

# Course Maths for Data Sciences

## TP 1. Introduction to Variational Auto Encoder.

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### Context and objective.

The goal of this practical session is to build some VAE to be applied to the MNIST database of digits, using the library Pytorch.

## 1 Building the VAE.

### 1.1 Encoding.

Given data  $x$ , the encoding part of the VAE is related to the construction of the so-called bottleneck distribution  $q_\phi(z|x)$  describing the transform of the data  $x$  into a latent variable  $z$ . It is modeled by a series of neural network layers taking  $x$  as input and resulting in the mean  $\mu_\phi(x)$  and covariance terms  $\Sigma_\phi(x)$  of the distribution  $q_\phi(z|x)$  (assumed to be multivariate Gaussian).

#### *Exercice 1.1*

Write pytorch modules that define the encoding part of the VAE. Two types of architectures are suggested:

- a first one exclusively made of successive dense layers,
- a second one made of convolutional layers.

Both models should end with dense layers given as output two vectors of  $\mu_\phi(x)$  and  $\Sigma_\phi(x)$  having the dimension of the latent variable. The specifications of the architectures can be freely set but should be discussed; the number of parameters should be reported.

### 1.2 Decoding.

Given latent variable  $z$ , the decoding part of the VAE is related to the reconstruction distribution  $p_\theta(z|x)$  describing the construction of the data  $x$  from the latent variable  $z$ . As for the encoding part, it is modeled by a series of neural network layers that usually reflect in a symmetric way the encoding part. This series of layers taking  $z$  as input and result in a reconstruction of  $x$ .

#### *Exercise 1.2*

Write pytorch modules that define the decoding part of the VAE. As for the encoding part, we may distinguish two types of architectures, those exclusively made of successive dense layers from those

having convolutional layers.

### 1.3 VAE

#### Exercise 1.3

Using the encoding and decoding modules, create another module to define the VAE that also includes a part for the simulation of the latent variable.

## 2 Training the VAE.

#### Exercise 2.1

Set the data for training using the MNIST database Define the optimization setting (criterion to minimize, minimization method, etc.) Define a script to learn model and apply it.

## 3 Experimenting the VAE.

#### Exercise 3.1

Generate and visualize new data from the learned models. Propose a graphical representation of the latent space.

#### Exercise 3.2

Compare two architectures (dense and convolutional). Evaluate the effect of the architecture design (number of layers, number of parameters, etc.)

#### Exercise 3.3

Evaluate the effect of the weighting  $\beta$  of the regularization term in the criterion.