Assignment 22

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The link to the solution is

https://github.com/Adarsh1310/EE5609

***Abstract*—This documents solves a problem based on Lagrange Interpolation**

1 Problem

Let $\mathbb{F}$ be the field of real numbers,

$$\mathbf{A} = \begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$p = (x-2)(x-3)(x-1)$$

1) Show that $p(\mathbf{A}) = 0$
2) Let P_1, P_2, P_3 be the Lagrange polynomials for $t_1 = 2, t_2 = 3, t_3 = 1$. Compute $E_i = P_i(\mathbf{A})$, i=1,2,3

2 Solution

1) Since $\mathbf{A}$ is a diagonal matrix, It's characteristic polynomial is,

$$\det(\mathbf{A} - x\mathbf{I}) = 0 \tag{2.0.1}$$

$$f(x) = (x-2)^2(x-3)(x-1) = 0 \tag{2.0.2}$$

From, (2.0.2) and using Cayley Hamilton Theorem,

$$(\mathbf{A}-2)^2(\mathbf{A}-3)(\mathbf{A}-1) = 0 \tag{2.0.3}$$

We can also see that (x-2)(x-3)(x-1) is a minimal polynomial for $\mathbf{A}$, Hence $p(\mathbf{A})$=0.

2) Using Lagrange Interpolation,

$$P_1(x) = \frac{(x-3)(x-1)}{(2-3)(2-1)} \tag{2.0.4}$$

$$= -(x-3)(x-1) \tag{2.0.5}$$

$$P_2(x) = \frac{(x-2)(x-1)}{(3-2)(3-1)} \tag{2.0.6}$$

$$= \frac{(x-2)(x-1)}{2} \tag{2.0.7}$$

$$P_3(x) = \frac{(x-2)(x-3)}{(1-2)(1-3)} \tag{2.0.8}$$

$$= \frac{(x-2)(x-3)}{2} \tag{2.0.9}$$

Now,Substituting the value of $\mathbf{A}$,

$$P_1(\mathbf{A}) = -\begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{2.0.10}$$

$$= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{2.0.11}$$

$$P_2(\mathbf{A}) = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{2.0.12}$$

$$= \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{2.0.13}$$

$$P_3(\mathbf{A}) = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}\begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2 \end{pmatrix} \tag{2.0.14}$$

$$= \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{2.0.15}$$