

Assignment 15

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The link to the solution is

<https://github.com/Adarsh1310/EE5609>

Abstract—This documents solves a problem based on vector subspaces.

1 PROBLEM

Let \mathbf{W} be the set of all $(x_1, x_2, x_3, x_4, x_5)$ in \mathbb{R}^5 which satisfy

$$2x_1 - x_2 + \frac{4}{3}x_3 - x_4 = 0$$

$$x_1 + \frac{2}{3}x_3 - x_5 = 0$$

$$9x_1 - 3x_2 + 6x_3 - 3x_4 - 3x_5 = 0$$

Find a finite set of vectors which spans \mathbf{W} .

2 SOLUTION

The above vectors can be written as,

$$\alpha_1 = (2, -1, \frac{4}{3}, -1, 0) \quad (2.0.1)$$

$$\alpha_2 = (1, 0, \frac{2}{3}, 0, -1) \quad (2.0.2)$$

$$\alpha_3 = (9, -3, 6, -3, -3) \quad (2.0.3)$$

Vector is in the subspace \mathbf{W} of \mathbb{R}^5 spanned by α_1, α_2 and α_3 if and only if there exist scalars c_1, c_2 as real numbers. We can see that α_3 is a linear combination of α_1 and α_2 . So,

$$\alpha = c_1\alpha_1 + c_2\alpha_2 \quad (2.0.4)$$

\mathbf{W} consists all vector of the form,

$$\alpha = (2c_1 + c_2, -1c_1, \frac{4}{3}c_1 + \frac{2}{3}c_2, -1c_1, -c_2) \quad (2.0.5)$$

where c_1, c_2 are scalar constant. Alternatively it can be written as

$$\alpha = (x_1, x_2, x_3, x_4, x_5) \quad (2.0.6)$$

with x_i in \mathbb{R}

$$2x_1 - x_2 + \frac{4}{3}x_3 - x_4 = 0 \quad (2.0.7)$$

$$x_1 + \frac{2}{3}x_3 - x_5 = 0 \quad (2.0.8)$$

which can be written as,

$$\begin{pmatrix} 2 & -1 & \frac{4}{3} & -1 & 0 \\ 1 & 0 & \frac{2}{3} & 0 & -1 \end{pmatrix} \mathbf{x} = 0 \quad (2.0.9)$$

Now, the augmented matrix,

$$\left(\begin{array}{ccccc|c} 2 & -1 & \frac{4}{3} & -1 & 0 & 0 \\ 1 & 0 & \frac{2}{3} & 0 & -1 & 0 \end{array} \right) \quad (2.0.10)$$

$$\xleftrightarrow{R_2=R_2-\frac{1}{2}R_1} \left(\begin{array}{ccccc|c} 2 & -1 & \frac{4}{3} & -1 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} & -1 & 0 \end{array} \right) \quad (2.0.11)$$

$$\xleftrightarrow{R_2=2R_2} \left(\begin{array}{ccccc|c} 2 & -1 & \frac{4}{3} & -1 & 0 & 0 \\ 0 & 1 & 0 & 1 & -2 & 0 \end{array} \right) \quad (2.0.12)$$

$$\xleftrightarrow{R_1=R_1+R_2} \left(\begin{array}{ccccc|c} 2 & 0 & \frac{4}{3} & 0 & -2 & 0 \\ 0 & 1 & 0 & 1 & -2 & 0 \end{array} \right) \quad (2.0.13)$$

So,

$$2x_1 + \frac{4}{3}x_3 - 2x_5 = 0 \quad (2.0.14)$$

$$x_2 + x_4 - 2x_5 = 0 \quad (2.0.15)$$

Solving the equations we get,

$$x_1 = -\frac{2}{3}x_3 + x_5 \quad (2.0.16)$$

$$x_2 = -x_4 + 2x_5 \quad (2.0.17)$$

From above we can see that x_3, x_4 and x_5 are free variables Hence (2.0.6) can be written as,

$$\alpha = (-\frac{2}{3}x_3 + x_5, -x_4 + 2x_5, x_3, x_4, x_5) \quad (2.0.18)$$

$$= (-\frac{2}{3}, 0, 1, 0, 0)x_3 + (0, -1, 0, 1, 0)x_4 + (1, 2, 0, 0, 1)x_5 \quad (2.0.19)$$

such that x_3, x_4 and $x_5 \in \mathbb{R}$. So, $(-\frac{2}{3}, 0, 1, 0, 0), (0, -1, 0, 1, 0)$ and $(1, 2, 0, 0, 1)$ will span \mathbf{W} .