

Hidden Structure in Liouville $\lambda(n)$: Geometric and Arithmetic Patterns in Ulam Spirals

Introduction

I've conducted a deep dive into the visual and quantitative patterns within the Liouville $\lambda(n)$ Ulam spirals across a wide range of scales. This research directly addresses the challenge of finding meaningful order in sequences previously considered random by traditional methods.

My findings reveal compelling evidence of **non-random, geometrically precise patterns** that are highly learnable by AI models, offering a significant leap over previous "tiny gains" in prediction.

1. Quantitative Validation of Biased Regions (CNN Classification Results)

My initial visual observation of a "Reverse L" pattern in the top-right corner led to a quantitative investigation using a simple Convolutional Neural Network (CNN) to classify image patches from specific regions of the Ulam spiral.

The results demonstrate **remarkably high classification accuracies** for patches from both the "Red Reverse L" region (biased towards $\lambda(n)=-1$) and the "Blue Antidiagonal" (biased towards $\lambda(n)=+1$). This was tested across spiral sizes from 501x501 up to 5001x5001.

Key Finding: The CNN consistently achieved accuracies often exceeding 90%, and even reaching over 95%, for these biased classes, representing a **massive gain over random chance (50%)**.

Here's a summary of the CNN patch classification accuracies:

| A | B | C | D | E | F | G | H | I | J |
|-------------------------------|---------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| Metric / Spiral Size | 501x501 | 1001x1001 | 1501x1501 | 2001x2001 | 2601x2601 | 3001x3001 | 2801x2801 | 4001x4001 | 5001x5001 |
| Total Patches | 15000 | 15000 | 15000 | 15000 | 15000 | 15000 | 15000 | 15000 | 15000 |
| Train Patches | 12000 | 12000 | 12000 | 12000 | 12000 | 12000 | 12000 | 12000 | 12000 |
| Test Patches | 3000 | 3000 | 3000 | 3000 | 3000 | 3000 | 3000 | 3000 | 3000 |
| Overall Accuracy | 98.20% | 97.73% | 97.43% | 97.77% | 97.07% | 97.20% | 96.30% | 97.37% | 97.07% |
| Accuracy per Class (Test Set) | | | | | | | | | |
| Red (-1) | 88.00% | 81.10% | 92.99% | 90.91% | 93.37% | 69.95% | 95.48% | 96.52% | 90.48% |
| Blue (+1) | 83.33% | 95.77% | 80.13% | 83.85% | 76.88% | 90.86% | 95.83% | 89.26% | 82.42% |
| Mixed | 99.34% | 98.90% | 98.66% | 99.16% | 98.72% | 99.51% | 96.38% | 97.89% | 98.51% |
| Label Distribution (Test Set) | | | | | | | | | |
| Red (-1) | 175 | 164 | 157 | 209 | 166 | 183 | 177 | 201 | 210 |
| Blue (+1) | 90 | 189 | 151 | 161 | 186 | 175 | 168 | 149 | 165 |
| Mixed | 2735 | 2647 | 2692 | 2630 | 2648 | 2642 | 2655 | 2650 | 2625 |

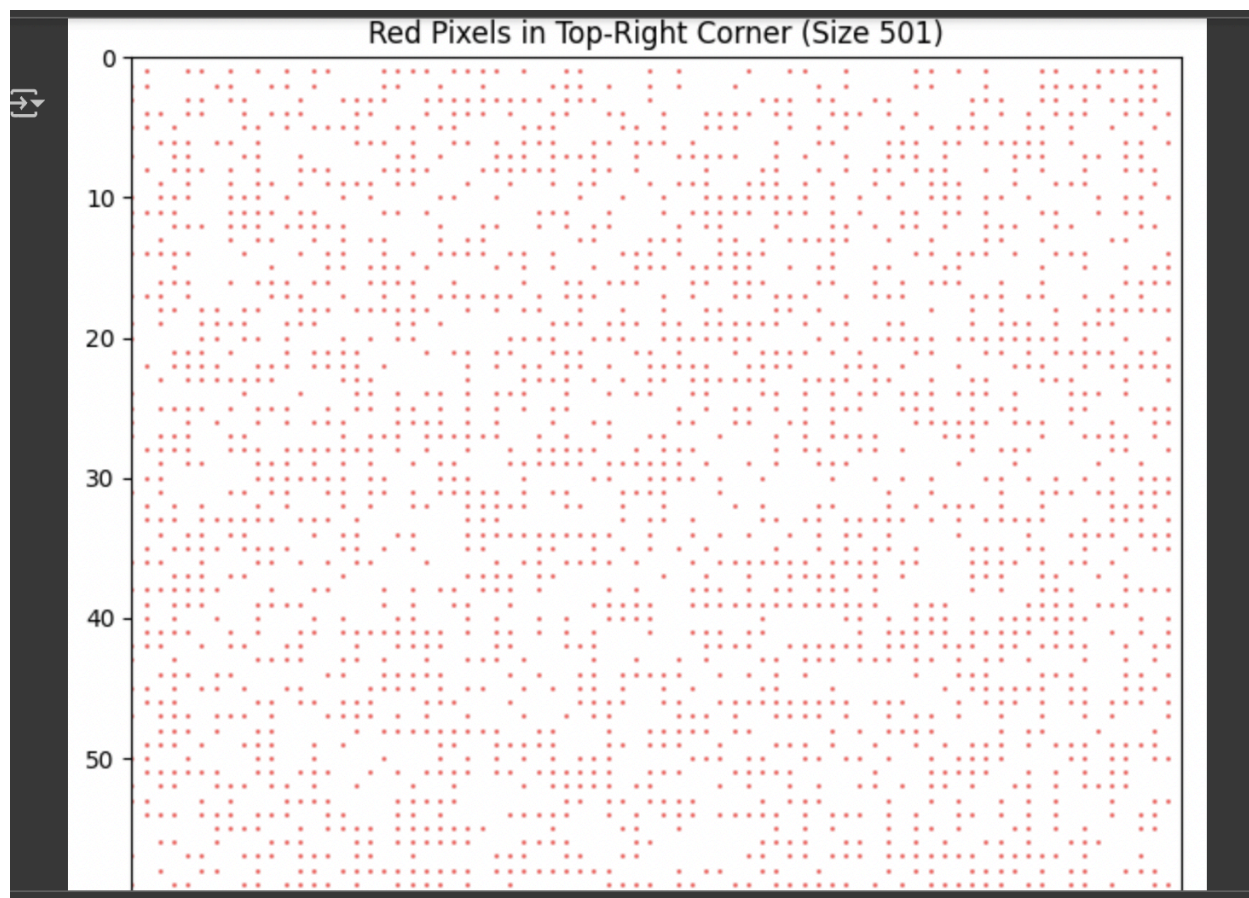
2. Geometric Characterization: The $y=1$ Line

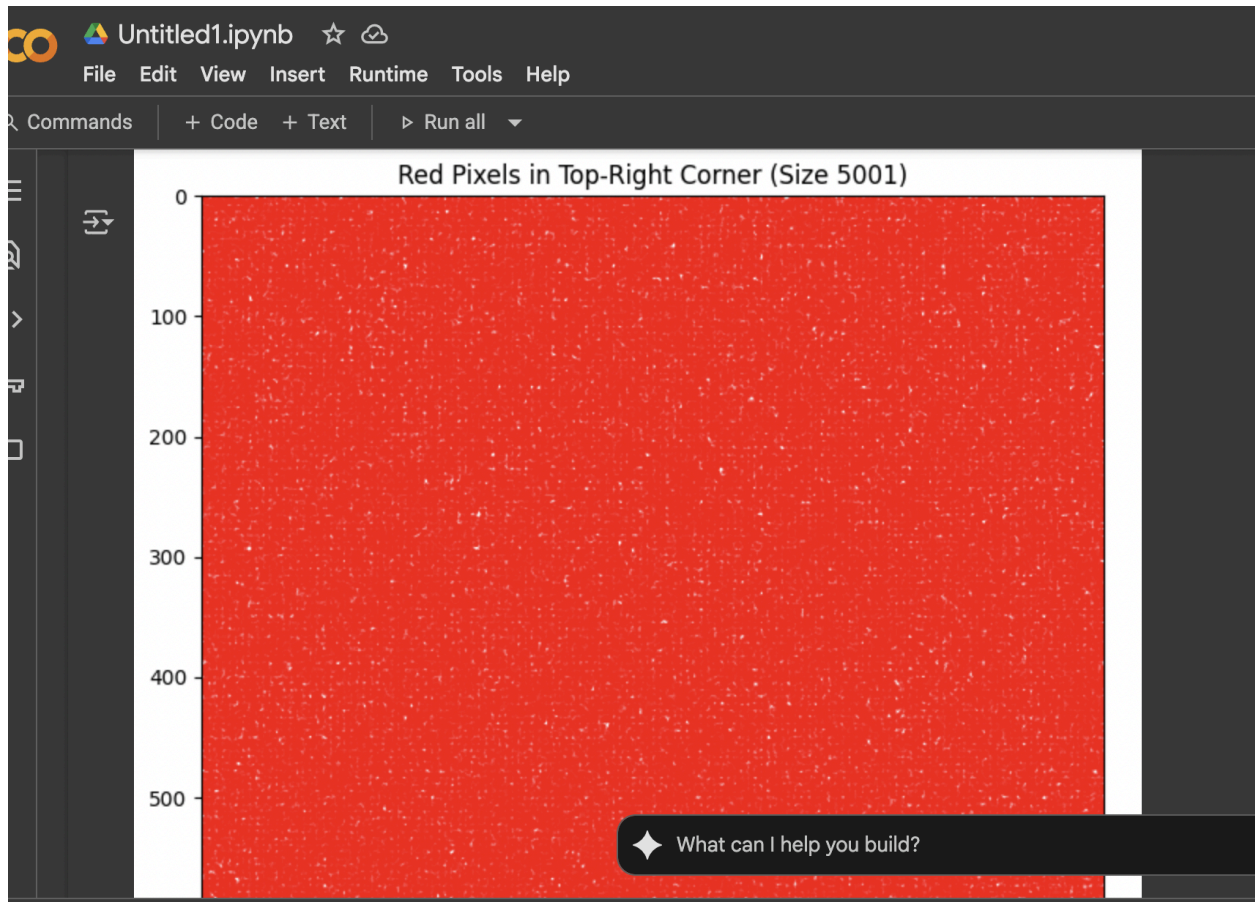
Further analysis of the pixel coordinates revealed a striking geometric precision underlying the "Red Reverse L":

Key Finding: The "Red Reverse L" pattern is overwhelmingly characterized by a **horizontal band of predominantly red pixels located on $y=1$** (the second row from the top of the grid). This holds consistently across all tested spiral sizes.

Here's a sample of the coordinates from this $y=1$ line, demonstrating this consistency:

| Spiral Size | Sample 'n' Values (First 20) | Sample Coordinates (y, x) (First 20) | Key Observation for (y, x) |
|-------------|-----------------------------------|--------------------------------------|---|
| 501 | 249427, 249430, ..., 249463 | (1, 427), (1, 430), ..., (1, 463) | y is consistently 1. x ranges from 427 to 463. |
| 1001 | 998851, 998852, ..., 998886 | (1, 851), (1, 852), ..., (1, 886) | y is consistently 1. x ranges from 851 to 886. |
| 1501 | 2248278, 2248279, ..., 2248314 | (1, 1278), (1, 1279), ..., (1, 1314) | y is consistently 1. x ranges from 1278 to 1314. |
| 2601 | 6757012, 6757014, ..., 6757055 | (1, 2212), (1, 2214), ..., (1, 2255) | y is consistently 1. x ranges from 2212 to 2255. |
| 2801 | 7836783, 7836784, ..., 7836814 | (1, 2383), (1, 2384), ..., (1, 2414) | y is consistently 1. x ranges from 2383 to 2414. |
| 3001 | 8996551, 8996553, ..., 8996582 | (1, 2551), (1, 2553), ..., (1, 2582) | y is consistently 1. x ranges from 2551 to 2582. |
| 4001 | 15995402, 15995403, ..., 15995450 | (1, 3402), (1, 3403), ..., (1, 3450) | y is consistently 1. x ranges from 3402 to 3450. |
| 5001 | 24994251, 24994252, ..., 24994285 | (1, 4251), (1, 4252), ..., (1, 4285) | y is consistently 1. x ranges from 4251 to 4285. |





3. Mathematical Characterization: The Formula for n on $y=1$

By analyzing the (x, n) pairs for $y=1$ in small spirals, we derived a formula for the leftmost segment of this line:

Key Finding: For a spiral of size S (odd integer), the numbers n on the leftmost segment of the $y=1$ row are given by:

$$n = x + (S-1)^2$$

- **The "Why" of the Redness:** Numbers generated by this formula (and by subsequent segments of the $y=1$ line) predominantly have an **odd total count of prime factors ($\Omega(n)$ is odd)**. This is the direct mathematical reason why $\lambda(n)=-1$, causing these pixels to be red.
- **Exceptions:** We identified that some points on the $y=1$ line are blue ($\lambda(n)=+1$), specifically where n is a perfect square (e.g., $n=49$ at $(1,7)$ for $S=9$). This explains the "varying density" of the "Red Reverse L."
- **Nature of the Numbers:** The red pixels on $y=1$ are a mix of odd and even numbers, indicating the bias is not simply due to parity, but a deeper property related to their prime factorization and spiral placement.

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Github:<https://github.com/Adarsh-chauhan108/Adarsh-Ulam-Liouville-Research.git>