

Assignment 14

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The link to the solution is

<https://github.com/Adarsh1310/EE5609>

Abstract—This documents solves a problem based on vector spaces.

1 PROBLEM

Let \mathbb{V} be the set of all complex-valued functions f on the real line such that

$$f(-t) = \overline{f(t)}$$

The bar denotes complex conjugation. Show that \mathbb{V} , with the operations

$$(f + g)(t) = f(t) + g(t)$$

$$(cf)(t) = cf(t)$$

is a vector space over the field of real numbers. Give an example of a function in \mathbb{V} which is not real valued.

2 SOLUTION

To prove that \mathbb{V} with the given operations is a vector space over the field of real numbers, we have to start by proving that additivity and homogeneity both hold true. So, we have to prove that $(cf+g)(t)$ is equal to $cf(t)+g(t)$.

$$(cf + g)(t) \quad (2.0.1)$$

$$= (cf)(t) + g(t) \quad (2.0.2)$$

$$= cf(t) + g(t) \quad (2.0.3)$$

Now, we know that $f(-t) = \overline{f(t)}$ and so $(cf+g)(t)$ should also satisfy the property,

$$(cf + g)(-t) \quad (2.0.4)$$

$$= cf(-t) + g(-t) \quad (2.0.5)$$

$$= \overline{cf(t)} + \overline{g(t)} \quad (2.0.6)$$

$$= \overline{cf(t) + g(t)} \quad (2.0.7)$$

$$= \overline{(cf + g)(t)} \quad (2.0.8)$$

3 EXAMPLE

Let's take $f(x)=a+ix$

$$f(1) = a + i \quad (3.0.1)$$

Hence, $f(x)$ is not real valued. Now,

$$f(x) = a + ix \quad (3.0.2)$$

$$f(-x) = a - ix \quad (3.0.3)$$

$$f(-x) = \overline{f(x)} \quad (3.0.4)$$

Since a, b and $x \in \mathbb{R}$, so $f \in \mathbb{V}$