# Assignment 5

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The link to the solution is

https://github.com/Adarsh1310/EE5609

***Abstract*—This documents solves a problem based on circles.**

## 1 Problem

Find the area of the region bounded by the circle $\mathbf{x}^{\mathbf{T}}\ \mathbf{x}= 4$ and $\left\|\mathbf{x}-\begin{pmatrix}2\\0\end{pmatrix}\right\| = 2.$

## 2 Solution

General equation of circle is $\mathbf{x}^T\mathbf{x}+2\mathbf{u}^T\mathbf{x}+$ f $=0$ Taking equation of the first circle to be,

$$\|\mathbf{x}\|^2+2\mathbf{u}_1^T\mathbf{x}+f_1=0 \tag{2.0.1}$$

$$\mathbf{x}^T\mathbf{x}-4=0 \tag{2.0.2}$$

$$\mathbf{u_1}=\begin{pmatrix}0\\0\end{pmatrix} \tag{2.0.3}$$

$$f_1=-4 \tag{2.0.4}$$

$$\mathbf{O_1}=\begin{pmatrix}0\\0\end{pmatrix} \tag{2.0.5}$$

Taking equation of the second circle to be,

$$\left\|\mathbf{x}-\begin{pmatrix}2\\0\end{pmatrix}\right\|^2=2^2 \tag{2.0.6}$$

$$\mathbf{x}^T\mathbf{x}+2\mathbf{u_2}^T\mathbf{x}=0 \tag{2.0.7}$$

$$\mathbf{u_2}=\begin{pmatrix}-2\\0\end{pmatrix} \tag{2.0.8}$$

$$f_2=0 \tag{2.0.9}$$

$$\mathbf{O_2}=\begin{pmatrix}2\\0\end{pmatrix} \tag{2.0.10}$$

Now,Subtracting equation (2.0.7) from (2.0.2) We get,

$$\mathbf{x}^T\mathbf{x}-2\mathbf{u_2}^T\mathbf{x}+f_1-\mathbf{x}^T\mathbf{x}=0 \tag{2.0.11}$$

$$2\mathbf{u}^T\mathbf{x}=-4 \tag{2.0.12}$$

$$\begin{pmatrix}-4 & 0\end{pmatrix}\mathbf{x}=-4 \tag{2.0.13}$$

Which can be written as:-

$$\begin{pmatrix}1 & 0\end{pmatrix}\mathbf{x}=1 \tag{2.0.14}$$

$$\mathbf{x}=\begin{pmatrix}1\\0\end{pmatrix}+\lambda\begin{pmatrix}0\\1\end{pmatrix} \tag{2.0.15}$$

$$\mathbf{x}=\mathbf{q}+\lambda\mathbf{m} \tag{2.0.16}$$

$$\mathbf{q}=\begin{pmatrix}1\\0\end{pmatrix} \tag{2.0.17}$$

$$\mathbf{m}=\begin{pmatrix}0\\1\end{pmatrix} \tag{2.0.18}$$

Substituting (2.0.16) in (2.0.1)

$$\|\mathbf{x}\|^2+2\mathbf{u}_1^T\mathbf{x}+f_1=0 \tag{2.0.19}$$

$$\|\mathbf{q}+\lambda\mathbf{m}\|^2+f_1=0 \tag{2.0.20}$$

$$(\mathbf{q}+\lambda\mathbf{m})^T(\mathbf{q}+\lambda\mathbf{m})+f_1=0 \tag{2.0.21}$$

$$\mathbf{q}^T(\mathbf{q}+\lambda\mathbf{m})+\lambda\mathbf{m}^T(\mathbf{q}+\lambda\mathbf{m})+f_1=0 \tag{2.0.22}$$

$$\|\mathbf{q}\|^2+\lambda\mathbf{q}^T\mathbf{m}+\lambda\mathbf{m}^T\mathbf{q}+\lambda^2\|\mathbf{m}\|^2+f_1=0 \tag{2.0.23}$$

$$\|\mathbf{q}\|^2+2\lambda\mathbf{q}^T\mathbf{m}+\lambda^2\|\mathbf{m}\|^2+f_1=0 \tag{2.0.24}$$

$$\lambda(\lambda\|\mathbf{m}\|^2+2\mathbf{q}^T\mathbf{m})=-f_1-\|\mathbf{q}\|^2 \tag{2.0.25}$$

$$\lambda^2\|\mathbf{m}\|^2=-f_1-\|\mathbf{q}\|^2 \tag{2.0.26}$$

$$\lambda^2=\frac{-f_1-\|\mathbf{q}\|^2}{\|\mathbf{m}\|^2} \tag{2.0.27}$$

$$\lambda^2=3 \tag{2.0.28}$$

$$\lambda=+\sqrt{3},-\sqrt{3} \tag{2.0.29}$$

Substituting the value of $\lambda$ in(2.0.16)

$$\mathbf{x}=\mathbf{q}+\lambda\mathbf{m} \tag{2.0.30}$$

$$\mathbf{A}=\begin{pmatrix}1\\\sqrt{3}\end{pmatrix} \tag{2.0.31}$$

$$\mathbf{B}=\begin{pmatrix}1\\-\sqrt{3}\end{pmatrix} \tag{2.0.32}$$

Now finding the direction vector $\mathbf{m}_{O_1A}$, $\mathbf{m}_{O_1B}$,$\mathbf{m}_{O_2A}$ and $\mathbf{m}_{O_2B}$.

$$\mathbf{m}_{O_1A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} - \begin{pmatrix} 1 \\ \sqrt{3} \end{pmatrix} = \begin{pmatrix} -1 \\ -\sqrt{3} \end{pmatrix} \tag{2.0.33}$$

$$\mathbf{m}_{O_1B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} - \begin{pmatrix} 1 \\ -\sqrt{3} \end{pmatrix} = \begin{pmatrix} -1 \\ \sqrt{3} \end{pmatrix} \tag{2.0.34}$$

$$\mathbf{m}_{O_2A} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} - \begin{pmatrix} 1 \\ \sqrt{3} \end{pmatrix} = \begin{pmatrix} 1 \\ -\sqrt{3} \end{pmatrix} \tag{2.0.35}$$

$$\mathbf{m}_{O_2B} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} - \begin{pmatrix} 1 \\ -\sqrt{3} \end{pmatrix} = \begin{pmatrix} 1 \\ \sqrt{3} \end{pmatrix} \tag{2.0.36}$$

Now finding the angle $\angle O_1AB$.

$$\mathbf{m}_{O_1A}^T \mathbf{m}_{O_1B} = \left\|\mathbf{m}_{O_1A}\right\| \left\|\mathbf{m}_{O_1B}\right\| \cos\theta_1 \tag{2.0.37}$$

$$\frac{\mathbf{m}_{O_1A}^T \mathbf{m}_{O_1B}}{\left\|\mathbf{m}_{O_1A}\right\| \left\|\mathbf{m}_{O_1B}\right\|} = \cos\theta_1 \tag{2.0.38}$$

$$\frac{-2}{4} = \cos\theta_1 \tag{2.0.39}$$

$$\frac{-1}{2} = \cos\theta_1 \tag{2.0.40}$$

$$\theta_1 = 120^\circ \tag{2.0.41}$$

Now finding the angle $\angle O_2AB$.

$$\mathbf{m}_{O_2A}^T \mathbf{m}_{O_2B} = \left\|\mathbf{m}_{O_2A}\right\| \left\|\mathbf{m}_{O_2B}\right\| \cos\theta_2 \tag{2.0.42}$$

$$\frac{\mathbf{m}_{O_2A}^T \mathbf{m}_{O_2B}}{\left\|\mathbf{m}_{O_2A}\right\| \left\|\mathbf{m}_{O_2B}\right\|} = \cos\theta_2 \tag{2.0.43}$$

$$\frac{-2}{4} = \cos\theta_2 \tag{2.0.44}$$

$$\frac{-1}{2} = \cos\theta_2 \tag{2.0.45}$$

$$\theta_2 = 120^\circ \tag{2.0.46}$$

Finding area of $\mathbf{O_1AB}$ and $\mathbf{O_2AB}$.

$$A_{O_1AB} = \frac{\theta_1}{360} r^2 - \frac{1}{2} 2\sqrt{3} \tag{2.0.47}$$

$$= \frac{120}{360} 4\pi - \frac{1}{2} 2\sqrt{3} \tag{2.0.48}$$

$$A_{O_2AB} = \frac{\pi\theta_2}{360} r^2 - \frac{1}{2} 2\sqrt{3} \tag{2.0.49}$$

$$= \frac{120}{360} 4\pi - \frac{1}{2} 2\sqrt{3} \tag{2.0.50}$$

Area of $\mathbf{O_1AO_2B}$

$$A_{O_1AO_2B} = \frac{120}{360} 4\pi - \frac{1}{2} 2\sqrt{3} + \frac{120}{360} 4\pi - \frac{1}{2} 2\sqrt{3} \tag{2.0.51}$$

$$= \frac{8\pi}{3} - 2\sqrt{3} \tag{2.0.52}$$

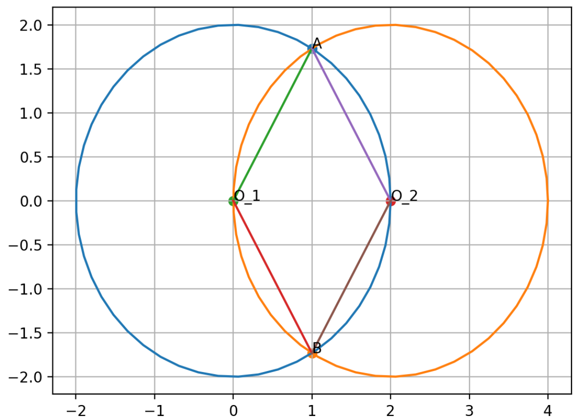


Fig. 0: Figure depicting intersection points of circle