Aix–Marseille Université
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Master M1 MAS
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# Introduction to Data Science

Homework 2: **Representation structured data**

## Exercise 1: Complex numbers

We consider the following complex numbers $z \in \mathbb{C}$:

$$3 + 2i, \quad \frac{2-i}{2-3i}, \quad \frac{1}{(3-i)^2}, \quad \exp(i2\pi), \quad \exp(i\pi), \quad i^n \ (n \in \mathbb{N})$$

Note that $i = \sqrt{-1}$. Decompose them into real and imaginary parts, give their polar representation ($z = re^{i\phi}$) and give a graphical illustration in $\mathbb{R}^2$.

## Exercise 2: Discrete signals and norms

We consider three discrete signals of finite length:

$$S_1 = \{3, -4, 5, 4, -1, 2, 4, 5, 0, -2, 3, -1, 5, 6, 7, 3, -1, 2, 1, 3\}$$

$$S_2 = \{1, 0, 0, 0, 7, 0, -2, 0, 0, 0, 1, 2, 1, 0, -1, 0, 0, 4, -4, 0\}$$

$$S_3 = \{0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}$$

a) Determine the length of each signal.

b) Compute the different norms $\ell^0, \ell^1, \ell^2$ and $\ell^\infty$ of the three signals. What do you observe?

c) What is the difference between the three signals? Which signal is sparser?

d) Measure the difference between the signals, i.e., compute $||S_i - S_j||_p$ for $p = 1, 2$ and $\infty$ and $i, j = 1, 2, 3$ and $i \neq j$.

The $\ell^0$ 'norm' is defined by $||x||_0 = \lim_{p \to 0} \sum_k |x|^p$ and counts the number of non zero entries of a vector or sequence. For $1 < p < \infty$ the corresponding $\ell^p$ norms are defined as $||x||_p = \sum_k |x_k|^p$ and for $p = \infty$ we have $||x||_\infty = \sup_k |x_k|$

## Exercise 3: Norms

Verify if $||x||_\alpha$ satisfies for $\alpha = 0, 1, 2$ and $\infty$ the properties of a norm.

Note that the $\ell^0$ 'norm' is defined by $||x||_0 = \lim_{p \to 0} \sum_k |x|^p$ and counts the number of non zero entries of a vector, here we define $0^0 = 0$. For $1 < p < \infty$ the corresponding $\ell^p$ norms are defined as $||x||_p = \sum_k |x_k|^p$ and for $p = \infty$ we have $||x||_\infty = \sup_k |x_k|$.

## Exercise 4: Norms

Give a graphical illustration of the set $||x||_\alpha = 1$ for $x \in \mathbb{R}^2$ and $\alpha = 1, 2$ and $\infty$.