CMSC 330: Regular Expression and Finite Automata Practice Problems

Disclaimer: Please let the TAs know if you find any problem or have any questions regarding the solutions below.

1. For each of the following regular expressions, write down three strings in the language generated by the expression, and give a short English description of the language. Assume $\sum = \{0, 1\}$.

   (a) $0^+(0 \cup 1)1^+$

   (b) 0*10*10*10*

   (c) 0*(100*)*1*

   (d) (0 ∪ 10)*1(1 ∪ 10)*

   Answer:

   (a) 001, 011, 0001, 0011; any string of length 3 or greater that is one or more 0's are followed by one or more 1's.

   (b) 111, 0111, 01011, 010101; any string that has at least three 1's.

   (c) 0, 1, 01, 0101; any string that has no substring 110.

   (d) 1, 01, 1011, 10110; any string that has no substring 00 after first 11 (if there is any 11 in the string).

2. Consider sets of binary strings $A = \{0, 00, 000\}$ and $B = \{11\}$. Show the language denoted by each of the following:

   (a) A^0

   (b) A^1

   (c) $A \cup A^2$

   (d) AB^2

   (e) $(AB)^2$

   (f) B^3

   (g) A^*

(h) $(A \cup B)^2$

Answer:

(a) $\{\epsilon\}$

(b) $\{0, 00, 000\}$

(c) $\{0, 00, 000, 0000, 00000, 000000\}$

(d) $\{01111, 001111, 0001111\}$

(e) $\{011011, 0110011, 01100011, 0011011, 00110011, 001100011, 00011011, 000110011, 0001100011\}$

(f) $\{111111\}$

(g) $\{\epsilon, 0, 00, 000, 0000, 00000, 000000, \ldots\}$

(h) $\{00, 000, 0000, 011, 00000, 0011, 000000, 00011, 110, 1100, 11000, 1111\}$

3. For each of the following problems construct a deterministic finite automaton which describes or recognizes the language given. The underlying alphabet is $\sum = \{0, 1\}$. Be sure to give DFAs and not NFAs. Do not use any notational conveniences or shortcuts given in lecture.

   (a) $\{\ w \mid w$ begins with 01 and ends with 01. $\}$

   (b) $\{\ w \mid w$ has an even number of 1's. $\}$

   (c) $\{\ w \mid w$ has two or three 1's. $\}$

   (d) $\{\ w \mid w$ has an even number of 0's, and $|w|$ is even. $\}$

   (e) $\{\ w \mid w$ has an even number of 0's and odd number of 1's. $\}$

   (f) $\{\ w \mid w$ contains the substring 110. $\}$

   (g) $\{\ w \mid w$ does not contain the substring 110. $\}$

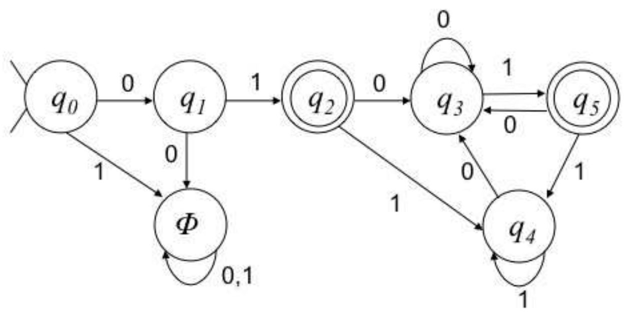
   (h) $\{\ w \mid w$ does not contain neither of the substrings 11 and 00. $\}$

   (i) $\{\ w \mid w$ has exactly one occurence of the substring 010. $\}$

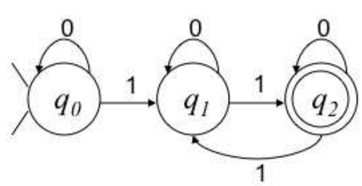
   (j) $\{\ w \mid w$ has n occurences of 0's where n mod 5 is 3. $\}$

   Answer:

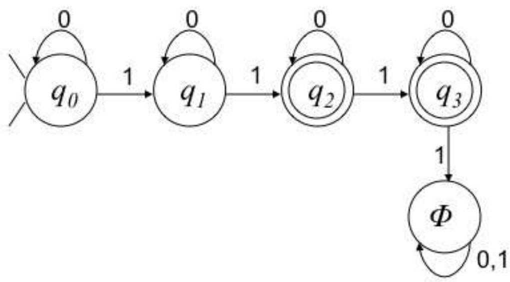
(a)



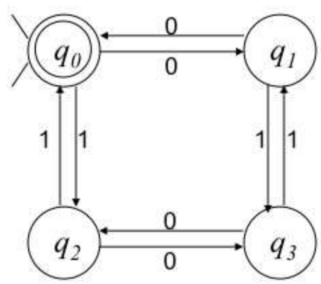
(b)



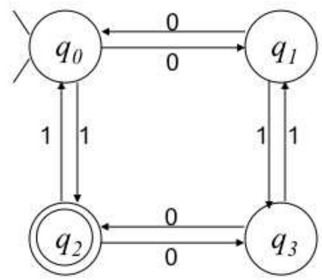
(c)



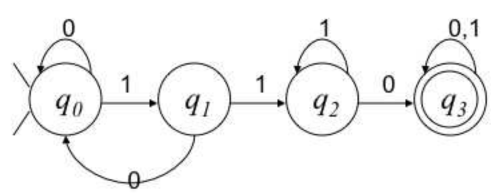
(d)



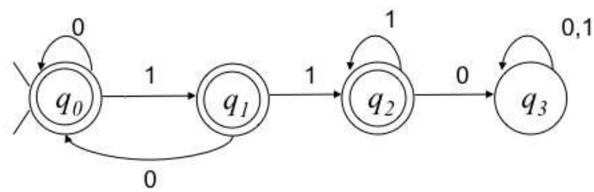
(e)



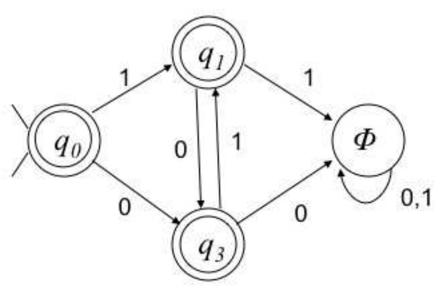
(f)



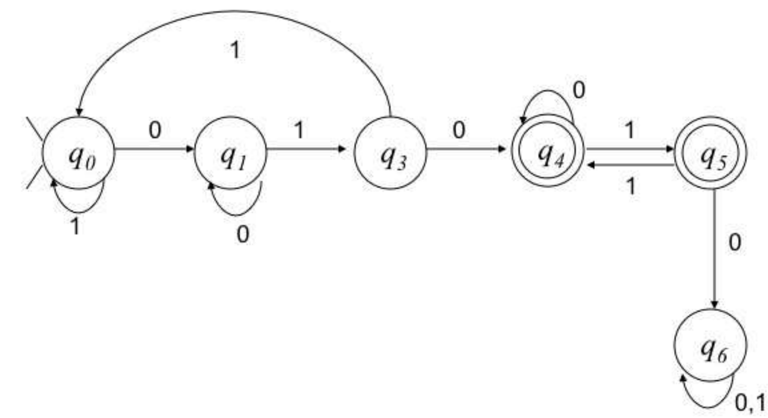
(g)



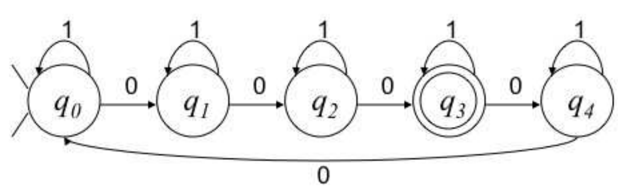
(h)



(i)

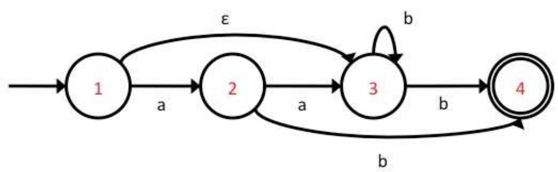


(j)



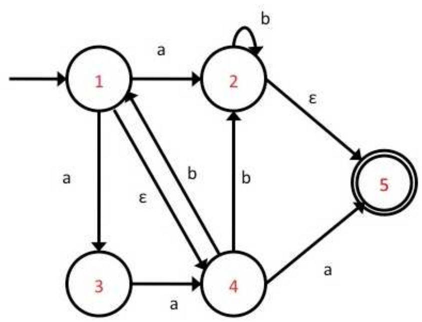
4. For each of the following problems, assume $\sum = \{a, b\}$.

(a) Convert the following NFA to a DFA.



(b) Write a regular expression that accepts the language defined by 4a.

(c) Conver the following NFA to a DFA.

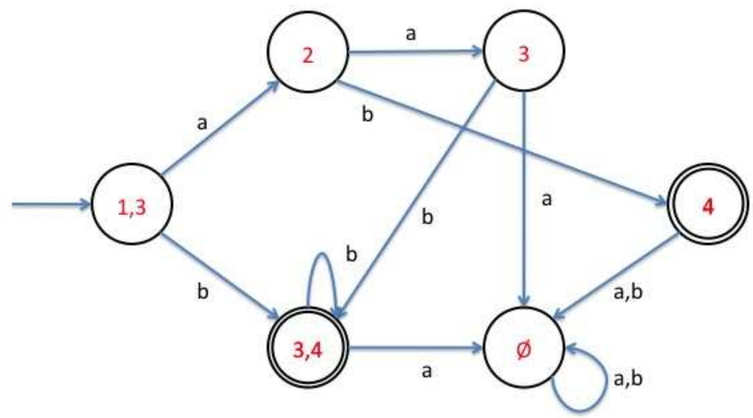


(d) For each of the following strings, determine whether it is recognized by 4c or not.

i. bab
ii. aababbb
iii. aabbaaaa
iv. aabaaa
v. bbaabbab
vi. aabba

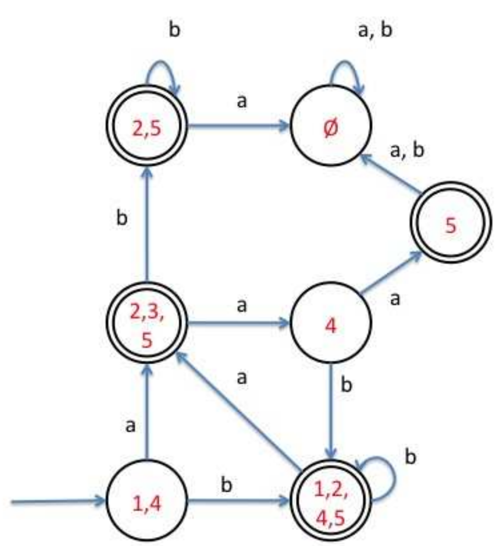
Answer:

(a)



(b) `(a(ab+|b)|b+)`

(c)



(d)

i. Yes (1-4-1-2-2-5 using the NFA)

ii. Yes (1-3-4-1-2-2-2-2-5)

iii. No

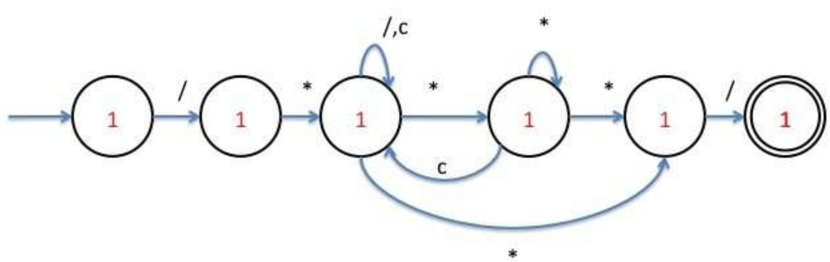
iv. Yes (1-3-4-1-3-4-5)

v. Yes (1-4-1-4-1-3-4-1-4-1-2-2-5)

vi. Yes (1-3-4-1-4-1-4-5)

5. Construct a NFA that accepts C-like comment delimited by `/*` and `*/`. Do not handle nested comments (assume they are not allowed). For simplicity, use $\sum = \{/, *, c\}$ where c is the only (non-comment) character in the language. Then, Write a regular expression for the NFA you contructed.

   Answer:



   The corresponding regular expression is `/\*([^*]|\*+[^/*])*\*+/` or `/\*((/|c)|\*+(c))*\*+/`

   (The character '`*`' is escaped using '\' to disambiguate it with Keene star.)

6. Let L be a regular language. Prove that R(L), strings in L reversed, is also a regular langauge.

   Answer: We prove R(L) is also a regular language by constructing a NFA that accepts R(L). Since L is a regular language, we can construct a NFA that accepts L. Given the NFA for L, add a new state δ_n and ϵ-transitions from the accepting states to δ_n. Change the accepting states into non-accepting states and the starting state into an accepting state. Lastly, reverse the direction of every transition.

For every string w in L, $\mathrm{rev}(w)$ is accepted by the newly constructed NFA. Hence, this proves that $\mathrm{R}(L)$ is also regular.