Assignment 8

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The link to the solution is

https://github.com/Adarsh1310/EE5609

***Abstract*—This documents solves a QR decomposition problem.**

1 Problem

Find QR decomposition of $\begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix}$

2 Solution

Let α and β be transpose of column vectors of the given matrix.

$$\alpha = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \tag{2.0.1}$$

$$\beta = \begin{pmatrix} -1 \\ 3 \end{pmatrix} \tag{2.0.2}$$

We can express these as

$$\alpha = k_1 \mathbf{u}_1 \tag{2.0.3}$$

$$\beta = r_1 \mathbf{u}_1 + k_2 \mathbf{u}_2 \tag{2.0.4}$$

where

$$k_1 = \|\alpha\| \tag{2.0.5}$$

$$\mathbf{u}_1 = \frac{\alpha}{k_1} \tag{2.0.6}$$

$$r_1 = \frac{\mathbf{u}_1^T \beta}{\|\mathbf{u}_1\|^2} \tag{2.0.7}$$

$$\mathbf{u}_2 = \frac{\beta - r_1 \mathbf{u}_1}{\|\beta - r_1 \mathbf{u}_1\|} \tag{2.0.8}$$

$$k_2 = \mathbf{u}_2^T \beta \tag{2.0.9}$$

From (2.0.3) and (2.0.4),

$$\begin{pmatrix} \alpha & \beta \end{pmatrix} = \begin{pmatrix} \mathbf{u}_1 & \mathbf{u}_2 \end{pmatrix} \begin{pmatrix} k_1 & r_1 \\ 0 & k_2 \end{pmatrix} \tag{2.0.10}$$

$$\begin{pmatrix} \alpha & \beta \end{pmatrix} = \mathbf{QR} \tag{2.0.11}$$

From above we can see that $\mathbf{R}$ is an upper triangular matrix and

$$\mathbf{Q}^T \mathbf{Q} = \mathbf{I} \tag{2.0.12}$$

Now by using equations (2.0.5) to (2.0.9)

$$k_1 = \sqrt{5} \tag{2.0.13}$$

$$\mathbf{u}_1 = \sqrt{\frac{1}{5}} \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \tag{2.0.14}$$

$$r_1 = \sqrt{5} \tag{2.0.15}$$

$$\mathbf{u}_2 = \sqrt{\frac{1}{5}} \begin{pmatrix} -2 \\ 1 \end{pmatrix} \tag{2.0.16}$$

$$k_2 = \sqrt{5} \tag{2.0.17}$$

Thus obtained QR decomposition is

$$\begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{5}} & -\frac{2}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{pmatrix} \begin{pmatrix} \sqrt{5} & \sqrt{5} \\ 0 & \sqrt{5} \end{pmatrix} \tag{2.0.18}$$