Assignment 19

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The link to the solution is

https://github.com/Adarsh1310/EE5609

***Abstract*—This documents solves a problem based on Linear Transformation.**

1 Problem

Let T be the linear transformation from $\mathbb{R}^3$ into $\mathbb{R}^2$ defined by,

$$T\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix} = \begin{pmatrix}x_1+x_2\\2x_3-x_1\end{pmatrix}$$

If β is the standard ordered basis for $\mathbb{R}^3$ and $\beta^{'}$ is the standard ordered basis for $\mathbb{R}^2$, what is the matrix of T relative to the pair β,$\beta^{'}$

2 Solution

We know that,

$$[T\alpha]_{\beta'} = \mathbf{A}[\alpha]_\beta \tag{2.0.1}$$

where $\mathbf{A}$ is called the matrix of T relative to ordered basis β,$\beta^{'}$ Using the ordered basis,

$$\beta = \left\{\begin{pmatrix}1\\0\\0\end{pmatrix},\begin{pmatrix}0\\1\\0\end{pmatrix},\begin{pmatrix}0\\0\\1\end{pmatrix}\right\} \tag{2.0.2}$$

$$\beta^{'} = \left\{\begin{pmatrix}1\\0\end{pmatrix},\begin{pmatrix}0\\1\end{pmatrix}\right\} \tag{2.0.3}$$

Using the given transformation,

$$T\begin{pmatrix}1\\0\\0\end{pmatrix} = \begin{pmatrix}1 & -1\end{pmatrix}\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix} \tag{2.0.4}$$

$$T\begin{pmatrix}0\\1\\0\end{pmatrix} = \begin{pmatrix}1 & 0\end{pmatrix}\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix} \tag{2.0.5}$$

$$T\begin{pmatrix}0\\0\\1\end{pmatrix} = \begin{pmatrix}0 & 2\end{pmatrix}\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix} \tag{2.0.6}$$

From (2.0.1) we can say that,

$$T[\alpha]_{\beta'} = \begin{pmatrix}T(\alpha_{\mathbf{1}}) & T(\alpha_{\mathbf{2}}) & T(\alpha_{\mathbf{3}})\end{pmatrix}[\alpha]_\beta \tag{2.0.7}$$

So, T relative to the pair β, $\beta^{'}$ will be,

$$\begin{pmatrix}1 & 1 & 0\\-1 & 0 & 2\end{pmatrix} \tag{2.0.8}$$