

Assignment 14

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The link to the solution is

<https://github.com/Adarsh1310/EE5609>

Abstract—This documents solves a problem based on vector spaces.

1 PROBLEM

Let \mathbb{V} be the set of all complex-valued functions f on the real line such that

$$f(-t) = \overline{f(t)}$$

The bar denotes complex conjugation. Show that \mathbb{V} , with the operations

$$(f + g)(t) = f(t) + g(t)$$

$$(cf)(t) = cf(t)$$

is a vector space over the field of real numbers. Give an example of a function in \mathbb{V} which is not real valued.

2 SOLUTION

To prove that \mathbb{V} with the given operations is a vector space over the field of real numbers, we have to start by proving that additivity and homogeneity both hold true. So, we have to prove that $(cf+g)(t)$ is equal to $cf(t)+g(t)$.

$$(cf + g)(t) \quad (2.0.1)$$

$$= (cf)(t) + g(t) \quad (2.0.2)$$

$$= cf(t) + g(t) \quad (2.0.3)$$

Now, for vector addition,

- 1) Since addition in \mathbb{V} is commutative

$$f(t) + g(t) = g(t) + f(t) \quad (2.0.4)$$

for all t , so the functions $f+g$ and $g+f$ are equal.

- 2) Since addition in \mathbb{V} is associative

$$(f(t) + g(t)) + a(t) = f(t) + (g(t) + a(t)) \quad (2.0.5)$$

for all t , so the functions $f+(g+a)$ and $(f+g)+a$ are equal.

- 3) The unique zero vector is the zero function which assigns to each element, the scalar 0 in \mathbb{V} .

- 4) For each f in \mathbb{V} , $(-f)$ is the function which is given by $(-f)(s) = -f(s)$.

Now, for scalar multiplication

- 1) Since scalar multiplication is associative

$$a(bf(t)) = ab(f(t)) \quad (2.0.6)$$

for all a and b , the functions $(ab)f$ and $a(bf)$ are equal.

- 2) Since scalar multiplication is distributive

$$a(f(t) + g(t)) = af(t) + ag(t) \quad (2.0.7)$$

for all a , the functions $a(f+g) = af+ag$.

- 3) Since scalar addition is distributive

$$(a + b)f(t) = af(t) + bf(t) \quad (2.0.8)$$

for all a and b , the functions $(a+b)f = af+bf$.

- 4) The unique scalar constant 1 which multiplied to any vector in \mathbb{V} will return the same vector.

3 EXAMPLE

Let's take $f(x) = a + ix$

$$f(1) = a + i \quad (3.0.1)$$

Hence, $f(x)$ is not real valued. Now,

$$f(x) = a + ix \quad (3.0.2)$$

$$f(-x) = a - ix \quad (3.0.3)$$

$$f(-x) = \overline{f(x)} \quad (3.0.4)$$

Since a, b and $x \in \mathbb{R}$, so $f \in \mathbb{V}$