Aix–Marseille Université
Faculté des Sciences
Master M1 MAS
Prof. Kai Schneider

1st semester 2024/2025

Introduction to Data Science

Homework 3: Introduction/Representation structured data

Exercise 1: Introduction to DS

Give a short definition of the following terms (5 lines max for each):

i) Data Science; ii) Data scientist; iii) Data analysis; iv) Machine learning; v) Big data;

Exercise 2: Projection and bases

A) We consider the unit vector $\phi = [1/\sqrt{2}, 1/\sqrt{2}]^T$ and the two vectors $x = [2,1]^T$ and $[-2,1]^T$ in $I\!R^2$.

a) Compute the orthogonal projection $\widehat{x} = \langle x, \phi \rangle \phi$ of x and y onto ϕ.

b) Show that $x - \widehat{x}$ is orthogonal to ϕ

c) Make a graphical illustration.

B) Show that the vectors $e_1 = [1,0,0]^T$, $e_2 = [0,1,0]^T$ and $e_3 = [0,0,1]^T \in I\!R^3$ are linearly independent, i.e. $\sum_{k=1}^{3} \alpha_k \, e_k = 0$, if and only if $\alpha_k = 0$ for $k = 1,2,3$. Show also that any vector $x \in I\!R^3$ can be represented using these three vectors.

C) We consider the set of vectors $\Phi = \{\phi_k\}_{k \in I\!N} \subset \mathbb{C}^{[-1/2,1/2]}$, where $\phi_0(t) = 1$ and $\phi_k(t) = \sqrt{2}\cos(2\pi k t)$ for $k = 1, 2, \ldots$.

a) Make a graphical illustration of ϕ_0, ϕ_1 and ϕ_2.

b) Show that the set of vectors Φ is orthogonal, i.e. $\langle \phi_k, \phi_\ell \rangle = \delta_{k,\ell}$.

c) Show that the set of vectors is orthogonal to the set of odd functions $S_{odd} = \{s | s(t) = -s(-t) \text{ for } t \in [-1/2, 1/2]\}$

Exercise 3: Projection onto a subspace

We consider two rectangular real-valued matrices A and B,

where $A = \frac{1}{2} \begin{bmatrix} 1 & 1 & -1 \\ -1 & 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix}$

a) Show that A is a left inverse of B.

b) Compute $P = BA$ and show that it is a projection operator.

c) Is P an orthogonal or oblique projection operator?

d) Compute Px for $x = [6 \quad 6 \quad 8]^t$ and make a graphical illustration of the two-dimensional range of P.

e) Determine the kernel (null space) of P.