Assignment 5

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The link to the solution is

https://github.com/Adarsh1310/EE5609

***Abstract*—This documents solves a problem based on circles.**

1 Problem

Find the area of the region bounded by the circle $\mathbf{x}^{\mathbf{T}}$ $\mathbf{x}$= 2 and $\left\|\mathbf{x}-\begin{pmatrix}2\\0\end{pmatrix}\right\|=2.$

2 Solution

$x^T\mathbf{x}-2(O)^T\mathbf{x}+\|\mathbf{O}\|^2-\mathbf{r}^2=0$

So from above equation we can say that,
Circle 1:
Equation of circle 1:

$$\mathbf{x}^{\mathbf{T}}\mathbf{x}=\mathbf{2} \tag{2.0.1}$$

radius=2
point of origin as (0,0)
Circle 2:
Equation of circle 2:

$$\left\|\mathbf{x}-\begin{pmatrix}2\\0\end{pmatrix}\right\|^2 \tag{2.0.2}$$

$$\mathbf{x}^T\mathbf{x}-2\begin{pmatrix}2\\0\end{pmatrix}^T\mathbf{x}+2-4=0 \tag{2.0.3}$$

$$\mathbf{x}^T\mathbf{x}-\begin{pmatrix}4\\0\end{pmatrix}^T\mathbf{x}-2=0 \tag{2.0.4}$$

radius=2
point of origin as (2,0)
Subtracting equation 2.0.4 from 2.0.1
We get,

$$\begin{pmatrix}4\\0\end{pmatrix}^T\mathbf{x}=0 \tag{2.0.5}$$

Now finding points of intersection:

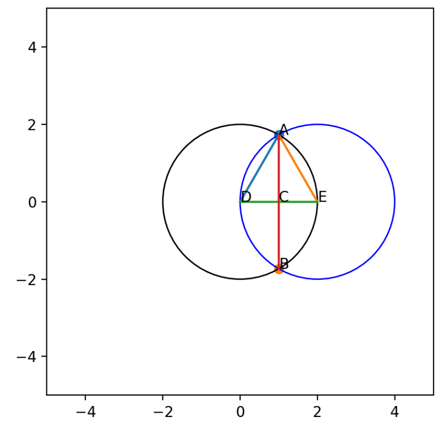

Fig. 0: Figure depicting intersection points of circle

Equation in general form is as follows:

$$x^2+y^2=4 \tag{2.0.6}$$

$$(x-2)^2+y^2=4 \tag{2.0.7}$$

Now Comparing equation 2.0.1 and 2.0.2:

$$(x-2)^2+(4-x^2)=4 \tag{2.0.8}$$

$$x^2+4-2x+4-x^2=4 \tag{2.0.9}$$

x comes out to be 1

Now,Substituting the value of x in equation 2.0.1:

y comes out to be $\sqrt{3}and-\sqrt{3}$

So the points of intersection of the two circles are $(1,\sqrt{3})and(1,-\sqrt{3})$

Now to find the area inclosed between these circles we have to find the integral of these point w.r.t the circles.For this we need to find the area of segment

ABE and double it to find the area of the entire overlapped region.

To find the area of segment ABE we need angle CDA as:
$Here,$
r=2 and $\theta = \angle CDA$
Now we have to find the angle

$\sin\theta = \frac{\mathbf{AC}}{\mathbf{AD}}$
$\theta = 30^\circ$
$\angle ADB = 2 * \theta$
$So, \angle ADB = 60^\circ$

$$Area = \frac{1}{2}(\frac{\angle ADB}{360} - \sin(\angle ADB))r^2 \tag{2.0.10}$$

(Area of Sector-Area of Triangle)

$$Area = \frac{1}{2}(\frac{\pi}{3} - \sin 60)r^2 \tag{2.0.11}$$

$$\tag{2.0.12}$$

$$= \frac{1}{2}(\frac{4\pi}{3} - \sqrt{3}) \tag{2.0.13}$$

$$Totalarea = 2 * Area \tag{2.0.14}$$

$$= \frac{4\pi}{3} - \sqrt{3} \tag{2.0.15}$$