1 Linear Equations in Two Variables

Let's study.

- Methods of solving linear equations in two variables – graphical method, Cramer's method
- Equations that can be transformed in linear equation in two variables
- Application of simultaneous equations

Let's recall.

Linear equation in two variables

An equation which contains two variables and the degree of each term containing variable is one, is called a linear equation in two variables.

$ax + by + c = 0$ is the general form of a linear equation in two variables; a, b, c are real numbers and a, b are not equal to zero at the same time.

Ex. $3x - 4y + 12 = 0$ is the general form of equation $3x = 4y - 12$

Activity : Complete the following table

No.	Equation	Is the equation a linear equation in 2 variables ?
1	$4m + 3n = 12$	Yes
2	$3x^2 - 7y = 13$	
3	$\sqrt{2}x - \sqrt{5}y = 16$	
4	$0x + 6y - 3 = 0$	
5	$0.3x + 0y - 36 = 0$	
6	$\frac{4}{x} + \frac{5}{y} = 4$	
7	$4xy - 5y - 8 = 0$	

Simultaneous linear equations

When we think about two linear equations in two variables at the same time, they are called simultaneous equations.

Last year we learnt to solve simultaneous equations by eliminating one variable. Let us revise it.

Ex. (1) Solve the following simultaneous equations.

(1) $5x - 3y = 8$; $3x + y = 2$

Solution :

Method I : $5x - 3y = 8$. . . (I)

$3x + y = 2$ . . . (II)

Multiplying both sides of equation (II) by 3.

$9x + 3y = 6$ . . . (III)

$5x - 3y = 8$. . . (I)

Now let us add equations (I) and (III)

$$\begin{array}{r} 5x - 3y = 8 \\ +\ 9x + 3y = 6 \\ \hline 14x = 14 \end{array}$$

$\therefore\ x = 1$

substituting $x = 1$ in equation (II)

$3x + y = 2$

$\therefore\ 3 \times 1 + y = 2$

$\therefore\ 3 + y = 2$

$\therefore\ y = -1$

solution is $x = 1, y = -1$; it is also written as $(x, y) = (1, -1)$

Method (II)

$5x - 3y = 8$. . . (I)

$3x + y = 2$ . . . (II)

Let us write value of y in terms of x from equation (II) as

$y = 2 - 3x$ . . . (III)

Substituting this value of y in equation (I).

$5x - 3y = 8$

$\therefore\ 5x - 3(2 - 3x) = 8$

$\therefore\ 5x - 6 + 9x = 8$

$\therefore\ 14x - 6 = 8$

$\therefore\ 14x = 8 + 6$

$\therefore\ 14x = 14$

$\therefore\ x = 1$

Substituting $x = 1$ in equation (III).

$y = 2 - 3x$

$\therefore\ y = 2 - 3 \times 1$

$\therefore\ y = 2 - 3$

$\therefore\ y = -1$

$x = 1, y = -1$ is the solution.

Ex. (2) Solve : $3x + 2y = 29$; $5x - y = 18$

Solution : $3x + 2y = 29$. . . (I) and $5x - y = 18$ . . . (II)

Let's solve the equations by eliminating 'y'. Fill suitably the boxes below.

Multiplying equation (II) by 2.

$\therefore\ 5x \times \boxed{} - y \times \boxed{} = 18 \times \boxed{}$

$\therefore\ 10x - 2y = \boxed{}$ . . . (III)

Let's add equations (I) and (III)

$$\begin{array}{r} 3x + 2y = 29 \\ + \quad \boxed{} - \boxed{} = \boxed{} \\ \hline \boxed{} = \boxed{} \end{array} \qquad \therefore\ x = \boxed{}$$

Substituting $x = 5$ in equation (I)

$3x + 2y = 29$

$\therefore\ 3 \times \boxed{} + 2y = 29$

$\therefore\ \boxed{} + 2y = 29$

$\therefore\ 2y = 29 - \boxed{}$

$\therefore\ 2y = \boxed{}$ $\qquad \therefore\ y = \boxed{}$

$(x, y) = (\boxed{}, \boxed{})$ is the solution.

Ex. (3) Solve : $15x + 17y = 21$; $17x + 15y = 11$

Solution : $15x + 17y = 21$. . . (I)

$17x + 15y = 11$ . . . (II)

In the two equations above, the coefficients of x and y are interchanged. While solving such equations we get two simple equations by adding and subtracting the given equations. After solving these equations, we can easily find the solution.

Let's add the two given equations.

$$\begin{array}{r} 15x + 17y = 21 \\ + \quad 17x + 15y = 11 \\ \hline 32x + 32y = 32 \end{array}$$

Dividing both sides of the equation by 32.

$$x + y = 1 \quad . . . \quad \text{(III)}$$

Now, let's subtract equation (II) from (I)

$$\begin{array}{rl} & 15x + 17y = 21 \\ - & 17x + 15y = 11 \\ & \underline{-\quad\;\; -\quad\;\; -} \\ & -2x + 2y = 10 \end{array}$$

dividing the equation by 2.

$$-x + y = 5 \quad . . . \quad \text{(IV)}$$

Now let's add equations (III) and (V).

$$\begin{array}{rl} & x + y = 1 \\ + & -x + y = 5 \\ \hline & \therefore \; 2y = 6 \qquad \therefore \; y = 3 \end{array}$$

Place this value in equation (III).

$x + y = 1$

$\therefore \; x + 3 = 1$

$\therefore \; x = 1 - 3 \qquad \therefore \; x = -2$

$(x, y) = (-2, 3)$ is the solution.

Practice Set 1.1

(1) Complete the following activity to solve the simultaneous equations.

$5x + 3y = 9$ -----(I)

$2x - 3y = 12$ ----- (II)

Let's add eqations (I) and (II).

$$\begin{array}{rl} & 5x + 3y = 9 \\ + & 2x - 3y = 12 \\ \hline & \boxed{}\, x = \boxed{} \end{array}$$

$x = \dfrac{\boxed{}}{\boxed{}} \qquad x = \boxed{}$

Place $x = 3$ in equation (I).

$5 \times \boxed{} + 3y = 9$

$3y = 9 - \boxed{}$

$3y = \boxed{}$

$y = \dfrac{\boxed{}}{3}$

$y = \boxed{}$

$\therefore$ Solution is $(x, y) = (\boxed{}, \boxed{})$.

2. Solve the following simultaneous equations.

(1) $3a + 5b = 26$; $a + 5b = 22$ (2) $x + 7y = 10$; $3x - 2y = 7$

(3) $2x - 3y = 9$; $2x + y = 13$ (4) $5m - 3n = 19$; $m - 6n = -7$

(5) $5x + 2y = -3$; $x + 5y = 4$ (6) $\frac{1}{3}x + y = \frac{10}{3}$; $2x + \frac{1}{4}y = \frac{11}{4}$

(7) $99x + 101y = 499$; $101x + 99y = 501$

(8) $49x - 57y = 172$; $57x - 49y = 252$

Let's recall.

Graph of a linear equation in two variables

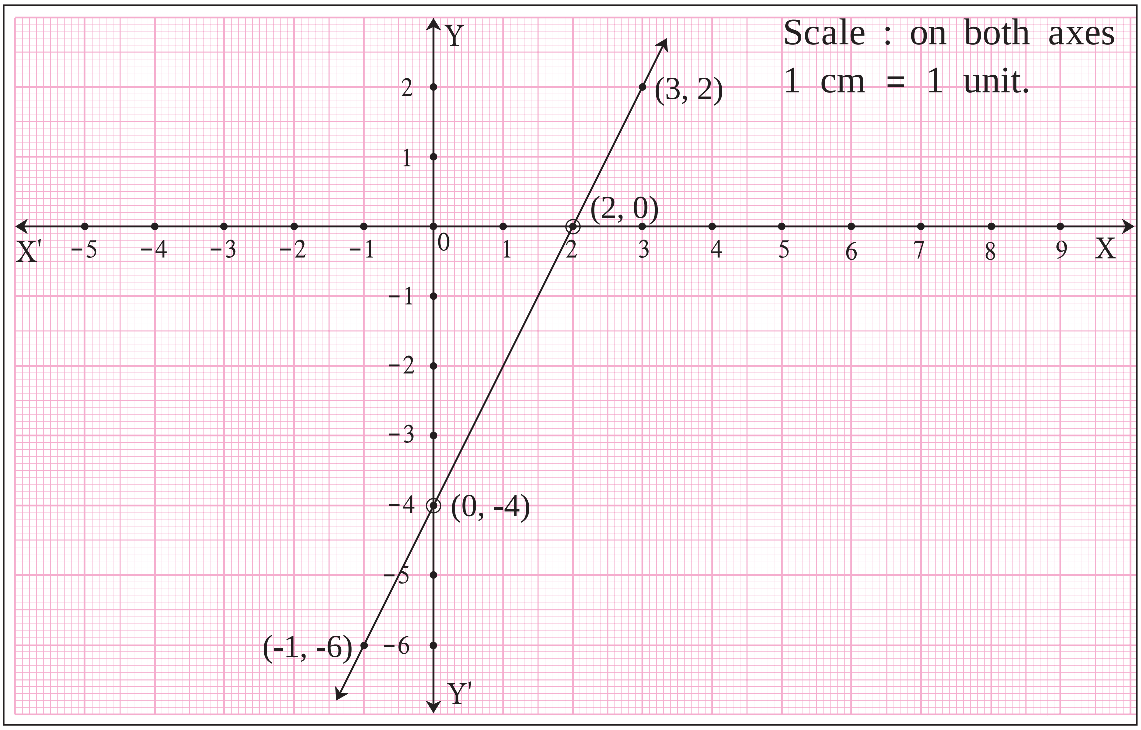
In the 9th standard we learnt that the graph of a linear equation in two variables is a straight line. The ordered pair which satisfies the equation is a solution of that equation. The ordered pair represents a point on that line.

Ex. Draw graph of $2x - y = 4$.

Solution : To draw a graph of the equation let's write 4 ordered pairs.

x	0	2	3	-1
y	-4	0	2	-6
(x, y)	$(0, -4)$	$(2, 0)$	$(3, 2)$	$(-1, -6)$

To obtain ordered pair by simple way let's take $x = 0$ and then $y = 0$.



Steps to follow for drawing a graph of linear equation in two variables.

Find at least 4 ordered pairs for given equation

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Draw X- axis, Y-axis on graph paper and plot the points

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See that all 4 points lie on one line

Two points are sufficient to represent a line, but if co-ordinates of one of the two points are wrong then you will not get a correct line.

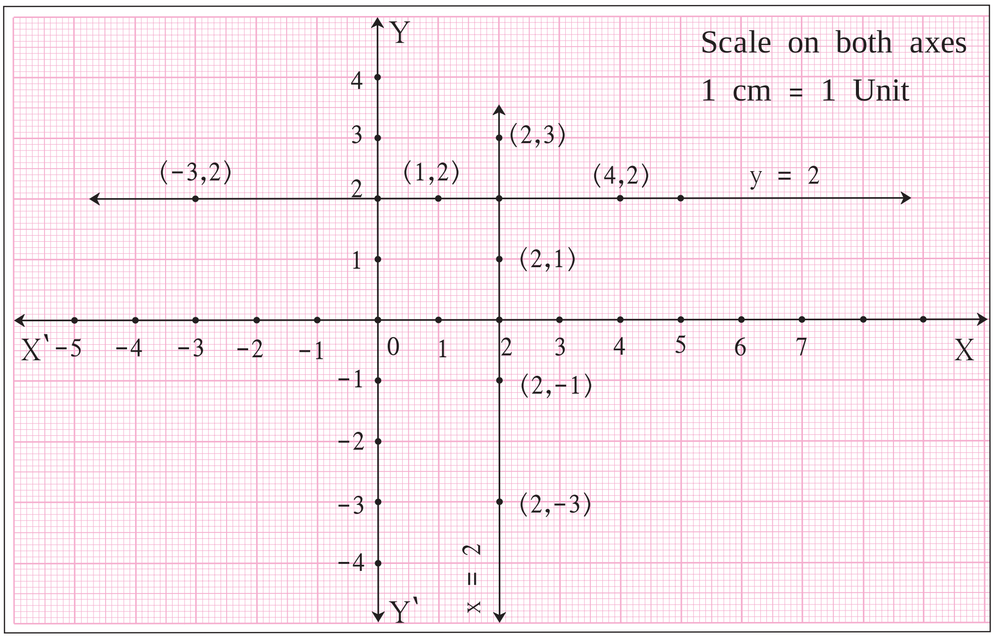
If you plot three points and if they are non collinear then it is understood that one of the points is wrongly plotted. But it is not easy to identify the incorrect point.

If we plot four points, it is almost certain that three of them will be collinear.

A linear equation $y = 2$ is also written as $0x + y = 2$. The graph of this line is parallel to X- axis; as for any value of x, y is always 2.

x	1	4	-3
y	2	2	2
(x, y)	$(1, 2)$	$(4, 2)$	$(-3, 2)$

Similarly equation $x = 2$ is written as $x + 0y = 2$ and its graph is parallel to Y-axis.



Let's learn.

Graphical method

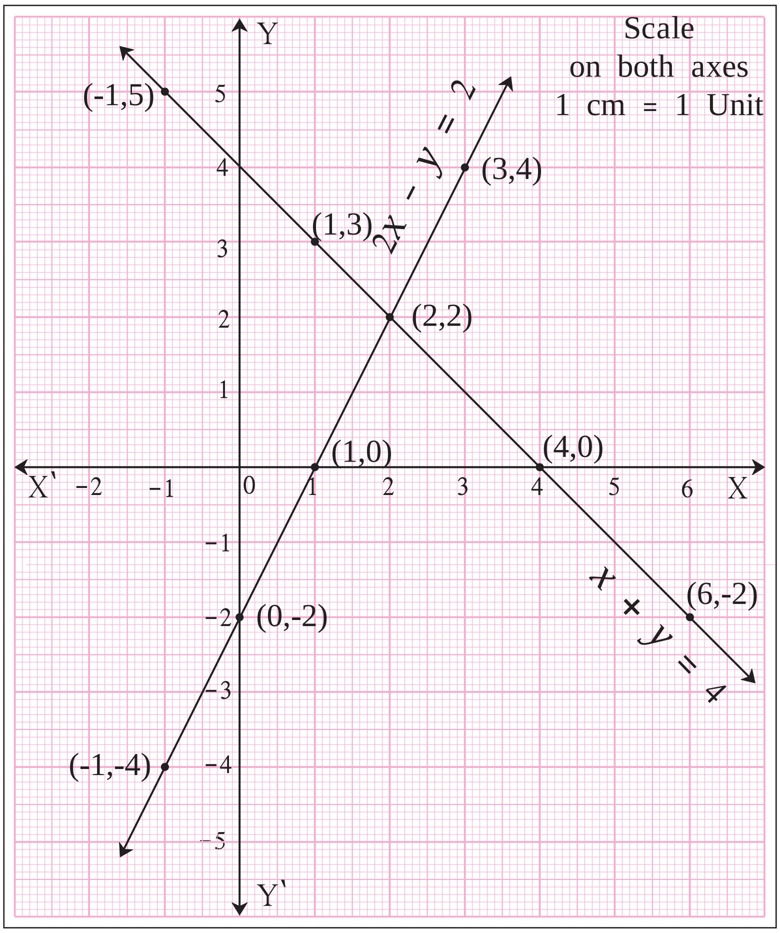
Ex. Let's draw graphs of $x + y = 4$, $2x - y = 2$ and observe them.

$x + y = 4$

x	-1	4	1	6
y	5	0	3	-2
(x,y)	$(-1,5)$	$(4,0)$	$(1,3)$	$(6,-2)$

$2x - y = 2$

x	0	1	3	-1
y	-2	0	4	-4
(x,y)	$(0,-2)$	$(1,0)$	$(3,4)$	$(-1,-4)$



Each point on the graph satisfies the equation. The two lines intersect each other at (2, 2).

Hence ordered pair (2, 2) i.e. $x = 2$, $y = 2$ satisfies the equations $x + y = 4$ and $2x - y = 2$.

The values of variables that satisfy the given equations, give the solution of given equations.

$\therefore$ the solution of given equations $x + y = 4$, $2x - y = 2$ is $x = 2$, $y = 2$.

Let's solve these equations by method of elimination.

$x + y = 4$ . . . (I)

$2x - y = 2$ . . . (II)

Adding equations (I) and (II) we get,

$3x = 6 \quad \therefore x = 2$

substituting this value in equation (I)

$x + y = 4$

$\therefore 2 + y = 4$

$\therefore y = 2$

Activity (I) : Solve the following simultaneous equations by graphical method.

- Complete the following tables to get ordered pairs.

$x - y = 1$

x	0		3	
y		0		-3
(x, y)				

$5x - 3y = 1$

x	2			-4
y		8	-2	
(x, y)				

- Plot the above ordered pairs on the same co-ordinate plane.
- Draw graphs of the equations. • Note the co-ordinates of the point of intersection of the two graphs. Write solution of these equations.

Activity II : Solve the above equations by method of elimination. Check your solution with the solution obtained by graphical method.

Let's think.

The following table contains the values of x and y co-ordinates for ordered pairs to draw the graph of $5x - 3y = 1$

x	0	$\frac{1}{5}$	1	-2
y	$-\frac{1}{3}$	0	$\frac{4}{3}$	$-\frac{11}{3}$
(x, y)	$\left(0, -\frac{1}{3}\right)$	$\left(\frac{1}{5}, 0\right)$	$\left(1, \frac{4}{3}\right)$	$\left(-2, -\frac{11}{3}\right)$

- Is it easy to plot these points ? • Which precaution is to be taken to find ordered pairs so that plotting of points becomes easy ?

Practice Set 1.2

1. Complete the following table to draw graph of the equations –

(I) $x + y = 3$ (II) $x - y = 4$

$x + y = 3$

x	3	$\square$	$\square$
y	$\square$	5	3
(x, y)	$(3, 0)$	$\square$	$(0, 3)$

$x - y = 4$

x	$\square$	-1	0
y	0	$\square$	-4
(x, y)	$\square$	$\square$	$(0, -4)$

2. Solve the following simultaneous equations graphically.

(1) $x + y = 6$; $x - y = 4$ (2) $x + y = 5$; $x - y = 3$

(3) $x + y = 0$; $2x - y = 9$ (4) $3x - y = 2$; $2x - y = 3$

(5) $3x - 4y = -7$; $5x - 2y = 0$ (6)★ $2x - 3y = 4$; $3y - x = 4$

Let's discuss.

To solve simultaneous equations $x + 2y = 4$; $3x + 6y = 12$ graphically, following are the ordered pairs.

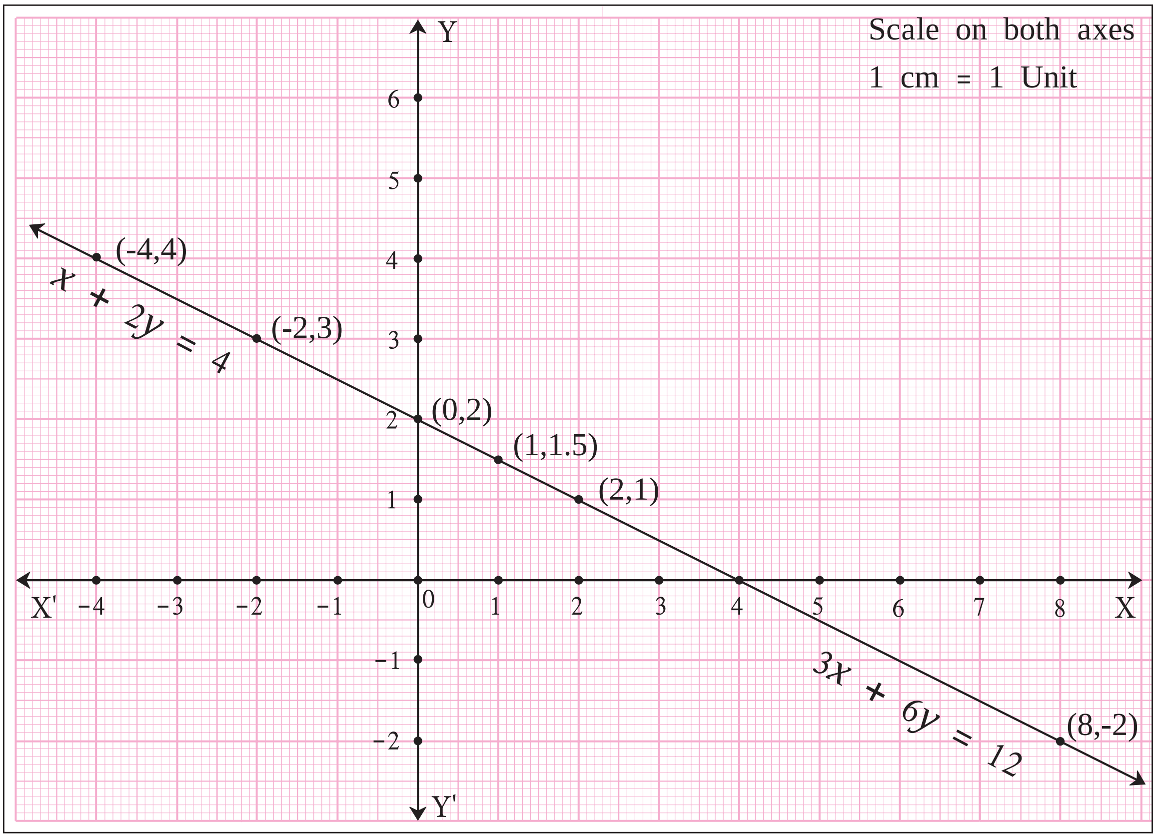
$x + 2y = 4$

x	-2	0	2
y	3	2	1
(x, y)	$(-2, 3)$	$(0, 2)$	$(2, 1)$

$3x + 6y = 12$

x	-4	1	8
y	4	1.5	-2
(x, y)	$(-4, 4)$	$(1, 1.5)$	$(8, -2)$

Plotting the above ordered pairs, graph is drawn. Observe it and find answers of the following questions.



(1) Are the graphs of both the equations different or same ?

(2) What are the solutions of the two equations $x + 2y = 4$ and $3x + 6y = 12$? How many solutions are possible ?

(3) What are the relations between coefficients of x, coefficients of y and constant terms in both the equations ?

(4) What conclusion can you draw when two equations are given but the graph is only one line ?

Now let us consider another example.

Draw graphs of $x - 2y = 4$, $2x - 4y = 12$ on the same co-ordinate plane. Observe it. Think of the realation between the coefficients of x, coefficients of y and the constant terms and draw the inference.

ICT Tools or Links.

Use Geogebra software, draw X- axis, Y-axis. Draw graphs of simultaneous equations.

Let's learn.

Determinant

$\begin{vmatrix} a & b \\ c & d \end{vmatrix}$ is a determinant. (a, b), (c, d) are rows and $\begin{pmatrix} a \\ c \end{pmatrix}$, $\begin{pmatrix} b \\ d \end{pmatrix}$ are columns.

Degree of this determinant is 2, because there are 2 elements in each column and 2 elements in each row. Determinant represents a number which is $(ad\text{-}bc)$.

i.e. $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad\text{-}bc$

$ad\text{-}bc$ is the value of determinant $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$

Determinants, usually, are represented with capital letters as A, B, C, D, ... etc.

Solved Examples

Ex. Find the values of the following determinants.

(1) $A = \begin{vmatrix} 5 & 3 \\ 7 & 9 \end{vmatrix}$ (2) $N = \begin{vmatrix} -8 & -3 \\ 2 & 4 \end{vmatrix}$ (3) $B = \begin{vmatrix} 2\sqrt{3} & 9 \\ 2 & 3\sqrt{3} \end{vmatrix}$

Solution :

(1) $A = \begin{vmatrix} 5 & 3 \\ 7 & 9 \end{vmatrix} = (5 \times 9) - (3 \times 7) = 45 - 21 = 24$

(2) $N = \begin{vmatrix} -8 & -3 \\ 2 & 4 \end{vmatrix} = [(-8) \times (4)] - [(-3\ ) \times 2)] = -32 - (-6)$

$= -32 + 6 = -26$

(3) $B = \begin{vmatrix} 2\sqrt{3} & 9 \\ 2 & 3\sqrt{3} \end{vmatrix} = [2\sqrt{3} \times 3\sqrt{3})] - [2 \times 9)] = 18 - 18 = 0$

Let's learn.

Determinant method (Cramer's Rule)

Using determinants, simultaneous equaions can be solved easily and in less space. This method is known as determinant method. This method was first given by a Swiss mathematician Gabriel Cramer, so it is also known as Cramer's method.

To use Cramer's method, the equations are written as $a_1x + b_1y = c_1$ and $a_2x + b_2y = c_2$.

$$a_1x + b_1y = c_1 \quad . . . \quad \text{(I)}$$

$$a_2x + b_2y = c_2 \quad . . . \quad \text{(II)}$$

Here x and y are variables, a_1, b_1, c_1 and a_2, b_2, c_2 are real numbers, $a_1b_2 - a_2b_1 \neq 0$

Now let us solve these equations.

Multiplying equation (I) by b_2.

$$a_1 b_2 x + b_1 b_2 y = c_1 b_2 \quad . . . \quad \text{(III)}$$

Multiplying equation (II) by b_1.

$$a_2 b_1 x + b_2 b_1 y = c_2 b_1 \quad . . . \quad \text{(IV)}$$

Subtracting equation (IV) from (III)

$$\begin{array}{rl} & a_1 b_2 x + b_1 b_2 y = c_1 b_2 \\ - & a_2 b_1 x \;\;-\; b_2 b_1 y = c_2 b_1 \\ & - \qquad\qquad\;\; - \\ \hline & (a_1 b_2 - a_2 b_1)\, x = c_1 b_2 - c_2 b_1 \end{array}$$

$$x = \frac{c_1 b_2 - c_2 b_1}{a_1 b_2 - a_2 b_1} \quad \ldots \quad \text{(V)}$$

Similarly $y = \dfrac{a_1 c_2 - a_2 c_1}{a_1 b_2 - a_2 b_1} \quad \ldots \quad$ (VI)

To remember and write the expressions

$c_1 b_2 - c_2 b_1$, $a_1 b_2 - a_2 b_1$, $a_1 c_2 - a_2 c_1$ we use the determinants.

Now $a_1 x + b_1 y = c_1$

and $a_2 x + b_2 y = c_2$

We can write 3 columns. $\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}, \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}, \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$

The values x, y in equation (V), (VI) are written using determinants as follows

$$x = \frac{c_1 b_2 - c_2 b_1}{a_1 b_2 - a_2 b_1} = \frac{\begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}},$$

$$y = \frac{a_1 c_2 - a_2 c_1}{a_1 b_2 - a_2 b_1} = \frac{\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

To remember let us denote $D = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$, $D_x = \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}$, $D_y = \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}$

$$\therefore\; x = \frac{D_x}{D}, \quad y = \frac{D_y}{D}$$

For writting D, Dx, Dy remember the order of columns $\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}, \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}, \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$.

From the equations,

$$a_1 x + b_1 y = c_1$$

and $a_2 x + b_2 y = c_2$ we get the columns $\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$, $\begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$, $\begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$.

- In D the column of constants $\begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$ is omitted.
- In D_x the column of the coefficients of x, $\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$ is replaced by $\begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$.
- In D_y the column of the coefficients of y, $\begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$ is replaced by $\begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$.

Let's remember!

Cramer's method to solve simultaneous equations.

Write given equations in the form $ax + by = c$.

↓

Find the values of determinants D, D_x and D_y

↓

Using, $x = \frac{D_x}{D}$ and $y = \frac{D_y}{D}$ find values of x, y.

Gabriel Cramer

(31 July, 1704 to 4 January, 1752)

This Swiss mathematician was born in Geneva. He was very well versed in mathematics, since childhood. At the age of eighteen, he got a doctorate. He was a professor in Geneva.

Solved Example

Ex. (1) Solve the following simultaneous equations using Cramer's Rule.

$5x + 3y = -11$; $2x + 4y = -10$

Solution : Given equations

$5x + 3y = -11$

$2x + 4y = -10$

$$D = \begin{vmatrix} 5 & 3 \\ 2 & 4 \end{vmatrix} = (5 \times 4) - (2 \times 3) = 20 - 6 = 14$$

$$D_x = \begin{vmatrix} -11 & 3 \\ -10 & 4 \end{vmatrix} = (-11) \times 4 - (-10) \times 3 = -44 - (-30)$$

$$= -44 + 30 = -14$$

$$D_y = \begin{vmatrix} 5 & -11 \\ 2 & -10 \end{vmatrix} = 5 \times (-10) - 2 \times (-11) = -50 - (-22)$$

$$= -50 + 22 = -28$$

$$x = \frac{D_x}{D} = \frac{-14}{14} = -1$$

and $$y = \frac{D_y}{D} = \frac{-28}{14} = -2$$

$\therefore$ $(x, y) = (-1, -2)$ is the solution.

Activity 1 : To solve the simultaneous equations by determinant method, fill in the blanks

$y + 2x - 19 = 0$; $2x - 3y + 3 = 0$

Solution : Write the given equations in the form $ax + by = c$

$2x + y = 19$

$2x - 3y = -3$

$$D = \begin{vmatrix} \boxed{} & \boxed{} \\ 2 & -3 \end{vmatrix} = [\boxed{} \times (-3)] - [2 \times (\boxed{})] = \boxed{} - (\boxed{})$$

$$= \boxed{} - \boxed{} = \boxed{}$$

$$D_x = \begin{vmatrix} 19 & \boxed{} \\ \boxed{} & -3 \end{vmatrix} = [19 \times (\boxed{})] - [(\boxed{}) \times (\boxed{})] = \boxed{} - \boxed{}$$

$$= \boxed{}$$

$$D_y = \begin{vmatrix} \square & 19 \\ 2 & \square \end{vmatrix} = [(\square) \times (\square)] - [(\square) \times (\square)]$$

$$= \square - \square = \square$$

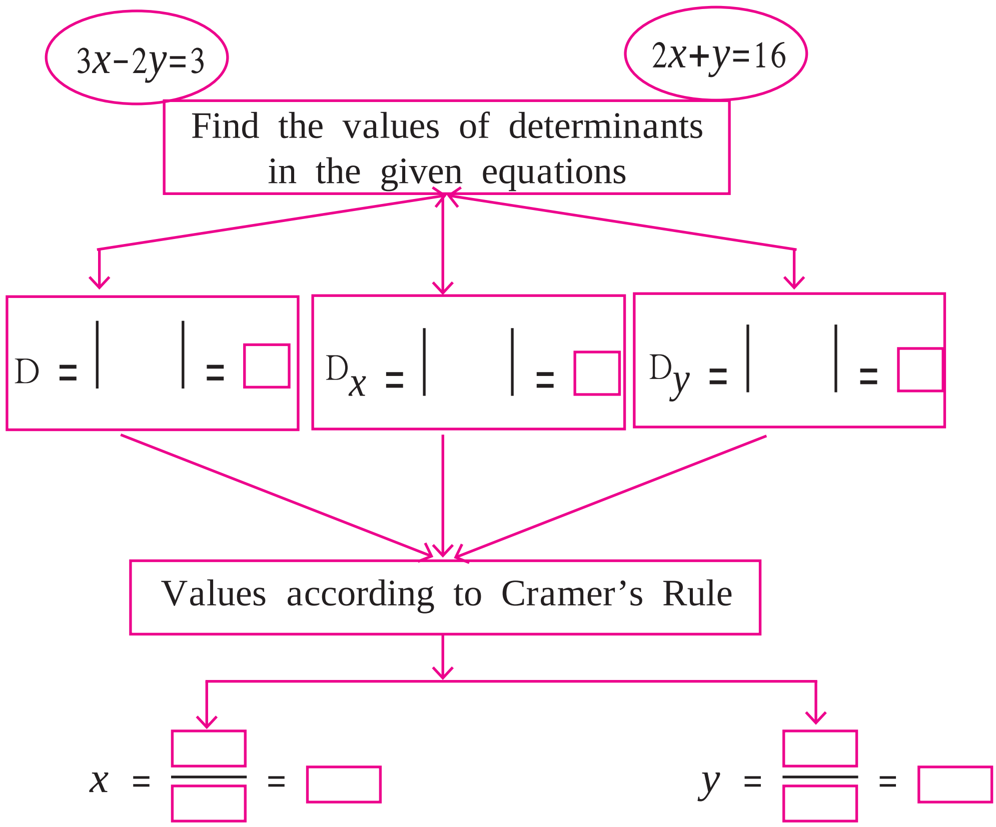
By Cramer's Rule –

$$x = \frac{D_x}{D} \qquad\qquad y = \frac{D_y}{D}$$

$$\therefore\ x = \frac{\square}{\square} = \square \qquad\qquad y = \frac{\square}{\square} = \square$$

$\therefore$ $(x, y) = (\square, \square)$ is the solution of the given equations.

Activity 2 : Complete the following activity –



$\therefore$ $(x, y) = (\square, \square)$ is the solution.

Let's think.

- What is the nature of solution if D = 0 ?
- What can you say about lines if common solution is not possible?

Practice Set 1.3

1. Fill in the blanks with correct number

$$\begin{vmatrix} 3 & 2 \\ 4 & 5 \end{vmatrix} = 3 \times \boxed{\quad} - \boxed{\quad} \times 4 \qquad = \boxed{\quad} - 8 \quad = \boxed{\quad}$$

2. Find the values of following determinants.

(1) $\begin{vmatrix} -1 & 7 \\ 2 & 4 \end{vmatrix}$ (2) $\begin{vmatrix} 5 & 3 \\ -7 & 0 \end{vmatrix}$ (3) $\begin{vmatrix} \frac{7}{3} & \frac{5}{3} \\ \frac{3}{2} & \frac{1}{2} \end{vmatrix}$

3. Solve the following simultaneous equations using Cramer's rule.

(1) $3x - 4y = 10$; $4x + 3y = 5$ (2) $4x + 3y - 4 = 0$; $6x = 8 - 5y$

(3) $x + 2y = -1$; $2x - 3y = 12$ (4) $6x - 4y = -12$; $8x - 3y = -2$

(5) $4m + 6n = 54$; $3m + 2n = 28$ (6) $2x + 3y = 2$; $x - \frac{y}{2} = \frac{1}{2}$

Let's learn.

Equations reducible to a pair of linear equations in two variables

Activity : Complete the following table.

Equation	No. of variables	whether linear or not
$\frac{3}{x} - \frac{4}{y} = 8$	2	Not linear
$\frac{6}{x-1} + \frac{3}{y-2} = 0$	$\boxed{\quad}$	$\boxed{\quad}$
$\frac{7}{2x+1} + \frac{13}{y+2} = 0$	$\boxed{\quad}$	$\boxed{\quad}$
$\frac{14}{x+y} + \frac{3}{x-y} = 5$	$\boxed{\quad}$	$\boxed{\quad}$

Let's think.

In the above table the equations are not linear. Can you convert the equations into linear equations ?

Let's remember!

We can create new variables making a proper change in the given variables. Substituting the new variables in the given non-linear equations, we can convert them in linear equations.

Also remember that the denominator of any fraction of the form $\frac{m}{n}$ cannot be zero.

Solved examples

Solve:

Ex. (1) $\frac{4}{x} + \frac{5}{y} = 7; \frac{3}{x} + \frac{4}{y} = 5$

Solution : $\frac{4}{x} + \frac{5}{y} = 7; \frac{3}{x} + \frac{4}{y} = 5$

$$4\left(\frac{1}{x}\right) + 5\left(\frac{1}{y}\right) = 7 \quad . . . \quad \text{(I)}$$

$$3\left(\frac{1}{x}\right) + 4\left(\frac{1}{y}\right) = 5 \quad . . . \quad \text{(II)}$$

Replacing $\left(\frac{1}{x}\right)$ by m and $\left(\frac{1}{y}\right)$ by n in equations (I) and (II), we get

$4m + 5n = 7 \quad . . . \quad$ (III)

$3m + 4n = 5 \quad . . . \quad$ (IV)

On solving these equations we get

$m = 3, n = -1$

Now, $m = \frac{1}{x} \quad \therefore 3 = \frac{1}{x} \quad \therefore x = \frac{1}{3}$

$n = \frac{1}{y} \quad \therefore -1 = \frac{1}{y} \quad \therefore y = -1$

$\therefore$ Solution of given simultaneous equations is $(x, y) = (\frac{1}{3}, -1)$

Ex.(2) $\dfrac{4}{x-y} + \dfrac{1}{x+y} = 3$; $\dfrac{2}{x-y} - \dfrac{3}{x+y} = 5$

Solution : $\dfrac{4}{x-y} + \dfrac{1}{x+y} = 3$; $\dfrac{2}{x-y} - \dfrac{3}{x+y} = 5$

$$4\left(\frac{1}{x-y}\right) + 1\left(\frac{1}{x+y}\right) = 3 \quad . . . \quad \text{(I)}$$

$$2\left(\frac{1}{x-y}\right) - 3\left(\frac{1}{x+y}\right) = 5 \quad . . . \quad \text{(II)}$$

Replacing $\left(\dfrac{1}{x-y}\right)$ by a and $\left(\dfrac{1}{x+y}\right)$ by b we get

$4a + b = 3$ . . . (III)

$2a - 3b = 5$ . . . (IV)

On solving these equations we get, $a = 1$ $b = -1$

But $a = \left(\dfrac{1}{x-y}\right)$, $b = \left(\dfrac{1}{x+y}\right)$

$\therefore \left(\dfrac{1}{x-y}\right) = 1, \left(\dfrac{1}{x+y}\right) = -1$

$\therefore$ $x - y = 1$ . . . (V)

$x + y = -1$ . . . (VI)

Solving equation (V) and (VI) we get $x = 0$, $y = -1$

$\therefore$ Solution of the given equations is $(x, y) = (0, -1)$

Let's think.

In the above examples the simultaneous equations obtained by transformation are solved by elimination method.

If you solve these equations by graphical method and by Cramer's rule will you get the same answers ? Solve and check it.

Activity : To solve given equations fill the boxes below suitably.

$$\frac{5}{x-1} + \frac{1}{y-2} = 2 \; ; \; \frac{6}{x-1} - \frac{3}{y-2} = 1$$

Replacing $\left(\frac{1}{x-1}\right)$ by m, $\left(\frac{1}{y-2}\right)$ by n

New equations

$\boxed{}$ $\quad$ $6m - 3n = 1$

On solving

$m = \boxed{}$, $n = \boxed{}$

Replacing m, n by their original values.

$\left(\frac{1}{x-1}\right) = \frac{1}{3}$ $\quad$ $\boxed{}$

On solving

$x = \boxed{}$, $y = \boxed{}$

$\therefore$ $\boxed{(x, y) = (\quad , \quad)}$ is the solution of the given simultaneous equations.

Practice Set 1.4

1. Solve the following simultaneous equations.

(1) $\frac{2}{x} - \frac{3}{y} = 15$; $\frac{8}{x} + \frac{5}{y} = 77$

(2) $\frac{10}{x+y} + \frac{2}{x-y} = 4$; $\frac{15}{x+y} - \frac{5}{x-y} = -2$

(3) $\frac{27}{x-2} + \frac{31}{y+3} = 85$; $\frac{31}{x-2} + \frac{27}{y+3} = 89$

(4) $\frac{1}{3x+y} + \frac{1}{3x-y} = \frac{3}{4}$; $\frac{1}{2(3x+y)} - \frac{1}{2(3x-y)} = -\frac{1}{8}$

Let's learn.

Application of Simultaneous equations

Activity : There are some instructions given below. Frame the equations from the information and write them in the blank boxes shown by arrows.

Sarthak's age is less by 8 than double the age of Sakshi

I am Sarthak

My Present age is x years.

4 years ago Sakshi's age was 3 years less than Sarthak's age at that time.

The sum of present ages of Sarthak and Sakshi is 25.

My Present age is y years.

I am Sakshi

Ex. (1) The perimeter of a rectangle is 40 cm. The length of the rectangle is more than double its breadth by 2. Find length and breadth.

Solution : Let length of rectangle be x cm and breadth be y cm.

From first condition –

$$2(x + y) = 40$$

$$x + y = 20 \quad \ldots \quad \text{(I)}$$

From 2^{nd} condition –

$$x = 2y + 2$$

$$\therefore\ x - 2y = 2 \quad \ldots \quad \text{(II)}$$

Let's solve eq. (I), (II) by determinant method

$$x + y = 20$$

$$x - 2y = 2$$