

# Assignment 22

Adarsh Srivastava

The link to the solution is

<https://github.com/Adarsh1310/EE5609>

**Abstract**—This documents solves a problem based on Lagrange Interpolation

$$= \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (2.0.4)$$

$$= \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (2.0.5)$$

## 1 PROBLEM

Let  $\mathbb{F}$  be the field of real numbers,

$$\mathbf{A} = \begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$p = (x - 2)(x - 3)(x - 1)$$

- 1) Show that  $p(\mathbf{A}) = 0$
- 2) Let  $P_1, P_2, P_3$  be the Lagrange polynomials for  $t_1 = 2, t_2 = 3, t_3 = 1$ . Compute  $E_i = P_i(\mathbf{A})$ ,  $i=1,2,3$

Hence we proved that  $p(\mathbf{A})=0$   
2) Using Lagrange Interpolation,

$$P_1(x) = \frac{(x - 3)(x - 1)}{(2 - 3)(2 - 1)} \quad (2.0.6)$$

$$= -(x - 3)(x - 1) \quad (2.0.7)$$

$$P_2(x) = \frac{(x - 2)(x - 1)}{(3 - 2)(3 - 1)} \quad (2.0.8)$$

$$= \frac{(x - 2)(x - 1)}{2} \quad (2.0.9)$$

$$P_3(x) = \frac{(x - 2)(x - 3)}{(1 - 2)(1 - 3)} \quad (2.0.10)$$

$$= \frac{(x - 2)(x - 3)}{2} \quad (2.0.11)$$

## 2 SOLUTION

- 1) We have been given that,

$$p(x) = (x - 2)(x - 3)(x - 1) \quad (2.0.1)$$

Now to find  $p(\mathbf{A})$ ,

$$p(\mathbf{A}) = (\mathbf{A} - 2)(\mathbf{A} - 3)(\mathbf{A} - 1) \quad (2.0.2)$$

$$= \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2 \end{pmatrix} \times \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (2.0.3)$$

Now,Substituting the value of  $\mathbf{A}$ ,

$$P_1(\mathbf{A}) = - \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (2.0.12)$$

$$= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (2.0.13)$$

$$P_2(\mathbf{A}) = \frac{1}{2} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (2.0.14)$$

$$= \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (2.0.15)$$

$$P_3(\mathbf{A}) = \frac{1}{2} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2 \end{pmatrix} \quad (2.0.16)$$

$$= \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (2.0.17)$$