Assignment 18

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The link to the solution is

https://github.com/Adarsh1310/EE5609

***Abstract*—This documents solves a problem based on Linear Transformation.**

1 Problem

Describe explicitly the linear transformation T from $\mathbb{F}^2$ into $\mathbb{F}^2$ such that T($\in_1$) = $(a, b), T(\in_2) = (c, d)$.

2 Solution

We are given a linear transformation,

$$T : \mathbb{F}^2 \to \mathbb{F}^2 \tag{2.0.1}$$

The transformation for $\in_1$ and $\in_2$ can be written as,

$$T(\in_1) = \begin{pmatrix} a \\ b \end{pmatrix} \tag{2.0.2}$$

$$T(\in_2) = \begin{pmatrix} c \\ d \end{pmatrix} \tag{2.0.3}$$

Now,let's assume $\in_1$ and $\in_2$ as linearly independent. So the linear transformation T for any vector $\mathbf{v}$ in two dimensional space will be,

$$T(\mathbf{v}) = \begin{pmatrix} T(\in_\mathbf{1}) & T(\in_\mathbf{2}) \end{pmatrix} \begin{pmatrix} k_1 \\ k_2 \end{pmatrix} \tag{2.0.4}$$

$$= \begin{pmatrix} a & c \\ b & d \end{pmatrix} \begin{pmatrix} k_1 \\ k_2 \end{pmatrix} \tag{2.0.5}$$

$$= \mathbf{Av} \tag{2.0.6}$$

Now, there can be two cases here, transformation of linearly independent vector can be independent or it can be dependent.Considering the first case and (2.0.5) we can say that,

$$Range(T) = \text{columnspace of} \begin{pmatrix} a & c \\ b & d \end{pmatrix} \tag{2.0.7}$$

Now, considering the case when linear transformation will be linearly dependent,

$$Range(T) = \text{columnspace of} \begin{pmatrix} a \\ b \end{pmatrix} \tag{2.0.8}$$

Now, considering that vectors $\in_1$ and $\in_2$ itself are linearly dependent.Let $\mathbf{v}=\in_1 + \in_2$

$$T(\mathbf{v}) = T(\in_1) + T(\in_2) \tag{2.0.9}$$

$$= T(\in_1) + T(k \in_1) \tag{2.0.10}$$

$$= (k+1)T(\in_1) \tag{2.0.11}$$

$$= (k+1) \begin{pmatrix} a \\ b \end{pmatrix} \tag{2.0.12}$$

We can see from above equation that when $\in_1$ and $\in_2$ as linearly dependent then the transformation T will be along the line only.

Vectors Independent	**Vectors Dependent**
T(v)=$\begin{pmatrix} a & c \\ b & d \end{pmatrix}\begin{pmatrix} k_1 \\ k_2 \end{pmatrix}$	T(v)=(k+1)$\begin{pmatrix} a \\ b \end{pmatrix}$
Output: On the plane	Output: On the line