

# Assignment 6

Adarsh Srivastava

**Abstract**—This document solves a question based on triangle.

All the codes for the figure in this document can be found at

[https://github.com/Adarsh1310/EE5609/tree/master/Assignment\\_6](https://github.com/Adarsh1310/EE5609/tree/master/Assignment_6)

## 1 PROBLEM

$\triangle ABC$  is an isosceles triangle in which altitudes  $BE$  and  $CF$  are drawn to equal sides  $AC$  and  $AB$  respectively. Show that these altitudes are equal.

## 2 SOLUTION

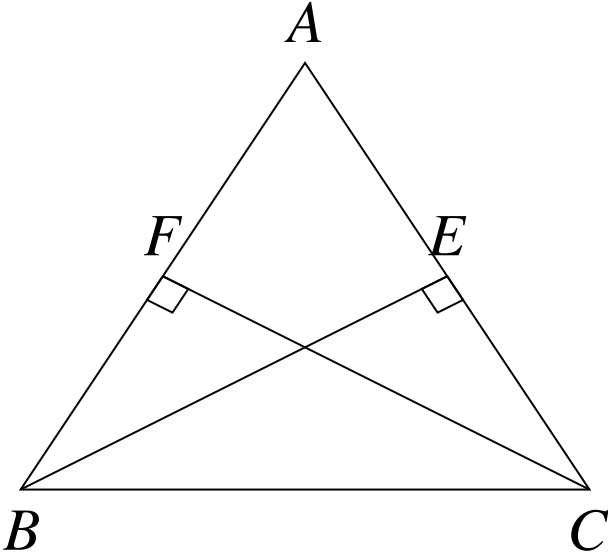


Fig. 1: Isosceles Triangle with altitudes drawn to equal sides

Let  $\mathbf{m}_{AC}$  and  $\mathbf{m}_{BE}$  be direction vector of side  $AC$  and altitude  $BE$  respectively.

$$\mathbf{m}_{AC} = \mathbf{A} - \mathbf{C} \quad (2.0.1)$$

$$\mathbf{m}_{BE} = \mathbf{B} - \mathbf{E} \quad (2.0.2)$$

Here,  $BE \perp AC$  because  $BE$  is the altitude to side  $AC$ . So,

$$\mathbf{m}_{AC}^T \mathbf{m}_{BE} = 0 \quad (2.0.3)$$

$$(\mathbf{A} - \mathbf{C})^T (\mathbf{B} - \mathbf{E}) = 0 \quad (2.0.4)$$

$$(\mathbf{A} - \mathbf{E} + \mathbf{E} - \mathbf{B} + \mathbf{B} - \mathbf{C})^T (\mathbf{B} - \mathbf{E}) = 0 \quad (2.0.5)$$

$$(\mathbf{A} - \mathbf{E})^T (\mathbf{B} - \mathbf{E}) + \|\mathbf{B} - \mathbf{E}\|^2 + (\mathbf{B} - \mathbf{C})^T (\mathbf{B} - \mathbf{E}) = 0 \quad (2.0.6)$$

$$\|\mathbf{B} - \mathbf{E}\|^2 + (\mathbf{B} - \mathbf{C})^T (\mathbf{B} - \mathbf{E}) = 0 \quad (2.0.7)$$

Let  $\mathbf{m}_{AB}$  and  $\mathbf{m}_{CF}$  be direction vector of side  $AB$  and altitude  $CF$  respectively.

$$\mathbf{m}_{AB} = \mathbf{A} - \mathbf{B} \quad (2.0.8)$$

$$\mathbf{m}_{CF} = \mathbf{C} - \mathbf{F} \quad (2.0.9)$$

Here,  $CF \perp AB$  because  $CF$  is the altitude to side  $AB$ . So,

$$\mathbf{m}_{AB}^T \mathbf{m}_{CF} = 0 \quad (2.0.10)$$

$$(\mathbf{A} - \mathbf{B})^T (\mathbf{C} - \mathbf{F}) = 0 \quad (2.0.11)$$

$$(\mathbf{A} - \mathbf{F} + \mathbf{F} - \mathbf{C} + \mathbf{C} - \mathbf{B})^T (\mathbf{C} - \mathbf{F}) = 0 \quad (2.0.12)$$

$$(\mathbf{A} - \mathbf{F})^T (\mathbf{C} - \mathbf{F}) + \|\mathbf{C} - \mathbf{F}\|^2 + (\mathbf{C} - \mathbf{B})^T (\mathbf{C} - \mathbf{F}) = 0 \quad (2.0.13)$$

$$\|\mathbf{C} - \mathbf{F}\|^2 + (\mathbf{C} - \mathbf{B})^T (\mathbf{C} - \mathbf{F}) = 0 \quad (2.0.14)$$

Comparing equation (2.0.14) and (2.0.7)

$$\|\mathbf{C} - \mathbf{F}\|^2 + (\mathbf{C} - \mathbf{B})^T (\mathbf{C} - \mathbf{F}) = \|\mathbf{B} - \mathbf{E}\|^2 + (\mathbf{B} - \mathbf{C})^T (\mathbf{B} - \mathbf{E}) \quad (2.0.15)$$

$$\|\mathbf{C} - \mathbf{F}\|^2 + (\mathbf{C} - \mathbf{B})^T (\mathbf{C} - \mathbf{A} + \mathbf{A} - \mathbf{F}) = \|\mathbf{B} - \mathbf{E}\|^2 + (\mathbf{B} - \mathbf{C})^T (\mathbf{B} - \mathbf{A} + \mathbf{A} - \mathbf{E}) \quad (2.0.16)$$

$$\|\mathbf{C} - \mathbf{F}\|^2 + (\mathbf{C} - \mathbf{B})^T (\mathbf{C} - \mathbf{A}) + (\mathbf{C} - \mathbf{B})^T (\mathbf{A} - \mathbf{F}) = \|\mathbf{B} - \mathbf{E}\|^2 + (\mathbf{B} - \mathbf{C})^T (\mathbf{B} - \mathbf{A}) + (\mathbf{B} - \mathbf{C})^T (\mathbf{A} - \mathbf{E}) \quad (2.0.17)$$

$$\begin{aligned} \|\mathbf{C} - \mathbf{F}\|^2 + 2(\mathbf{C} - \mathbf{B})^T(\mathbf{C} - \mathbf{A}) = \\ \|\mathbf{B} - \mathbf{E}\|^2 + 2(\mathbf{B} - \mathbf{C})^T(\mathbf{B} - \mathbf{A}) \end{aligned} \quad (2.0.18)$$

$$\begin{aligned} \|\mathbf{C} - \mathbf{F}\|^2 + 2(\|\mathbf{C} - \mathbf{B}\| \|\mathbf{C} - \mathbf{A}\|) \cos \theta = \\ \|\mathbf{B} - \mathbf{E}\|^2 + 2(\|\mathbf{B} - \mathbf{C}\| \|\mathbf{B} - \mathbf{A}\|) \cos \theta \end{aligned} \quad (2.0.19)$$

$$\|\mathbf{C} - \mathbf{F}\|^2 = \|\mathbf{B} - \mathbf{E}\|^2 \quad (2.0.20)$$

$$\|\mathbf{C} - \mathbf{F}\| = \|\mathbf{B} - \mathbf{E}\| \quad (2.0.21)$$

Hence, the altitudes drawn to equal sides of isosceles triangle is equal.