

# Assignment 17

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The link to the solution is

<https://github.com/Adarsh1310/EE5609>

**Abstract**—This documents solves a problem based on basis and dimensions.

## 1 PROBLEM

Let  $\mathbf{V}$  be the real vector space of all polynomial functions from  $\mathbb{R}$  to  $\mathbb{R}$  of degree 2 or less, i.e, the space of all functions  $f$  of the form,

$$f(x) = c_0 + c_1x + c_2x^2$$

Let  $t$  be a fixed real number and define

$$g_1(x) = 1, g_2(x) = x + t, g_3(x) = (x + t)^2$$

Prove that  $\beta = \{g_1, g_2, g_3\}$  is a basis for  $\mathbf{V}$ . If

$$f(x) = c_0 + c_1x + c_2x^2$$

what are the coordinates of  $f$  in the ordered basis  $\beta$

## 2 SOLUTION

Let's start by proving that  $\{g_1, g_2, g_3\}$  are linearly independent,

$$\begin{pmatrix} a & b & c \end{pmatrix} \begin{pmatrix} g_1 \\ g_2 \\ g_3 \end{pmatrix} = 0 \quad (2.0.1)$$

$$\begin{pmatrix} a & b & c \end{pmatrix} \begin{pmatrix} 1 \\ x + t \\ (x + t)^2 \end{pmatrix} = 0 \quad (2.0.2)$$

$$\begin{pmatrix} a & b & c \end{pmatrix} \left( \begin{pmatrix} 1 \\ x \\ x^2 \end{pmatrix} + \begin{pmatrix} 0 \\ t \\ 2t \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ t^2 \end{pmatrix} \right) = 0 \quad (2.0.3)$$

$$(2.0.4)$$

We know that  $\begin{pmatrix} 1 \\ x \\ x^2 \end{pmatrix}$  is independent hence the values of  $a, b$  and  $c$  will be 0. Hence  $\{g_1, g_2, g_3\}$  are linearly

independent. Now, equating (2.0.2) and  $f(x)$

$$f(x) = \begin{pmatrix} a & b & c \end{pmatrix} \begin{pmatrix} 1 \\ x + t \\ (x + t)^2 \end{pmatrix} \quad (2.0.5)$$

$$\begin{pmatrix} c_0 & c_1 & c_2 \end{pmatrix} \begin{pmatrix} 1 \\ x \\ x^2 \end{pmatrix} = \begin{pmatrix} a & b & c \end{pmatrix} \begin{pmatrix} 1 \\ x + t \\ (x + t)^2 \end{pmatrix} \quad (2.0.6)$$

$$\begin{pmatrix} c_0 & c_1 & c_2 \end{pmatrix} \begin{pmatrix} 1 \\ x \\ x^2 \end{pmatrix} = \begin{pmatrix} a & b & c \end{pmatrix} \left( \begin{pmatrix} 1 \\ x \\ x^2 \end{pmatrix} + \begin{pmatrix} 0 \\ t \\ 2t \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ t^2 \end{pmatrix} \right) \quad (2.0.7)$$

$$\begin{pmatrix} c_0 & c_1 & c_2 \end{pmatrix} \begin{pmatrix} 1 \\ x \\ x^2 \end{pmatrix} = \begin{pmatrix} a + bt + ct^2 & b + 2ct & c \end{pmatrix} \begin{pmatrix} 1 \\ x \\ x^2 \end{pmatrix} \quad (2.0.8)$$

Hence,

$$c = c_2 \quad (2.0.9)$$

$$b = c_1 - 2c_2t \quad (2.0.10)$$

$$a = c_0 - c_1t + c_2t^2 \quad (2.0.11)$$

So, finally the coordinates of  $f$  in ordered basis of  $\beta$ ,

$$\begin{pmatrix} c_0 - c_1t + c_2t^2 \\ c_1 - 2c_2t \\ c_2 \end{pmatrix} \quad (2.0.12)$$