Assignment 3

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The link to the solution is

https://github.com/Adarsh1310/EE5609

***Abstract*—This documents depicts that matrix multiplication is non commutative.**

1 Problem

Show That

$$\begin{pmatrix}1 & 2 & 3\\0 & 1 & 0\\1 & 1 & 0\end{pmatrix}\begin{pmatrix}-1 & 1 & 0\\-1 & 1 & 0\\2 & 3 & 4\end{pmatrix} \neq \begin{pmatrix}-1 & 1 & 0\\-1 & 1 & 0\\2 & 3 & 4\end{pmatrix}\begin{pmatrix}1 & 2 & 3\\0 & 1 & 0\\1 & 1 & 0\end{pmatrix}$$

2 Explanation

· If Matrices $\mathbf{M}$ and $\mathbf{N}$ are rectangular then we can directly say that multiplication of these matrices can't be commutative.

· If Matrices $\mathbf{M}$ and $\mathbf{N}$ are square matrices then they are generally not commutative because:-

1. While computing $\mathbf{MN}$, row element of $\mathbf{M}$ will be Multiplied with columns element of $\mathbf{N}$ to find the elements of resultant matrix.

2. While computing $\mathbf{NM}$, rows elements of $\mathbf{N}$ will be multiplied with column elements of $\mathbf{M}$ and then summed to find the elements of resultant matrix.

Note:There are certain special cases in which matrix multiplication becomes commutative such as:
1. If one of the matrix is $\mathbf{I}$.

2. If one matrix is scalar multiple of other.

3. If one matrix is power of another.

3 Solution

Let's name the ,matrices as:-

$$\mathbf{M} = \begin{pmatrix}1 & 2 & 3\\0 & 1 & 0\\1 & 1 & 0\end{pmatrix} \text{ and } \mathbf{N} = \begin{pmatrix}-1 & 1 & 0\\-1 & 1 & 0\\2 & 3 & 4\end{pmatrix}$$

To prove that multiplication is non commutative we have to show that

$$\mathbf{MN} \neq \mathbf{NM} \tag{3.0.1}$$

Solving L.H.S of Equation 3.0.1

$$\mathbf{MN} = \begin{pmatrix}1 & 2 & 3\\0 & 1 & 0\\1 & 1 & 0\end{pmatrix}\begin{pmatrix}-1 & 1 & 0\\-1 & 1 & 0\\2 & 3 & 4\end{pmatrix} \tag{3.0.2}$$

$$= \begin{pmatrix}1\times-1+2\times-1+3\times2 & 1\times1+2\times1+3\times3 & 1\times0+2\times0+3\times4\\0\times-1+1\times-1+0\times2 & 0\times1+1\times1+0\times3 & 0\times0+1\times0+0\times4\\1\times-1+1\times-1+0\times2 & 1\times1+1\times1+0\times3 & 1\times0+1\times0+0\times4\end{pmatrix} \tag{3.0.3}$$

$$\mathbf{MN} = \begin{pmatrix}3 & 12 & 12\\-1 & 1 & 0\\-2 & 2 & 0\end{pmatrix} \tag{3.0.4}$$

Solving R.H.S of Equation 3.0.1

$$\mathbf{NM} = \begin{pmatrix}-1 & 1 & 0\\-1 & 1 & 0\\2 & 3 & 4\end{pmatrix}\begin{pmatrix}1 & 2 & 3\\0 & 1 & 0\\1 & 1 & 0\end{pmatrix} \tag{3.0.5}$$

$$= \begin{pmatrix}-1\times1+1\times0+0\times1 & -1\times2+1\times1+0\times1 & -1\times3+1\times0+0\times0\\-1\times1+1\times0+0\times1 & -1\times2+1\times1+0\times1 & -1\times3+1\times0+0\times0\\2\times1+3\times0+4\times1 & 2\times2+3\times1+4\times1 & 2\times3+3\times0+4\times0\\ & & \end{pmatrix} \tag{3.0.6}$$

$$\mathbf{NM} = \begin{pmatrix}-1 & -1 & -3\\-1 & -1 & -3\\6 & 11 & 6\end{pmatrix} \tag{3.0.7}$$

From Equations 3.0.4 and 3.0.7 we can clearly see that R.H.S $\neq$ L.H.S and Hence, Matrix multiplication is non commutative