Assignment 11

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The link to the solution is

https://github.com/Adarsh1310/EE5609

***Abstract*—This documents show a method to perform row exchange using elementary row operations.**

1 Problem

Prove that the interchange of two rows of a matrix can be accomplished by a finite sequence of elementary row operations of the other two types.

2 Solution

Let A be a 3×3 matrix with having row vectors $\mathbf{a}_1$,$\mathbf{a}_2$ and $\mathbf{a}_3$.

$$\mathbf{A} = \begin{pmatrix} \mathbf{a}_1 \\ \mathbf{a}_2 \\ \mathbf{a}_3 \end{pmatrix} \tag{2.0.1}$$

Let's exchange row $\mathbf{a}_1$ and $\mathbf{a}_2$. Let's call this elementary operation $\mathbf{E_1}$.

$$\begin{pmatrix} \mathbf{a}_2 \\ \mathbf{a}_1 \\ \mathbf{a}_3 \end{pmatrix} \tag{2.0.2}$$

Now, to prove that same matrix can be obtained by elementary operations let's call them $\mathbf{E_2}$ and $\mathbf{E_3}$. We will first perform elementary operation $\mathbf{E_2}$ by adding row 1 to row 2.

$$\begin{pmatrix} \mathbf{a}_1 + \mathbf{a}_2 \\ \mathbf{a}_2 \\ \mathbf{a}_3 \end{pmatrix} \tag{2.0.3}$$

Using elementary operation $\mathbf{E_2}$ we will subtract row 1 from row 2.

$$\begin{pmatrix} \mathbf{a}_1 + \mathbf{a}_2 \\ -\mathbf{a}_1 \\ \mathbf{a}_3 \end{pmatrix} \tag{2.0.4}$$

Using elementary operation $\mathbf{E_2}$ we will add row 2 to row 1.

$$\begin{pmatrix} \mathbf{a}_2 \\ -\mathbf{a}_1 \\ \mathbf{a}_3 \end{pmatrix} \tag{2.0.5}$$

Using elementary operation $\mathbf{E_3}$ we will multiply row 2 by -1.

$$\begin{pmatrix} \mathbf{a}_2 \\ \mathbf{a}_1 \\ \mathbf{a}_3 \end{pmatrix} \tag{2.0.6}$$

Hence (2.0.2) is obtained using $\mathbf{E_2}$ and $\mathbf{E_3}$.

3 Example

Let us assume an elementary matrix $\mathbf{A}$

$$\mathbf{A} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{3.0.1}$$

Let's exchange row $\mathbf{a}_1$ and $\mathbf{a}_2$ by applying operation $\mathbf{E_1}$.

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{3.0.2}$$

We will first perform elementary operation $\mathbf{E_2}$ by adding row 1 to row 2.

$$\begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{3.0.3}$$

Using elementary operation $\mathbf{E_2}$ we will subtract row 1 from row 2.

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{3.0.4}$$

Using elementary operation $\mathbf{E_2}$ we will add row 2 to row 1.

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{3.0.5}$$

Using elementary operation $\mathbf{E_3}$ we will multiply row 2 by -1.

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{3.0.6}$$

Hence, (3.0.2) was obtained from other two operations.