Assignment 6

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***Abstract*—This document solves a question based on triangle.**

All the codes for the figure in this document can be found at

https://github.com/Adarsh1310/EE5609/tree/master/Assignment_6

1 Problem

$\triangle ABC$ is an isosceles triangle in which altitudes **BE** and **CF** are drawn to equal sides **AC** and **AB** respectively. Show that these altitudes are equal.

2 Solution

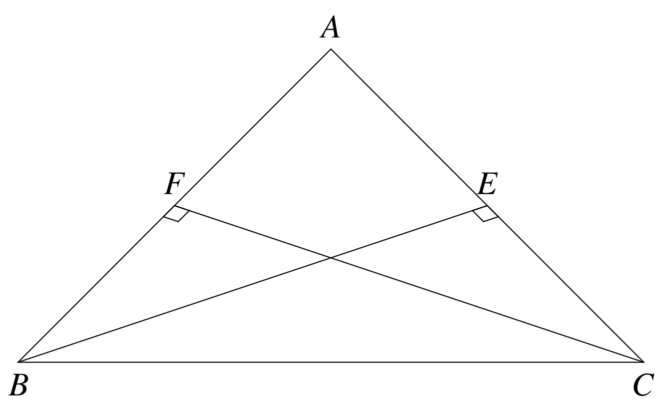

Fig. 1: Isoceles Triangle with BE and CF as altitude

Let $\mathbf{m}_{AC}$ and $\mathbf{m}_{BE}$ be direction vector of side **AC** and altitude **BE** respectively.

$$\mathbf{m}_{AC} = \mathbf{A} - \mathbf{C} \tag{2.0.1}$$
$$\mathbf{m}_{BE} = \mathbf{B} - \mathbf{E} \tag{2.0.2}$$

Here, $\mathbf{BE} \perp \mathbf{AC}$ because **BE** is the altitude to side $\mathbf{AC}.So,$

$$A_{\triangle CEB} = \frac{1}{2}\left\|\mathbf{A} - \mathbf{C}\right\|\left\|\mathbf{B} - \mathbf{E}\right\|\sin\ 90^\circ \tag{2.0.3}$$
$$A_{\triangle CEB} = \frac{1}{2}\left\|\mathbf{A} - \mathbf{C}\right\|\left\|\mathbf{B} - \mathbf{E}\right\| \tag{2.0.4}$$

Let $\mathbf{m}_{AB}$ and $\mathbf{m}_{CF}$ be direction vector of side **AB** and altitude **CF** respectively.

$$\mathbf{m}_{AB} = \mathbf{A} - \mathbf{B} \tag{2.0.5}$$
$$\mathbf{m}_{CF} = \mathbf{C} - \mathbf{F} \tag{2.0.6}$$

Here, $\mathbf{CF} \perp \mathbf{AB}$ because **CF** is the altitude to side **AB**.So,

$$A_{\triangle BFC} = \frac{1}{2}\left\|\mathbf{A} - \mathbf{B}\right\|\left\|\mathbf{C} - \mathbf{F}\right\|\sin\ 90^\circ \tag{2.0.7}$$
$$A_{\triangle BFC} = \frac{1}{2}\left\|\mathbf{A} - \mathbf{B}\right\|\left\|\mathbf{C} - \mathbf{F}\right\| \tag{2.0.8}$$

Comparing $\triangle CEB$ and $\triangle BFC$

$$\angle ECB = \angle FBC \tag{2.0.9}$$
$$\angle CEB = \angle BFC \tag{2.0.10}$$
$$BC = BC \tag{2.0.11}$$

By, AAS property $\triangle CEB \cong \triangle BFC$ and hence they will have equal area.Now by using (2.0.4) and (2.0.8)

$$\frac{1}{2}\left\|\mathbf{A} - \mathbf{C}\right\|\left\|\mathbf{B} - \mathbf{E}\right\| = \frac{1}{2}\left\|\mathbf{A} - \mathbf{B}\right\|\left\|\mathbf{C} - \mathbf{F}\right\| \tag{2.0.12}$$
$$\left\|\mathbf{B} - \mathbf{E}\right\| = \left\|\mathbf{C} - \mathbf{F}\right\| [\because \left\|\mathbf{m}_{AB}\right\| = \left\|\mathbf{m}_{AC}\right\|] \tag{2.0.13}$$

Hence, the altitudes drawn to equal sides of isosceles triangle are equal.