

# Assignment 14

Adarsh Srivastava

The link to the solution is

<https://github.com/Adarsh1310/EE5609>

**Abstract**—This documents solves a problem based on vector spaces.

## 1 PROBLEM

Let  $\mathbb{V}$  be the set of all complex-valued functions  $f$  on the real line such that

$$f(-t) = \overline{f(t)}$$

The bar denotes complex conjugation. Show that  $\mathbb{V}$ , with the operations

$$(f + g)(t) = f(t) + g(t)$$

$$(cf)(t) = cf(t)$$

is a vector space over the field of real numbers. Give an example of a function in  $\mathbb{V}$  which is not real valued.

## 2 SOLUTION

Let's start by showing that scalar multiplication and vector addition is defined on set  $\mathbb{V}$ . Let's take  $c \in \mathbb{R}$

$$\implies (cf)(-t) \quad (2.0.1)$$

$$= cf(-t) \quad (2.0.2)$$

$$= c\overline{f(t)} \quad (2.0.3)$$

$$= \overline{cf(t)} \quad (2.0.4)$$

Now for vector addition, Let's take  $f(-t)=f(t)$  and  $g(-t)=g(t)$  then  $(f+g)$  should also show the property  $(f+g)(-t)=(f+g)(t)$

$$\implies (f + g)(-t) \quad (2.0.5)$$

$$= f(-t) + g(-t) \quad (2.0.6)$$

$$= \overline{f(t)} + \overline{g(t)} \quad (2.0.7)$$

$$= \overline{f(t) + g(t)} \quad (2.0.8)$$

Hence both scalar multiplication and vector addition hold true. Now we have to prove that the functions  $\in$

$\mathbb{V}$  hold the property for additivity and homogeneity. So, we have to prove that  $(cf+g)(t)$  is equal to  $cf(t)+g(t)$ .

$$(cf + g)(t) \quad (2.0.9)$$

$$= (cf)(t) + g(t) \quad (2.0.10)$$

$$= cf(t) + g(t) \quad (2.0.11)$$

## 3 EXAMPLE

Let's take  $f(x)=a+ix$

$$f(1) = a + i \quad (3.0.1)$$

Hence,  $f(x)$  is not real valued. Now,

$$f(x) = a + ix \quad (3.0.2)$$

$$f(-x) = a - ix \quad (3.0.3)$$

$$f(-x) = \overline{f(x)} \quad (3.0.4)$$

Since  $a, b$  and  $x \in \mathbb{R}$ , so  $f \in \mathbb{V}$