

Assignment 13

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The link to the solution is

<https://github.com/Adarsh1310/EE5609>

Abstract—This documents solves a problem based on invertible matrix.

1 PROBLEM

Suppose \mathbf{A} is a 2×1 matrix and \mathbf{B} is 1×2 matrix. Prove that $\mathbf{C} = \mathbf{AB}$ is non invertible.

2 SOLUTION

Let's assume,

$$\mathbf{A} = \begin{pmatrix} a \\ b \end{pmatrix} \quad (2.0.1)$$

$$\mathbf{B} = \begin{pmatrix} c & d \end{pmatrix} \quad (2.0.2)$$

Now,

$$\mathbf{C} = \mathbf{AB} \quad (2.0.3)$$

$$= \begin{pmatrix} a \\ b \end{pmatrix} \begin{pmatrix} c & d \end{pmatrix} \quad (2.0.4)$$

$$= \begin{pmatrix} ac & ad \\ bc & bd \end{pmatrix} \quad (2.0.5)$$

Let,

$$\mathbf{U} = \begin{pmatrix} ac \\ bc \end{pmatrix} \quad (2.0.6)$$

$$\mathbf{V} = \begin{pmatrix} ad \\ bd \end{pmatrix} \quad (2.0.7)$$

For (2.0.5) to be non invertible

$$\mathbf{Cx} = 0 \quad (2.0.8)$$

Should have a non zero solution.

$$\begin{pmatrix} \mathbf{U} & \mathbf{V} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0 \quad (2.0.9)$$

$$x_1 \mathbf{U} = -x_2 \mathbf{V} \quad (2.0.10)$$

This shows that the two columns are a scalar multiple of each other and hence (2.0.8) will have a non trivial solution. So, we can say that \mathbf{C} is non invertible.