1.9 KIRCHHOFF'S VOLTAGE LAW

Kirchhoff's voltage law states that the algebraic sum of all branch voltages around any closed path in a circuit is always zero at all instants of time. When the current passes through a resistor, there is a loss of energy and, therefore, a voltage drop. In any element, the current always flows from higher potential to lower potential. Consider the circuit in Fig. 1.11. It is customary to take the direction of current I as indicated in the figure, i.e. it leaves the positive terminal of the voltage source and enters into the negative terminal.

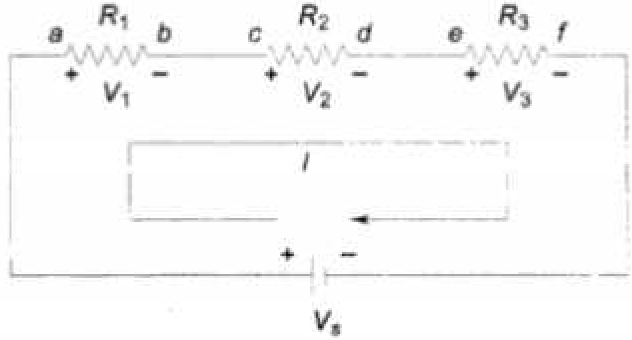

Fig. 1.11

As the current passes through the circuit, the sum of the voltage drop around the loop is equal to the total voltage in that loop. Here the polarities are attributed to the resistors to indicate that the voltages at points a, c and e are more than the voltages at b, d and f, respectively, as the current passes from a to f.

$$\therefore \qquad V_s = V_1 + V_2 + V_3$$

Consider the problem of finding out the current supplied by the source V in the circuit shown in Fig. 1.12.

Our first step is to assume the reference current direction and to indicate the polarities for different elements. (See Fig. 1.13).

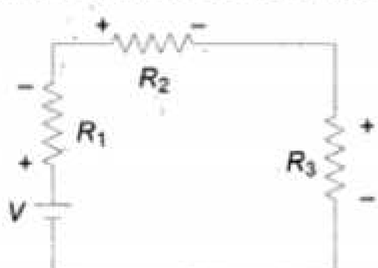

Fig. 1.12

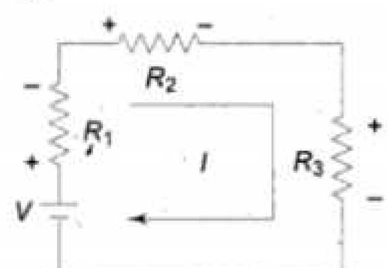

Fig. 1.13

By using Ohm's law, we find the voltage across each resistor as follows.

$$V_{R1} = IR_1,\ V_{R2} = IR_2,\ V_{R3} = IR_3$$

where V_{R1}, V_{R2} and V_{R3} are the voltages across R_1, R_2 and R_3, respectively. Finally, by applying Kirchhoff's law, we can form the equation

$$V = V_{R1} + V_{R2} + V_{R3}$$

$$V = IR_1 + IR_2 + IR_3$$

From the above equation the current delivered by the source is given by

$$I = \frac{V}{R_1 + R_2 + R_3}$$

1.10 VOLTAGE DIVISION

The series circuit acts as a voltage divider. Since the same current flows through each resistor, the voltage drops are proportional to the values of resistors. Using this principle, different voltages can be obtained from a single source, called a voltage divider. For example, the voltage across a 40 Ω resistor is twice that of 20 Ω in a series circuit shown in Fig. 1.19.

In general, if the circuit consists of a number of series resistors, the total current is given by the total voltage divided by equivalent resistance. This is shown in Fig. 1.20.

Fig. 1.19

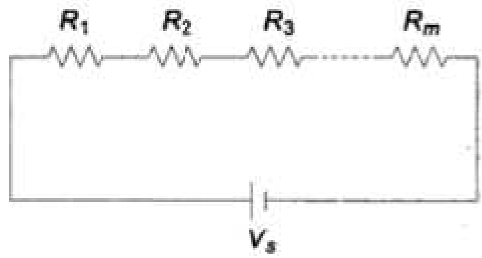

Fig. 1.20

The current in the circuit is given by $I = V_s/(R_1 + R_2 + \ldots + R_m)$. The voltage across any resistor is nothing but the current passing through it, multiplied by that particular resistor.

Therefore,

$$V_{R1} = IR_1$$

$$V_{R2} = IR_2$$

$$V_{R3} = IR_3$$

$$\vdots$$

$$V_{Rm} = IR_m$$

or
$$V_{Rm} = \frac{V_s(R_m)}{R_1 + R_2 + \ldots + R_m}$$

From the above equation, we can say that the voltage drop across any resistor, or a combination of resistors, in a series circuit is equal to the ratio of that resistance value to the total resistance, multiplied by the source voltage, i.e.

$$V_m = \frac{R_m}{R_T} V_s$$

where V_m is the voltage across mth resistor,
R_m is the resistance across which the voltage is to be determined and R_T is the total series resistance.

1.12 KIRCHHOFF'S CURRENT LAW

Kirchhoff's current law states that the sum of the currents entering into any node is equal to the sum of the currents leaving that node. The node may be an interconnection of two or more branches. In any parallel circuit, the node is a junction point of two or more branches. The total current entering into a node is equal to the current leaving that node. For example, consider the circuit shown in Fig. 1.24, which contains two nodes A and B. The total current I_T entering node A is divided into I_1, I_2 and I_3. These currents flow out of node A. According to Kirchhoff's current law, the current into node A is equal to the total current out of node A: that is, $I_T = I_1 + I_2 + I_3$. If we consider node B, all three currents I_1, I_2, I_3 are entering B, and the total current I_T is leaving node B, Kirchhoff's current law formula at this node is therefore the same as at node A.

$$I_1 + I_2 + I_3 = I_T$$

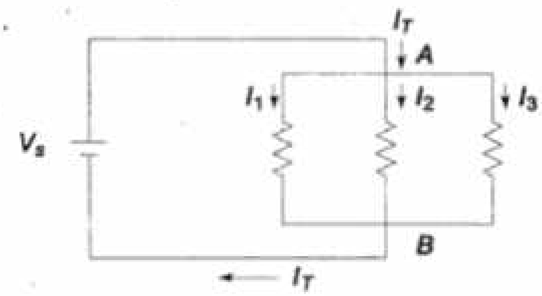

Fig. 1.24

In general, sum of the currents entering any point or node or junction equal to sum of the currents leaving from that point or node or junction as shown in Fig. 1.25.

$$I_1 + I_2 + I_4 + I_7 = I_3 + I_5 + I_6$$

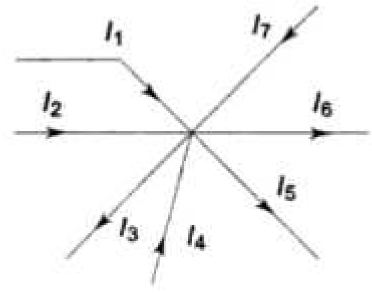

Fig. 1.25

If all of the terms on the right side are brought over to the left side, their signs change to negative and a zero is left on the right side, i.e.

$$I_1 + I_2 + I_4 + I_7 - I_3 - I_5 - I_6 = 0$$

This means that the algebraic sum of all the currents meeting at a junction is equal to zero.

1.14 CURRENT DIVISION

In a parallel circuit, the current divides in all branches. Thus, a parallel circuit acts as a current divider. The total current entering into the parallel branches is divided into the branches currents according to the resistance values. The branch having higher resistance allows lesser current, and the branch with lower resistance allows more current. Let us find the current division in the parallel circuit shown in Fig. 1.32.

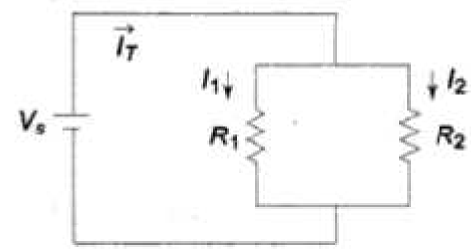

Fig. 1.32

The voltage applied across each resistor is V_s. The current passing through each resistor is given by

$$I_1 = \frac{V_s}{R_1},\ I_2 = \frac{V_s}{R_2}$$

If R_T is the total resistance, which is given by $R_1R_2/(R_1 + R_2)$,

Total current $$I_T = \frac{V_s}{R_T} = \frac{V_s}{R_1 R_2}\ (R_1 + R_2)$$

or $$I_T = \frac{I_1 R_1}{R_1 R_2}\ (R_1 + R_2) \text{ since } V_s = I_1 R_1$$

$$I_1 = I_T \cdot \frac{R_2}{R_1 + R_2}$$

Similarly,

$$I_2 = I_T \cdot \frac{R_1}{R_1 + R_2}$$

From the above equations, we can conclude that the current in any branch is equal to the ratio of the opposite branch resistance to the total resistance value, multiplied by the total current in the circuit. In general, if the circuit consists of *m* branches, the current in any branch can be determined by

$$I_i = \frac{R_T}{R_i + R_T} I_T$$

where I_i represents the current in the *i*th branch
R_i is the resistance in the *i*th branch
R_T is the total parallel resistance to the *i*th branch and
I_T is the total current entering the circuit.