Assignment 15

Adarsh Srivastava

The link to the solution is

https://github.com/Adarsh1310/EE5609

***Abstract*—This documents solves a problem based on vector subspaces.**

1 Problem

Let $\mathbf{W}$ be the set of all $(x_1, x_2, x_3, x_4, x_5)$ in $\mathbb{R}^5$ which satisfy

$$2x_1 - x_2 + \frac{4}{3}x_3 - x_4 = 0$$
$$x_1 + \frac{2}{3}x_3 - x_5 = 0$$
$$9x_1 - 3x_2 + 6x_3 - 3x_4 - 3x_5 = 0$$

Find a finite set of vectors which spans $\mathbf{W}$.

2 Solution

The given equations are,

$$2x_1 - x_2 + \frac{4}{3}x_3 - x_4 = 0 \tag{2.0.1}$$
$$x_1 + \frac{2}{3}x_3 - x_5 = 0 \tag{2.0.2}$$
$$9x_1 - 3x_2 + 6x_3 - 3x_4 - 3x_5 = 0 \tag{2.0.3}$$

which can be written as,

$$\begin{pmatrix} 2 & -1 & \frac{4}{3} & -1 & 0 \\ 1 & 0 & \frac{2}{3} & 0 & -1 \\ 9 & -3 & 6 & -3 & -3 \end{pmatrix} \mathbf{x} = 0 \tag{2.0.4}$$

Now, the augmented matrix,

$$\left(\begin{array}{ccccc|c} 2 & -1 & \frac{4}{3} & -1 & 0 & 0 \\ 1 & 0 & \frac{2}{3} & 0 & -1 & 0 \\ 9 & -3 & 6 & -3 & -3 & 0 \end{array}\right) \tag{2.0.5}$$

$$\xleftrightarrow{R_3 = R_3 - 3R_1 - 3R_2} \left(\begin{array}{ccccc|c} 2 & -1 & \frac{4}{3} & -1 & 0 & 0 \\ 1 & 0 & \frac{2}{3} & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right) \tag{2.0.6}$$

$$\xleftrightarrow{R_2 = R_2 - \frac{1}{2}R_1} \left(\begin{array}{ccccc|c} 2 & -1 & \frac{4}{3} & -1 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right) \tag{2.0.7}$$

$$\xleftrightarrow{R_2 = 2R_2} \left(\begin{array}{ccccc|c} 2 & -1 & \frac{4}{3} & -1 & 0 & 0 \\ 0 & 1 & 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right) \tag{2.0.8}$$

$$\xleftrightarrow{R_1 = R_1 + R_2} \left(\begin{array}{ccccc|c} 2 & 0 & \frac{4}{3} & 0 & -2 & 0 \\ 0 & 1 & 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right) \tag{2.0.9}$$

So,

$$2x_1 + \frac{4}{3}x_3 - 2x_5 = 0 \tag{2.0.10}$$
$$x_2 + x_4 - 2x_5 = 0 \tag{2.0.11}$$

Solving the equations we get,

$$x_1 = -\frac{2}{3}x_3 + x_5 \tag{2.0.12}$$
$$x_2 = -x_4 + 2x_5 \tag{2.0.13}$$

which can be written as,

$$\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} \tag{2.0.14}$$

$$= \begin{pmatrix} -\frac{2}{3}x_3 + x_5 \\ -x_4 + 2x_5 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} \tag{2.0.15}$$

$$= x_3 \begin{pmatrix} -\frac{2}{3} \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} + x_4 \begin{pmatrix} 0 \\ -1 \\ 0 \\ 1 \\ 0 \end{pmatrix} + x_5 \begin{pmatrix} 1 \\ 2 \\ 0 \\ 0 \\ 1 \end{pmatrix} \tag{2.0.16}$$

where x_3,x_4 and $x_5 \in \mathbb{R}$.Hence, the vectors $\begin{pmatrix} -\frac{2}{3} \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 0 \\ -1 \\ 0 \\ 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 2 \\ 0 \\ 0 \\ 1 \end{pmatrix}$ will span $\mathbf{W}$