

Assignment 8

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The link to the solution is

<https://github.com/Adarsh1310/EE5609>

Abstract—This documents solves a QR decomposition problem.

1 PROBLEM

Find QR decomposition of $\begin{pmatrix} -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{pmatrix}$

2 SOLUTION

Let α and β be transpose of column vectors of the given matrix.

$$\alpha = \begin{pmatrix} -\frac{1}{2} \\ \frac{1}{2} \end{pmatrix} \quad (2.0.1)$$

$$\beta = \begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \end{pmatrix} \quad (2.0.2)$$

We can express these as

$$\alpha = k_1 \mathbf{u}_1 \quad (2.0.3)$$

$$\beta = r_1 \mathbf{u}_1 + k_2 \mathbf{u}_2 \quad (2.0.4)$$

where

$$k_1 = \|\alpha\|, \mathbf{u}_1 = \frac{\alpha}{k_1} \quad (2.0.5)$$

$$r_1 = \frac{\mathbf{u}_1^T \beta}{\|\mathbf{u}_1\|^2} \quad (2.0.6)$$

$$\mathbf{u}_2 = \frac{\beta - r_1 \mathbf{u}_1}{\|\beta - r_1 \mathbf{u}_1\|} \quad (2.0.7)$$

$$k_2 = \mathbf{u}_2^T \beta \quad (2.0.8)$$

From (2.0.3) and (2.0.4),

$$\begin{pmatrix} \alpha & \beta \end{pmatrix} = \begin{pmatrix} \mathbf{u}_1 & \mathbf{u}_2 \end{pmatrix} \begin{pmatrix} k_1 & r_1 \\ 0 & k_2 \end{pmatrix} \quad (2.0.9)$$

$$\begin{pmatrix} \alpha & \beta \end{pmatrix} = \mathbf{Q} \mathbf{R} \quad (2.0.10)$$

From above we can see that \mathbf{R} is an upper triangular matrix and $\mathbf{Q}^T \mathbf{Q} = \mathbf{I}$

$$k_1 = \sqrt{\frac{1}{2}}, \mathbf{u}_1 = \sqrt{2} \begin{pmatrix} -\frac{1}{2} \\ \frac{1}{2} \end{pmatrix}, \quad (2.0.11)$$

$$r_1 = -\sqrt{\frac{1}{2}} \quad (2.0.12)$$

$$\mathbf{u}_2 = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (2.0.13)$$

$$k_2 = 0 \quad (2.0.14)$$

Thus obtained QR decomposition is

$$\begin{pmatrix} -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{pmatrix} = \begin{pmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ 0 & 0 \end{pmatrix} \quad (2.0.15)$$