

Assignment 5

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The link to the solution is

<https://github.com/Adarsh1310/EE5609>

Abstract—This documents solves a problem based on circles.

1 PROBLEM

Find the area of the region bounded by the circle $\mathbf{x}^T \mathbf{x} = 2$ and $\left\| \mathbf{x} - \begin{pmatrix} 2 \\ 0 \end{pmatrix} \right\| = 2$.

2 SOLUTION

$$[\mathbf{x}^T \mathbf{x} - 2(\mathbf{O})^T \mathbf{x} + \|\mathbf{O}\|^2 - \mathbf{r}^2 = 0]$$

So from above equation we can say that

Circle 1:

radius=2

point of origin as (0,0)

Circle 2:

radius=2

point of origin as (2,0)

Now finding points of intersection:

Equation in general form is as follows:

$$x^2 + y^2 = 4 \quad (2.0.1)$$

$$(x - 2)^2 + y^2 = 4 \quad (2.0.2)$$

Now Comparing equation 2.0.1 and 2.0.2:

$$(x - 2)^2 + (4 - x^2) = 4 \quad (2.0.3)$$

$$x^2 + 4 - 2x^2 + 4 - x^2 = 4 \quad (2.0.4)$$

x comes out to be 1

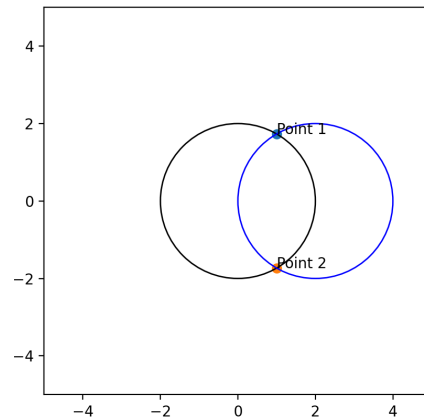


Fig. 0: Figure depicting intersection points of circle

Now, Substituting the value of x in equation 2.0.1:

y comes out to be $\sqrt{3}$ and $-\sqrt{3}$

Now to find the area inclosed between these circles we have to find the integral of these point w.r.t the circles.

$$\begin{aligned} \text{Area} &= 2 \int_1^2 \sqrt{4 - x^2} \, dx \\ &= 2 \left[\frac{x}{2} \sqrt{4 - x^2} + 2 \frac{x}{2} \right]_1^2 \\ &= -\sqrt{3} + \frac{4\pi}{3} \end{aligned}$$