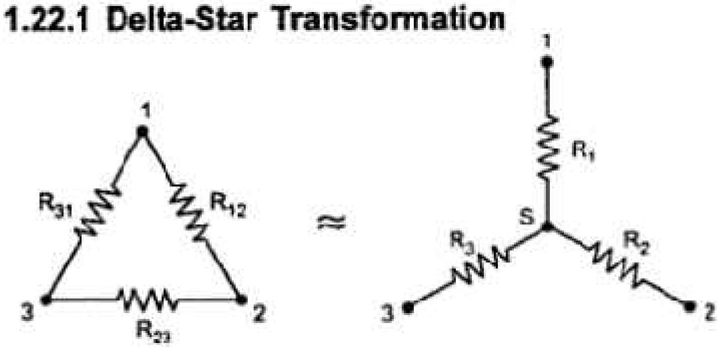
1.22.1 Delta-Star Transformation



Given Delta Fig. 1.49 Equivalent Star

Consider the three resistances R_{12}, R_{23}, R_{31} connected in Delta as shown in the Fig. 1.49. The terminals between which these are connected in Delta are named as 1, 2 and 3.

Now it is always possible to replace these Delta connected resistances by three equivalent Star connected resistances R_1, R_2, R_3 between the same terminals 1, 2, and 3. Such a Star is shown inside the Delta in the Fig. 1.49 which is called **equivalent Star of Delta connected resistances.**

> **Key Point** : *Now to call these two arrangements as equivalent, the resistance between any two terminals must be same in both the types of connections.*

Let us analyse Delta connection first, shown in the Fig. 1.50 (a).

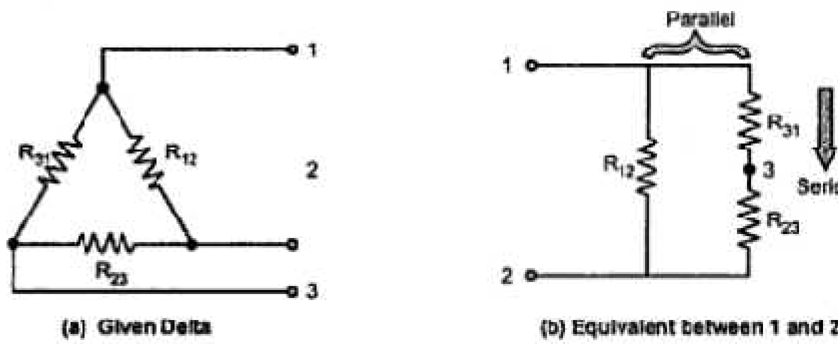


Fig. 1.50

Now consider the terminals (1) and (2). Let us find equivalent resistance between (1) and (2). We can redraw the network as viewed from the terminals (1) and (2), without considering terminal (3). This is shown in the Fig. 1.50 (b).

Now terminal '3' we are not considering, so between terminals (1) and (2) we get the combination as,

R_{12} parallel with $(R_{31}+R_{23})$ as R_{31} and R_{23} are in series.

$\therefore$ Between (1) and (2) the resistance is,

$$= \frac{R_{12}(R_{31}+R_{23})}{R_{12}+(R_{31}+R_{23})} \qquad \text{...(a)}$$

[using $\frac{R_1 R_2}{R_1+R_2}$ for parallel combination]

Now consider the same two terminals of equivalent Star connection shown in the Fig. 1.51.

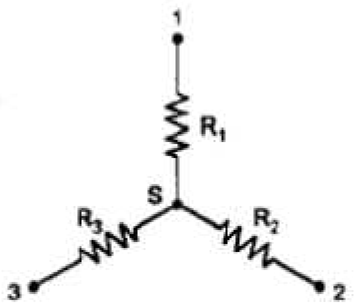


Fig. 1.51 Star connection

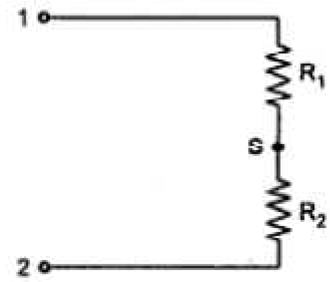


Fig. 1.52 Equivalent between 1 and 2

Now as viewed from terminals (1) and (2) we can see that terminal (3) is not getting connected anywhere and hence is not playing any role in deciding the resistance as viewed from terminals (1) and (2).

And hence we can redraw the network as viewed through the terminals (1) and (2) as shown in the Fig. 1.52.

$\therefore$ Between (1) and (2) the resistance is $= R_1+R_2$ ... (b)

This is because, two of them found to be in series across the terminals 1 and 2 while 3 found to be open.

Now to call this Star as equivalent of given Delta it is necessary that the resistances calculated between terminals (1) and (2) in both the cases should be equal and hence equating equations (a) and (b),

$$\frac{R_{12}(R_{31}+R_{23})}{R_{12}+(R_{23}+R_{31})} = R_1+R_2 \qquad \text{...(c)}$$

Similarly if we find the equivalent resistance as viewed through terminals (2) and (3) in both the cases and equating, we get,

$$\frac{R_{23}(R_{31}+R_{12})}{R_{12}+(R_{23}+R_{31})} = R_2+R_3 \qquad \text{...(d)}$$

Similarly if we find the equivalent resistance as viewed through terminals (3) and (1) in both the cases and equating, we get,

$$\frac{R_{31}(R_{12}+R_{23})}{R_{12}+(R_{23}+R_{31})} = R_3+R_1 \quad \text{...(e)}$$

Now we are interested in calculating what are the values of R_1, R_2, R_3 interms of known values R_{12}, R_{23}, and R_{31}.

Subtracting euqation (d) from euqation (c),

$$\frac{R_{12}(R_{31}+R_{23})-R_{23}(R_{31}+R_{12})}{(R_{12}+R_{23}+R_{31})} = R_1+R_2-R_2-R_3$$

$$\therefore \quad R_1-R_3 = \frac{R_{12}R_{31}-R_{23}R_{31}}{R_{12}+R_{23}+R_{31}} \quad \text{...(f)}$$

Adding euqation (f) and euqation (e),

$$\frac{R_{12}R_{31}-R_{23}R_{31}+R_{31}(R_{12}+R_{23})}{R_{12}+R_{23}+R_{31}} = R_1+R_3+R_1-R_3$$

$$\therefore \quad \frac{R_{12}R_{31}-R_{23}R_{31}+R_{31}R_{12}+R_{31}R_{23}}{R_{12}+R_{23}+R_{31}} = 2R_1$$

$$2R_1 = \frac{2R_{12}R_{31}}{R_{12}+R_{23}+R_{31}}$$

$$\boxed{R_1 = \frac{R_{12}R_{31}}{R_{12}+R_{23}+R_{31}}}$$

Similarly by using another combinations of subtraction and addition with equations (c), (d) and (e) we can get,

$$\boxed{R_2 = \frac{R_{12}R_{23}}{R_{12}+R_{23}+R_{31}}}$$

and

$$\boxed{R_3 = \frac{R_{23}R_{31}}{R_{12}+R_{23}+R_{31}}}$$

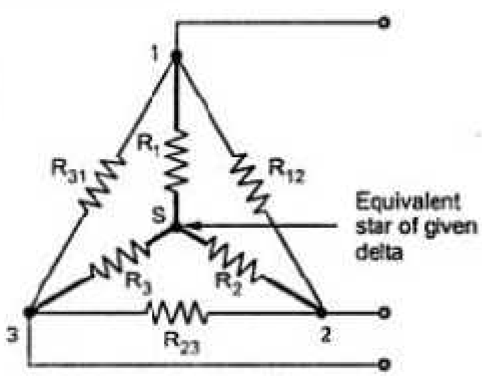
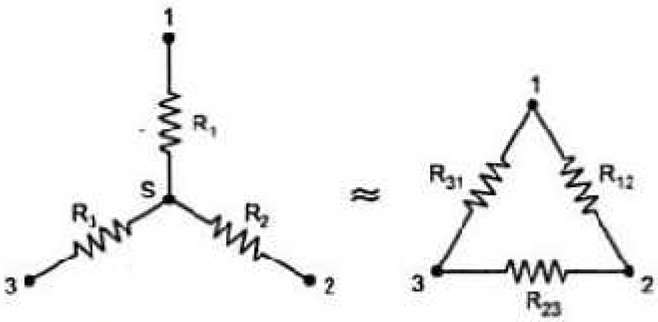


Fig. 1.53 Delta and equivalent Star

Easy way of remembering the result :

The equivalent star resistance between any terminal and star point is equal to the product of the two resistances in delta, which are connected to same terminal, divided by the sum of all three delta connected resistances.

1.22.2 Star-Delta Transformation



Given Star **Fig. 1.54** **Equivalent Delta**

Consider the three resistances R_1, R_2 and R_3 connected in Star as shown in Fig. 1.54.

Now by Star-Delta conversion, it is always possible to replace these Star connected resistances by three equivalent Delta connected resistances R_{12}, R_{23} and R_{31}, between the same terminals. This is called **equivalent Delta of the given star.**

Now we are interested in finding out values of R_{12}, R_{23} and R_{31} interms of R_1, R_2 and R_3.

For this we can use set of equations derived in previous article. From the result of Delta-Star transformation we know that,

$$R_1 = \frac{R_{12}\, R_{31}}{R_{12}+R_{23}+R_{31}} \quad \text{...(g)}$$

$$R_2 = \frac{R_{12}\, R_{23}}{R_{12}+R_{23}+R_{31}} \quad \text{...(h)}$$

$$R_3 = \frac{R_{23}\, R_{31}}{R_{12}+R_{23}+R_{31}} \quad \text{...(i)}$$

Now multiply (g) and (h), (h) and (i), (i) and (g) to get following three equations.

$$R_1 R_2 = \frac{R_{12}^2\, R_{31}\, R_{23}}{(R_{12}+R_{23}+R_{31})^2} \quad \text{...(j)}$$

$$\therefore \quad R_2 R_3 = \frac{R_{23}^2\, R_{12}\, R_{31}}{(R_{12}+R_{23}+R_{31})^2} \quad \text{...(k)}$$

$$R_3 R_1 = \frac{R_{31}^2\, R_{12}\, R_{23}}{(R_{12}+R_{23}+R_{31})^2} \quad \text{...}(l)$$

Now add equations (j) ,(k) and (l)

$$\therefore \qquad R_1R_2 + R_2R_3 + R_3R_1 = \frac{R_{12}^2 R_{31} R_{23} + R_{23}^2 R_{12} R_{31} + R_{31}^2 R_{12} R_{23}}{(R_{12} + R_{23} + R_{31})^2}$$

$$\therefore \qquad R_1R_2 + R_2R_3 + R_3R_1 = \frac{R_{12} R_{31} R_{23} (R_{12} + R_{23} + R_{31})}{(R_{12} + R_{23} + R_{31})^2}$$

$$\therefore \qquad R_1R_2 + R_2R_3 + R_3R_1 = \frac{R_{12} R_{31} R_{23}}{R_{12} + R_{23} + R_{31}}$$

But $\dfrac{R_{12} R_{31}}{R_{12} + R_{23} + R_{31}} = R_1$ From equation (g)

$\therefore$ Substituting in above in R.H.S. we get,

$$\therefore \qquad R_1R_2 + R_2R_3 + R_3R_1 = R_1 R_{23}$$

$$\therefore \qquad \boxed{R_{23} = R_2 + R_3 + \frac{R_2 R_3}{R_1}}$$

Similarly substituting in R.H.S., remaining values, we can write relations for remaining two resistances.

$$\boxed{R_{12} = R_1 + R_2 + \frac{R_1 R_2}{R_3}}$$

and

$$\boxed{R_{31} = R_3 + R_1 + \frac{R_1 R_3}{R_2}}$$

Easy way of remembering the result :

The equivalent delta connected resistance to be connected between any two terminals is sum of the two resistances connected between the same two terminals and star point respectively in star, plus the product of the same two star resistances divided by the third star resistance.