

Assignment 21

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The link to the solution is

<https://github.com/Adarsh1310/EE5609>

Abstract—This documents solves a problem based on Polynomial of Linear Transformation.

Hence the polynomial $f(\mathbf{A})$ can be written using the characteristic function of \mathbf{A} as follows,

$$f(\mathbf{A}) = -\mathbf{A}^3 + 2\mathbf{I} \quad (2.0.10)$$

$$= 2\mathbf{I} - \mathbf{A} + 2\mathbf{I} \quad (2.0.11)$$

$$= \mathbf{A} \quad (2.0.12)$$

1 PROBLEM

Let T be the linear operator on \mathbb{R}^3 defined by

$$T(x_1, x_2, x_3) = (x_1, x_3, -2x_2 - x_3) \quad (1.0.1)$$

Let f be the polynomial over \mathbb{R} defined by $f = -x^3 + 2$

Hence,

$$f(T)(\mathbf{x}) = \mathbf{A}\mathbf{x} \quad (2.0.13)$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -2 & -1 \end{pmatrix} \mathbf{x} \quad (2.0.14)$$

2 SOLUTION

The given transformation can be written as,

$$T(\mathbf{x}) = \mathbf{A}\mathbf{x} \quad (2.0.1)$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -2 & -1 \end{pmatrix} \mathbf{x} \quad (2.0.2)$$

Hence,

$$\mathbf{A} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -2 & -1 \end{pmatrix} \quad (2.0.3)$$

Now the characteristic equation of \mathbf{A} is given by,

$$\det(\mathbf{A} - \lambda\mathbf{I}) = 0 \quad (2.0.4)$$

$$= \begin{vmatrix} 1-\lambda & 0 & 0 \\ 0 & -\lambda & 1 \\ 0 & -2 & -1-\lambda \end{vmatrix} \quad (2.0.5)$$

$$\implies (1-\lambda)(\lambda^2 + \lambda + 2) = 0 \quad (2.0.6)$$

Where \mathbf{I} is 3×3 Identity matrix. Now using Cayley Hamilton Theorem we get from (2.0.6) the following,

$$(\mathbf{I} - \mathbf{A})(\mathbf{A}^2 + \mathbf{A} + 2\mathbf{I}) = 0 \quad (2.0.7)$$

$$\mathbf{A}^2 + \mathbf{A} + 2\mathbf{I} - \mathbf{A}^3 - \mathbf{A}^2 - 2\mathbf{A} = 0 \quad (2.0.8)$$

$$\mathbf{A}^3 = 2\mathbf{I} - \mathbf{A} \quad (2.0.9)$$