# Assignment 15

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The link to the solution is

https://github.com/Adarsh1310/EE5609

***Abstract*—This documents solves a problem based on vector subspaces.**

## 1 Problem

Let **W** be the set of all $(x_1, x_2, x_3, x_4, x_5)$ in $\mathbb{R}^5$ which satisfy

$$2x_1 - x_2 + \frac{4}{3}x_3 - x_4 = 0$$
$$x_1 + \frac{2}{3}x_3 - x_5 = 0$$
$$9x_1 - 3x_2 + 6x_3 - 3x_4 - 3x_5 = 0$$

Find a finite set of vectors which spans **W**.

## 2 Solution

The above vectors can be written as,

$$\alpha_{\mathbf{1}} = \begin{pmatrix} 2 \\ -1 \\ \frac{4}{3} \\ -1 \\ 0 \end{pmatrix} \tag{2.0.1}$$

$$\alpha_{\mathbf{2}} = \begin{pmatrix} 1 \\ 0 \\ \frac{2}{3} \\ 0 \\ -1 \end{pmatrix} \tag{2.0.2}$$

$$\alpha_{\mathbf{3}} = \begin{pmatrix} 9 \\ -3 \\ 6 \\ -3 \\ -3 \end{pmatrix} \tag{2.0.3}$$

Vector is in the subspace **W** of $\mathbb{R}^5$ spanned by $\alpha_{\mathbf{1}}$, $\alpha_{\mathbf{2}}$ and $\alpha_{\mathbf{3}}$ if and only if there exist scalars $c_1$,$c_2$ as real numbers.We can see that $\alpha_{\mathbf{3}}$ is a linear combination of $\alpha_{\mathbf{1}}$ and $\alpha_{\mathbf{2}}$.

$$\alpha_3 = 3\begin{pmatrix} 2 \\ -1 \\ \frac{4}{3} \\ -1 \\ 0 \end{pmatrix} + 3\begin{pmatrix} 1 \\ 0 \\ \frac{2}{3} \\ 0 \\ -1 \end{pmatrix} \tag{2.0.4}$$

So,

$$\alpha = c_1\alpha_{\mathbf{1}} + c_2\alpha_{\mathbf{2}} \tag{2.0.5}$$

**W** consists all vector of the form,

$$\alpha = \begin{pmatrix} 2c_1 + c_2 \\ -1c_1 \\ \frac{4}{3}c_1 + \frac{2}{3}c_2 \\ -1c_1 \\ -c_2 \end{pmatrix} \tag{2.0.6}$$

where $c_1$,$c_2$ are scalar constant. Alternatively,

$$2x_1 - x_2 + \frac{4}{3}x_3 - x_4 = 0 \tag{2.0.7}$$

$$x_1 + \frac{2}{3}x_3 - x_5 = 0 \tag{2.0.8}$$

which can be written as,

$$\begin{pmatrix} 2 & -1 & \frac{4}{3} & -1 & 0 \\ 1 & 0 & \frac{2}{3} & 0 & -1 \end{pmatrix}\mathbf{x} = 0 \tag{2.0.9}$$

Now, the augmented matrix,

$$\left(\begin{array}{ccccc|c} 2 & -1 & \frac{4}{3} & -1 & 0 & 0 \\ 1 & 0 & \frac{2}{3} & 0 & -1 & 0 \end{array}\right) \tag{2.0.10}$$

$$\xleftrightarrow{R_2=R_2-\frac{1}{2}R_1} \left(\begin{array}{ccccc|c} 2 & -1 & \frac{4}{3} & -1 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} & -1 & 0 \end{array}\right) \tag{2.0.11}$$

$$\xleftrightarrow{R_2=2R_2} \left(\begin{array}{ccccc|c} 2 & -1 & \frac{4}{3} & -1 & 0 & 0 \\ 0 & 1 & 0 & 1 & -2 & 0 \end{array}\right) \tag{2.0.12}$$

$$\xleftrightarrow{R_1=R_1+R_2} \left(\begin{array}{ccccc|c} 2 & 0 & \frac{4}{3} & 0 & -2 & 0 \\ 0 & 1 & 0 & 1 & -2 & 0 \end{array}\right) \tag{2.0.13}$$

So,

$$2x_1 + \frac{4}{3}x_3 - 2x_5 = 0 \tag{2.0.14}$$

$$x_2 + x_4 - 2x_5 = 0 \tag{2.0.15}$$

Solving the equations we get,

$$x_1 = -\frac{2}{3}x_3 + x_5 \tag{2.0.16}$$

$$x_2 = -x_4 + 2x_5 \tag{2.0.17}$$

which can be written as,

$$\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} \tag{2.0.18}$$

$$= \begin{pmatrix} -\frac{2}{3}x_3 + x_5 \\ -x_4 + 2x_5 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} \tag{2.0.19}$$

$$= x_3 \begin{pmatrix} -\frac{2}{3} \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} + x_4 \begin{pmatrix} 0 \\ -1 \\ 0 \\ 1 \\ 0 \end{pmatrix} + x_5 \begin{pmatrix} 1 \\ 2 \\ 0 \\ 0 \\ 1 \end{pmatrix} \tag{2.0.20}$$

where $x_3$,$x_4$ and $x_5 \in \mathbb{R}$.Hence, the vectors $\begin{pmatrix} -\frac{2}{3} \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 0 \\ -1 \\ 0 \\ 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 2 \\ 0 \\ 0 \\ 1 \end{pmatrix}$ will span $\mathbf{W}$