Assignment 16

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The link to the solution is

https://github.com/Adarsh1310/EE5609

***Abstract*—This documents solves a problem based on basis and dimensions.**

1 Problem

Let $\mathbf{V}$ be a vector space over a subfield $\mathbf{F}$ of complex numbers. Suppose α, β and γ are linearly independent vectors in $\mathbf{V}$. Prove that (α+β),(β+γ) and (γ+α) are linearly independent.

2 Solution

Let α, β and γ be three n$\times$ 1 dimensional vectors. We know that (α+β),(β+γ) and (γ+α) will also have the same number of rows as α, β and γ so, we need to prove that,

$$\begin{pmatrix} \alpha+\beta & \beta+\gamma & \gamma+\alpha \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0 \tag{2.0.1}$$

will only have a trivial solution.

$$x_1(\alpha+\beta) + x_2(\beta+\gamma) + x_3(\gamma+\alpha) = 0 \tag{2.0.2}$$

$$x_1\alpha + x_1\beta + x_2\beta + x_2\gamma + x_3\gamma + x_3\alpha = 0 \tag{2.0.3}$$

$$(x_1 + x_3)\alpha + (x_1 + x_2)\beta + (x_2 + x_3)\gamma = 0 \tag{2.0.4}$$

Since, α, β and γ are independent then all the scalar multiplication to these will be zero hence, (α+β),(β+γ) and (γ+α) are linearly independent.