Representation of sinusoidal waveforms

THE SINE WAVE

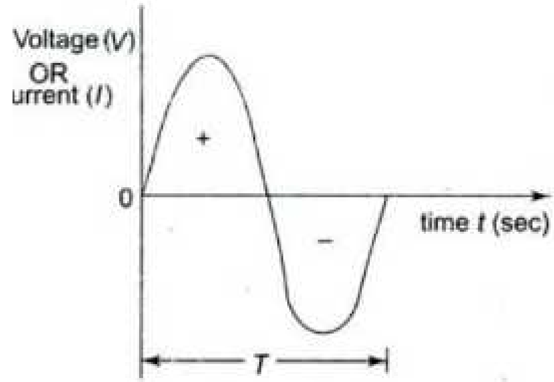

Fig. 4.1

Many a time, alternating voltages and currents are represented by a sinusoidal wave, or simply a sinusoid. It is a very common type of alternating current (ac) and alternating voltage. The sinusoidal wave is generally referred to as a sine wave. Basically an alternating voltage (current) waveform is defined as the voltage (current) that fluctuates with time periodically, with change in polarity and direction.

The shape of a sinusoidal waveform is shown The complete positive and negative portion of the wave is one cycle of the sine wave. Time is designated by t. The time taken for any wave to complete one full cycle is called the period (T).

The period can be measured in the following different ways (See Fig. 4.3).

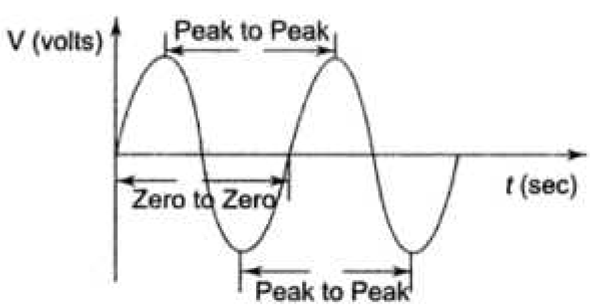

Fig. 4.3

1. From zero crossing of one cycle to zero crossing of the next cycle.
2. From positive peak of one cycle to positive peak of the next cycle, and
3. From negative peak of one cycle to negative peak of the next cycle.

The frequency of a wave is defined as the number of cycles that a wave completes in one second. The relation between time period and frequency is given by

$$f = \frac{1}{T}$$

4.3 THE SINE WAVE EQUATION

A sine wave is graphically represented as shown in Fig. 4.10 (a). The amplitude of a sine wave is represented on vertical axis. The angular measurement (in degrees or radians) is represented on horizontal axis. Amplitude A is the maximum value of the voltage or current on the Y-axis.

In general, the sine wave is represented by the equation

$$v(t) = V_m \sin \omega t$$

When a sine wave is shifted to the left of the reference wave by a certain angle ϕ, as shown in Fig. 4.10 (b), the general expression can be written as

$$v(t) = V_m \sin(\omega t + \phi)$$

When a sine wave is shifted to the right of the reference wave by a certain angle ϕ, as shown in Fig. 4.10 (c), the general expression is

$$v(t) = V_m \sin(\omega t - \phi)$$

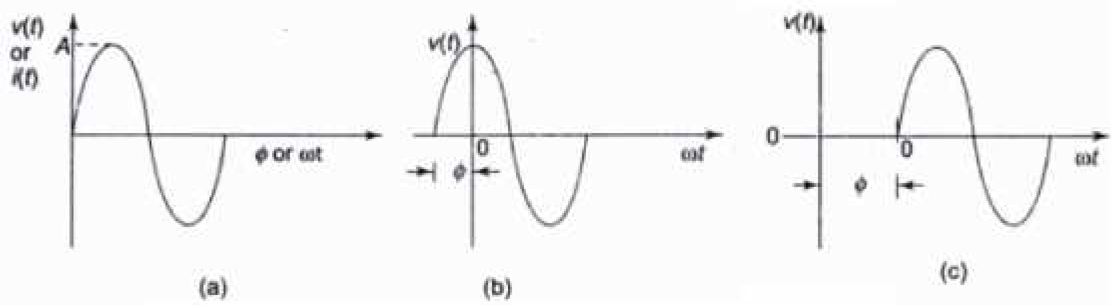

Fig. 4.10

4.4 VOLTAGE AND CURRENT VALUES OF A SINE WAVE

As the magnitude of the waveform is not constant, the waveform can be measured in different ways. These are instantaneous, peak, peak to peak, root mean square (*rms*) and average values.

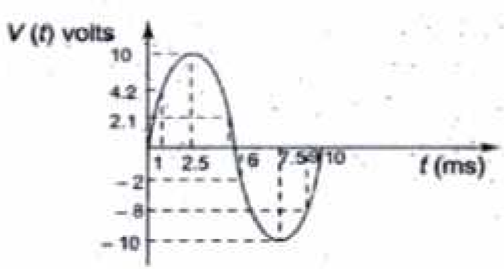

Fig. 4.12

Instantaneous Value

Consider the sine wave shown in Fig. 4.12. At any given time, it has some instantaneous value. This value is different at different points along the waveform.

In Fig. 4.12 during the positive cycle, the instantaneous values are positive and during the negative cycle, the instantaneous values are negative. In Fig. 4.12 shown at time 1 ms, the value is 4.2 V; the value is 10 V at 2.5 ms, –2 V at 6 ms and –10 V at 7.5 and so on.

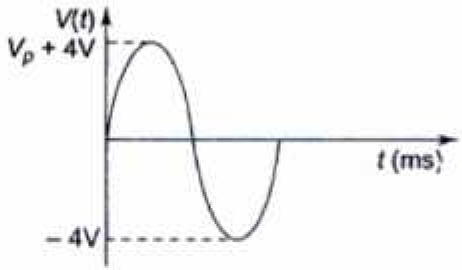

Fig. 4.13

Peak Value

The peak value of the sine wave is the maximum value of the wave during positive half cycle, or maximum value of wave during negative half cycle. Since the value of these two are equal in magnitude, a sine wave is characterised by a single peak value. The peak value of the sine wave is shown in Fig. 4.13; here the peak value of the sine wave is 4 V.

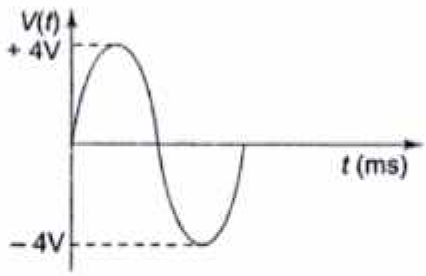

Fig. 4.14

Peak to Peak Value

The peak to peak value of a sine wave is the value from the positive to the negative peak as shown in Fig. 4.14. Here the peak to peak value is 8 V.

Average Value

In general, the average value of any function $v(t)$, with period T is given by

$$v_{av} = \frac{1}{T}\int_0^T v(t)dt$$

That means that the average value of a curve in the X-Y plane is the total area under the complete curve divided by the distance of the curve. The average value of a sine wave over one complete cycle is always zero. So the average value of a sine wave is defined over a half-cycle, and not a full cycle period.

The average value of the sine wave is the total area under the half-cycle curve divided by the distance of the curve.

The average value of the sine wave

$v(t) = V_P \sin \omega t$ is given by

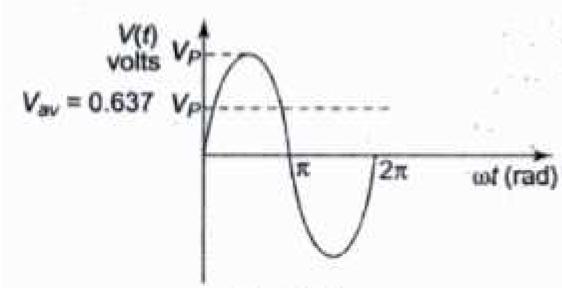

Fig. 4.15

$$v_{av} = \frac{1}{\pi}\int_0^{\pi} V_P \sin \omega t \, d(\omega t)$$

$$= \frac{1}{\pi}\left[-V_P \cos \omega t\right]_0^{\pi}$$

$$= \frac{2V_P}{\pi} = 0.637\, V_P\,.$$

The average value of a sine wave is shown by the dotted line in Fig. 4.15.

Root Mean Square Value or Effective Value

The root mean square (*rms*) value of a sine wave is a measure of the heating effect of the wave. When a resistor is connected across a dc voltage source as shown in Fig. 4.17(a), a certain amount of heat is produced in the resistor in a given time. A similar resistor is connected across an ac voltage source for the same time as shown in Fig. 4.17 (b). The value of the ac voltage is adjusted such that the same amount of heat is produced in the resistor as in the case of the dc source. This value is called the *rms* value.

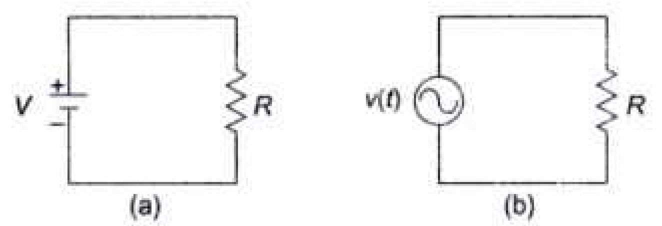

Fig. 4.17

That means the *rms* value of a sine wave is equal to the dc voltage that produces the same heating effect. In general, the *rms* value of any function with period T has an effective value given by

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T \overline{v(t)}^2\, dt}$$

Consider a function $v(t) = V_P \sin \omega t$

$$\text{The } rms \text{ value, } V_{rms} = \sqrt{\frac{1}{T}\int_0^T (V_P \sin\, \omega t)^2\, d(\omega t)}$$

$$= \sqrt{\frac{1}{T}\int_0^{2\pi} V_P^2 \left[\frac{1-\cos 2\omega t}{2}\right] d(\omega t)}$$

$$= \frac{V_P}{\sqrt{2}} = 0.707\ V_P$$

If the function consists of a number of sinusoidal terms, that is

$$v(t) = V_0 + (V_{c1} \cos\ \omega t + V_{c2} \cos 2\ \omega t + \cdots) + (V_{s1} \sin\ \omega t + V_{s2} \sin 2\ \omega t + \cdots)$$

The rms, or effective value is given by

$$V_{rms} = \sqrt{V_0^2 + \frac{1}{2}(V_{c1}^2 + V_{c2}^2 + \cdots) + \frac{1}{2}(V_{s1}^2 + V_{s2}^2 + \cdots)}$$