeel, daar is 1 bediener, kliënte in die tou volg algemene toudissipline, daar is beperkte ruimte in die stelsel vir slegs N kliënte en die kliënte populasie bevat slegs N lede.

"$M/M/1$ with finite source" is actually $M/M/1/GD/N/N$

Interarrival times and service times are exponentially distributed, there is 1 server, clients in the queue follow the general queuing discipline, there is space for only N clients in the system and the client contains N members.

Voorbeelde / Examples

- Vloot diensfasiliteit.
- Smousoutomate.
- Numeries beheerde masjien werskwinkel.
- Klein span pakkers in pakhuis.
- Vloot wipbakvragmotors wat by 'n uitgrawing materiaal wegry.

- Fleet service facility.
- Vending machines.
- Numerically controlled machine workshop.
- Small team of packers in a warehouse.
- Fleet tipper trucks removing material at an excavation.

'n Voorbeeld oefening / An example exercise

Joel Ebbs het 'n droogdok vir sy vloot van 10 skepe. Elkeen van die skepe benodig herstelwerk en onderhoud (h&o) 1 keer in 4 jaar. Dit neem gemiddeld 6 maande om h&o uit te voer op 'n skip in die droogdok. Deur uitbreiding van die onderhoudsvermoë, teen R12 mil/jaar kan die h&o tyd afgebring word na 4 maande per skip. Behoort dit gedoen te word? 'n Skip op die see verdien vir Joel R11 mil per jaar.

Joel Ebbs owns a drydock for his fleet of 10 ships. Each one of the ships require repairs and maintenance (r&m) once in 4 years. It takes on average 6 months to carry out r&m on a ship in the drydock. By expanding his maintenace capacity, at R12 mil/year, the r&m time can be brought down to 4 months. Should it be done? A ship on the ocean earns Joel R11 mil per year.

Joel wil weet / Joel wants to know

1. Waarskynlikheid dat die droogdok ledig is.
2. Gemiddelde toulengte van skepe by die droogdok.
3. Gemiddelde aantal skepe buite aksie.
4. Gemiddelde wagtyd in die tou.
5. Gemiddelde tyd wat 'n skip buite aksie is per geleentheid.
6. Waarskynlikheid dat n-aantal skepe by die droogdok lê.

1. Probability of the drydock being idle.
2. Average quelength of ships at the drydock.
3. Average number of ships out of action.
4. Average waiting time in the queue.
5. Average time a ship is out of action per occurrence.
6. Probability of n-number of ships at the dry-dock.

Joel wil weet / Joel wants to know

1. Waarskynlikheid dat die droogdok ledig is. P_0
2. Gemiddelde toulengte van skepe by die droogdok. L_q
3. Gemiddelde aantal skepe buite aksie. L
4. Gemiddelde wagtyd in die tou. W_q
5. Gemiddelde tyd wat 'n skip buite aksie is per geleentheid. W
6. Waarskynlikheid van n aantal skepe by die droogdok lê . $P_n, \quad n \in \{1, 2, .., N\}$

1. Probability of the drydock being idle. P_0
2. Average quelength of ships at the drydock. L_q
3. Average number of ships out of action. L
4. Average waiting time in the queue. W_q
5. Average time a ship is out of action per occurrence. W
6. Probability of n-number of ships at the dry-dock. $P_n, \quad n \in \{1, 2, .., N\}$

$M/M/1/GD/N/N$- toustaanstelsel / queueing system

Aannames

1. Kliënte in die tou volg die algemene tou-dissilpline (GD) wat 'n eerste in eerste bedien logika is.
2. Kliënte draai nie om voordat hulle in die tou inval nie.
3. Kliënte bly in die tou tot hulle diens ontvang, daar is met ander woorde geen uitval uit die tou nie.
4. Die aantal aankomste per tydseenheid vir een kliënt is Poisson verdeel met parameter λ bekend.

Assumptions

1. Clients in the queue follow the general queueing discipline (GD) which is a first come first serve logic.
2. Clients do not turn around before joining the queue, thus no balking takes place
3. Clients remain in the queue until they receive service, thus no reneging takes place.
4. The number of arrivals per time unit for one client is Poisson distributed with λ known.

$M/M/1/GD/N/N$- toustaanstelsel / queueing system

5. Dienstye is onderling onafhanklik en volg die identiese eksponensiële verdeling met dienstempo μ bekend.
6. Die besoekende populasie bevat N kliënte.
7. Daar is slegs ruimte vir N kliënte in die stelsel.

5. Service times are mutually independent and follow the identical exponential distribution with service rate μ known.
6. The calling population contains N clients.
7. There is only space for N clients in the system.

Formules / Formulas

$$P_0 = \frac{1}{\sum_{n=0}^{N} \frac{N!}{(N-n)!} \left(\frac{\lambda}{\mu}\right)^n} \tag{31}$$

$$L_q = N - \left(\frac{\lambda + \mu}{\lambda}\right)(1 - P_0) \tag{32}$$

$$L = L_q + (1 - P_0) \tag{33}$$

$$W_q = \frac{L_q}{(N - L)\,\lambda} \tag{34}$$

$$W = W_q + \frac{1}{\mu} \tag{35}$$

$$P_n = \frac{N!}{(N-n)!} \left(\frac{\lambda}{\mu}\right)^n P_0 \;\forall\; n \;\in \{0, 1, 2, \ldots, N\} \tag{36}$$

$$\bar{\lambda} = (N - L)\,\lambda \tag{37}$$

Konstante dienstyd model $M/D/1$ / Constant service time model $M/D/1$

Die $M/D/1$-model is in werklikheid 'n $M/D/1/GD/\infty/\infty$-model.
Klas verskaf 'n uiteensetting van die Kendall-Lee-notasie.

The $M/D/1$-model is in reality a $M/D/1/GD/\infty/\infty$-model.
Class to provide an exposition of Kendall Lee notation.

Voorbeelde van konstante dienstyd modelle / Examples of constant service time models

- Werksaamhede wat plaasvind volgens konstante tydhouer - plaatkoekie-oond in 'n ontbyt-salon.
- Ingang deur 'n draai-hek.
- Automatiese motor-wassery.
- Sekuriteitskandering van 'n houer deur 'n masjien.
- Standaard inspeksie.

- Operations occurring according to a constant timer - flapjack oven in a breakfast lounge.
- Entrance through a turnstile.
- Automatic car wash.
- Security scanning of a container by a machine.
- Standard inspection.

$M/D/1/GD/\infty/\infty$- toustaanstelsel aannames / queueing system assumptions

1. Kliënte in die tou volg die algemene tou-dissipline (GD) wat 'n eerste in eerste bedien logika is.
2. Kliënte draai nie om voordat hulle in die tou inval nie.
3. Kliënte bly in die tou tot hulle diens ontvang, daar is met ander woorde geen uitval uit die tou nie.
4. Tussenaankomstye is onderling onafhanklik en volg die identiese eksponensiële verdeling met aankomstempo (λ) bekend.

1. Clients in the queue follow the general queueing discipline (GD) which is a first come first serve logic.
2. Clients do not turn around before joining the queue, thus no balking takes place
3. Clients remain in the queue until they receive service, thus no reneging takes place.
4. Interarrival times are mutually independent and follow the identical exponential distribution with arrival rate (λ) known.

$M/D/1/GD/\infty/\infty$- toustaanstelsel / queueing system

Aannames vervolg

5. Dienstye is konstant en die dienstempo (μ) is bekend.
6. Die besoekende populasie is oneindig groot.
7. Daar is geen ruimtebeperking in die stelsel nie.
8. Die aankomstempo is kleiner as die dienstempo, met ander woorde $\lambda < \mu \equiv \frac{\lambda}{\mu} < 1$.

Assumptions continued

5. Service times are are constant and the service rate μ is known.
6. The calling population is infinitely large.
7. There is no space constraint in the system
8. The arrival rate is less than the service rate, in other words $\lambda < \mu \equiv \frac{\lambda}{\mu} < 1$.

Houer x-straal inspeksie / Container x-ray inspection

Houers wat 'n hawe binnegebring word, moet eers x-straal skandering ondergaan voordat dit na die bergingsruimte gaan. Die x-straal beeld word af-lyn ge-ëvalueer en aksie word slegs geneem indien anomalieë waargeneem word, derhalwe is die skandeertyd van 90 sekondes per houer, vir alle houers presies dieselfde. Tussenaankomstye van houers is eksponensieel verdeel en arriveer teen 'n tempo van 800 houers per dag. Bepaal die standaard toustaan-karakteristieke van hierdie stelsel as aanvaar kan word dat al die aannames geld vir 'n $M/D/1$-toustaanmodel.

Containers brought into a port must first be x-ray scanned before being stowed. The x-ray image is evaluated off-line and action is only taken if anomalies are observed, therefore the scanning time of 90 seconds per container, is exactly the same for all containers. Interarrival times of containers are exponential distributed and arriving at a rate of 800 containers per day. Determine the standard queueing characteristics of this system if it can be assumed that all the assumptions for a $M/D/1$-queueing model hold.

Formulas

$$L_q = \frac{\lambda^2}{2\mu(\mu - \lambda)} \tag{38}$$

$$L_s = \frac{\lambda}{\mu} \tag{39}$$

$$L = L_q + L_s \tag{40}$$

$$W_q = \frac{\lambda}{2\mu(\mu - \lambda)} \tag{41}$$

$$W_s = \frac{1}{\mu} \tag{42}$$

$$W = W_q + W_s \tag{43}$$

$$\rho = \frac{\lambda}{\mu} \tag{44}$$