Assignment 13

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The link to the solution is

https://github.com/Adarsh1310/EE5609

***Abstract*—This documents solves a problem based on invertible matrix.**

1 Problem

Suppose **A** is a 2×1 matrix and **B** is 1×2 matrix.Prove that **C**=**AB** is non invertible.

2 Solution

Let's take,

$$\mathbf{A} = \begin{pmatrix} a \\ b \end{pmatrix} \tag{2.0.1}$$

$$\mathbf{B} = \begin{pmatrix} c & d \end{pmatrix} \tag{2.0.2}$$

assuming **A** and **B** to be non zero vectors. Let $\mathbf{x}^{'}$ be the nullspace of **B**,

$$\mathbf{B}\mathbf{x}^{'} = 0 \tag{2.0.3}$$

$$\begin{pmatrix} c & d \end{pmatrix} \mathbf{x}^{'} = 0 \tag{2.0.4}$$

From above we can see that,

$$\mathbf{x}^{'} = k \begin{pmatrix} -d \\ c \end{pmatrix} \tag{2.0.5}$$

where k is a scalar constant. Now,we know that for **C** to be non invertible $\mathbf{C}\mathbf{x} = 0$ should have a non trivial solution.So,

$$\mathbf{C}\mathbf{x} = 0 \tag{2.0.6}$$

$$\implies \mathbf{A}\mathbf{B}\mathbf{x} = 0 \tag{2.0.7}$$

We know that the nullspace of **B** will be a subset of the nullspace of **AB**. Substituting (2.0.3)

$$\mathbf{A}\mathbf{B}\mathbf{x} = 0 \tag{2.0.8}$$

$$\implies \mathbf{A}\mathbf{B}\mathbf{x}^{'} = 0 \tag{2.0.9}$$

$$k\mathbf{A}\mathbf{B} \begin{pmatrix} -d \\ c \end{pmatrix} = 0 \tag{2.0.10}$$

From (2.0.3) we can see that (2.0.7) has a non trivial solution and hence is invertible.