1.14 Series Circuit

A **series** circuit is one in which several resistances are connected one after the other. Such connection is also called **end to end** connection or **cascade** connection. There is only one path for the flow of current.

1.14.1 Resistors in Series

Current same voltage division

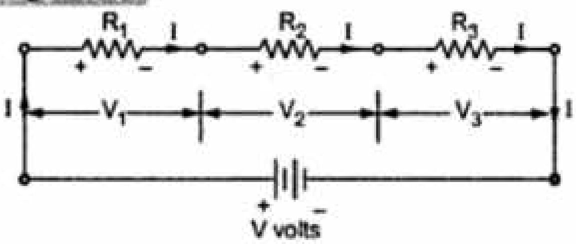


Fig. 1.21 A series circuit

Consider the resistances shown in the Fig. 1.21.

The resistance R_1, R_2 and R_3 are said to be in series. The combination is connected across a source of voltage V volts. Naturally the current flowing through all of them is same indicated as I amperes. e.g. the chain of small lights, used for the decoration purposes is good example of series combination.

Now let us study the **voltage distribution.**

Let V_1, V_2 and V_3 be the voltages across the terminals of resistances R_1, R_2 and R_3 respectively

Then, $$V = V_1 + V_2 + V_3$$

Now according to Ohm's law, $$V_1 = I R_1\ ,\ V_2 = I R_2\ ,\ V_3 = I R_3$$

Current through all of them is same i.e. I

$$\therefore \qquad V = I R_1 + I R_2 + I R_3 = I(R_1 + R_2 + R_3)$$

Applying Ohm's law to overall circuit,

$$V = I R_{eq}$$

where R_{eq} = Equivalent resistance of the circuit. By comparison of two equations,

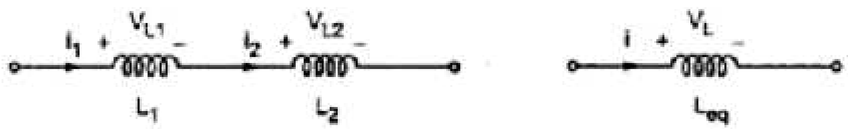
$$R_{eq} = R_1 + R_2 + R_3$$

i.e. total or **equivalent resistance** of the series circuit is arithmetic sum of the resistances connected in series.

For n resistances in series, $$\boxed{R = R_1 + R_2 + R_3 + \ldots\ldots + R_n}$$

1.14.2 Inductors in Series

Consider the Fig. 1.22 (a). Two inductors L_1 and L_2 are connected in series. The currents flowing through L_1 and L_2 are i_1 and i_2 while voltages developed across L_1 and L_2 are V_{L1} and V_{L2} respectively. The equivalent circuit is shown in the Fig. 1.22 (b).



(a) Fig. 1.22 (b)

We have, $V_{L1} = L_1 \frac{di_1}{dt}$ and $V_{L2} = L_2 \frac{di_2}{dt}$ while $V_L = L_{eq} \frac{di}{dt}$

For series combination,

$$i = i_1 = i_2$$

and $$V_L = V_{L1} + V_{L2}$$

$$\therefore \quad L_{eq} \frac{di}{dt} = L_1 \frac{di}{dt} + L_2 \frac{di}{dt}$$

$$\therefore \quad L_{eq} \frac{di}{dt} = (L_1 + L_2) \frac{di}{dt}$$

$$\therefore \quad L_{eq} = L_1 + L_2$$

That means, equivalent inductance of the series combination of two inductances is the sum of inductances connected in series.

The total equivalent inductance of the series circuit is sum of the inductances connected in series.

For n inductances in series,

$$\boxed{L_{eq} = L_1 + L_2 + L_3 + \ldots + L_n}$$

1.14.3 Capacitor in Series

Consider the Fig. 1.23 (a). Two capacitors C_1 and C_2 are connected in series. The currents flowing through and voltages developed across C_1 and C_2 are i_1, i_2 and V_{C1} and V_{C2} respectively. The equivalent circuit is shown in the Fig. 1.23 (b).

(a) Fig. 1.23 (b)

We have, $V_{C1} = \frac{1}{C_1}\int_{-\infty}^{t} i_1 dt$, $V_{C2} = \frac{1}{C_2}\int_{-\infty}^{t} i_2 dt$ while $V = \frac{1}{C_{eq}}\int_{-\infty}^{t} i dt$

For series combination,

$$i = i_1 = i_2 \quad \text{and}$$

$$V_C = V_{C1} + V_{C2}$$

$$\frac{1}{C_{eq}}\int_{-\infty}^{t} i dt = \frac{1}{C_1}\int_{-\infty}^{t} i_1 dt + \frac{1}{C_2}\int_{-\infty}^{t} i_2 dt$$

But $\qquad i = i_1 = i_2$

$\therefore$
$$\frac{1}{C_{eq}}\int_{-\infty}^{t} i dt = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)\int_{-\infty}^{t} i\, dt$$

$\therefore$
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

$\therefore$
$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$$

That means, reciprocal of equivalent capacitor of the series combination is the sum of the reciprocal of individual capacitances.

The reciprocal of the total equivalent capacitor of the series combination is the sum of the reciprocols of the individual capacitors, connected in series.

For n capacitors in series,

$$\boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots \frac{1}{C_n}}$$

1.15 Parallel Circuits

The **parallel** circuit is one in which several resistances are connected across one another in such a way that one terminal of each is connected to form a junction point while the remaining ends are also joined to form another junction point.

1.15.1 Resistors in Parallel

Consider a parallel circuit shown in the Fig. 1.24.

In the parallel connection shown, the three resistances R_1, R_2 and R_3 are connected in parallel and combination is connected across a source of voltage 'V '.

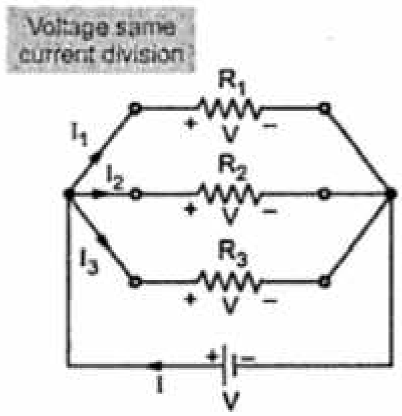


Fig. 1.24 A parallel circuit

In parallel circuit current passing through each resistance is different. Let total current drawn is say ' I ' as shown. There are 3 paths for this current, one through R_1, second through R_2 and third through R_3. Depending upon the values of R_1, R_2 and R_3 the appropriate fraction of total current passes through them. These individual currents are shown as I_1, I_2 and I_3. While the voltage across the two ends of each resistances R_1, R_2 and R_3 is the same and equals the supply voltage V.

Now let us study current distribution. Apply Ohm's law to each resistance.

$$V = I_1 R_1, \quad V = I_2 R_2, \quad V = I_3 R_3$$

$$I_1 = \frac{V}{R_1}, \quad I_2 = \frac{V}{R_2}, \quad I_3 = \frac{V}{R_3}$$

$$I = I_1 + I_2 + I_3 = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

$$= V\left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right] \quad \text{... (1)}$$

For overall circuit if Ohm's law is applied,

$$V = I R_{eq}$$

and $$I = \frac{V}{R_{eq}} \quad \text{... (2)}$$

where R_{eq} = Total or **equivalent resistance** of the circuit.

Comparing the two equations,

$$\frac{1}{R_{eq}} = \frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}$$

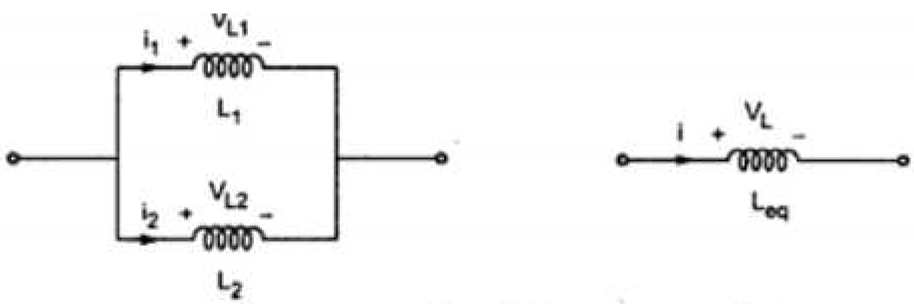
where R is the equivalent resistance of the parallel combination.

In general if 'n' resistances are connected in parallel,

$$\boxed{\frac{1}{R} = \frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}+......+\frac{1}{R_n}}$$

1.15.2 Inductors in Parallel

Consider the Fig. 1.25 (a). Two inductors L_1 and L_2 are connected in parallel. The currents flowing through L_1 and L_2 are i_1 and i_2 respectively. The voltage developed across L_1 and L_2 are V_{L1} and V_{L2} respectively. The equivalent circuit is shown in Fig. 1.25 (b)



(a) Fig. 1.25 (b)

For inductor we have,

$$i_1 = \frac{1}{L_1}\int_{-\infty}^{t} V_{L1}\,dt, \qquad i_2 = \frac{1}{L_2}\int_{-\infty}^{t} V_{L2}\,dt, \text{ while } i = \frac{1}{L_{eq}}\int_{-\infty}^{t} V_L\,dt$$

For parallel combination,

$$V_L = V_{L1} = V_{L2} \quad \text{and}$$

$$i = i_1 + i_2$$

$$\therefore \quad \frac{1}{L_{eq}}\int_{-\infty}^{t} V_L dt = \frac{1}{L_1}\int_{-\infty}^{t} V_L\,dt + \frac{1}{L_2}\int_{-\infty}^{t} V_L\,dt = \left(\frac{1}{L_1}+\frac{1}{L_2}\right)\int_{-\infty}^{t} V_L\,dt$$

$$\therefore \quad \frac{1}{L_{eq}} = \frac{1}{L_1}+\frac{1}{L_2}$$

That means, reciprocal of equivalent inductance of the parallel combination is the sum of reciprocals of the individual inductances.

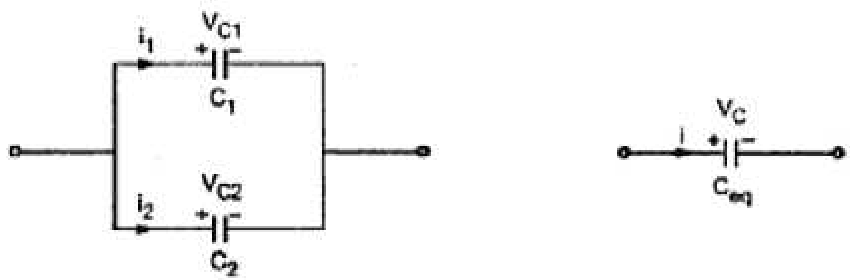
For n inductances in parallel,

$$\therefore \quad \boxed{\frac{1}{L_{eq}} = \frac{1}{L_1}+\frac{1}{L_2}+....\frac{1}{L_n}}$$

1.15.3 Capacitors in Parallel

Consider the Fig. 1.26 (a). Two capacitors C_1 and C_2 are connected in parallel. The currents flowing through C_1 and C_2 are i_1 and i_2 respectively and voltages developed across C_1, C_2 are V_{C1} and V_{C2} respectively.

The equivalent circuit is shown in the Fig. 1.26 (b).



(a) Fig. 1.26 (b)

For capacitor we have, $i_1 = C_1 \frac{dV_{C1}}{dt}$, $i_2 = C_2 \frac{dV_{C2}}{dt}$, while $i = C_{eq} \frac{dV_C}{dt}$

For parallel combination,

$$V_{C1} = V_{C2} = V_C \quad \text{and}$$

$$i = i_1 + i_2$$

$$C_{eq} \frac{dV_C}{dt} = C_1 \frac{dV_{C1}}{dt} + C_2 \frac{dV_{C2}}{dt}$$

$$\therefore \quad C_{eq} \frac{dV_C}{dt} = (C_1 + C_2) \frac{dV_C}{dt}$$

$$\therefore \quad C_{eq} = C_1 + C_2$$

That means, equivalent capacitance of the parallel combination of the capacitances is the sum of the individual capacitances connected in series.

For n capacitors in parallel,

$$\therefore \quad \boxed{C_{eq} = C_1 + C_2 + \ldots + C_n}$$