Assignment 6

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***Abstract*—This document solves a question based on triangle.**

All the codes for the figure in this document can be found at

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https://github.com/Adarsh1310/EE5609/tree/
    master/Assignment_6
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1 Problem

$\triangle ABC$ is an isosceles triangle in which altitudes BE and CF are drawn to equal sides AC and AB respectively. Show that these altitudes are equal.

2 Solution

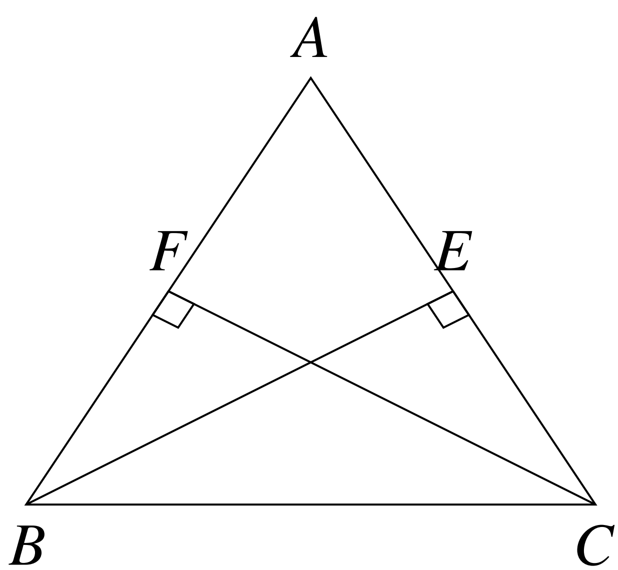

Fig. 1: Isosceles Triangle with altitudes drawn to equal sides

Let $\mathbf{m}_{AC}$ and $\mathbf{m}_{BE}$ be direction vector of side AC and altitude BE respectively.

$$\mathbf{m}_{AC} = \mathbf{A} - \mathbf{C} \tag{2.0.1}$$
$$\mathbf{m}_{BE} = \mathbf{B} - \mathbf{E} \tag{2.0.2}$$

Here, BE $\perp$ AC because BE is the altitude to side AC.So,

$$\mathbf{m}_{AC}^T\mathbf{m}_{BE} = 0 \tag{2.0.3}$$
$$(\mathbf{A} - \mathbf{C})^T(\mathbf{B} - \mathbf{E}) = 0 \tag{2.0.4}$$
$$(\mathbf{A} - \mathbf{E} + \mathbf{E} - \mathbf{B} + \mathbf{B} - \mathbf{C})^T(\mathbf{B} - \mathbf{E}) = 0 \tag{2.0.5}$$
$$(\mathbf{A} - \mathbf{E})^T(\mathbf{B} - \mathbf{E}) + ||\mathbf{B} - \mathbf{E}||^2 + (\mathbf{B} - \mathbf{C})^T(\mathbf{B} - \mathbf{E}) = 0 \tag{2.0.6}$$
$$||\mathbf{B} - \mathbf{E}||^2 + (\mathbf{B} - \mathbf{C})^T(\mathbf{B} - \mathbf{E}) = 0 \tag{2.0.7}$$

Let $\mathbf{m}_{AB}$ and $\mathbf{m}_{CF}$ be direction vector of side AB and altitude CF respectively.

$$\mathbf{m}_{AB} = \mathbf{A} - \mathbf{B} \tag{2.0.8}$$
$$\mathbf{m}_{CF} = \mathbf{C} - \mathbf{F} \tag{2.0.9}$$

Here, CF $\perp$ AB because CF is the altitude to side AB.So,

$$\mathbf{m}_{AB}^T\mathbf{m}_{CF} = 0 \tag{2.0.10}$$
$$(\mathbf{A} - \mathbf{B})^T(\mathbf{C} - \mathbf{F}) = 0 \tag{2.0.11}$$
$$(\mathbf{A} - \mathbf{F} + \mathbf{F} - \mathbf{C} + \mathbf{C} - \mathbf{B})^T(\mathbf{C} - \mathbf{F}) = 0 \tag{2.0.12}$$
$$(\mathbf{A} - \mathbf{F})^T(\mathbf{C} - \mathbf{F}) + ||\mathbf{C} - \mathbf{F}||^2 + (\mathbf{C} - \mathbf{B})^T(\mathbf{C} - \mathbf{F}) = 0 \tag{2.0.13}$$
$$||\mathbf{C} - \mathbf{F}||^2 + (\mathbf{C} - \mathbf{B})^T(\mathbf{C} - \mathbf{F}) = 0 \tag{2.0.14}$$

Comparing equation (2.0.7) and (2.0.14)

$$||\mathbf{C} - \mathbf{F}||^2 + (\mathbf{C} - \mathbf{B})^T(\mathbf{C} - \mathbf{F}) = ||\mathbf{B} - \mathbf{E}||^2 + (\mathbf{B} - \mathbf{C})^T(\mathbf{B} - \mathbf{E}) \tag{2.0.15}$$

$$||\mathbf{C} - \mathbf{F}||^2 + (\mathbf{C} - \mathbf{B})^T(\mathbf{C} - \mathbf{A} + \mathbf{A} - \mathbf{F}) = ||\mathbf{B} - \mathbf{E}||^2 + (\mathbf{B} - \mathbf{C})^T(\mathbf{B} - \mathbf{A} + \mathbf{A} - \mathbf{E}) \tag{2.0.16}$$

$$||\mathbf{C} - \mathbf{F}||^2 + (\mathbf{C} - \mathbf{B})^T(\mathbf{C} - \mathbf{A}) + (\mathbf{C} - \mathbf{B})^T(\mathbf{A} - \mathbf{F}) = ||\mathbf{B} - \mathbf{E}||^2 + (\mathbf{B} - \mathbf{C})^T(\mathbf{B} - \mathbf{A}) + (\mathbf{B} - \mathbf{C})^T(\mathbf{A} - \mathbf{E}) \tag{2.0.17}$$

$$\|\mathbf{C}-\mathbf{F}\|^2 + 2(\mathbf{C}-\mathbf{B})^T(\mathbf{C}-\mathbf{A}) = \\ \|\mathbf{B}-\mathbf{E}\|^2 + 2(\mathbf{B}-\mathbf{C})^T(\mathbf{B}-\mathbf{A}) \quad (2.0.18)$$

$$\|\mathbf{C}-\mathbf{F}\|^2 + 2(\|\mathbf{C}-\mathbf{B}\|\,\|\mathbf{C}-\mathbf{A}\|)\cos\theta = \\ \|\mathbf{B}-\mathbf{E}\|^2 + 2(\|\mathbf{B}-\mathbf{C}\|\,\|\mathbf{B}-\mathbf{A}\|)\cos\theta \quad (2.0.19)$$

$$\|\mathbf{C}-\mathbf{F}\|^2 = \|\mathbf{B}-\mathbf{E}\|^2 \quad (2.0.20)$$

$$\|\mathbf{C}-\mathbf{F}\| = \|\mathbf{B}-\mathbf{E}\| \quad (2.0.21)$$

Hence, the altitudes drawn to equal sides of isosceles triangle is equal.