Assignment 5

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The link to the solution is

https://github.com/Adarsh1310/EE5609

***Abstract*—This documents solves a problem based on circles.**

1 Problem

Find the area of the region bounded by the circle $\mathbf{x^T}$ $\mathbf{x}$= 2 and $\left\|\mathbf{x}-\begin{pmatrix}2\\0\end{pmatrix}\right\| = 2$.

2 Solution

$\mathrm{x}^T\mathbf{x}-2(O)^T\mathbf{x}+\|\mathbf{O}\|^2-\mathbf{r}^2 = 0$
So from above equation we can say that,
Circle 1:
Equation of circle 1:

$$\mathbf{x^T x = 2} \tag{2.0.1}$$

radius=2
point of origin as (0,0)
Circle 2:
Equation of circle 2:

$$\left\|\mathbf{x}-\begin{pmatrix}2\\0\end{pmatrix}\right\|^2 \tag{2.0.2}$$

$$\mathbf{x}^T\mathbf{x}-2\begin{pmatrix}2\\0\end{pmatrix}^T\mathbf{x}+2-4 = 0 \tag{2.0.3}$$

$$\mathbf{x}^T\mathbf{x}-\begin{pmatrix}0\\4\end{pmatrix}^T\mathbf{x}-2 = 0 \tag{2.0.4}$$

radius=2
point of origin as (2,0)

Subtracting equation 2.0.4 from 2.0.1
We get,

$$\begin{pmatrix}1 & 0\end{pmatrix}\mathbf{x} = 0 \tag{2.0.5}$$

$$\mathbf{x} = \lambda\mathbf{m}, where\mathbf{m} = \begin{pmatrix}0\\1\end{pmatrix} \tag{2.0.6}$$

$$\mathbf{x} = \lambda\begin{pmatrix}0\\1\end{pmatrix} \tag{2.0.7}$$

$$\tag{2.0.8}$$

Substituting 2.0.7 in 2.0.1

$$\lambda = \sqrt{2}\begin{pmatrix}0\\1\end{pmatrix} \tag{2.0.9}$$

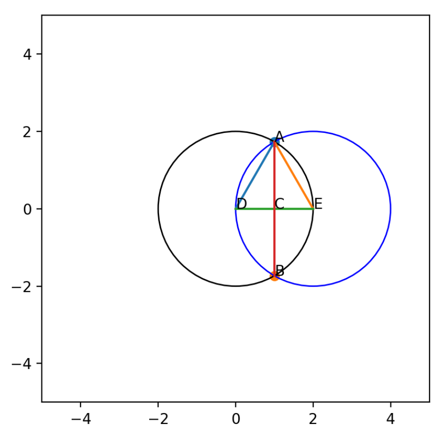

Fig. 0: Figure depicting intersection points of circle

Now finding points of intersection:

Equation in general form is as follows:

$$x^2+y^2 = 4 \tag{2.0.10}$$

$$(x-2)^2+y^2 = 4 \tag{2.0.11}$$

Now Comparing equation 2.0.1 and 2.0.2:

$$(x-2)^2 + (4-x^2) = 4 \tag{2.0.12}$$
$$x^2 + 4 - 2x + 4 - x^2 = 4 \tag{2.0.13}$$

x comes out to be 1

Now,Substituting the value of x in equation 2.0.1:
y comes out to be $\sqrt{3} and - \sqrt{3}$
So the points of intersection of the two circles are $(1, \sqrt{3}) and (1, -\sqrt{3})$
Now to find the area inclosed between these circles we have to find the integral of these point w.r.t the circles.For this we need to find the area of segment **ABE** and double it to find the area of the entire overlapped region.
To find the area of segment ABE we need angle CDA as:
$Here,$
r=2 and $\theta = \angle CDA$
Now we have to find the angle
$\sin\theta = \frac{\mathbf{AC}}{\mathbf{AD}}$
$\theta = 30^\circ$
$\angle ADB = 2 * \theta$
$So, \angle ADB = 60^\circ$

$$Area = \frac{1}{2}(\frac{\angle ADB}{360} - \sin(\angle ADB))r^2 \tag{2.0.14}$$
(Area of Sector-Area of Triangle)
$$Area = \frac{1}{2}(\frac{\pi}{3} - \sin 60)r^2 \tag{2.0.15}$$
$$= \frac{1}{2}(\frac{4\pi}{3} - \sqrt{3}) \tag{2.0.16}$$
$$Totalarea = 2 * Area \tag{2.0.17}$$
$$= \frac{4\pi}{3} - \sqrt{3} \tag{2.0.18}$$