

Assignment 5

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The link to the solution is

<https://github.com/Adarsh1310/EE5609>

Abstract—This documents solves a problem based on circles.

1 PROBLEM

Find the area of the region bounded by the circle $\mathbf{x}^T \mathbf{x} = 4$ and $\left\| \mathbf{x} - \begin{pmatrix} 2 \\ 0 \end{pmatrix} \right\| = 2$.

2 SOLUTION

$$\mathbf{x}^T \mathbf{x} + 2\mathbf{u}^T \mathbf{x} + f = 0$$

So from above equation we can say that,

2.1 Circle 1

Taking equation of the first circle to be,

$$\mathbf{x}^T \mathbf{x} + 2\mathbf{u}_1^T \mathbf{x} + f_1 = 0 = 0 \quad (2.1.1)$$

$$\mathbf{x}^T \mathbf{x} - 4 = 0 \text{ (given)} \quad (2.1.2)$$

$$\mathbf{u}_1 = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (2.1.3)$$

$$f_1 = -4 \quad (2.1.4)$$

2.2 Circle 2

Taking equation of the second circle to be,

$$\left\| \mathbf{x} - \begin{pmatrix} 2 \\ 0 \end{pmatrix} \right\|^2 = 2^2 \text{ (given)} \quad (2.2.1)$$

$$\mathbf{x}^T \mathbf{x} + 2\mathbf{u}_2^T \mathbf{x} = 0 \quad (2.2.2)$$

$$\mathbf{u}_2 = \begin{pmatrix} -2 \\ 0 \end{pmatrix} \quad (2.2.3)$$

$$f_2 = 0 \quad (2.2.4)$$

Now, Subtracting equation (2.2.2) from (2.1.2) We get,

$$\mathbf{x}^T \mathbf{x} - 2\mathbf{u}_2^T \mathbf{x} + f_1 - \mathbf{x}^T \mathbf{x} = 0 \quad (2.2.5)$$

$$2\mathbf{u}_2^T \mathbf{x} = -4 \quad (2.2.6)$$

$$\begin{pmatrix} -4 & 0 \end{pmatrix} \mathbf{x} = -4 \quad (2.2.7)$$

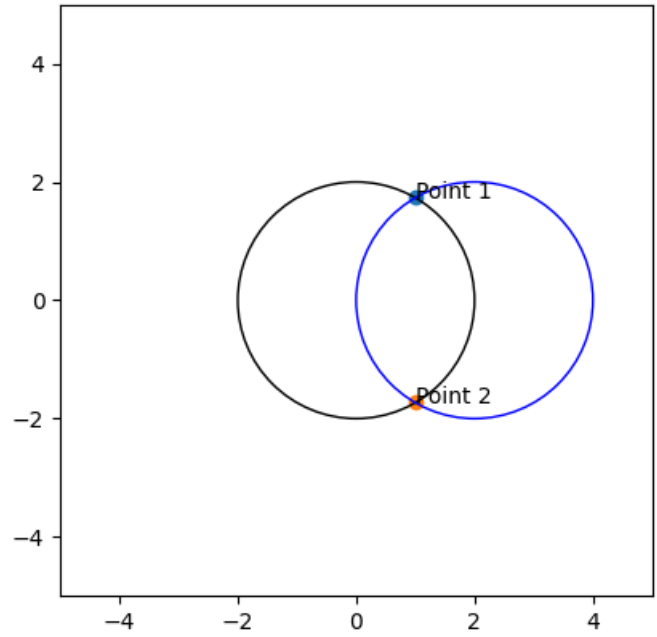


Fig. 0: Figure depicting intersection points of circle

Which can be written as:-

$$\begin{pmatrix} -1 & 0 \end{pmatrix} \mathbf{x} = -1 \quad (2.2.8)$$

$$\begin{pmatrix} -1 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = -1 \text{ (Let } \mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \text{)} \quad (2.2.9)$$

$$-x_1 = -1 \quad (2.2.10)$$

$$x_1 = 1 \quad (2.2.11)$$

$$\mathbf{x} = \begin{pmatrix} 1 \\ x_2 \end{pmatrix} \quad (2.2.12)$$

Substituting (2.2.12) in (2.1.2)

$$\mathbf{x}^T \mathbf{x} - 4 = 0 \quad (2.2.13)$$

$$\|\mathbf{x}\| = 4 \quad (2.2.14)$$

$$1 + x_2^2 = 4 \quad (2.2.15)$$

$$x_2 = +\sqrt{3}, -\sqrt{3} \quad (2.2.16)$$

$$\mathbf{x} = \begin{pmatrix} 1 \\ \sqrt{3} \end{pmatrix}, \begin{pmatrix} 1 \\ -\sqrt{3} \end{pmatrix} \quad (2.2.17)$$

Substituting value of λ in 2.2.10, Points of intersection come out to be $(1, \sqrt{3})$ and $(1, -\sqrt{3})$, Now

finding the direction vector between point of intersection and origin of two circle. Subtracting Point 1 from point 2 direction vector comes out to be $k_1 \begin{pmatrix} 0 \\ \sqrt{3} \end{pmatrix}$ Subtracting both the origin then the vector come out to be $k_2 \begin{pmatrix} -2 \\ 0 \end{pmatrix}$ Now using these to find angle between two vectors by finding inner product:-

$$\mathbf{a} \cdot \mathbf{b} = \|\mathbf{a}\| \|\mathbf{b}\| \cos \theta \quad (2.2.18)$$

$$\frac{0}{\|\mathbf{a}\| \|\mathbf{b}\|} = \cos \theta \quad (2.2.19)$$

$$\theta = 90 \quad (2.2.20)$$

θ gives the angle segment. We have to Double that to find out area.

$$Area = Area of sector - Area of Triangle \quad (2.2.21)$$

$$Area = \frac{\pi \theta}{360} r^2 - \frac{1}{2} 2 \sqrt{3} \quad (2.2.22)$$

$$Total area = 2 * Area \quad (2.2.23)$$

$$= 2\pi - 2 \sqrt{3} \quad (2.2.24)$$