Assignment 24

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The link to the solution is

https://github.com/Adarsh1310/EE5609

***Abstract*—This documents solves a problem based on Jordan Form.**

1 Problem

If $\mathbf{N}$ is a nilpotent 3 X 3 matrix over C, prove that $\mathbf{A} = \mathbf{I} + \frac{1}{2}\mathbf{N}$ - $\frac{1}{8}\mathbf{N}^2$ satisfies $\mathbf{A}^2 = \mathbf{I} + \mathbf{N}$, i.e., $\mathbf{A}$ is a square root of $\mathbf{I} + \mathbf{N}$. Use the binomial series for $(1+t)^{\frac{1}{2}}$ to obtain a similar formula for a square root of $\mathbf{I} + \mathbf{N}$, where $\mathbf{N}$ is any nilpotent n X n matrix over C.

2 Solution

We know that $\mathbf{N}^3$=0 since the minimal polynomial of $\mathbf{N}$ is x^3,So,

$$\mathbf{A}^2 = \left(\mathbf{I} + \frac{1}{2}\mathbf{N} - \frac{1}{8}\mathbf{N}^2\right)\left(\mathbf{I} + \frac{1}{2}\mathbf{N} - \frac{1}{8}\mathbf{N}^2\right) \quad (2.0.1)$$

$$= \mathbf{I} + \frac{1}{2}\mathbf{N} - \frac{1}{8}\mathbf{N}^2 + \frac{1}{4}\mathbf{N}^2 - \frac{1}{8}\mathbf{N}^2 \quad (2.0.2)$$

$$= \mathbf{I} + \mathbf{N} \quad (2.0.3)$$

Now using Taylor's Formula on $(1 + t)^{1/2}$,

$$= 1 + \sum_{i=1} \frac{1}{i!}[(1 + t)^{1/2}]^i t^i \quad (2.0.4)$$

$$= 1 + \sum_{i=1} (-1)^{i+1} \frac{(2i-3)t^i}{i!2^i} \quad (2.0.5)$$

So square root of $\mathbf{I} + \mathbf{N}$ where $\mathbf{N}$ is n $\times$ n nilpotent matrix can be,

$$= \mathbf{I} + \sum_{i=1}^{n-1} (-1)^{i+1} \frac{(2i-3)\mathbf{N}^i}{i!2^i} \quad (2.0.6)$$