Assignment 20

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The link to the solution is

https://github.com/Adarsh1310/EE5609

***Abstract*—This documents solves a problem based on Polynomial of Linear Transformation.**

1 Problem

Let T be the linear operator on $\mathbb{R}^3$ defined by

$$T(x_1, x_2, x_3) = (x_1, x_3, -2x_2 - x_3) \tag{1.0.1}$$

Let f be the polynomial over $\mathbb{R}$ defined by f= $-x^3+2$

2 Solution

The given transformation can be written as,

$$T(\mathbf{x}) = \mathbf{A}\mathbf{x} \tag{2.0.1}$$
$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -2 & -1 \end{pmatrix} \mathbf{x} \tag{2.0.2}$$

Hence,

$$\mathbf{A} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -2 & -1 \end{pmatrix} \tag{2.0.3}$$

Now the characteristic equation of $\mathbf{A}$ is given by,

$$\det(\mathbf{A} - \lambda \mathbf{I}) = 0 \tag{2.0.4}$$
$$= \begin{pmatrix} 1-\lambda & 0 & 0 \\ 0 & -\lambda & 1 \\ 0 & -2 & -1-\lambda \end{pmatrix} \tag{2.0.5}$$
$$\implies (1-\lambda)(\lambda^2 + \lambda + 2) = 0 \tag{2.0.6}$$

Where $\mathbf{I}$ is 3×3 Identity matrix. Now using Cayley Hamilton Theorem we get from (2.0.6) the following,

$$(\mathbf{I} - \mathbf{A})(\mathbf{A}^2 + \mathbf{A} + 2) = 0 \tag{2.0.7}$$
$$\mathbf{A}^2 + \mathbf{A} + 2 - \mathbf{A}^3 - \mathbf{A}^2 - 2\mathbf{A} = 0 \tag{2.0.8}$$
$$\mathbf{A}^3 = 2\mathbf{I} - \mathbf{A} \tag{2.0.9}$$

Hence the polynomial $f(\mathbf{A})$ can be written using the characteristic function of $\mathbf{A}$ as follows,

$$f(\mathbf{A}) = -\mathbf{A}^3 + 2\mathbf{I} \tag{2.0.10}$$
$$= 2\mathbf{I} - \mathbf{A} + 2\mathbf{I} \tag{2.0.11}$$
$$= \mathbf{A} \tag{2.0.12}$$

Hence,

$$f(T)(\mathbf{x}) = \mathbf{A}\mathbf{x} \tag{2.0.13}$$
$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -2 & -1 \end{pmatrix} \mathbf{x} \tag{2.0.14}$$