Assignment 5

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The link to the solution is

https://github.com/Adarsh1310/EE5609

***Abstract*—This documents solves a problem based on circles.**

1 Problem

Find the area of the region bounded by the circle $\mathbf{x^T}\ \mathbf{x}= 4$ and $\left\|\mathbf{x}-\begin{pmatrix}2\\0\end{pmatrix}\right\| = 2.$

2 Solution

$\|\mathbf{x}\|^2 + 2\mathbf{u}^T\mathbf{x} + f = 0$
$\mathbf{x}^T\mathbf{x} + 2\mathbf{u}^T\mathbf{x} + f = 0$
So from above equation we can say that,

2.1 Circle 1

Taking equation of the first circle to be,

$$\|\mathbf{x}\|^2 + 2\mathbf{u}_1^T\mathbf{x} + f_1 = 0 \tag{2.1.1}$$

$$\mathbf{x}^T\mathbf{x} - 4 = 0(\text{given}) \tag{2.1.2}$$

$$\mathbf{u_1} = \begin{pmatrix}0\\0\end{pmatrix} \tag{2.1.3}$$

$$f_1 = -4 \tag{2.1.4}$$

2.2 Circle 2

Taking equation of the second circle to be,

$$\left\|\mathbf{x}-\begin{pmatrix}2\\0\end{pmatrix}\right\|^2 = 2^2(\text{given}) \tag{2.2.1}$$

$$\mathbf{x}^T\mathbf{x} + 2\mathbf{u_2}^T\mathbf{x} = 0 \tag{2.2.2}$$

$$\mathbf{u_2} = \begin{pmatrix}-2\\0\end{pmatrix} \tag{2.2.3}$$

$$f_2 = 0 \tag{2.2.4}$$

Now,Subtracting equation (2.2.2) from (2.1.2) We get,

$$\mathbf{x}^T\mathbf{x} - 2\mathbf{u_2}^T\mathbf{x} + f_1 - \mathbf{x}^T\mathbf{x} = 0 \tag{2.2.5}$$

$$2\mathbf{u}^T\mathbf{x} = -4 \tag{2.2.6}$$

$$\begin{pmatrix}-4 & 0\end{pmatrix}\mathbf{x} = -4 \tag{2.2.7}$$

Which can be written as:-

$$\begin{pmatrix}1 & 0\end{pmatrix}\mathbf{x} = 1 \tag{2.2.8}$$

$$\mathbf{x} = \begin{pmatrix}1\\0\end{pmatrix} + \lambda\begin{pmatrix}0\\1\end{pmatrix} \tag{2.2.9}$$

$$\mathbf{x} = \mathbf{q} + \lambda\mathbf{m} \tag{2.2.10}$$

$$\mathbf{q} = \begin{pmatrix}1\\0\end{pmatrix} \tag{2.2.11}$$

$$\mathbf{m} = \begin{pmatrix}0\\1\end{pmatrix} \tag{2.2.12}$$

Substituting (2.2.10) in (2.1.1)

$$\|\mathbf{x}\|^2 + 2\mathbf{u}_1^T\mathbf{x} + f_1 = 0 \tag{2.2.13}$$

$$\|\mathbf{q} + \lambda\mathbf{m}\|^2 + f_1 = 0 \tag{2.2.14}$$

$$\left\|\begin{pmatrix}1\\\lambda\end{pmatrix}\right\|^2 - 4 = 0 \tag{2.2.15}$$

$$\lambda^2 + 1 - 4 = 0 \tag{2.2.16}$$

$$\lambda = +\sqrt{3}, -\sqrt{3} \tag{2.2.17}$$

$$\tag{2.2.18}$$

Substituting the value of λ in(2.2.10)

$$\mathbf{x} = \mathbf{q} + \lambda\mathbf{m} \tag{2.2.19}$$

$$For\lambda = \sqrt{3}, \mathbf{x} = \begin{pmatrix}1\\\sqrt{3}\end{pmatrix} \tag{2.2.20}$$

$$For\lambda = -\sqrt{3}, \mathbf{x} = \begin{pmatrix}1\\-\sqrt{3}\end{pmatrix} \tag{2.2.21}$$

Points of intersection come out to be$(1, \sqrt{3}) and (1, -\sqrt{3})$,Now finding the direction vector between point of intersection and origin of two circle. Subtracting Point 1 from point 2 direction vector comes out to be $k_1\begin{pmatrix}0\\\sqrt{3}\end{pmatrix}$Subtracting both the origin then the vector come out to be $k_2\begin{pmatrix}2\\0\end{pmatrix}$Now using these to find angle between two vectors by finding inner product:-

$$\mathbf{ab} = \|a\| \, \|b\| \cos\theta \tag{2.2.22}$$
$$\frac{0}{\|a\| \, \|b\|} = \cos\theta \tag{2.2.23}$$
$$\theta = 90 \tag{2.2.24}$$

θ gives the angle segment. We have to Double that to find out area.

$$Area = Areaofsector - AreaofTriangle \tag{2.2.25}$$
$$Area = \frac{\pi\theta}{360} r^2 - \frac{1}{2} 2\sqrt{3} \tag{2.2.26}$$
$$Totalarea = 2 * Area \tag{2.2.27}$$
$$= 2\pi - 2\sqrt{3} \tag{2.2.28}$$

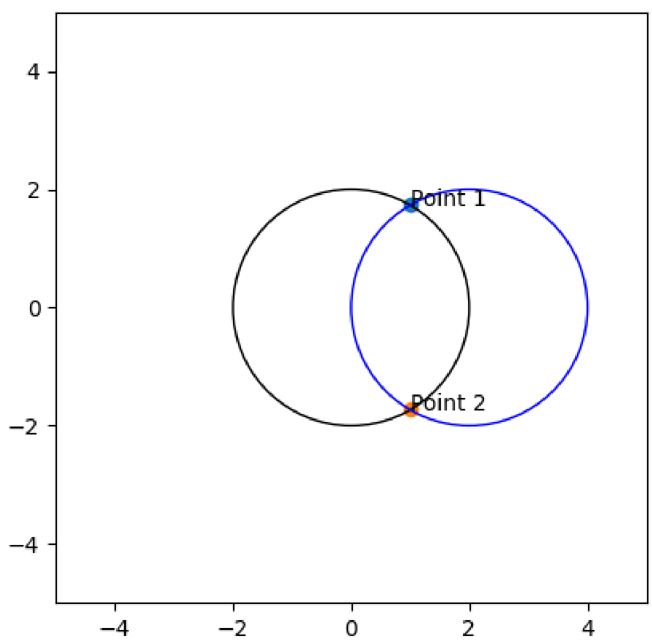

Fig. 0: Figure depicting intersection points of circle