# Determination of the Electron to it's Mass Ratio

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This paper will cover the determination of the Electron to mass ratio using the e/m Apparatus. The known value of e/m is $1.759 * 10^{11}\ c/kg$, where c is Columbus and kg is kilograms. Electrons have a charge and a mass, the ratio between these values can be obtained using the apparatus. I determined the value at its closest to be $1.85 * 10^{11}\ c/kg$ which is within 5% accuracy of the the true value. With the best line fit, I achieved an accuracy of 3.9% for the lowest, and an accuracy of 28.2% for the highest. Concluding that a electric field can bend electrons which have been found to have a negative charge though this experiment.

## I. BACKGROUND INFORMATION AND INTRODUCTION

In this day and age, it's a well-known fact that electrons carry a negative charge and have a ridiculously small mass, but proving this was a troubling task. Joseph J. Thomson discovered the negative charge of an electron through an experiment known as the Cathode Ray Tube experiment, where electrons were shot out of an electron gun and were attracted to the positively charged anode in the tube. This leads to what we have today to show his experiment using the Lorentz Force Demonstrator, or in other words, the e/m apparatus. We use a Helmholtz coil to create a magnetic field to bend the electrons, and we control the accelerating voltage of the electrons, giving us a bend in their path. Measuring the radius when the electrons bend far enough to give us a circle gives us all the components we need to evaluate the e/m ratio. J. J. Thomson's work is extremely important because everything we interact with in the world has electrons. We need these electrons to balance out all the atoms around us. Furthermore, electrons are the carriers of electrical charges. Without J. J. Thomson's work, we wouldn't have computers, or electricity, or light, or any of the conveniences we use today.

## II. PROCESS AND THE E/M APPARATUS

before going through the raw data, I want to explain every part of the equipment used today to retrieve the e/m ratio. Firstly, we essentially do the same experiment as J. J. Thomson where we shoot electrons out of an electron gun which contains a heater to heat the cathode, which allows the electrons to be emitted. A cathode and an anode are there to allow the electrons to be accelerated by a potential, and a grid is there to stabilize between the cathode and anode, which helps to focus the shot out electrons. With just the electron gun, we can achieve a beam if it is preformed in the right setting. For the experiment performed here, we used a helium-filled vacuum tube with ideally $10^{-2}$ mm of helium. We did not have the ideal setting, which will show in the results achieved in the report. Furthermore, we will need a magnetic field. The e/m apparatus contains the Helmholtz coil to create an artificial magnetic field. The need for this is to bend the electrons to form a circular path with the right amount of accelerating voltage and the current through the Helmholtz coil.

### A. Figure 1: The e/m Apparatus

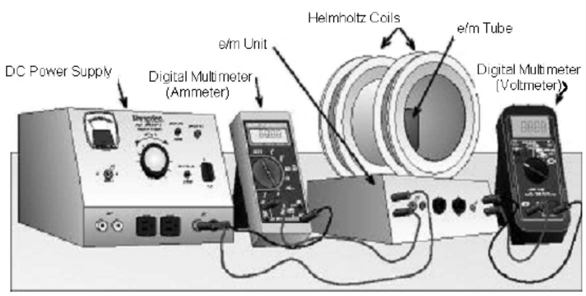


*Figure 1: the general setup for the experiment from [2], nearly identical to what was actually performed in the experiment. Two different voltmeters one for the current through the Helmholtz coils, and one for the accelerating voltage given to the electrons. And a power supply to control the output of the energy.*

There are appropriate voltages and current for the experiment that should not be exceeded to not damage the e/m apparatus. For the electron gun, the coils of the cathode should not exceed 6.3 volts of output. Exceeding it further will damage the electron gun. For the tests performed here, all data sets used a voltage of 6.29 volts for the heater. The range to perform a circular path is from 150–300 volts for the accelerating voltage and a current of 1-2 Amps through the Helmholtz coils. Additionally, the geometry of the Helmholtz coils and, with a little mathematical work, we achieve a constant for the value of the magnetic field, which comes out to be $7.8 * 10^{-4}\ T$ This value was already provided to us with the e/m apparatus. To prove it, however, it is necessary to know the number of turns that the Helmholtz coils make and the radius of the coils.

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$$B = ((N\mu_0)/(5/4)^{3/2}a) * I \quad (1)$$

where B is the magnetic field, N is the number of turns performed by the Helmholtz coils, for this apparatus it is 130 turns per second, $\mu_0$ is the permeability constant of $4\pi * 10^{-7}$, a is the radius of the Helmholtz constant which is 15 cm, and I is the current through the Helmholtz coils.

## III. RAW DATA ANALYSIS

with all the known value of B, we can start recording data sets at different voltages with a constant current, and different current with a constant voltage to obtain different values of the ratio between electron charge, and the mass of the electron. In other words, the e/m ratio.

$$e/m = (2V)/(B^2r^2) \quad (2)$$

where V is the accelerating voltage of the electron, B is the magnetic field constant which is $7.8 * 10^{-4}\ T$ multiplied by the current through the coils, and r is the radius observed from the circular path of the electrons.

First I set a constant accelerating voltage of 150 volts and changed the current through the coils from 1 amps to 1.966 amps, this is to get the most diverse data and to later accurately determine the e/m ratio. With that in mind here is the table and graph demonstrating the the radius's of different current's at the same accelerating voltage.

### A. Table 1: 150 Accelerating Voltage at Different Current's

| I | r | $I^{-2}$ | $(r * 10^{-2})^2$ |
|---|---|---|---|
| 1.000 | 4.375 | 1.000 | 0.00191 |
| 1.066 | 4.075 | 0.880 | 0.00166 |
| 1.133 | 3.850 | 0.779 | 0.00148 |
| 1.200 | 3.725 | 0.694 | 0.00139 |
| 1.266 | 3.575 | 0.624 | 0.00128 |
| 1.333 | 3.225 | 0.563 | 0.00104 |
| 1.400 | 2.925 | 0.510 | 0.00086 |
| 1.466 | 2.750 | 0.465 | 0.00076 |
| 1.533 | 2.500 | 0.426 | 0.00063 |
| 1.600 | 2.350 | 0.391 | 0.00055 |
| 1.666 | 2.000 | 0.360 | 0.00040 |
| 1.733 | 1.900 | 0.333 | 0.00036 |
| 1.800 | 1.775 | 0.309 | 0.00032 |
| 1.866 | 1.675 | 0.287 | 0.00028 |
| 1.933 | 1.550 | 0.268 | 0.00024 |

Where I is the current through the Helmholtz, r is the radius recorded in cm, $I^{-2}$ is just the inverse of the current squared, and the final row represents the radius in $m^2$.

Those data sets mean more when put into a graph, but first it is necessary to organize the data sets to find what we really need, which in this case is the difference in the radius compared to the difference in the current through the Helmholtz coils. I take the inverse of the current and square it, and the reason for this is that in equation (2), B is squared and is in the denominator. Then we just covert the radius from cm to m by multiplying by a magnitude of 2, and then squaring it. We do not need to inverse it since we need to rise over the run of radius vs. current. With all those steps accomplished, we achieve the following graph:

### B. Figure 2: Table 1 Information Graphed

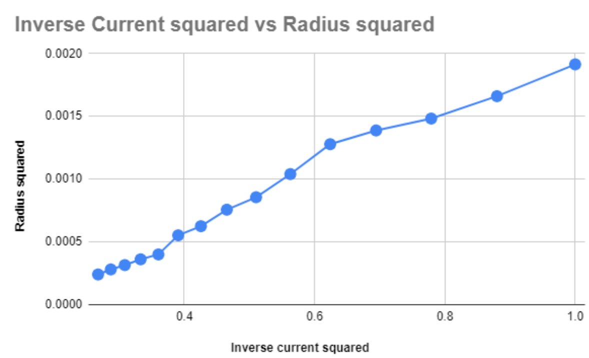


Figure 2: the slope here is 408.12 $m^{-2}A^{-2}$ and the y-intercept is 0.1681

we then plug in the slope to equation (2) for e/m and achieve a general e/m for a accelerating voltage of 150 V, the targeted accuracy here is to be within 25% anything less then that would show a sufficient experiment. Since the current is already squared and the radius is already squared and both are already in-versed, just plug in the known values and the slope as shown in the following formula:

$$e/m = (2 * 150)/(7.8 * 10^{-4})^2 * 408.12 \quad (3)$$

and achieve a value of $2.0124 * 10^{11}\ c/kg$ comparing this to the true value and we notice a %difference of 14.41% which is good, but I can do much better if I change the accelerating voltage.

I hypothesized after recording this data that an increased constant accelerating voltage would result in a greater diversity of the graph and decreased general error. Notice how the far left and far right have a great difference in the number of data sets. Before I recorded the second data set, it was necessary to know what each variable does to the circle, and why increasing the accelerating voltage matters. Analysing the first table, we notice that an increase in current resulted in a decrease in the radius. And from testing the accelerating voltage, I noticed that an increase in the voltage resulted in an increase in the circle size. Also, an increase in the accelerating voltage resulted in increased accuracy. This is due to the helium tube. More collisions would happen if the electrons were slower. Increasing the speed, aka the accelerating voltage, helps to minimize collisions. I needed diversity in the data, so less change in the radius. Increasing the accelerating voltage to the highest possible while maintaining a circle was the key here. To maintain a circle and retrieve a good amount of data, I started with 1.5 amps, which allows us to have less change in the current, which should result in less change in the radius. Essentially, focusing more on a section of Figure 2's broad graph and adding more data to that section The results of this are shown in the following table:

### C. Table 2: 286.1 Accelerating Voltage at Different Current's

| I | r | $I^{-2}$ | $(r*10^{-2})^2$ |
|---|---|---|---|
| 1.500 | 4.463 | 0.444 | 0.00199 |
| 1.533 | 4.363 | 0.426 | 0.00190 |
| 1.566 | 4.275 | 0.408 | 0.00183 |
| 1.600 | 4.175 | 0.391 | 0.00174 |
| 1.633 | 4.025 | 0.375 | 0.00162 |
| 1.666 | 3.950 | 0.360 | 0.00156 |
| 1.700 | 3.875 | 0.346 | 0.00150 |
| 1.733 | 3.750 | 0.333 | 0.00141 |
| 1.766 | 3.675 | 0.321 | 0.00135 |
| 1.800 | 3.550 | 0.309 | 0.00126 |
| 1.833 | 3.525 | 0.298 | 0.00124 |
| 1.866 | 3.475 | 0.287 | 0.00121 |
| 1.900 | 3.363 | 0.277 | 0.00113 |
| 1.933 | 3.325 | 0.268 | 0.00111 |
| 1.966 | 3.275 | 0.259 | 0.00107 |
| 2.000 | 3.150 | 0.250 | 0.00099 |

Notice the difference in the radius between the first data set and last data set is only 1.5 cm, compared to the Table 1 where the difference was nearly 3 cm.

Next I graphed the data as I did in the previous section with the same idea. And the result are shown in the following figure:

### D. Figure 3: Table 2 Information Graphed

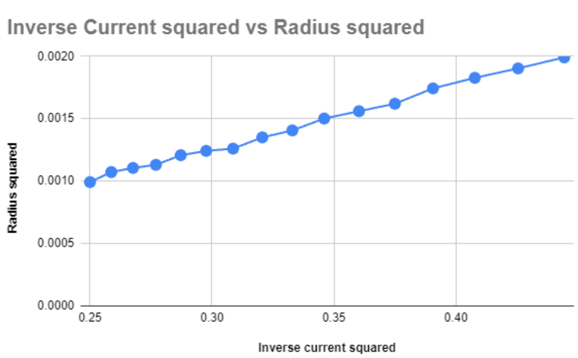


Figure 2: the slope here is 194.46 $m^{-2}A^{-2}$ and the y-intercept is 0.0559

Plug in to formula (2) from the previous page where e/m was defined, similar to (3) and you achieve:

$$e/m = (2*286.1)/(7.8*10^{-4})^2*194.46 \tag{4}$$

Where e/m is $1.829*10^{11}$ $c/kg$. When compared to the true value of e/m we get a %difference of 3.97%.

I will now keep the current constant whitelist, changing the accelerating velocity of the electron. As stated earlier, increasing the voltage will increase the radius. We will notice an increase in the radius rather than the decrease from the previous sections. Next, I want to show that increasing the accelerating voltage resulted in an increased accuracy for e/m by taking the e/m of each data set. I set the current of the Helmholtz coil to 1.488 A and increased the voltage by 10, from 150 to 290. Similar to the first data set, this is meant to get a broad picture to get a better understanding of what happens as we increase the voltage by a staggering amount. For now, here is the data set from this experiment:

### E. Table 3: 1.488 A of Current at Different Accelerating Voltages.

| V | r | e/m | $(r*10^{-2})^2$ |
|---|---|---|---|
| 150 | 2.750 | $2.945*10^{11}$ | 0.00076 |
| 160 | 2.825 | $2.977*10^{11}$ | 0.00080 |
| 170 | 3.000 | $2.804*10^{11}$ | 0.00090 |
| 180 | 3.175 | $2.651*10^{11}$ | 0.00101 |
| 190 | 3.325 | $2.552*10^{11}$ | 0.00111 |
| 200 | 3.438 | $2.513*10^{11}$ | 0.00118 |
| 210 | 3.700 | $2.277*10^{11}$ | 0.00137 |
| 220 | 3.850 | $2.204*10^{11}$ | 0.00148 |
| 230 | 4.025 | $2.108*10^{11}$ | 0.00162 |
| 240 | 4.225 | $1.996*10^{11}$ | 0.00178 |
| 250 | 4.325 | $1.984*10^{11}$ | 0.00187 |
| 260 | 4.400 | $1.994*10^{11}$ | 0.00294 |
| 270 | 4.565 | $1.915*10^{11}$ | 0.00209 |
| 280 | 4.675 | $1.902*10^{11}$ | 0.00213 |
| 290 | 4.825 | $1.849*10^{11}$ | 0.00233 |

Table 3: V is the Accelerating voltage, r is radius in cm, e/m is the ratio for each individual data set, and the last row is the radius in $m^2$ Notice how the e/m ratio increases in accuracy as the accelerating voltage increases.

I'm going to perform the same operation as the previous two data sets to get a general e/m for the data by using the slope. It's essentially the same operation. I don't have to manipulate the voltage since it's already in the nominator and is to the power of 1. However, the radius will be altered in the same matter. With that, I will graph it for V vs $r^2$ in the following graph:

### F. Figure 4: Table 3 Information Graphed

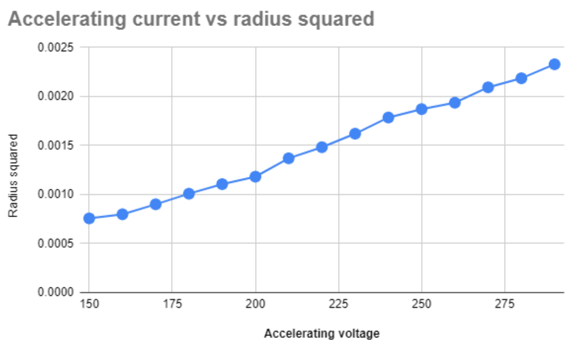


Figure 2: the slope here is 85032.87 $Vr^{-2}$ and the y-intercept is 92.91

plug the values into equation (2), and we get:

$$e/m = (2 * 85032.87)/(7.8 * 10^{-4} * 1.488)^2 \tag{5}$$

The general e/m here is $1.262 * 10^{11}$ $c/kg$ which gives us a %difference of 28.23% This is on the inaccurate side, and here is why: It is not because the math here is wrong, nor is it because the graph was incorrect. It is because of the usage of the graph. We don't need to graph this in the same matter; a basic analysis of this should tell you all you need. From Table 3, we notice an increase in the accuracy of the e/m ratio as I increase in voltage. Which means that increasing the voltage will result in the electrons' having fewer collisions with ambient particles within the helium tube. In comparing this result to the previous two, we can see a general trend that they all lead to e/m in some way, shape, or form. In other words, they all agree with one another.

## IV. CONCLUSION AND FINAL NOTES

To wrap all the data together, we notice that the higher the voltage, the more accurate it is. This is noted to be the ambient particle collisions within the helium tube. Ideally, we want the tube to allow the electrons to move freely through the tube. This would have resulted in an increase in accuracy. The current through the Helmholtz coils essentially increases the magnetic field, which results in a bend of the electrons. The larger the current, the greater the bend, in this case, the smaller the circle. For the first data set, it was setup to get a view of how the graph would look and how to tackle the variables to achieve a better fit of the slope. I first noted that the radius change was great, and that the graphing for e/m slope gave us an uneven distribution of data sets. I wanted to increase the frequency of the data sets, so I focused my energy on getting a smaller section of the broad graph. Then I noted that increasing voltage gave us a smaller circle, which means that if I wanted a smaller section, I'm going to have to get a smaller difference in the radius. which resulted in the second graph, which generally gave us an increased accuracy of e/m. Finally, I hypothesized earlier that increasing the voltage resulted in an increased e/m ratio. My hypothesis came to be true after the final data set, which gave us an e/m ratio accuracy increase as the voltage increased. The final graph confirmed the e/m ratio and when testing the difference between the 290 volts and the found e/m value in the second data set, I got a difference of 1.12%, which means that the two different data sets agree with one another.

The importance of this experiment in today's world is underappreciated. As stated in the introduction, everything we use has electrons. They carry electrical charge, compose all the atoms around us, and so much more. Knowing how they function is the key to understanding how energy works and how to increase the efficiency of everything around us.

[1] D. W. Preston and E. R. Dietz, *The art of Experimental Physics* (1991).
[2] Determination of e/m for the electron, College physics labs electricity and magnetism (2011).
[3] M. Shuttleworth, Cathode ray experiment, (2008).
[4] J. Tega, What are electrons (2010), importance of electrons.
[5] B. Klaus, Electron on the scale (2014).