Department of Computer Engineering MNIT, Jaipur

IT-309 Data Networks

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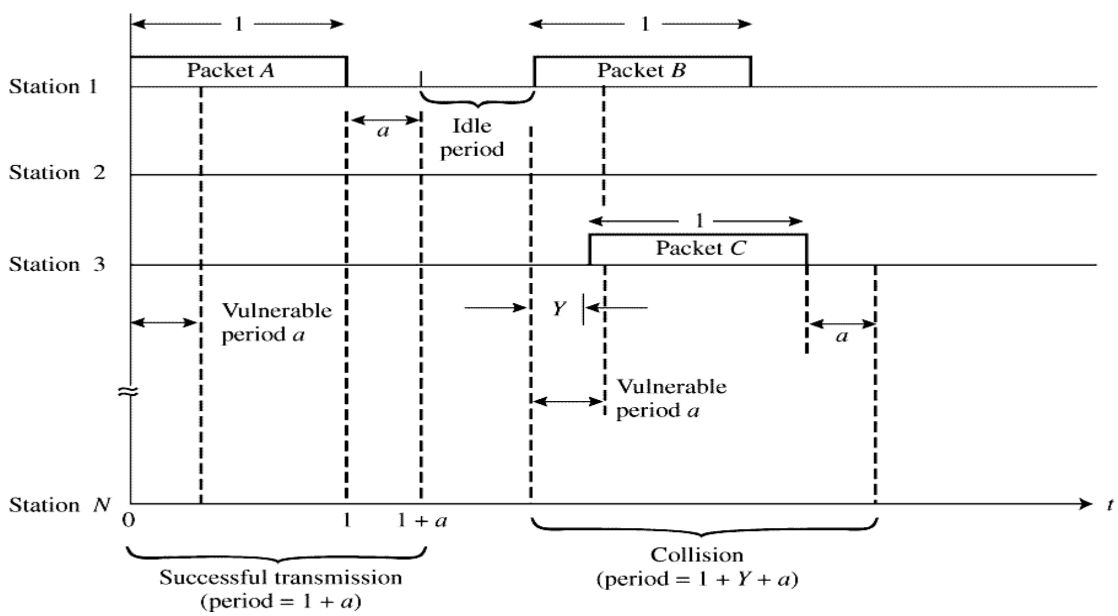
Lecture Notes

Throughput Analysis for CSMA Protocol

Here I am presenting the throughput analysis of for nonpersistent CSMA and slotted nonpersistent CSMA. The analyses of the throughput for 1-persistent CSMA, nonpersistent CSMA/CD, and 1-persistent CSMA /CD are quite similar.
In order to understand the following throughput a few assumptions are required to be made for the sake of simplicity. The following assumptions on which these analyses are based:

1. A station may not transmit and receive simultaneously.
2. The state of the channel can be sensed instantaneously.
3. All packets are of constant length t_p.
4. The channel is noiseless (i.e., message error resulting from random noise are negligible compared to errors caused by overlapping packets).
5. Any fractional overlap of two packets results in destructive interference so that both must be retransmitted.
6. The propagation delay is the same between all source-destination pairs and is small compared to the packet transmission time.
7. The generation of packets (both new ones and retransmitted ones) from an infinite source of users follows a position distribution. Each user generates traffic at an infinitesimally small rate so that the average channel traffic sums to G packets per packets time $t_{p.}$

1 NONPERSISTENT CSMA



To find the channel throughput S, let us assume that E[B] be the expected duration of the busy period, E[I] the expected length of the idle period, and E[U] be the average time during the cycle that the channel is used without collisions. Then the throughput S is given by

$$S = E[U] / E[B] + E[I] \qquad (1)$$

This equation is based on the facts that all cycles are statistically similar, assuming steady-state conditions, and that the throughput is the ratio of the average successful packet transmission time for a cycle to the total cycle time.

To find E[U], we note that having a successful transmission during a busy period is the probability that no station transmits during the first a time units of the period, so that from Eq. (1) we have

$$E[U] = e^{-aG} \qquad (2)$$

The idle period is just the time interval between the end of a busy period and the next arrival to the network. Hence looking at the end of a busy period and noting that the packet arrivals follow a Poisson distribution, the average duration of an idle period is given by

$$E[I] = 1/G \qquad (3)$$

Now let us examine the busy periods. The time duration B of s transmission attempt is given by the random length (see Figure)

$$B = 1 + a + Y \qquad (4)$$

Where Y = 0 for successful transmissions. The average value of B then is

$$E[B] = 1 + E[Y] + a \qquad (5)$$

Since Y is only random quantity on the right-hand side of Eq. (5)

Equation states that the last packet which collides with the one sent from station 1 arrives on the average E[Y] times units after the busy period begins, spends one time unit in transmission, and finally clears the channel a times units later. The probability density function of Y is the probability that no packet arrival occurs in an interval of length (a – y). Thus we have

$$f(y) = Ge^{-G(a-y)} \qquad \text{for } 0 <= y <= a$$

So that

$$E[Y] = \int_0^a y f(y)\, dy$$

$$= a - 1/G\,(1 - e^{-aG}) \qquad (6)$$

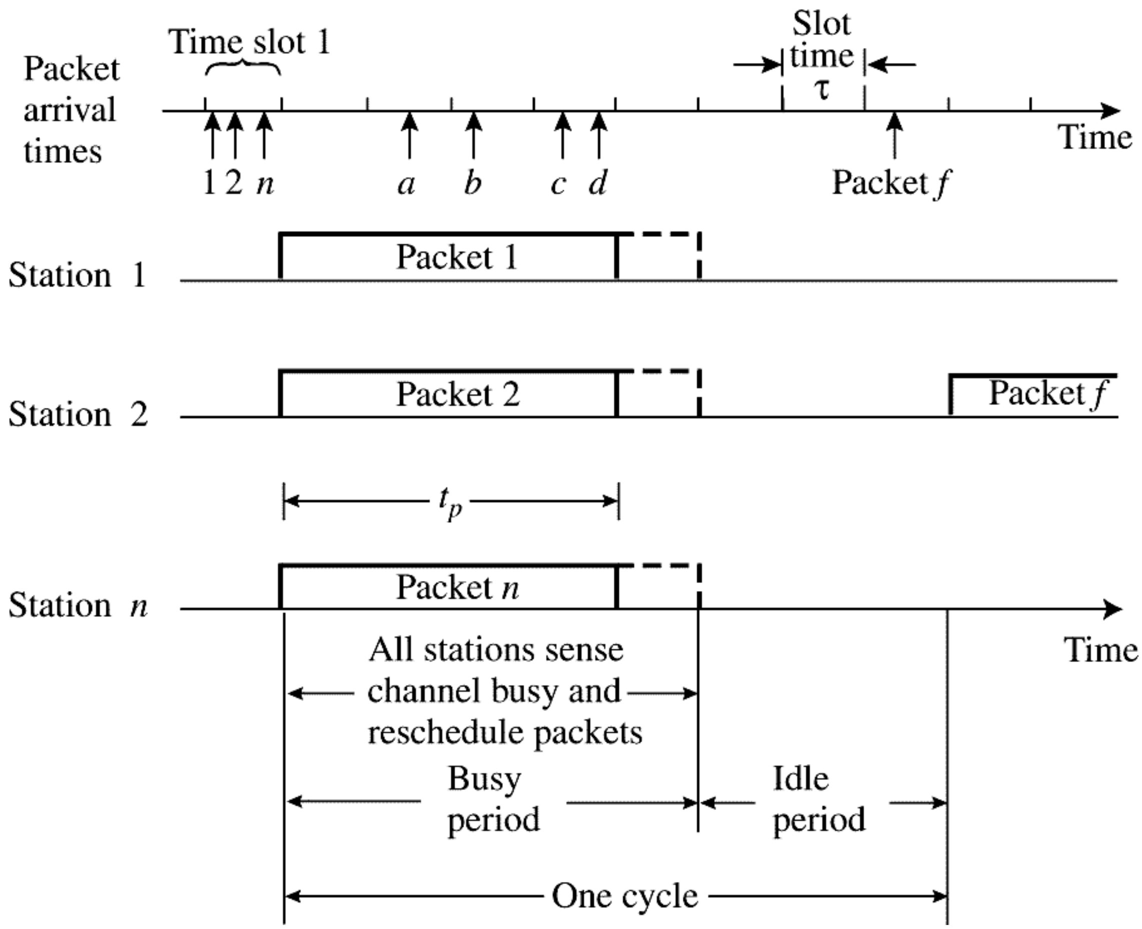
The expected duration of the busy period E[B] then becomes

$$E[B] = 1 + 2a - 1/G\,(1 - e^{-aG}) \qquad (7)$$

Substituting Eq. 2, 3, and 7 into Eq. (1), we have for nonpersistent CSMA

$$\boxed{S = Ge^{-aG} / G(1 + 2a) + e^{-aG}}$$

2 SLOTTED NONPERSISTENT CSMA



The analysis for the slotted nonpersistent CSMA case parallels that of the unslotted case in that we calculate S using Eq.1. First let us find E [U], the average time during a cycle that a transmission is successful. In normalized time units this is given by

$$\mathbf{E[U] = Ps} \qquad \text{--- x}$$

Where the conditional probability Ps is the fraction of time that a transmission is good. This is given by

Ps = P{one packet arrives in slot a or some arrival occurs}
= P{one packet arrives in slot a and some arrival occurs} / P{some arrival occurs}
= P{one packet arrives in slot a} / P{some arrival occurs}

Using Poisson arrival statistics, we then have

$$P\{\text{one packet arrives in slot a}\} = aGe^{-aG}$$

and

$$P\{\text{some arrival occurs}\} = 1 – e^{-aG}$$

So that Equation(x) becomes

$$E[U] = aGe^{-aG} / 1 – e^{-aG}$$

Since in normalized time units the busy period is always 1 + a, its average value is simply

$$E[B] = 1 + a$$

Now let us look at the average value of the idle period E[I]. First we need to recall the characteristics of the idle period. For slotted nonpersistent CSMA an idle period always consists of an integral number of time slots I => 0. If a packet arrives during the last time slot of a busy period, then the next slot immediately starts a new busy period so that I = 0. When there are no arrivals during the last slot of busy period, then the next I – 1 slot will be empty until there is an arrival in the final I^{th} slot. This then marks the beginning of a new busy period.
To find E[I], we first consider the case I = 0. The probability p of this occurring is merely the probability of some packet arriving in the interval a, which is given by Eq.   that is,

$$P\{I = 0\} = p = 1\text{-}e^{-aG}$$

Next we look at the case I = 1. The probability of this is the joint probability that no arrival occurs in the last slot of the busy period and that some arrival occurs in the next time slot. This is given by

$$P\{I = 1\} = (1 - p)p$$

Extending this argument to an idle period of length I = I, we have the probability that for no arrivals in I consecutive time slots followed by some arrival in the next slot is

$$P\{I = i\} = (1 - p)^I p$$

This describes a geometrically distributed random variable V with a mean value of

$$E[V] = \Sigma^{\infty}_{i=0}\, i(1 - p)^i p = 1 - p /p$$

The average length of the idle period then is a times E[V] so that from this we have

$$E[I] = aE[V] = ae^{-aG} / 1 - e^{-aG}$$

Substituting values in Eq. (1) we have for the slotted nonpersistent CSMA case

$$\boxed{S = aGe^{-aG} / 1 - e^{-aG} + a}$$

Reference Book:

Local Area Networks 2nd Edition, Gerd Keiser: TMH