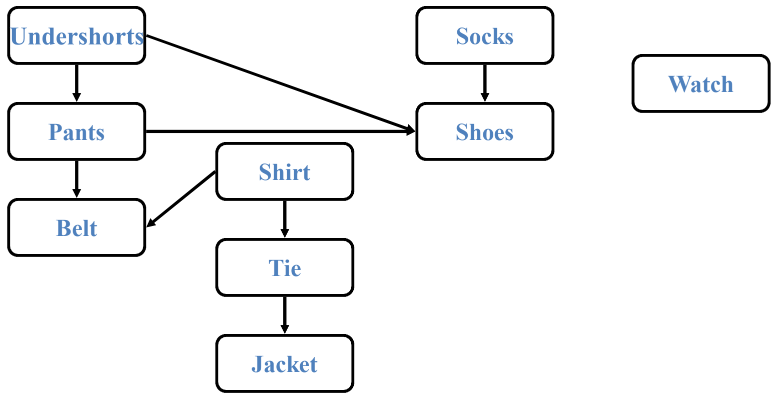
Application of DFS

- Topological Sort
- Detection of a cycles
- Strongly Connected Components
  - Component graph
  - Transpose of directed graph
  - Algorithm to compute SCC
- Biconnectivity
- Articulation Points
- Bridges

Topological Sort

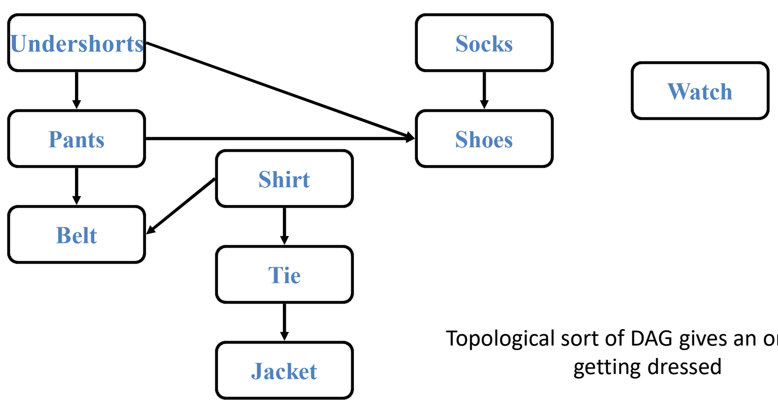
- *DFS used to perform topological sort of a DAG*
- *Topological sort* of a DAG:
  - Linear ordering of all vertices in graph G such that vertex u appears before vertex v *in the ordering* if edge $(u, v) \in G$
- Example: getting dressed

Getting Dressed

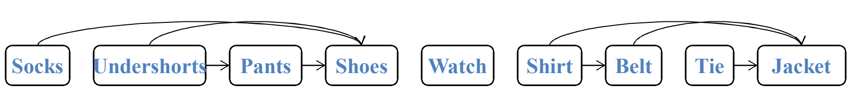


-directed edge (u,v) indicates that garment u must be put on before garment
-certain garments put on before others
-other items may put on in any order

Getting Dressed



Topological sort of DAG gives an order for getting dressed

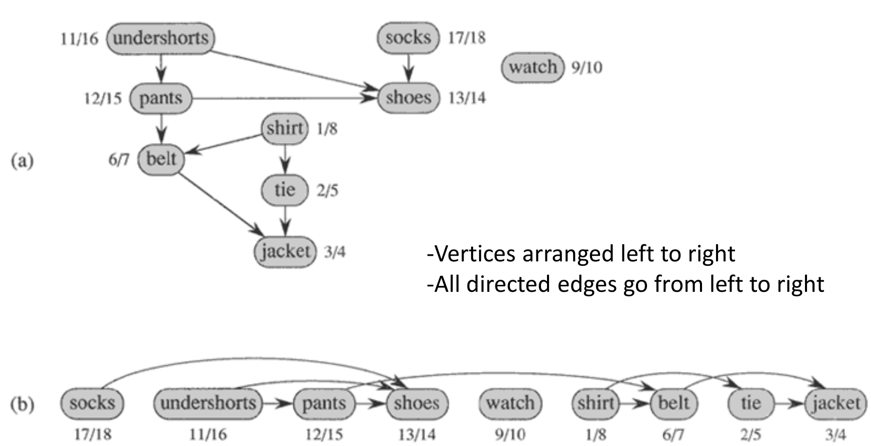


Topological Sort Algorithm

```
Topological-Sort(G)
{
1.Call DFS(G) to compute finishing times f(v)
  for each vertex v
2. As each vertex is finished, insert it onto
  the front of linked list
3.Return the linked list of vertices
}
```

Topological sort performed in Time: O(V+E)

Topologically sorted graph

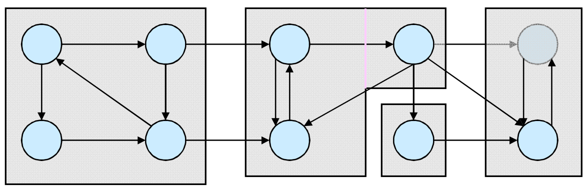


-Vertices arranged left to right
-All directed edges go from left to right

-topologically sorted vertices appears in reverse order of their finishing times

Strongly Connected Components

- G is strongly connected if every pair (u, v) of vertices in G is reachable from one another
- A **strongly connected component** (***SCC***) of G is a maximal set of vertices $C \subseteq V$ such that for all $u, v \in C$, both $u \rightsquigarrow v$ and $v \rightsquigarrow u$ exist
  - i.e. vertices u and v are reachable from each other

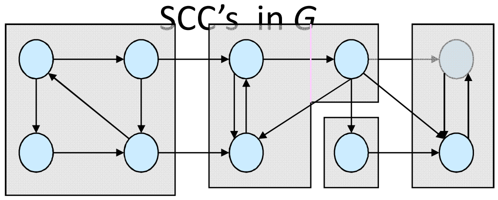


Component Graph

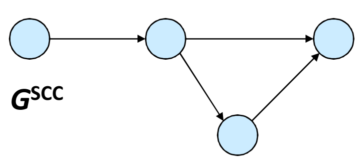
Component graph $G^{SCC} = (V^{SCC}, E^{SCC})$

V^{SCC} → has one vertex for each SCC in G

E^{SCC} →has an edge if there's an edge between the corresponding SCC's in G



G



G^{SCC}

Transpose of a Directed Graph

- **Transpose** of directed $G(V,E)$ *is* G^T
  - $G^T = (V, E^T)$,
    - where $E^T = \{(u, v) : (v, u) \in E\}$.
  - G^T is G with all its edges reversed

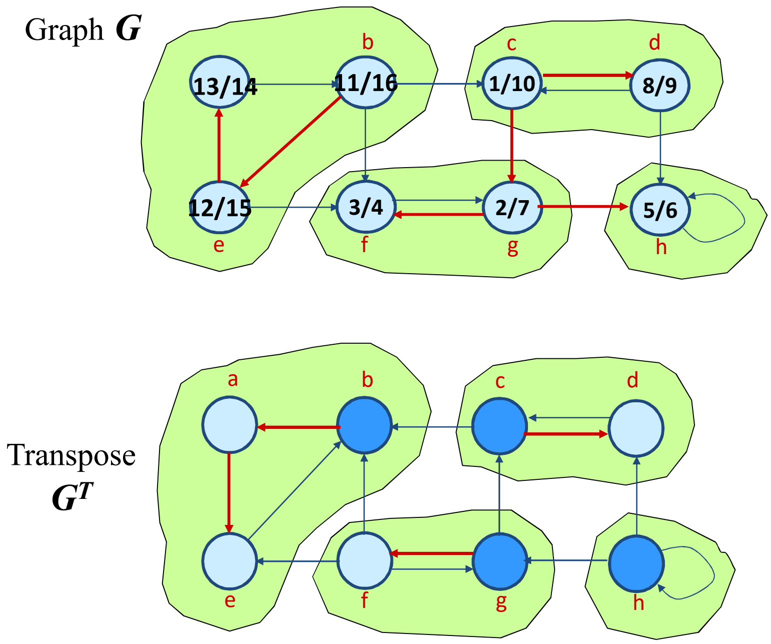
- Using Adjacency list of G
  - G^T created in $\Theta(V + E)$ time
- G and G^T have the same *strongly connected components*
  - u and v are reachable from each other in G if and only if reachable from each other in G^T

Strongly_Connected_Components(G)

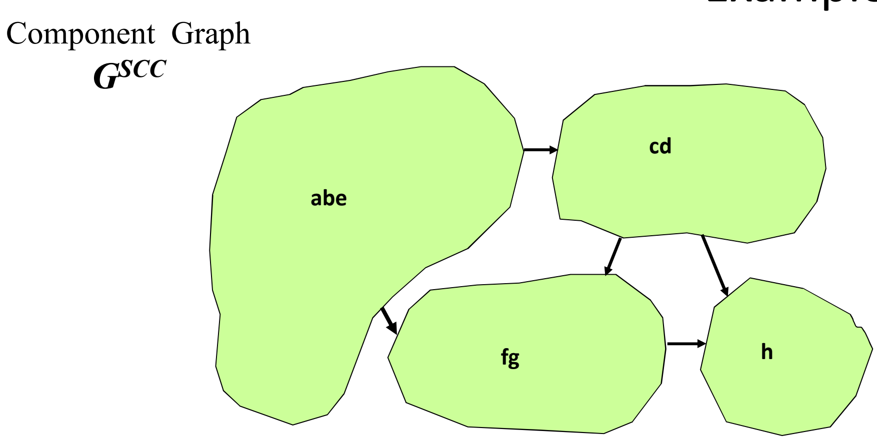
- Call DFS(G) to compute finishing times $f[u]$ for all u
- Compute G^T
- Call DFS(G^T), but in the main loop, consider vertices
  in order of decreasing $f[u]$ (as computed in first DFS)
- Output the vertices in each tree of the depth-first forest formed in second DFS as a separate SCC

Linear running Time: $\Theta(V + E)$

Example



Example



Point to Note:

→By considering vertices in second DFS in decreasing order of finishing times from first DFS

→actually visiting vertices of the component graph in topologically sorted order

→Since running DFS on G^T, we will not be visiting any v from a u, where v and u are in different components

Lemma

Let C and C' be distinct SCC's in $G = (V, E)$.

Suppose there is an edge $(u, v) \in E$ such that $u \in C$ and $v \in C'$

Then $f(C) > f(C')$.

Corollary

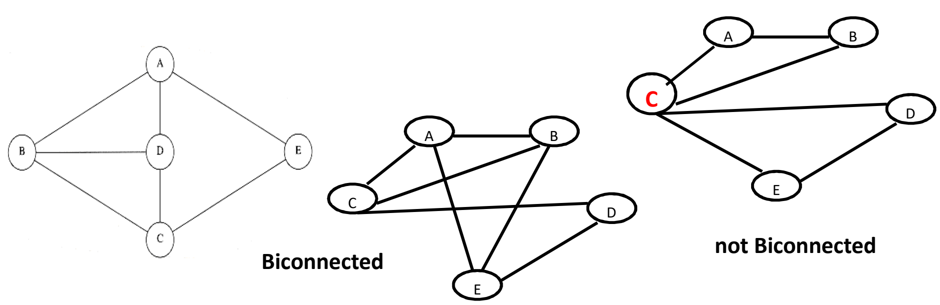
Let C and C' be distinct strongly connected components in $G = (V, E)$.

Suppose there is an edge $(u, v) \in E^T$ such that $u \in C$ and $v \in C'$

Then $f(C) < f(C')$.

Biconnectivity

- A connected undirected graph is **biconnected** if there are no vertices whose removal will disconnects the rest of the graph



Example :

--Computer→ Nodes and Links →Edges

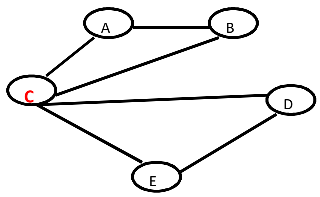
--if any computer goes down, mailing service of network mail unaffected, except at the down computer.

Articulation Points

- **Articulation point** or cut-vertex : A vertex whose removal (along with its attached arcs) makes the graph disconnected

  → graph is not biconnected

Eg. **C** is an articulation point



--Relevant to computer networks
--Represent vulnerabilities in a network
--In many applications these nodes are critical

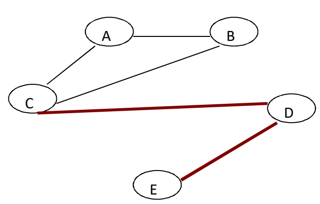
Articulation points can be computed using **depth-first search** and a special numbering of the vertices in the order of their visiting

Bridges

Bridge: An edge in a graph whose removal disconnects the graph

- a bridge is any edge in a graph, that does not lie on a cycle
- a bridge has at least one articulation point at its end
- however an articulation point is not necessarily linked in a bridge

Eg. (C,D) and (E,D)



No articulation points and No bridges→ **Biconnected graph**

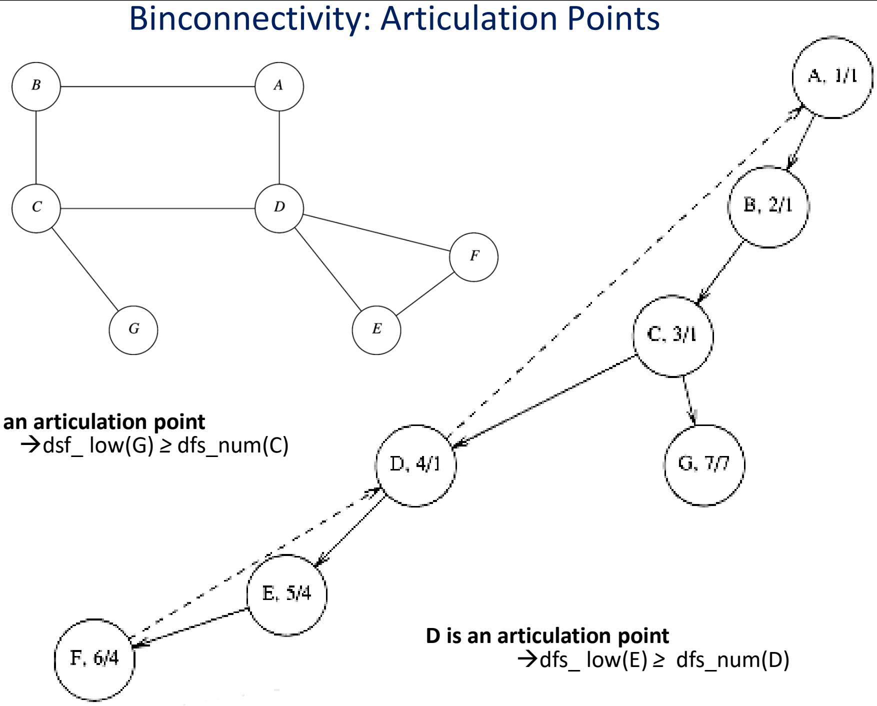
Binconnectivity: Articulation Points

In a connected graph all articulation points can computed using DFS in a linear time:

1. Perform a DFS starting at any vertex
   - number the nodes as they are visited say: ***dfs_num(v)*** as visiting sequence of vertices ***v*** and
2. For every vertex v in the depth-first spanning tree
   - compute ***dfs_low(v) :*** the lowest-numbered vertex, ***dfs_num*** that is reachable from ***v*** through a path by taking ***zero or more tree edges*** and then possibly followed by ***zero or one back edge***
3. Vertex ***v*** is an articulation point of G if and only if either
   - ***v*** is the root of DFS tree and has at least two children
   - ***v*** is not the root of DFS tree and has a child ***u*** for which no vertex in subtree rooted with ***u*** has a back edge to one of the ancestors (in DFS tree) of ***v***

   i.e. vertex v is an articulation point if and only if **v** has

Binconnectivity: Articulation Points



C is an articulation point
→dsf_ low(G) ≥ dfs_num(C)

D is an articulation point
→dfs_ low(E) ≥ dfs_num(D)