שאלה 4 - א'.

נסמן:

n_i - מספר הדגימות במחלקה i.

X_k^i - ה-k דגימות ממחלקה i.

כלל ההחלטה בבייסיאני $\Rightarrow$ $P(w_i|x) \geq P(w_j|x)$

לפי משפט בייס

$$= \frac{P(x|w_i)\cdot P(w_i)}{\cancel{P(x)}} \geq \frac{P(x|w_j)\cdot P(w_j)}{\cancel{P(x)}}$$

$$= P(x|w_i)\cdot P(w_i) \geq P(x|w_j)\cdot P(w_j)$$

נחשב את $P(x|w_i)$:

$$P(x|w_i) = \frac{1}{n_i \cdot h} \sum_{i=k=1}^{n_i} \phi\left(\frac{X_i^k - x}{h}\right)$$

לכן נקבל:

$$\frac{1}{n_i \cdot h} \sum_{k=1}^{n_i} \phi\left(\frac{X_i^k - x}{h}\right)\cdot P(w_i) \geq \frac{1}{n_j \cdot h} \sum_{k=1}^{n_j} \phi\left(\frac{X_j^k - x}{h}\right)\cdot P(w_j)$$

נכפיל את שני האגפים ב- $n_i \cdot n_j \cdot h$ נקבל:

$$n_j \cdot P(w_i) \cdot \sum_{k=1}^{n_i} \phi\left(\frac{X_i^k - x}{h}\right) \geq n_i \cdot P(w_j) \cdot \sum_{k=1}^{n_j} \phi\left(\frac{X_j^k - x}{h}\right)$$

$P(w_i) = P(w_j)$

$$= \sum_{k=1}^{n_i} \phi\left(\frac{X_i^k - x}{h}\right) \geq \sum_{k=1}^{n_j} \phi\left(\frac{X_j^k - x}{h}\right)$$

2. You are given some samples from an unknown distribution, $D = \{1, -3, 2, 4, 5, -8, 0, -1, -2, -4\}$. Given $h = 4$, use the 0-1 window function and build the full distribution. Attach only the drawn graph (the pdf function). No coding required.

(a) According to the graph, what is the name of the real distribution of the data?

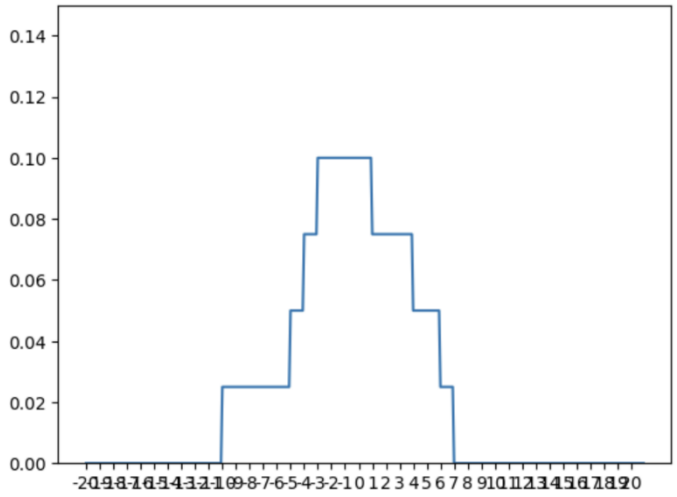
First, we will write the density estimation function,

As given at the question : V=h=4 ,we are at 1d , and n=10.

$$P(x) = \frac{1}{10 \cdot 4^1} \cdot \left(\emptyset\left(\frac{(x-1)}{4}\right) + \emptyset\left(\frac{(x+3)}{4}\right) + \ldots\ldots + \emptyset\left(\frac{(x+4)}{4}\right)\right)$$

We use the parzen window code from the tutorial:

```
p(x=1) = 0.1
p(x=-3) = 0.1
p(x=2) = 0.1
p(x=4) = 0.075
p(x=5) = 0.05
p(x=-8) = 0.025
p(x=0) = 0.125
p(x=-1) = 0.125
p(x=-2) = 0.125
p(x=-4) = 0.075
```



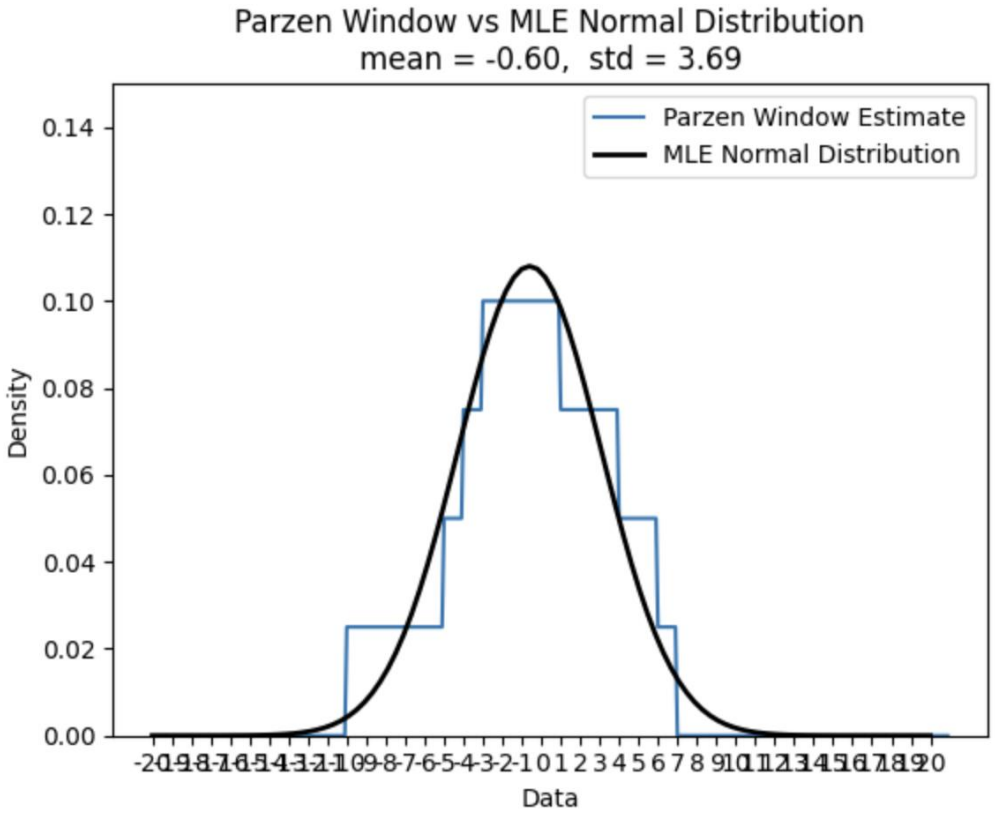
The resulting plot appears to be symmetric and bell-shaped, which is typical of a normal distribution.

(b) Use MLE to estimate the distribution parameters and attach the drawing of the real distribution. Show the similarity between both graphs.

Firstly, we will calculate the MLE estimates for the normal distribution parameters as we learned:

$$mean = \frac{1}{10} \cdot \sum_{i=1}^{10} X_i = \frac{1}{10} \cdot -6 = -0.6$$

$$standard\ deviation = \sqrt{\frac{1}{10} \cdot \sum_{i=1}^{10} (X_i + 0.6)^2} = \ldots = 3.69$$



Here we use a python code that generates the comparison plot between the parzen window estimate and the normal distribution estimated using MLE.

The comparison of the two graphs reveals a strong similarity, indicating that the data follows a normal distribution.