Assignment 5

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The link to the solution is

https://github.com/Adarsh1310/EE5609

Abstract—**This documents solves a problem based on circles.**

1 PROBLEM

Find the area of the region bounded by the circle $\mathbf{x^T}\ \mathbf{x} = 4$ and $\left\| \mathbf{x} - \begin{pmatrix} 2 \\ 0 \end{pmatrix} \right\| = 2$.

2 SOLUTION

$\mathbf{x}^T\mathbf{x} + 2\mathbf{u}^T\mathbf{x} + f = 0$

So from above equation we can say that,

2.1 Circle 1

Taking equation of the first circle to be,

$$\mathbf{x}^T\mathbf{x} + 2\mathbf{u_1}^T\mathbf{x} + f_1 = 0 = 0 \tag{2.1.1}$$

$$\mathbf{x}^T\mathbf{x} - 4 = 0 \text{(given)} \tag{2.1.2}$$

$$\mathbf{u_1} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{2.1.3}$$

$$f_1 = -4 \tag{2.1.4}$$

2.2 Circle 2

Taking equation of the second circle to be,

$$\left\| \mathbf{x} - \begin{pmatrix} 2 \\ 0 \end{pmatrix} \right\|^2 = 2^2 \text{(given)} \tag{2.2.1}$$

$$\mathbf{x}^T\mathbf{x} + 2\mathbf{u_2}^T\mathbf{x} = 0 \tag{2.2.2}$$

$$\mathbf{u_2} = \begin{pmatrix} -2 \\ 0 \end{pmatrix} \tag{2.2.3}$$

$$f_2 = 0 \tag{2.2.4}$$

Now,Subtracting equation (2.2.2) from (2.1.2) We get,

$$\mathbf{x}^T\mathbf{x} - 2\mathbf{u_2}^T\mathbf{x} + f_1 - \mathbf{x}^T\mathbf{x} = 0 \tag{2.2.5}$$

$$2\mathbf{u}^T\mathbf{x} = -4 \tag{2.2.6}$$

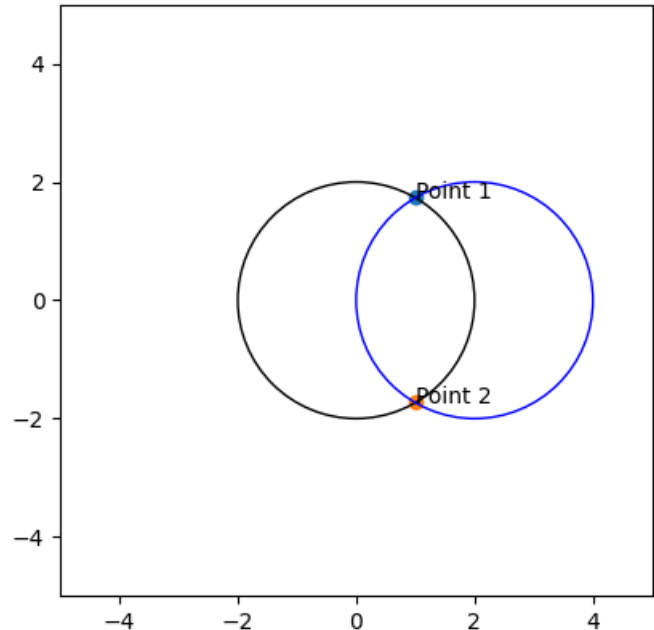$$\begin{pmatrix} -4 & 0 \end{pmatrix}\mathbf{x} = -4 \tag{2.2.7}$$

Fig. 0: Figure depicting intersection points of circle

Which can be written as:-

$$\begin{pmatrix} -1 & 0 \end{pmatrix}\mathbf{x} = -1 \tag{2.2.8}$$

$$\begin{pmatrix} -1 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = -1 \text{(Let } \mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}) \tag{2.2.9}$$

$$-x_1 = -1 \tag{2.2.10}$$

$$x_1 = 1 \tag{2.2.11}$$

$$x = \begin{pmatrix} 1 \\ x_2 \end{pmatrix} \tag{2.2.12}$$

Substituting (2.2.12) in (2.1.2)

$$\mathbf{x}^T\mathbf{x} - 4 = 0 \tag{2.2.13}$$

$$\|x\| = 4 \tag{2.2.14}$$

$$1 + x_2^2 = 4 \tag{2.2.15}$$

$$x_2 = +\sqrt{3}, -\sqrt{3} \tag{2.2.16}$$

$$x = \begin{pmatrix} 1 \\ \sqrt{3} \end{pmatrix}, \begin{pmatrix} 1 \\ -\sqrt{3} \end{pmatrix} \tag{2.2.17}$$

Points of intersection come out to be$(1, \sqrt{3})$ and $(1, -\sqrt{3})$,Now finding the direction

vector between point of intersection and origin of two circle. Subtracting Point 1 from point 2 direction vector comes out to be $k_1 \begin{pmatrix} 0 \\ \sqrt{3} \end{pmatrix}$ Subtracting both the origin then the vector come out to be $k_2 \begin{pmatrix} 2 \\ 0 \end{pmatrix}$ Now using these to find angle between two vectors by finding inner product:-

$$\mathbf{ab} = \|a\| \|b\| \cos\theta \qquad (2.2.18)$$

$$\frac{0}{\|a\| \|b\|} = \cos\theta \qquad (2.2.19)$$

$$\theta = 90 \qquad (2.2.20)$$

θ gives the angle segment. We have to Double that to find out area.

$$Area = Area\,of\,sector - Area\,of\,Triangle \quad (2.2.21)$$

$$Area = \frac{\pi\theta}{360}r^2 - \frac{1}{2}2\sqrt{3} \quad (2.2.22)$$

$$Total\,area = 2 * Area \quad (2.2.23)$$

$$= 2\pi - 2\sqrt{3} \quad (2.2.24)$$