Assignment 19

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The link to the solution is

https://github.com/Adarsh1310/EE5609

Abstract—**This documents solves a problem based on Linear Transformation.**

1 PROBLEM

Let T be the linear transformation from $\mathbb{R}^3$ into $\mathbb{R}^2$ defined by,

$$T\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_1 + x_2 \\ 2x_3 - x_1 \end{pmatrix}$$

If β is the standard ordered basis for $\mathbb{R}^3$ and β' is the standard ordered basis for $\mathbb{R}^2$, what is the matrix of T relative to the pair β,β'

2 SOLUTION

We know that,

$$[T\alpha]_\beta = \mathbf{A}[\alpha]_{\beta'} \qquad (2.0.1)$$

where $\mathbf{A}$ is called the matrix of T relative to ordered basis β,β' Using the ordered basis,

$$\beta = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad (2.0.2)$$

$$\beta' = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad (2.0.3)$$

Now, Let's consider the equation given in the question,

$$T\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_1 + x_2 \\ 2x_3 - x_1 \end{pmatrix} \qquad (2.0.4)$$

R.H.S of the equation can be considered as a product of 2×3 and 3×1 matrices,

$$= \begin{pmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \qquad (2.0.5)$$

Hence the transformation matrix is,

$$\begin{pmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \end{pmatrix} \qquad (2.0.6)$$

Now, since the transformation has to be found relative to the pair β, β' we should row reduce,

$$\left(\begin{array}{cc|ccc} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & -1 & 0 & 2 \end{array}\right) \qquad (2.0.7)$$

but from here we can see that β' is already an identity matrix and hence no row reduction is required. So by using (2.0.1) we can say that,

$$[T\alpha]_\beta = \begin{pmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \end{pmatrix}[\alpha]_{\beta'} \qquad (2.0.8)$$