Assignment 19

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The link to the solution is

https://github.com/Adarsh1310/EE5609

Abstract—**This documents solves a problem based on Linear Transformation.**

1 PROBLEM

Let T be the linear transformation from $\mathbb{R}^3$ into $\mathbb{R}^2$ defined by,

$$T\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_1 + x_2 \\ 2x_3 - x_1 \end{pmatrix}$$

If β is the standard ordered basis for $\mathbb{R}^3$ and β' is the standard ordered basis for $\mathbb{R}^2$, what is the matrix of T relative to the pair β,β'

2 SOLUTION

We know that,

$$[T\alpha]_{\beta'} = \mathbf{A}[\alpha]_{\beta} \qquad (2.0.1)$$

where $\mathbf{A}$ is called the matrix of T relative to ordered basis β,β' Using the ordered basis,

$$\beta = \left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\} \qquad (2.0.2)$$

$$\beta' = \left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\} \qquad (2.0.3)$$

Using the given transformation,

$$T\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 & -1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad (2.0.4)$$

$$T\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad (2.0.5)$$

$$T\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 & 2 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad (2.0.6)$$

From (2.0.1) we can say that,

$$[T\alpha]_{\beta'} = \begin{pmatrix} T(\alpha_1) & T(\alpha_2) & T(\alpha_3) \end{pmatrix} [\alpha]_{\beta} \qquad (2.0.7)$$

So, T relative to the pair β, β' will be,

$$\begin{pmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \end{pmatrix} \qquad (2.0.8)$$