

Assignment 5

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The link to the solution is

<https://github.com/Adarsh1310/EE5609>

Abstract—This documents solves a problem based on circles.

1 PROBLEM

Find the area of the region bounded by the circle $\mathbf{x}^T \mathbf{x} = 4$ and $\left\| \mathbf{x} - \begin{pmatrix} 2 \\ 0 \end{pmatrix} \right\| = 2$.

2 SOLUTION

$$\mathbf{x}^T \mathbf{x} + 2\mathbf{u}^T \mathbf{x} + f = 0$$

So from above equation we can say that,

2.1 Circle 1

Taking equation of the first circle to be,

$$\|\mathbf{x}\|^2 + 2\mathbf{u}_1^T \mathbf{x} + f_1 = 0 \quad (2.1.1)$$

$$\mathbf{x}^T \mathbf{x} - 4 = 0 \text{ (given)} \quad (2.1.2)$$

$$\mathbf{u}_1 = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (2.1.3)$$

$$f_1 = -4 \quad (2.1.4)$$

2.2 Circle 2

Taking equation of the second circle to be,

$$\left\| \mathbf{x} - \begin{pmatrix} 2 \\ 0 \end{pmatrix} \right\|^2 = 2^2 \text{ (given)} \quad (2.2.1)$$

$$\mathbf{x}^T \mathbf{x} + 2\mathbf{u}_2^T \mathbf{x} = 0 \quad (2.2.2)$$

$$\mathbf{u}_2 = \begin{pmatrix} -2 \\ 0 \end{pmatrix} \quad (2.2.3)$$

$$f_2 = 0 \quad (2.2.4)$$

Now, Subtracting equation (2.2.2) from (2.1.2) We get,

$$\mathbf{x}^T \mathbf{x} - 2\mathbf{u}_2^T \mathbf{x} + f_1 - \mathbf{x}^T \mathbf{x} = 0 \quad (2.2.5)$$

$$2\mathbf{u}^T \mathbf{x} = -4 \quad (2.2.6)$$

$$\begin{pmatrix} -4 & 0 \end{pmatrix} \mathbf{x} = -4 \quad (2.2.7)$$

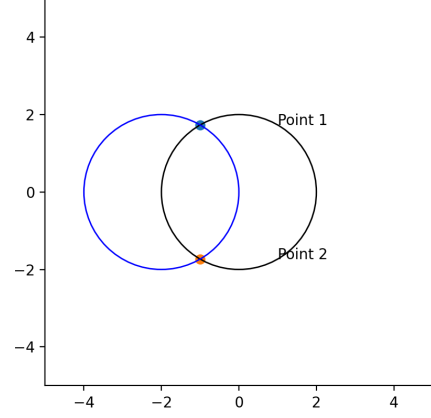


Fig. 0: Figure depicting intersection points of circle

Which can be written as:-

$$\begin{pmatrix} -1 & 0 \end{pmatrix} \mathbf{x} = -1 \quad (2.2.8)$$

$$\mathbf{x} = \lambda \mathbf{m}, \text{ where } \mathbf{m} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (2.2.9)$$

$$\mathbf{x} = \lambda \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (2.2.10)$$

Substituting (2.2.9) in (2.2.2)

$$\mathbf{x}^T \mathbf{x} + 2\mathbf{u}_2^T \mathbf{x} = 0 \quad (2.2.11)$$

$$(\lambda \mathbf{m})^T (\lambda \mathbf{m}) + 2 * \mathbf{u}^T \lambda \mathbf{m} = 0 \quad (2.2.12)$$

$$(2.2.13)$$

Substituting the value of λ, \mathbf{u} and solving

$$\lambda^2 - 4\lambda = 0 \quad (2.2.14)$$

$$\lambda = 0, 4 \quad (2.2.15)$$