Assignment 16

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The link to the solution is

https://github.com/Adarsh1310/EE5609

***Abstract*—This documents solves a problem based on basis and dimensions.**

1 Problem

Let $\mathbf{V}$ be a vector space over a subfield $\mathbf{F}$ of complex numbers. Suppose α, β and γ are linearly independent vectors in $\mathbf{V}$. Prove that $(\alpha+\beta)$,$(\beta+\gamma)$ and $(\gamma+\alpha)$ are linearly independent.

2 Solution

Let α, β and γ be three n$\times$ 1 dimensional vectors. We need to prove that,

$$\begin{pmatrix} \alpha+\beta & \beta+\gamma & \gamma+\alpha \end{pmatrix} \mathbf{x} = 0 \tag{2.0.1}$$

will only have a trivial solution.The above equation can be written as

$$\begin{pmatrix} \alpha & \beta & \gamma \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix} \mathbf{x} = 0 \tag{2.0.2}$$

$$\mathbf{x}^T \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} \alpha^T \\ \beta^T \\ \gamma^T \end{pmatrix} = 0 \tag{2.0.3}$$

Since, α, β and γ are independent.

$$\mathbf{x}^T \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix} = 0 \tag{2.0.4}$$

In the above equation we can see that the 3×3 matrix has linearly independent rows and hence will have a trivial solution.So,$\mathbf{x}$ is a zero vector.Hence, $(\alpha+\beta)$,$(\beta+\gamma)$ and $(\gamma+\alpha)$ are linearly independent.