Assignment 21

Adarsh Srivastava

The link to the solution is

https://github.com/Adarsh1310/EE5609

***Abstract*—This documents solves a problem based on Polynomial of Linear Transformation.**

1 Problem

Let T be the linear operator on $\mathbb{R}^3$ defined by

$$T(x_1, x_2, x_3) = (x_1, x_3, -2x_2 - x_3) \tag{1.0.1}$$

Let f be the polynomial over $\mathbb{R}$ defined by f= $-x^3+2$

2 Solution

The given transformation can be written as,

$$T(\mathbf{x}) = \mathbf{A}\mathbf{x} \tag{2.0.1}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -2 & -1 \end{pmatrix} \mathbf{x} \tag{2.0.2}$$

Hence,

$$\mathbf{A} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -2 & -1 \end{pmatrix} \tag{2.0.3}$$

Now the characteristic equation of $\mathbf{A}$ is given by,

$$\det(\mathbf{A} - \lambda\mathbf{I}) = 0 \tag{2.0.4}$$

$$= \begin{pmatrix} 1-\lambda & 0 & 0 \\ 0 & -\lambda & 1 \\ 0 & -2 & -1-\lambda \end{pmatrix} \tag{2.0.5}$$

$$\implies (1-\lambda)(\lambda^2 + \lambda + 2) = 0 \tag{2.0.6}$$

Now, simplifying the above equation,

$$(1-\lambda)(\lambda^2 + \lambda + 2) = 0 \tag{2.0.7}$$

$$\lambda^2 + \lambda + 2 - \lambda^3 - \lambda^2 - 2\lambda = 0 \tag{2.0.8}$$

$$\lambda^3 = 2 - \lambda \tag{2.0.9}$$

Now using Cayley Hamilton Theorem we get,

$$\mathbf{A}^3 = 2\mathbf{I} - \mathbf{A} \tag{2.0.10}$$

Hence the polynomial $f(\mathbf{A})$ can be written using the characteristic function of $\mathbf{A}$ as follows,

$$f(\mathbf{A}) = -\mathbf{A}^3 + 2\mathbf{I} \tag{2.0.11}$$

$$= 2\mathbf{I} - \mathbf{A} + 2\mathbf{I} \tag{2.0.12}$$

$$= \mathbf{A} \tag{2.0.13}$$

Hence,

$$f(T)(\mathbf{x}) = \mathbf{A}\mathbf{x} \tag{2.0.14}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -2 & -1 \end{pmatrix} \mathbf{x} \tag{2.0.15}$$