Assignment 19

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The link to the solution is

https://github.com/Adarsh1310/EE5609

***Abstract*—This documents solves a problem based on Linear Transformation.**

1 Problem

Let T be the linear transformation from $\mathbb{R}^3$ into $\mathbb{R}^2$ defined by,

$$T\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix} = \begin{pmatrix}x_1+x_2\\2x_3-x_1\end{pmatrix}$$

If β is the standard ordered basis for $\mathbb{R}^3$ and $\beta^{'}$ is the standard ordered basis for $\mathbb{R}^2$, what is the matrix of T relative to the pair β, $\beta^{'}$

2 Solution

We know that,

$$[T\alpha]_{\beta'} = \mathbf{A}[\alpha]_\beta \tag{2.0.1}$$

where $\mathbf{A}$ is called the matrix of T relative to ordered basis β, $\beta^{'}$ and α is any vector in the space formed using basis vectors β. Using the ordered basis,

$$\beta = \begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix} \tag{2.0.2}$$

$$\beta^{'} = \begin{pmatrix}1&0\\0&1\end{pmatrix} \tag{2.0.3}$$

Now, Let's consider the equation given in the question,

$$T\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix} = \begin{pmatrix}x_1+x_2\\2x_3-x_1\end{pmatrix} \tag{2.0.4}$$

R.H.S of the equation can be written as a product of 2×3 and 3×1 matrices,

$$= \begin{pmatrix}1&1&0\\-1&0&2\end{pmatrix}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix} \tag{2.0.5}$$

Hence the transformation matrix is,

$$\begin{pmatrix}1&1&0\\-1&0&2\end{pmatrix} \tag{2.0.6}$$

Now, since the transformation has to be found relative to the pair β, $\beta^{'}$ we should row reduce,

$$\left(\begin{array}{cc|ccc}1&0&1&1&0\\0&1&-1&0&2\end{array}\right) \tag{2.0.7}$$

but from here we can see that $\beta^{'}$ is already an identity matrix and hence no row reduction is required. So by using (2.0.1) we can say that,

$$[T\alpha]_{\beta'} = \begin{pmatrix}1&1&0\\-1&0&2\end{pmatrix}[\alpha]_\beta \tag{2.0.8}$$