

Assignment 6

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Abstract—This document solves a question based on triangle.

All the codes for the figure in this document can be found at

https://github.com/Adarsh1310/EE5609/tree/master/Assignment_6

1 PROBLEM

$\triangle ABC$ is an isosceles triangle in which altitudes **BE** and **CF** are drawn to equal sides **AC** and **AB** respectively. Show that these altitudes are equal.

2 RESULT USED

$$FB=EC$$

3 SOLUTION

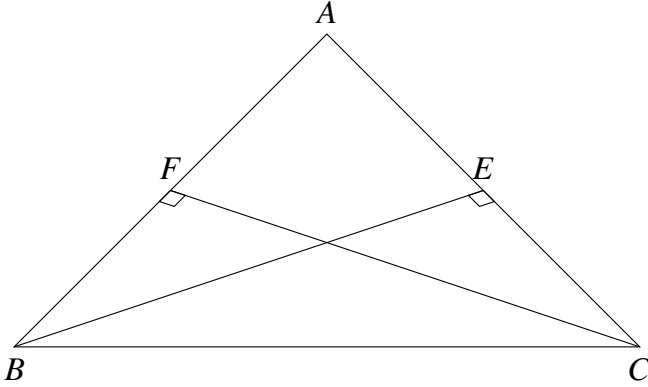


Fig. 1: Isosceles Triangle with BE and CF as altitude

Let \mathbf{m}_{AC} and \mathbf{m}_{BE} be direction vector of side **AC** and altitude **BE** respectively.

$$\mathbf{m}_{AC} = \mathbf{A} - \mathbf{C} \quad (3.0.1)$$

$$\mathbf{m}_{BE} = \mathbf{B} - \mathbf{E} \quad (3.0.2)$$

Here, $\mathbf{BE} \perp \mathbf{AC}$ because **BE** is the altitude to side **AC**. So, Let \mathbf{m}_{AB} and \mathbf{m}_{CF} be direction vector of side **AB** and altitude **CF** respectively.

$$\mathbf{m}_{AB} = \mathbf{A} - \mathbf{B} \quad (3.0.3)$$

$$\mathbf{m}_{CF} = \mathbf{C} - \mathbf{F} \quad (3.0.4)$$

Here, $\mathbf{CF} \perp \mathbf{AB}$ because **CF** is the altitude to side **AB**. So,

$$\mathbf{m}_{AB}^T \mathbf{m}_{CF} = 0 \quad (3.0.5)$$

$$(\mathbf{A} - \mathbf{B})^T (\mathbf{C} - \mathbf{F}) = 0 \quad (3.0.6)$$

$$\Rightarrow (\mathbf{A} - \mathbf{E} + \mathbf{E} - \mathbf{B} + \mathbf{B} - \mathbf{C})^T (\mathbf{B} - \mathbf{E}) = 0 \quad (3.0.7)$$

$$\Rightarrow (\mathbf{A} - \mathbf{E})^T (\mathbf{B} - \mathbf{E}) + \|\mathbf{B} - \mathbf{E}\| + (\mathbf{B} - \mathbf{C})^T (\mathbf{B} - \mathbf{E}) = 0 \quad (3.0.8)$$

$$\Rightarrow \|\mathbf{B} - \mathbf{E}\| + (\mathbf{B} - \mathbf{C})^T (\mathbf{B} - \mathbf{E}) = 0 \quad (3.0.9)$$

Comparing equation (??) and (3.0.9)

$$\begin{aligned} \|\mathbf{C} - \mathbf{F}\| + (\mathbf{C} - \mathbf{B})^T \mathbf{C} - \mathbf{F} &= \\ \|\mathbf{B} - \mathbf{E}\| + (\mathbf{B} - \mathbf{C})^T \mathbf{B} - \mathbf{E} &\quad (3.0.10) \end{aligned}$$

$$\begin{aligned} \|\mathbf{C} - \mathbf{F}\| + \|\mathbf{C} - \mathbf{B}\| \|\mathbf{C} - \mathbf{F}\| \cos \theta &= \\ \|\mathbf{B} - \mathbf{E}\| + \|\mathbf{B} - \mathbf{C}\| \|\mathbf{B} - \mathbf{E}\| \cos \theta &\quad (3.0.11) \end{aligned}$$

$$\begin{aligned} \|\mathbf{C} - \mathbf{F}\| (1 + \|\mathbf{C} - \mathbf{B}\| \|\mathbf{C} - \mathbf{F}\| \cos \theta) &= \\ \|\mathbf{B} - \mathbf{E}\| (1 + \|\mathbf{B} - \mathbf{C}\| \|\mathbf{B} - \mathbf{E}\| \cos \theta) &\quad (3.0.12) \end{aligned}$$

$$\|\mathbf{C} - \mathbf{F}\| = \|\mathbf{B} - \mathbf{C}\| \quad (3.0.13)$$

Hence, the altitudes drawn to equal sides of isosceles triangle is equal.