INSTANTANEOUS POWER

In a purely resistive circuit, all the energy delivered by the source is dissipated in the form of heat by the resistance.
In a purely reactive(inductive or capacitive) circuit, all the energy delivered by the source is stored by the inductor or capacitor in its magnetic or electric field during a portion of the voltage cycle, and then is returned to the source during another portion of the cycle, so that no net energy is transferred.

When there is complex impedance in a circuit, a part of the energy is alternately stored and returned by the reactive part, and part of it is dissipated by the resistance. The amount of energy dissipated is determined by the relative values of resistance and reactance.

Consider a circuit having complex impedance. Let $v(t) = V_m \cos \omega t$ be the voltage applied to the circuit and let $i(t) = I_m \cos(\omega t + \theta)$ be the corresponding current flowing through the circuit. Then the power at any instant of time is

$$P(t) = v(t)\, i(t)$$
$$= V_m \cos \omega t\, I_m \cos(\omega t + \theta) \qquad (6.1)$$

From Eq. (6.1), we get

$$P(t) = \frac{V_m I_m}{2}\left[\cos(2\omega t + \theta) + \cos\theta\right] \qquad (6.2)$$

Equation (6.2) represents *instantaneous power*. It consists of two parts. One is a fixed part, and the other is time-varying which has a frequency twice that of the voltage or current waveforms.

If θ becomes zero in Eq. (6.1), we get

$$P(t) = v(t)\, i(t)$$
$$= V_m I_m \cos^2 \omega t$$
$$= \frac{V_m I_m}{2}(1 + \cos 2\omega t) \qquad (6.3)$$

AVERAGE POWER

To find the average value of any power function, we have to take a particular time interval from t_1 to t_2; by integrating the function from t_1 to t_2 and dividing the result by the time interval $t_2 - t_1$, we get the average power.

$$\text{Average power } P = \frac{1}{t_2 - t_1}\int_{t_1}^{t_2} P(t)\,dt \tag{6.4}$$

In general, the average value over one cycle is

$$P_{av} = \frac{1}{T}\int_0^T P(t)\,dt \tag{6.5}$$

By integrating the instantaneous power $P(t)$ in Eq. (6.5) over one cycle, we get average power

$$P_{av} = \frac{1}{T}\int_0^T \left\{\frac{V_m I_m}{2}[\cos(2\omega t + \theta) + \cos\theta]\,dt\right\}$$

$$= \frac{1}{T}\int_0^T \frac{V_m I_m}{2}[\cos(2\omega t + \theta)]\,dt + \frac{1}{T}\int_0^T \frac{V_m I_m}{2}\cos\theta\,dt \tag{6.6}$$

In Eq. (6.6), the first term becomes zero, and the second term remains. The average power is therefore

$$P_{av} = \frac{V_m I_m}{2}\cos\theta \text{ W} \tag{6.7}$$

We can write Eq. 6.7 as

$$P_{av} = \left(\frac{V_m}{\sqrt{2}}\right)\left(\frac{I_m}{\sqrt{2}}\right)\cos\theta \tag{6.8}$$

In Eq. 6.8, $V_m/\sqrt{2}$ and $I_m/\sqrt{2}$ are the effective values of both voltage and current.

$\therefore \qquad P_{av} = V_{eff} I_{eff} \cos\theta$

To get average power, we have to take the product of the effective values of both voltage and current multiplied by cosine of the phase angle between voltage and the current.

If we consider a purely resistive circuit, the phase angle between voltage and current is zero. Hence, the average power is

$$P_{av} = \frac{1}{2} V_m I_m = \frac{1}{2} I_m^2 R$$

If we consider a purely reactive circuit (i.e. purely capacitive or purely inductive), the phase angle between voltage and current is 90°. Hence, the average power is zero or $P_{av} = 0$.

If the circuit contains complex impedance, the average power is the power dissipated in the resistive part only.

APPARENT POWER AND POWER FACTOR

The power factor is useful in determining useful power (true power) transferred to a load. The highest power factor is 1, which indicates that the current to a load is in phase with the voltage across it (i.e. in the case of resistive load). When the power factor is 0, the current to a load is 90° out of phase with the voltage (i.e. in case of reactive load).

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Consider the following equation

$$P_{av} = \frac{V_m I_m}{2} \cos\theta \text{ W} \tag{6.9}$$

In terms of effective values

$$P_{av} = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos\theta$$

$$= V_{eff} I_{eff} \cos\theta \text{ W} \tag{6.10}$$

The average power is expressed in watts. It means the useful power transferred from the source to the load, which is also called true power. If we consider a dc source applied to the network, true power is given by the product of the voltage and the current. In case of sinusoidal voltage applied to the circuit, the product of voltage and current is not the true power or average power. This product is called *apparent power*. The apparent power is expressed in volt amperes, or simply VA.

$$\therefore \qquad \text{Apparent power} = V_{\text{eff}} I_{\text{eff}}$$

In Eq. 6.10, the average power depends on the value of cos θ, this is called the *power factor* of the circuit.

$$\therefore \qquad \text{Power factor (pf)} = \cos\theta = \frac{P_{\text{av}}}{V_{\text{eff}} I_{\text{eff}}}$$

Therefore, power factor is defined as the ratio of average power to the apparent power, whereas apparent power is the product of the effective values of the current and the voltage. Power factor is also defined as the factor with which the volt amperes are to be multiplied to get true power in the circuit.

In the case of sinusoidal sources, the power factor is the cosine of the phase angle between voltage and current

$$pf = \cos\theta$$

As the phase angle between voltage and total current increases, the power factor decreases. The smaller the power factor, the smaller the power dissipation. The power factor varies from 0 to 1. For purely resistive circuits, the phase angle between voltage and current is zero, and hence the power factor is unity. For purely reactive circuits, the phase angle between voltage and current is 90°, and hence the power factor is zero. In an RC circuit, the power factor is referred to as *leading* power factor because the current leads the voltage. In an RL circuit, the power factor is referred to as lagging power factor because the current lags behind the voltage.

6.4 REACTIVE POWER

We know that the average power dissipated is

$$P_{av} = V_{eff}\,[I_{eff}\cos\theta] \tag{6.11}$$

From the impedance triangle shown in Fig. 6.4

$$\cos\theta = \frac{R}{|Z|} \tag{6.12}$$

and $$V_{eff} = I_{eff}Z \tag{6.13}$$

If we substitute Eqs (6.12) and (6.13) in Eq. (6.11), we get

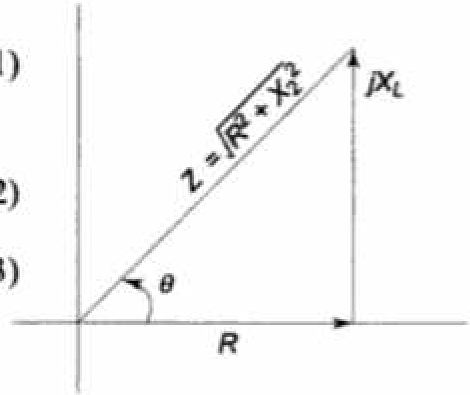

Fig. 6.4

$$P_{av} = I_{eff}Z\left[I_{eff}\frac{R}{Z}\right]$$
$$= I_{eff}^2 R \text{ watts} \tag{6.14}$$

This gives the average power dissipated in a resistive circuit.

If we consider a circuit consisting of a pure inductor, the power in the inductor

$$P_r = iv_L \tag{6.15}$$
$$= iL\frac{di}{dt}$$

Consider $$i = I_m \sin(\omega t + \theta)$$

Then $$P_r = I_m^2 \sin(\omega t + \theta)\, L\omega\cos(\omega t + \theta)$$

$$= \frac{I_m^2}{2}(\omega L)\sin 2(\omega t + \theta)$$

$$\therefore \quad P_r = I_{eff}^2(\omega L)\sin 2(\omega t + \theta) \tag{6.16}$$

From the above equation, we can say that the average power delivered to the circuit is zero. This is called *reactive* power. It is expressed in volt-amperes reactive (VAR).

$$P_r = I_{eff}^2 X_L \text{ VAR} \tag{6.17}$$

From Fig. 6.4, we have

$$X_L = Z\sin\theta \tag{6.18}$$

Substituting Eq. (6.1.) in Eq. (6.1.), we get

$$P_r = I_{eff}^2 Z\sin\theta$$
$$= (I_{eff}Z)I_{eff}\sin\theta$$
$$= V_{eff}I_{eff}\sin\theta \text{ VAR}$$