Section 6.3 Selected Homework Solutions

5. Ww want to see how many strings there are that don't have consecutive S's. So, if we arrange the other 8 letters, there are $\frac{8!}{2!}$ ways to do this. Then, we have to place the S's. If we allow a single S to be placed between any two of the other letters, before the first or after the last, then there are 9 positions we could place an S and we need 4 of them to cover all of the S's. So, there are ${}_9C_4$ ways to select where the S's will go. Since the arrangement of the other letters has no bearing on the S's (just the positions), we use the multiplication rule to get that the total number of ways is

$$\frac{{}_9C_4 8!}{2!}$$

23. We are looking for the number of solutions to $x_1 + x_2 + x_3 = 15$ where each variable must be at least 1. If there were no restrictions, then we would have a generalized combination with 15 1's and two dividers, so it would be ${}_{17}C_{15}$. But, since each must be at least 1, we have 12 1's to arrange with the two dividers, so we have ${}_{14}C_{12}$ ways.

24. Since $x_1 = 1$, this is really a problem of two variables. So

$$x_1 + x_2 + x_3 = 15$$

reduces in this case to

$$x_2 + x_3 = 14$$

Here, we have 14 1's and 1 divider to place, so we would have ${}_{15}C_{14}$ ways.

26. Here we will use a difference - we will get rid of what we don't want. So first, if there are no restrictions, then there are ${}_{17}C_{15}$ ways to find a solution. But a bad thing for us would be if any more than 6 were already assigned to x_1. So, if we assigned 7, then there would be 8 more 1's, which no matter where they went, would be a bad thing, an 3 dividers to place, giving the number of bad arrangements as ${}_{10}C_8$. So, the bottom line is that the number of solutions are

$${}_{17}C_{15} -_{10} C_8$$

27. We will look at this in parts:

- There are ${}_{16}C_{14}$ solutions if we have to have x_2 with a minimum of 1, leaving 14 more 1's to arrange with the 3 dividers.
- Similar the last problem, we do not want 6 assigned to x_1, and anywhere we place the other 9 1's is a bad arrangement, so ${}_{10}C_8$ of these arrangements that have $x_1 \geq 6$. Remember, we already placed one 1 in the 'bin' for x_2, so we have 14 more to place. This is why we get the same number of ways to do this as in the last problem even though here we have $x_1 < 6$ but before it was $x_1 \leq 6$.

- If we need additionally for $x_2 < 9$ then if we placed 9 or more in the 'bin' for x_2, then we would have a bad arrangement. (Note: this is the bin we used for the first 1, so 9 total is bad, making 6 more to place). So, there would be ${}_8C_6$ ways. And, there is one way - when $x_1 = 6$ and $x_2 = 9$ - that violates both conditions, so we apply the Principle of Inclusion-Exclusion as well. Final answer for the number of solutions here is

$$ {}_{16}C_{14} - ({}_{10}C_8 +_8 C_6 - 1) $$

37. Since balls of the same color are considered identical, we really only have 3 types to consider instead of 20 balls. Think of the colors as the bins and we want to select 5 balls. Based on their color tells us where to put each one. So, we would have ${}_7C_5$ ways to do this.

38. Since we know how many of each color we need to select, each one is a combination in itself because the balls are indistinguishable.

    - Since we want two red ones, there are ${}_6C_2$ ways to select these.
    - Since we want three green ones, there are ${}_6C_3$ ways to select these.
    - Since we want two purple ones, there are ${}_8C_2$ ways to select these.

    Since how we select those of one color have no bearing on how we select those of another color, we apply the multiplication principle to get

$$ {}_6C_2 \cdot_6 C_3 \cdot_8 C1_2 $$

    ways.

40. Since all balls are considered indistinguishable here, we forget that there are colors and just worry about the fact that there are 20 balls. So, there are ${}_{20}C_5$ ways to select the first 5 balls. Since we are not replacing, there are 15 left and we want 5 more, so there are ${}_{15}C_5$ ways to select the next 5. And, since there is no bearing on the second 5 as to which first 5 are selected, we apply the multiplication principle to get

$$ {}_{20}C_5 \cdot_{15} C_5 $$

    ways.