Assignment 6

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***Abstract*—This document solves a question based on triangle.**

All the codes for the figure in this document can be found at

```
https://github.com/Adarsh1310/EE5609/tree/
    master/Assignment_6
```

1 Problem

$\triangle ABC$ is an isosceles triangle in which altitudes **BE** and **CF** are drawn to equal sides **AC** and **AB** respectively. Show that these altitudes are equal.

2 Solution

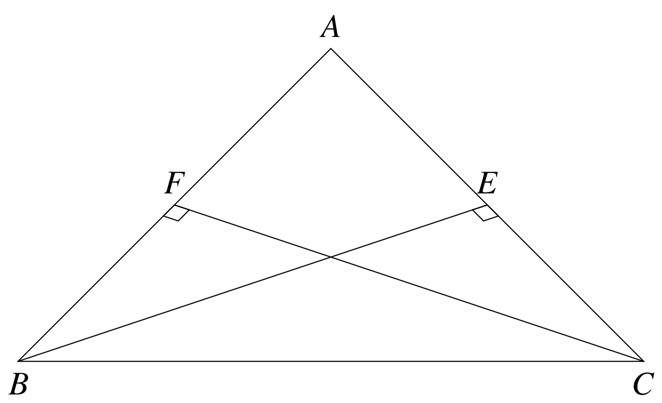

Fig. 1: Isoceles Triangle with BE and CF as altitude

Let $\mathbf{m}_{AC}$ and $\mathbf{m}_{BE}$ be direction vector of side **AC** and altitude **BE** respectively.

$$\mathbf{m}_{AC} = \mathbf{A} - \mathbf{C} \tag{2.0.1}$$

$$\mathbf{m}_{BE} = \mathbf{B} - \mathbf{E} \tag{2.0.2}$$

Here,BE$\perp$ **AC** because **BE** is the altitude to side **AC**.So,

$$\mathbf{m}_{AC}^T\mathbf{m}_{BE} = 0 \tag{2.0.3}$$

$$(\mathbf{A} - \mathbf{C})^T(\mathbf{B} - \mathbf{E}) = 0 \tag{2.0.4}$$

$$(\mathbf{A} - \mathbf{E} + \mathbf{E} - \mathbf{B} + \mathbf{B} - \mathbf{C})^T(\mathbf{B} - \mathbf{E}) = 0 \tag{2.0.5}$$

$$\begin{aligned}(\mathbf{A} - \mathbf{E})^T(\mathbf{B} - \mathbf{E}) + \|\mathbf{B} - \mathbf{E}\|^2 +\\ (\mathbf{B} - \mathbf{C})^T(\mathbf{B} - \mathbf{E}) = 0\end{aligned} \tag{2.0.6}$$

$$\|\mathbf{B} - \mathbf{E}\|^2 + (\mathbf{B} - \mathbf{C})^T(\mathbf{B} - \mathbf{E}) = 0 \tag{2.0.7}$$

Letm_{AB} and $\mathbf{m}_{CF}$ be direction vector of side **AB** and altitude **CF** respectively.

$$\mathbf{m}_{AB} = \mathbf{A} - \mathbf{B} \tag{2.0.8}$$

$$\mathbf{m}_{CF} = \mathbf{C} - \mathbf{F} \tag{2.0.9}$$

Here, **CF** $\perp$ **AB** because **CF** is the altitude to side **AB**.So,

$$\mathbf{m}_{AB}^T\mathbf{m}_{CF} = 0 \tag{2.0.10}$$

$$(\mathbf{A} - \mathbf{B})^T(\mathbf{C} - \mathbf{F}) = 0 \tag{2.0.11}$$

$$(\mathbf{A} - \mathbf{F} + \mathbf{F} - \mathbf{C} + \mathbf{C} - \mathbf{B})^T(\mathbf{C} - \mathbf{F}) = 0 \tag{2.0.12}$$

$$\begin{aligned}(\mathbf{A} - \mathbf{F})^T(\mathbf{C} - \mathbf{F}) + \|\mathbf{C} - \mathbf{F}\|^2 +\\ (\mathbf{C} - \mathbf{B})^T(\mathbf{C} - \mathbf{F}) = 0\end{aligned} \tag{2.0.13}$$

$$\|\mathbf{C} - \mathbf{F}\|^2 + (\mathbf{C} - \mathbf{B})^T(\mathbf{C} - \mathbf{F}) = 0 \tag{2.0.14}$$

Comparing equation (2.0.14) and (2.0.7)

$$\begin{aligned}\|\mathbf{C} - \mathbf{F}\|^2 + (\mathbf{C} - \mathbf{B})^T(\mathbf{C} - \mathbf{F}) =\\ \|\mathbf{B} - \mathbf{E}\|^2 + (\mathbf{B} - \mathbf{C})^T(\mathbf{B} - \mathbf{E})\end{aligned} \tag{2.0.15}$$

$$\begin{aligned}\|\mathbf{C} - \mathbf{F}\|^2 + (\mathbf{C} - \mathbf{B})^T(\mathbf{C} - \mathbf{A} + \mathbf{A} - \mathbf{F}) =\\ \|\mathbf{B} - \mathbf{E}\|^2 + (\mathbf{B} - \mathbf{C})^T(\mathbf{B} - \mathbf{A} + \mathbf{A} - \mathbf{E})\end{aligned} \tag{2.0.16}$$

$$\begin{aligned}\|\mathbf{C} - \mathbf{F}\|^2 + (\mathbf{C} - \mathbf{B})^T(\mathbf{C} - \mathbf{A}) + (\mathbf{C} - \mathbf{B})^T(\mathbf{A} - \mathbf{F}) =\\ \|\mathbf{B} - \mathbf{E}\|^2 + (\mathbf{B} - \mathbf{C})^T(\mathbf{B} - \mathbf{A}) + (\mathbf{B} - \mathbf{C})^T(\mathbf{A} - \mathbf{E})\end{aligned} \tag{2.0.17}$$

$$\|\mathbf{C}-\mathbf{F}\|^2 + 2(\mathbf{C}-\mathbf{B})^T(\mathbf{C}-\mathbf{A}) = \\ \|\mathbf{B}-\mathbf{E}\|^2 + 2(\mathbf{B}-\mathbf{C})^T(\mathbf{B}-\mathbf{A}) \quad (2.0.18)$$

$$\|\mathbf{C}-\mathbf{F}\|^2 + 2(\|\mathbf{C}-\mathbf{B}\|\,\|\mathbf{C}-\mathbf{A}\|)\cos\theta = \\ \|\mathbf{B}-\mathbf{E}\|^2 + 2(\|\mathbf{B}-\mathbf{C}\|\,\|\mathbf{B}-\mathbf{A}\|)\cos\theta \quad (2.0.19)$$

$$\|\mathbf{C}-\mathbf{F}\|^2 = \|\mathbf{B}-\mathbf{E}\|^2 \quad (2.0.20)$$

$$\|\mathbf{C}-\mathbf{F}\| = \|\mathbf{B}-\mathbf{E}\| \quad (2.0.21)$$

Hence, the altitudes drawn to equal sides of isosceles triangle is equal.