Assignment 5

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The link to the solution is

https://github.com/Adarsh1310/EE5609

***Abstract*—This documents solves a problem based on circles.**

1 Problem

Find the area of the region bounded by the circle $\mathbf{x^T}$ $\mathbf{x}$= 4 and $\left\|\mathbf{x} - \begin{pmatrix} 2 \\ 0 \end{pmatrix}\right\| = 2$.

2 Solution

$||\mathbf{x}||^2 + 2\mathbf{u}^T\mathbf{x} + f = 0$
So from above equation we can say that,

2.1 Circle 1

Taking equation of the first circle to be,

$$||\mathbf{x}||^2 + 2\mathbf{u}_1^T\mathbf{x} + f_1 = 0 \quad (2.1.1)$$

$$\mathbf{x}^T\mathbf{x} - 4 = 0(\text{given}) \quad (2.1.2)$$

$$\mathbf{u_1} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (2.1.3)$$

$$f_1 = -4 \quad (2.1.4)$$

2.2 Circle 2

Taking equation of the second circle to be,

$$\left\|\mathbf{x} - \begin{pmatrix} 2 \\ 0 \end{pmatrix}\right\|^2 = 2^2(\text{given}) \quad (2.2.1)$$

$$\mathbf{x}^T\mathbf{x} + 2\mathbf{u_2}^T\mathbf{x} = 0 \quad (2.2.2)$$

$$\mathbf{u_2} = \begin{pmatrix} -2 \\ 0 \end{pmatrix} \quad (2.2.3)$$

$$f_2 = 0 \quad (2.2.4)$$

Now,Subtracting equation (**??**) from (**??**) We get,

$$\mathbf{x}^T\mathbf{x} - 2\mathbf{u_2}^T\mathbf{x} + f_1 - \mathbf{x}^T\mathbf{x} = 0 \quad (2.2.5)$$

$$2\mathbf{u}^T\mathbf{x} = -4 \quad (2.2.6)$$

$$\begin{pmatrix} -4 & 0 \end{pmatrix}\mathbf{x} = -4 \quad (2.2.7)$$

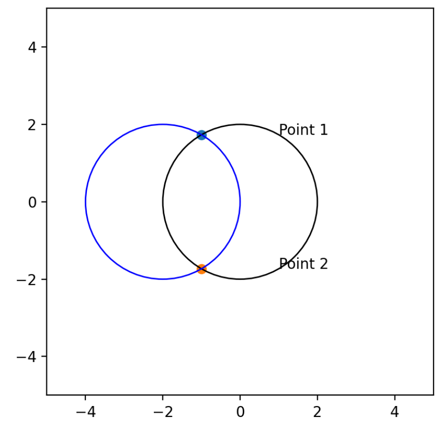

Fig. 0: Figure depicting intersection points of circle

Which can be written as:-

$$\begin{pmatrix} -1 & 0 \end{pmatrix}\mathbf{x} = -1 \quad (2.2.8)$$

$$\mathbf{x} = \lambda\mathbf{m} + q, \text{where}\mathbf{m} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} - 1 \quad (2.2.9)$$

$$\mathbf{x} = \lambda\begin{pmatrix} 0 \\ 1 \end{pmatrix} - 1 \quad (2.2.10)$$

Substituting 2.2.7 in 2.2.2

$$\mathbf{x}^T\mathbf{x} + 2\mathbf{u_2}^T\mathbf{x} = 0 \quad (2.2.11)$$

$$(q + \mathbf{m})^2(q + m) + 2\mathbf{u}^T(q + \mathbf{m}) = 0 \quad (2.2.12)$$

$$(2.2.13)$$

Substituting the value of λ,$\mathbf{u}$ and q and solving

$$\lambda^2 - 4 + 1 = 0 \quad (2.2.14)$$

$$\lambda^2 - 3 = 0 \quad (2.2.15)$$

$$\lambda^2 = 3 \quad (2.2.16)$$

$$\lambda = \sqrt{3}, -\sqrt{3} \quad (2.2.17)$$

Substituting value of in ,Points of intersection come out to be(-1, $\sqrt{3}$)$and$$(-1, -\sqrt{3})$,Now finding the direction vector between point of intersection and origin of two circle.

Subtracting Point 1 from point 2 direction vector comes out to be $k_1\begin{pmatrix}0\\\sqrt{3}\end{pmatrix}$
Subtracting both the origin then the vector come out to be $k_2\begin{pmatrix}-2\\0\end{pmatrix}$
Now using these to find angle between two vectors by finding inner product:-

$$\mathbf{ab} = ||a||\,||b||\cos\theta \tag{2.2.18}$$
$$\frac{0}{||a||\,||b||} = \cos\theta \tag{2.2.19}$$
$$\theta = 90 \tag{2.2.20}$$

θ gives the angle segment. We have to Double that to find out area.

$$\sin\theta = \frac{\mathbf{P}}{\mathbf{H}}$$
$$\theta = 90^\circ$$
$$Now finding area. Total angle = 180^\circ$$

$$Area = Area of sector - Area of Triangle \tag{2.2.21}$$
$$Area = \frac{\pi\theta}{360}r^2 - \frac{1}{2}2\sqrt{3} \tag{2.2.22}$$
$$Total area = 2 * Area \tag{2.2.23}$$
$$= 2\pi - 2\sqrt{3} \tag{2.2.24}$$