

Assignment 5

Adarsh Srivastava

The link to the solution is

<https://github.com/Adarsh1310/EE5609>

Abstract—This documents solves a problem based on circles.

1 PROBLEM

Find the area of the region bounded by the circle $\mathbf{x}^T \mathbf{x} = 2$ and $\left\| \mathbf{x} - \begin{pmatrix} 2 \\ 0 \end{pmatrix} \right\| = 2$.

2 SOLUTION

$$[\mathbf{x}^T \mathbf{x} - 2(\mathbf{O})^T \mathbf{x} + \|\mathbf{O}\|^2 - r^2 = 0]$$

So from above equation we can say that

Circle 1:

radius=2
point of origin as (0,0)

Circle 2:

radius=2
point of origin as (2,0)

Now finding points of intersection:

Equation in general form is as follows:

$$x^2 + y^2 = 4 \quad (2.0.1)$$

$$(x - 2)^2 + y^2 = 4 \quad (2.0.2)$$

Now Comparing equation 2.0.1 and 2.0.2:

$$(x - 2)^2 + (4 - x^2) = 4 \quad (2.0.3)$$

$$x^2 + 4 - 2x + 4 - x^2 = 4 \quad (2.0.4)$$

x comes out to be 1

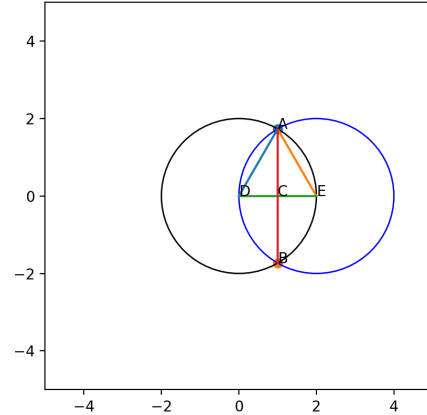


Fig. 0: Figure depicting intersection points of circle

Now, Substituting the value of x in equation 2.0.1:

y comes out to be $\sqrt{3}$ and $-\sqrt{3}$

Now to find the area enclosed between these circles we have to find the integral of these point w.r.t the circles. For this we need to find the area of segment **ABE** and double it to find the area of the entire overlapped region.

To find the area of segment ABE we need angle CDA as:

Here,

$$r = 2 \text{ and } \theta = \angle CDA$$

$$\sin \theta = \frac{AC}{AD}$$

$$\theta = 60$$

$$\text{Area} = \frac{1}{2} \left(\frac{\pi}{3} - \sin 60 \right) r^2 \quad (2.0.5)$$

$$(\text{Area of Sector} - \text{Area of Triangle})$$

$$= \frac{1}{2} \left(\frac{4\pi}{3} - \sqrt{3} \right) \quad (2.0.6)$$

$$\text{Total area} = 2 * \text{Area} \quad (2.0.7)$$

$$= \frac{4\pi}{3} - \sqrt{3} \quad (2.0.8)$$