Subtraction Using Nine's and Ten's Complements

Date: 05/27/2000 at 01:24:37
From: Katie Artus
Subject: Method of complements

I am trying to figure out why the method below, called the "method of complements" by my university professor, will give me the correct answer all of the time. It is used in place of regrouping in subtraction problems.

```
 623     999
-465    -465
-----   -----
 158     534
        +623
        -----
        1157

        /157     (Cross off the 1 in the thousands column and add

          +1      this 1 to the ones column.
        -----
         158
```

Nines must always be used, and the order of operations depicted above must be followed. If these conditions are met, the problem will always work no matter how many numbers are in the problem. This will also work with standard subtraction when regrouping is not necessary. I would like to know why this method works. What mathematics are lurking behind the scenes? I wish to explain this to a third grade child I am student teaching.

Thank you,
Katie Artus

Date: 05/29/2000 at 14:44:11
From: Doctor TWE
Subject: Re: Method of complements

Hi Katie - thanks for writing to Dr. Math.

The method of complements is perhaps most famous in its binary variations (called one's complement and two's complement) because those are the methods that most computers use to subtract. What your professor showed you is the decimal equivalent of one's complement, sometimes called nine's complement.

As to why it works, let's examine what we're doing algebraically. Let's call the minuend (623 in your example) X and the subtrahend (465 in your example) Y. We wish to find X-Y.

In the first step, we subtract Y from 99...9 (the number of 9's equaling the number of digits in the larger of X and Y). In your

example, 999. So we have:

```
999-Y
```

Next, we add X to that to get:

```
999-Y+X
```

As long as X > Y, we can see that the result will be greater than or equal to 1000. When we cross out the leading digit (the 1000's digit in this example), we are in fact subtracting 1000 from our result. Thus we have:

```
999-Y+X-1000
```

Finally, we add 1 to the units digit. So we have:

```
999-Y+X-1000+1
```

Can you see how algebraically this equals X-Y?

If I wanted to explain this to a third grader, I'd probably explain the ten's complement method instead. Ten's complement works similar to nine's complement with two slight differences. First, instead of subtracting the subtrahend (Y) from 99...9, you subtract it from 100...0 instead. [I write this as 99...(10) so I don't have to borrow.] The second difference is that you don't have to add the 1 at the end. Can you see why this method produces the same results as nine's complement?

Here's how I'd explain why it works:

Imagine you want to make some money by selling widgets (pick the student's favorite thing-a-ma-bob). First, you have to buy the materials to make widgets. This costs $465. Then you can sell them for $623. Now you want to figure out how much money you'll make.

But there's one problem - you have to buy the materials BEFORE you can sell the widgets. So you borrow $1000 from Mom and Dad. From that $1000 you spend $465 to buy the materials. So you have:

```
$1000
- 465
 ----
$ 535
```

Now you sell the widgets for $623, you now have:

```
$1000
- 465
 ----
$ 535
+ 623
 ----
$1158
```

Finally, you have to pay back Mom and Dad the $1000 you borrowed to

start up. So your final profit is:

```
     $1000
     - 465
      ----
     $ 535
     + 623
      ----
     $/158
```

Which is the selling price ($623) less the cost of materials ($465).

A final word of caution: While nine's complement and ten's complement produce the same results when X > Y, they need to be handled differently when X <= Y. In the cases where X < Y (and where X = Y for nine's complement), there will not be a "final carry" to cross out. For example, to subtract 235-687:

```
     9'S Comp.     10'S Comp.
       999           1000
     - 687         -  687
       ---           ----
       312            313
     + 235         +  235
       ---           ----
       547            548
```

How do we interpret this? With nine's complement, we subtract the result from 99...9 and add a negative sign. With ten's complement, we subtract the result from 100...0 and add a negative sign, like so:

```
     9'S Comp.     10'S Comp.
       999           1000
     - 547         -  548
       ---           ----
    (-)452        (-)452
```

I hope this helps. If you have any more questions, write back.

- Doctor TWE, The Math Forum
  http://mathforum.org/dr.math/