Assignment 14

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The link to the solution is

https://github.com/Adarsh1310/EE5609

***Abstract*—This documents solves a problem based on vector spaces.**

1 Problem

Let $\mathbb{V}$ be the set of all complex-valued functions f on the real line such that

$$f(-t) = \overline{f(t)}$$

The bar denotes complex conjugation.Show that $\mathbb{V}$,with the operations

$$(f+g)(t) = f(t) + g(t)$$
$$(cf)(t) = cf(t)$$

is a vector space over the field of real numbers. Give an example of a function in V which is not real valued.

2 Solution

To prove that $\mathbb{V}$ with the given operations is a vector space over the field of real numbers, we have to start by proving that additivity and homogeneity both hold true. So, we have to prove that (cf+g)(t) is equal to cf(t)+g(t).

$$(cf+g)(t) \tag{2.0.1}$$
$$= (cf)(t) + g(t) \tag{2.0.2}$$
$$= cf(t) + g(t) \tag{2.0.3}$$

Now, we know that f(-t)= $\overline{f(-t)}$ and so (cf+g)(t) should also satisfy the property,

$$(cf+g)(-t) \tag{2.0.4}$$
$$= cf(-t) + g(-t) \tag{2.0.5}$$
$$= c\overline{f(t)} + \overline{g(t)} \tag{2.0.6}$$
$$= \overline{cf(t) + g(t)} \tag{2.0.7}$$
$$= \overline{(cf+g)(t)} \tag{2.0.8}$$

Now,for vector addition,

1) Since addition in $\mathbb{V}$ is commutative
$$f(t) + g(t) = g(t) + f(t) \tag{2.0.9}$$
for all t, so the functions f+g and g+f are equal.
2) Since addition in $\mathbb{V}$ is associative
$$(f(t) + g(t)) + a(t) = f(t) + (g(t) + a(t)) \tag{2.0.10}$$
for all t, so the functions f+(g+a) and (f+g)+a are equal.
3) The unique zero vector is the zero function which assigns to each element, the scalar 0 in $\mathbb{V}$.
4) For each f in $\mathbb{V}$,(-f) is the function which is given by (-f)(s)=-f(s).

Now, for scalar multiplication

1) Since scalar multiplication is associative
$$a(bf(t)) = ab(f(t)) \tag{2.0.11}$$
for all a and b, the functions (ab)f and a(bf) are equal.
2) Since scalar multiplication is distributive
$$a(f(t) + g(t)) = af(t) + ag(t) \tag{2.0.12}$$
for all a, the functions a(f+g)=af+ag.
3) Since scalar addition is distributive
$$(a+b)f(t) = af(t) + bf(t) \tag{2.0.13}$$
for all a and b, the functions (a+b)f=af+bf.
4) The unique scalar constant 1 which multiplied to any vector in $\mathbb{V}$ will return the same vector.

3 Example

Let's take f(x)=a+ix

$$f(1) = a + i \tag{3.0.1}$$

Hence, f(x) is not real valued.Now,

$$f(x) = a + ix \tag{3.0.2}$$
$$f(-x) = a - ix \tag{3.0.3}$$
$$f(-x) = \overline{f(x)} \tag{3.0.4}$$

Since a,b and $x \in \mathbb{R}$, so $f \in \mathbb{V}$