Assignment 14

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The link to the solution is

https://github.com/Adarsh1310/EE5609

***Abstract*—This documents solves a problem based on vector spaces.**

1 Problem

Let $\mathbb{V}$ be the set of all complex-valued functions f on the real line such that

$$f(-t) = \overline{f(t)}$$

The bar denotes complex conjugation.Show that $\mathbb{V}$,with the operations

$$(f+g)(t) = f(t) + g(t)$$
$$(cf)(t) = cf(t)$$

is a vector space over the field of real numbers. Give an example of a function in V which is not real valued.

2 Solution

Let's start by showing that scalar multiplication and vector addition is defined on set $\mathbb{V}$ Let's take c $\in \mathbb{R}$

$$\implies (cf)(-t) \tag{2.0.1}$$
$$= cf(-t) \tag{2.0.2}$$
$$= c\overline{f(t)} \tag{2.0.3}$$
$$= \overline{cf(t)} \tag{2.0.4}$$

Now for vector addition, Let's take f(-t)=f(t) and g(-t)=g(t) then (f+g) should also show the property (f+g)(-t)=(f+g)(t)

$$\implies (f+g)(-t) \tag{2.0.5}$$
$$= f(-t) + g(-t) \tag{2.0.6}$$
$$= \overline{f(t)} + \overline{g(t)} \tag{2.0.7}$$
$$= \overline{f(t) + g(t)} \tag{2.0.8}$$

Hence both scalar multiplication and vector addition hold true.Now we have to prove that the functions $\in \mathbb{V}$ hold the following properties,

1) Addition should be commutative

$$(f+g)(t) = f(t) + g(t) \tag{2.0.9}$$
$$= g(t) + g(t) \tag{2.0.10}$$
$$= (g+f)(t) \tag{2.0.11}$$

2) Addition is associative

$$((f+g)+a)(t) = (f+g)(t) + a(t) \tag{2.0.12}$$
$$= f(t) + g(t) + a(t) \tag{2.0.13}$$
$$= (f(t) + g(t)) + a(t) \tag{2.0.14}$$
$$= f(t) + (g(t) + a(t)) \tag{2.0.15}$$

3) Additive Identity exists

$$(f+0)(t) = f(t) + 0(t) \tag{2.0.16}$$
$$f(t) + 0 \tag{2.0.17}$$
$$f(t) \tag{2.0.18}$$

Zero function is in $\mathbb{V}$ since -0=$\overline{0}$.

4) Additive inverse exists

$$f + (-f) = f(t) + (-f) \tag{2.0.19}$$
$$= f(t) - f(t) \tag{2.0.20}$$
$$= 0 \tag{2.0.21}$$

5) Multiplicative identity exists

$$1.f = f \tag{2.0.22}$$

for all f $\in$ V

6) Scalar multiplication is associative

$$(ab).f = ((ab).f)(t) \tag{2.0.23}$$
$$= a(bf(t)) \tag{2.0.24}$$
$$= a(b.f) \tag{2.0.25}$$
$$= a.(b.f) \tag{2.0.26}$$

a,b $\in$ R

7) Scalar constant is distributive

$$a(f+g) = af + ag \tag{2.0.27}$$

a $\in$ R

8) Scalar addition is distributive

$$(a+b)f = af + bf \tag{2.0.28}$$

a,b $\in$ R

3 Example

Let's take f(x)=a+ix

$$f(1) = a + i \tag{3.0.1}$$

Hence, f(x) is not real valued.Now,

$$f(x) = a + ix \tag{3.0.2}$$

$$f(-x) = a - ix \tag{3.0.3}$$

$$f(-x) = \overline{f(x)} \tag{3.0.4}$$

Since a,b and x $\in \mathbb{R}$, so f $\in \mathbb{V}$