Assignment 22

Adarsh Srivastava

The link to the solution is

https://github.com/Adarsh1310/EE5609

Abstract—This documents solves a problem based on Lagrange Interpolation

1 PROBLEM

Let $\mathbb{F}$ be the field of real numbers,

$$\mathbf{A} = \begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$p = (x-2)(x-3)(x-1)$$

1) Show that $p(\mathbf{A}) = 0$
2) Let P_1, P_2, P_3 be the Lagrange polynomials for $t_1 = 2, t_2 = 3, t_3 = 1$. Compute $E_i = P_i(\mathbf{A})$, i=1,2,3

2 SOLUTION

1) We have been given that,

$$p(x) = (x-2)(x-3)(x-1) \quad (2.0.1)$$

Now to find p(**A**),

$$p(\mathbf{A}) = (\mathbf{A}-2)(\mathbf{A}-3)(\mathbf{A}-1) \quad (2.0.2)$$

$$= \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2 \end{pmatrix} \times$$
$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (2.0.3)$$

$$= \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (2.0.4)$$

$$= \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (2.0.5)$$

Hence we proved that p(**A**)=0
2) Using Lagrange Interpolation,

$$P_1(x) = \frac{(x-3)(x-1)}{(2-3)(2-1)} \quad (2.0.6)$$

$$= -(x-3)(x-1) \quad (2.0.7)$$

$$P_2(x) = \frac{(x-2)(x-1)}{(3-2)(3-1)} \quad (2.0.8)$$

$$= \frac{(x-2)(x-1)}{2} \quad (2.0.9)$$

$$P_3(x) = \frac{(x-2)(x-3)}{(1-2)(1-3)} \quad (2.0.10)$$

$$= \frac{(x-2)(x-3)}{2} \quad (2.0.11)$$

Now,Substituting the value of **A**,

$$P_1(\mathbf{A}) = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{2.0.12}$$

$$= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{2.0.13}$$

$$P_2(\mathbf{A}) = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{2.0.14}$$

$$= \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{2.0.15}$$

$$P_3(\mathbf{A}) = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}\begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2 \end{pmatrix} \tag{2.0.16}$$

$$= \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{2.0.17}$$