Shadow Assignment

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The link to the solution is

https://github.com/Adarsh1310/EE5609

***Abstract*—This documents finds shadow of an object on a plane.**

1 Problem

Find the shadow of an object on the plane described by the given orthonormal vectors.

2 Solution

Let $\mathbf{O}$ be the plane whose projection has to be obtained and P be the final projection on the given plane.We have been provided with two orthonormal vector, let us call them $\mathbf{a}_1$,$\mathbf{a}_2$ and a direction vector of the light $\mathbf{d}$.Using these vectors we can find the plane as,

$$\mathbf{A} = \begin{pmatrix} \mathbf{a}_1 & \mathbf{a}_2 \end{pmatrix} \tag{2.0.1}$$

Let,

$$\mathbf{e} = \mathbf{O} - \mathbf{P} \tag{2.0.2}$$

This vector will be perpendicular to the $\mathbf{P}$ and opposite to the direction of $\mathbf{d}$.
Now,$\mathbf{P}$ is just some linear combinations of $\mathbf{a}_1$, $\mathbf{a}_2$ and can be written as

$$\mathbf{P} = x_1\mathbf{a}_1 + x_2\mathbf{a}_2 \tag{2.0.3}$$
$$= \mathbf{Ax} \tag{2.0.4}$$

Our goal is to find $\mathbf{x}$.We know that the vector $\mathbf{e}$ obtained in (2.0.2) is perpendicular to the planes.Hence,

$$\mathbf{a}_1{}^T(\mathbf{O} - \mathbf{Ax}) = 0 \tag{2.0.5}$$
$$\mathbf{a}_2{}^T(\mathbf{O} - \mathbf{Ax}) = 0 \tag{2.0.6}$$
$$\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}(\mathbf{O} - \mathbf{Ax}) = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{2.0.7}$$
$$\mathbf{A}^T(\mathbf{O} - \mathbf{Ax}) = 0 \tag{2.0.8}$$

Form (2.0.8) we can see that $\mathbf{e}$ is in the nullspace of $\mathbf{A}^T$ which means that $\mathbf{e}$ is perpendicular to column space of $\mathbf{A}^T$.

$$\mathbf{A}^T\mathbf{Ax} = \mathbf{A}^T\mathbf{O} \tag{2.0.9}$$

From equation (2.0.8) we can find the value of x

$$\mathbf{x} = (\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T\mathbf{O} \tag{2.0.10}$$

Substituting (2.0.10) in (2.0.4)

$$\mathbf{P} = \mathbf{A}(\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T\mathbf{O} \tag{2.0.11}$$

Hence, We can find the projection vector by multiplying $\mathbf{A}(\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T$ to the given space $\mathbf{O}$