# Assignment 10

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The link to the solution is

https://github.com/Adarsh1310/EE5609

***Abstract*—This documents solves a problem based on fields.**

## 1 Problem

Let $\mathbb{F}$ be a set which contains exactly two elements,0 and 1.Define an addition and multiplication by tables.

| + | 0 | 1 |
|---|---|---|
| 0 | 0 | 1 |
| 1 | 1 | 0 |

| · | 0 | 1 |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 0 | 1 |

Verify that the set $\mathbb{F}$, together with these two operations, is a field.

## 2 Solution

To prove that ($\mathbb{F}$,+,·) is a field we need to satisfy the following,

1) + and · should be closed
   - For any a and b in $\mathbb{F}$, a+b $\in \mathbb{F}$ and a · b $\in \mathbb{F}$.For example 0+0=0 and 0·0=0.
2) + and · should be commutative
   - For any a and b in $\mathbb{F}$, a+b = b+a and a · b = b· a. For example 0+1=1+0 and 0*1=1*0.
3) + and · should be associative
   - For any a and b in $\mathbb{F}$, a+(b+c) = (a+b)+c and a· (b · c) = (a · b) · c. For example 0+(1+0)=(0+1)+0 and 0·(1·0)=(0·1)·0.
4) + and · operations should have an identity element
   - If we perform a + 0 then for any value of a from $\mathbb{F}$ the result will be a itself. Hence 0 is an identity element of + operation.If we perform a · 1 then for any value of a from $\mathbb{F}$ the result will be a itself. Hence 1 is an identity element of · operation.
5) $\forall$ a $\in \mathbb{F}$ there exists an additive inverse
   - For additive inverse to exist, $\forall$ a in $\mathbb{F}$ a+(-a)=0. For example. 1-1=0 and 0-0=0.
6) $\forall$ a $\in \mathbb{F}$ such that a is non zero there exists a multiplicative inverse
   - For multiplicative inverse to exist, $\forall$ a such that a is non zero in $\mathbb{F}$, a·a$^{-1}$=1.For example 1·1$^{-1}$ = 1.
7) + and · should hold distributive property
   - For any a,b and c in $\mathbb{F}$ the proberty a·(b+c)=a·b+a·c should always hold true.For example 0·(1+2)=0·1+0·2.

## 3 Result

Since the above properties are satisfied we can say that ($\mathbb{F}$,+,·) is a field.