Assignment 6

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***Abstract*—This document solves a question based on triangle.**

All the codes for the figure in this document can be found at

https://github.com/Adarsh1310/EE5609/tree/master/Assignment_6

1 Problem

$\triangle ABC$ is an isosceles triangle in which altitudes **BE** and **CF** are drawn to equal sides **AC** and **AB** respectively. Show that these altitudes are equal.

2 Result used

FB=EC

3 Solution

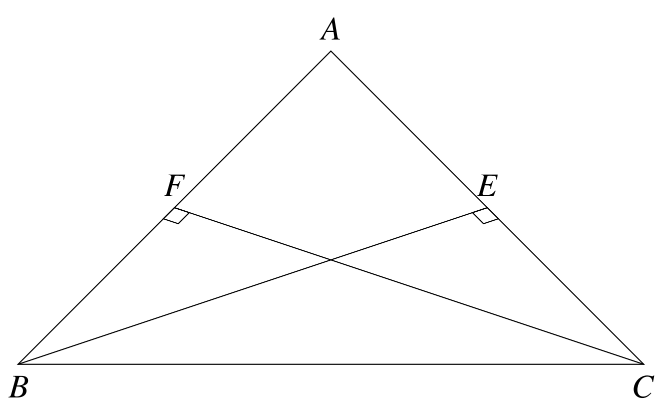

Fig. 1: Isoceles Triangle with BE and CF as altitude

Let $\mathbf{m}_{AC}$ and $\mathbf{m}_{BE}$ be direction vector of side **AC** and altitude **BE** respectively.

$$\mathbf{m}_{AC} = \mathbf{A} - \mathbf{C} \tag{3.0.1}$$
$$\mathbf{m}_{BE} = \mathbf{B} - \mathbf{E} \tag{3.0.2}$$

Here,BE$\perp$ **AC** because **BE** is the altitude to side **AC**.So,

$$\mathbf{m}_{AC}^T\mathbf{m}_{BE} = 0 \tag{3.0.3}$$
$$(\mathbf{A} - \mathbf{C})^T(\mathbf{B} - \mathbf{E}) = 0 \tag{3.0.4}$$
$$\implies (\mathbf{A} - \mathbf{E} + \mathbf{E} - \mathbf{B} + \mathbf{B} - \mathbf{C})^T(\mathbf{B} - \mathbf{E}) = 0 \tag{3.0.5}$$
$$\implies (\mathbf{A} - \mathbf{E})^T(\mathbf{B} - \mathbf{E}) + ||\mathbf{B} - \mathbf{E}|| + (\mathbf{B} - \mathbf{C})^T(\mathbf{B} - \mathbf{E}) = 0 \tag{3.0.6}$$
$$\implies ||\mathbf{B} - \mathbf{E}|| + (\mathbf{B} - \mathbf{C})^T(\mathbf{B} - \mathbf{E}) = 0 \tag{3.0.7}$$

Letm_{AB} and $\mathbf{m}_{CF}$ be direction vector of side **AB** and altitude **CF** respectively.

$$\mathbf{m}_{AB} = \mathbf{A} - \mathbf{B} \tag{3.0.8}$$
$$\mathbf{m}_{CF} = \mathbf{C} - \mathbf{F} \tag{3.0.9}$$

Here, **CF** $\perp$ **AB** because **CF** is the altitude to side **AB**.So,

$$\mathbf{m}_{AB}^T\mathbf{m}_{CF} = 0 \tag{3.0.10}$$
$$(\mathbf{A} - \mathbf{B})^T(\mathbf{C} - \mathbf{F}) = 0 \tag{3.0.11}$$
$$\implies (\mathbf{A} - \mathbf{F} + \mathbf{F} - \mathbf{C} + \mathbf{C} - \mathbf{B})^T(\mathbf{C} - \mathbf{F}) = 0 \tag{3.0.12}$$
$$\implies (\mathbf{A} - \mathbf{F})^T(\mathbf{C} - \mathbf{F}) + ||\mathbf{C} - \mathbf{F}|| + (\mathbf{C} - \mathbf{B})(\mathbf{C} - \mathbf{F}) = 0 \tag{3.0.13}$$
$$\implies ||\mathbf{C} - \mathbf{F}|| + (\mathbf{C} - \mathbf{B})(\mathbf{C} - \mathbf{F}) = 0 \tag{3.0.14}$$

Comparing equation (3.0.14) and (3.0.7)

$$||\mathbf{C} - \mathbf{F}|| + (\mathbf{C} - \mathbf{B})^T(\mathbf{C} - \mathbf{F}) = ||\mathbf{B} - \mathbf{E}|| + (\mathbf{B} - \mathbf{C})^T(\mathbf{B} - \mathbf{E}) \tag{3.0.15}$$

$$||\mathbf{C} - \mathbf{F}|| + ||\mathbf{C} - \mathbf{B}||\,||\mathbf{C} - \mathbf{F}|| \cos\theta = ||\mathbf{B} - \mathbf{E}|| + ||\mathbf{B} - \mathbf{C}||\,||(\mathbf{B} - \mathbf{E})|| \cos\theta \tag{3.0.16}$$

$$\|\mathbf{C}-\mathbf{F}\|\left(1+\|\mathbf{C}-\mathbf{B}\|\cos\theta\right) = \\ \|\mathbf{B}-\mathbf{E}\|\left(1+\|\mathbf{B}-\mathbf{C}\|\cos\theta\right) \quad (3.0.17)$$

$$\|\mathbf{C}-\mathbf{F}\| = \|\mathbf{B}-\mathbf{E}\| \quad (3.0.18)$$

Hence, the altitudes drawn to equal sides of isosceles triangle is equal.