4.5 PHASE RELATION IN A PURE RESISTOR

When a sinusoidal voltage of certain magnitude is applied to a resistor, a certain amount of sine wave current passes through it. We know the relation between $v(t)$ and $i(t)$ in the case of a resistor. The voltage/current relation in case of a resistor is linear,

i.e. $v(t) = i(t)R$

Consider the function

$$i(t) = I_m \sin \omega t = IM\left[I_m e^{j\omega t}\right] \text{ or } I_m \angle 0^\circ$$

If we substitute this in the above equation, we have

$$v(t) = I_m R \sin \omega t = V_m \sin \omega t$$

$$= IM\left[V_m e^{j\omega t}\right] \text{ or } V_m \angle 0^\circ$$

where $V_m = I_m R$

If we draw the waveform for both voltage and current as shown in Fig. 4.18, there is no phase difference between these two waveforms. The amplitudes of the waveform may differ according to the value of resistance.

As a result, in pure resistive circuits, the voltages and currents are said to be in phase. Here the term impedance is defined as the ratio of voltage to current function.

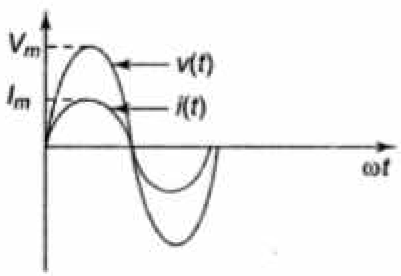

Fig. 4.18

With ac voltage applied to elements, the ratio of exponential voltage to the corresponding current (impedance) consists of magnitude and phase angles. Since the phase difference is zero in case of a resistor, the phase angle is zero. The impedance in case of resistor consists only of magnitude, i.e.

$$Z = \frac{V_m \angle 0^\circ}{I_m \angle 0^\circ} = R$$

4.6 PHASE RELATION IN A PURE INDUCTOR

Ad discussed earlier in Chapter 1, the voltage current relation in the case of an inductor is given by

$$v(t) = L\frac{di}{dt}$$

Consider the function $i(t) = I_m \sin \omega t = IM\left[I_m e^{j\omega t}\right]$ or $I_m \angle 0°$

$$v(t) = L\frac{d}{dt}(I_m \sin \omega t)$$
$$= L\omega I_m \cos \omega t = \omega L I_m \cos \omega t$$
$$v(t) = V_m \cos \omega t, \quad \text{or} \quad V_m \sin(\omega t + 90°)$$
$$= IM\left[V_m e^{j(\omega t + 90°)}\right] \quad \text{or} \quad V_m \angle 90°$$

where $V_m = \omega L I_m = X_L I_m$
and $e^{j90°} = j = 1\angle 90°$

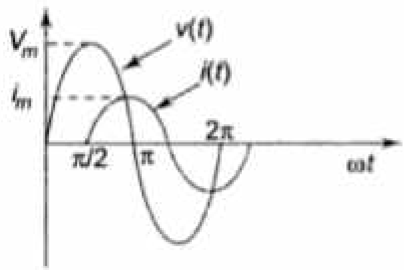

Fig. 4.19

If we draw the waveforms for both, voltage and current, as shown in Fig. 4.19, we can observe the phase difference between these two waveforms.

As a result, in a pure inductor the voltage and current are out of phase. The current lags behind the voltage by 90° in a pure inductor as shown in Fig. 4.20.

The impedance which is the ratio of exponential voltage to the corresponding current, is given by

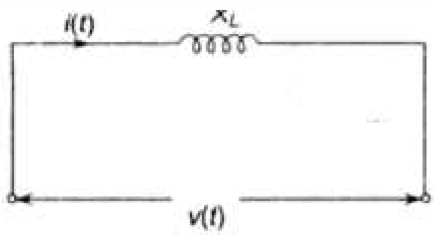

Fig. 4.20

$$Z = \frac{V_m \sin(\omega t + 90°)}{I_m \sin \omega t}$$

where $V_m = \omega L I_m$

$$= \frac{I_m \omega L \sin(\omega t + 90°)}{I_m \sin \omega t}$$
$$= \frac{\omega L I_m \angle 90°}{I_m \angle 0°}$$
$$\therefore \quad Z = j\omega L = jX_L$$

where $X_L = \omega L$ and is called the inductive reactance.
Hence, a pure inductor has an impedance whose value is ωL.

4.7 PHASE RELATION IN A PURE CAPACITOR

As discussed in Chapter 1, the relation between voltage and current is given by

$$v(t) = \frac{1}{C}\int i(t)\,dt$$

Consider the function $i(t) = I_m \sin\omega t = IM\left[I_m e^{j\omega t}\right]$ or $I_m\angle 0°$

$$v(t) = \frac{1}{C}\int I_m \sin\omega t\, d(t)$$

$$= \frac{1}{\omega C} I_m[-\cos\omega t]$$

$$= \frac{I_m}{\omega C}\sin(\omega t - 90°)$$

$$\therefore \quad v(t) = V_m \sin(\omega t - 90°)$$

$$= IM\left[I_m e^{j(\omega t - 90°)}\right] \text{ or } V_m\angle -90°$$

where $\qquad V_m = \dfrac{I_m}{\omega C}$

$$\therefore \quad \frac{V_m\angle -90°}{I_m\angle 0°} = Z = \frac{-j}{\omega C}$$

Hence, the impedance is $Z = \dfrac{-j}{\omega C} = -jX_C$

where $X_C = \dfrac{1}{\omega C}$ and is called the capacitive reactance.

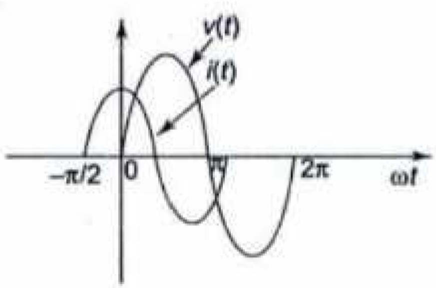

Fig. 4.21

If we draw the waveform for both, voltage and current, as shown in Fig. 4.21, there is a phase difference between these two waveforms.

As a result, in a pure capacitor, the current leads the voltage by 90°. The impedance value of a pure capacitor

$$X_C = \frac{1}{\omega C}$$