Assignment 17

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The link to the solution is

https://github.com/Adarsh1310/EE5609

***Abstract*—This documents solves a problem based on coordinates.**

1 Problem

Let **V** be the real vector space of all polynomial functions from $\mathbb{R}$ to $\mathbb{R}$ of degree 2 or less, i.e, the space of all functions f of the form,

$$f(x) = c_0 + c_1 x + c_2 x^2$$

Let t be a fixed real number and define

$$g_1(x) = 1, g_2(x) = x + t, g_3(x) = (x + t)^2$$

Prove that β={g1,g2,g3} is a basis for V. If

$$f(x) = c_0 + c_1 x + c_2 x^2$$

what are the coordinates of f in the ordered basis β

2 Solution

We start by taking,

$$\mathbf{f} = \begin{pmatrix} 1 \\ x \\ x^2 \end{pmatrix} \tag{2.0.1}$$

Let's start by proving that **g** is linearly independent.

$$\mathbf{g} = \mathbf{B}\mathbf{f} \tag{2.0.2}$$

where,

$$\mathbf{B} = \begin{pmatrix} 1 & 0 & 0 \\ t & 1 & 0 \\ t^2 & 2t & 1 \end{pmatrix} \tag{2.0.3}$$

Now,

$$\mathbf{v}^T\mathbf{g} = 0 \tag{2.0.4}$$

$$\implies \mathbf{v}^T\mathbf{B}\mathbf{f} = 0 \tag{2.0.5}$$

Since **f** is linearly independent,

$$\mathbf{v}^T\mathbf{B} = 0 \tag{2.0.6}$$

We can se that the rows of **B** are linearly independent and hence $\mathbf{v}^T$ will be zero vector. Now, to find the coordinates,

$$f(x) = \mathbf{w}^T\mathbf{g} \tag{2.0.7}$$

where,

$$\mathbf{w} = \begin{pmatrix} a \\ b \\ c \end{pmatrix} \tag{2.0.8}$$

So,

$$\mathbf{c}\mathbf{f}^T = \begin{pmatrix} a \\ 0 \\ 0 \end{pmatrix}\mathbf{f}^T + \begin{pmatrix} bt \\ b \\ 0 \end{pmatrix}\mathbf{f}^T + \begin{pmatrix} ct^2 \\ 2ct \\ c \end{pmatrix}\mathbf{f}^T \tag{2.0.9}$$

$$\mathbf{c}\mathbf{f}^T = \left(\begin{pmatrix} a \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} bt \\ b \\ 0 \end{pmatrix} + \begin{pmatrix} ct^2 \\ 2ct \\ c \end{pmatrix}\right)\mathbf{f}^T \tag{2.0.10}$$

$$\begin{pmatrix} c_0 \\ c_1 \\ c_2 \end{pmatrix}\mathbf{f} = \begin{pmatrix} a + bt + ct^2 \\ b + 2ct \\ c \end{pmatrix}\mathbf{f}^T \tag{2.0.11}$$

From above we can see that,

$$c_2 = c \tag{2.0.12}$$

$$c_1 = b + 2ct \tag{2.0.13}$$

$$c_0 = a + bt + ct^2 \tag{2.0.14}$$

Solving the above equation we get,

$$c = c_2 \tag{2.0.15}$$

$$b = c_1 - 2c_2 t \tag{2.0.16}$$

$$a = c_0 - c_1 t + c_2 t^2 \tag{2.0.17}$$

So, finally the coordinates will be,

$$\begin{pmatrix} c_0 - c_1 t + c_2 t^2 \\ c_1 - 2c_2 t \\ c_2 \end{pmatrix} \tag{2.0.18}$$