

# Assignment 17

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The link to the solution is

<https://github.com/Adarsh1310/EE5609>

**Abstract**—This documents solves a problem based on coordinates.

## 1 PROBLEM

Let  $\mathbf{V}$  be the real vector space of all polynomial functions from  $\mathbb{R}$  to  $\mathbb{R}$  of degree 2 or less, i.e, the space of all functions  $f$  of the form,

$$f(x) = c_0 + c_1x + c_2x^2$$

Let  $t$  be a fixed real number and define

$$g_1(x) = 1, g_2(x) = x + t, g_3(x) = (x + t)^2$$

Prove that  $\beta = \{g_1, g_2, g_3\}$  is a basis for  $\mathbf{V}$ . If

$$f(x) = c_0 + c_1x + c_2x^2$$

what are the coordinates of  $f$  in the ordered basis  $\beta$

## 2 SOLUTION

We start by taking,

$$\mathbf{f} = \begin{pmatrix} 1 \\ x \\ x^2 \end{pmatrix} \quad (2.0.1)$$

Let's start by proving that  $\mathbf{g}$  is linearly independent.

$$\mathbf{g} = \mathbf{B}\mathbf{f} \quad (2.0.2)$$

where,

$$\mathbf{B} = \begin{pmatrix} 1 & 0 & 0 \\ t & 1 & 0 \\ t^2 & 2t & 1 \end{pmatrix} \quad (2.0.3)$$

Now,

$$\mathbf{v}^T \mathbf{g} = 0 \quad (2.0.4)$$

$$\implies \mathbf{v}^T \mathbf{B}\mathbf{f} = 0 \quad (2.0.5)$$

Since  $\mathbf{f}$  is linearly independent,

$$\mathbf{v}^T \mathbf{B} = 0 \quad (2.0.6)$$

$$\mathbf{B}^T \mathbf{v} = 0 \quad (2.0.7)$$

Since  $\mathbf{B}^T$  is an upper triangular matrix with non zero values in principal diagonal, it is invertible matrix and hence  $\mathbf{v}$  will be zero vector. Now, to find the coordinates,

$$f(x) = \mathbf{w}^T \mathbf{g} \quad (2.0.8)$$

So,

$$\mathbf{c}^T \mathbf{f} = \mathbf{w}^T \mathbf{g} \quad (2.0.9)$$

$$\mathbf{c}^T \mathbf{f} = \mathbf{w}^T \mathbf{B}\mathbf{f} \quad (2.0.10)$$

$$(\mathbf{c}^T - \mathbf{w}^T \mathbf{B})\mathbf{f} = 0 \quad (2.0.11)$$

Since,  $\mathbf{f}$  is linearly independent,

$$\mathbf{c}^T - \mathbf{w}^T \mathbf{B} = 0 \quad (2.0.12)$$

$$\mathbf{c}^T = \mathbf{w}^T \mathbf{B} \quad (2.0.13)$$

$$\mathbf{c}^T \mathbf{B}^{-1} = \mathbf{w}^T \quad (2.0.14)$$

$$(\mathbf{B}^{-1})^T \mathbf{c} = \mathbf{w} \quad (2.0.15)$$

Hence,

$$\mathbf{w} = \begin{pmatrix} 1 & -t & t^2 \\ 0 & 1 & -2t \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_0 \\ c_1 \\ c_2 \end{pmatrix} \quad (2.0.16)$$