Assignment 5

Adarsh Srivastava

The link to the solution is

https://github.com/Adarsh1310/EE5609

***Abstract*—This documents solves a problem based on circles.**

1 Problem

Find the area of the region bounded by the circle $\mathbf{x}^{\mathbf{T}}\ \mathbf{x}= 4$ and $\left\|\mathbf{x}-\begin{pmatrix}2\\0\end{pmatrix}\right\| = 2.$

2 Solution

$\|\mathbf{x}\|^2 + 2\mathbf{u}^T\mathbf{x} + f = 0$
So from above equation we can say that,

2.1 Circle 1

Taking equation of the first circle to be,

$$\|\mathbf{x}\|^2 + 2\mathbf{u}_1^T\mathbf{x} + f_1 = 0 \tag{2.1.1}$$

$$\mathbf{x}^T\mathbf{x} - 4 = 0(\text{given}) \tag{2.1.2}$$

$$\mathbf{u_1} = \begin{pmatrix}0\\0\end{pmatrix} \tag{2.1.3}$$

$$f_1 = -4 \tag{2.1.4}$$

2.2 Circle 2

Taking equation of the second circle to be,

$$\left\|\mathbf{x}-\begin{pmatrix}2\\0\end{pmatrix}\right\| = 2(\text{given}) \tag{2.2.1}$$

$$\mathbf{x}^T\mathbf{x} + 2\mathbf{u_2}^T\mathbf{x} = 0 \tag{2.2.2}$$

$$\mathbf{u_2} = \begin{pmatrix}2\\0\end{pmatrix} \tag{2.2.3}$$

$$f_2 = 0 \tag{2.2.4}$$

Now,Subtracting equation (**??**) from (**??**) We get,

$$\mathbf{x}^T\mathbf{x} - 2\mathbf{u_2}^T\mathbf{x} + f_1 - \mathbf{x}^{\mathbf{T}}\mathbf{x} = \mathbf{0} \tag{2.2.5}$$

$$2\mathbf{u}^T\mathbf{x} = 4 \tag{2.2.6}$$

$$\begin{pmatrix}4 & 0\end{pmatrix}\mathbf{x} = 4(\text{Substituting } (\mathbf{??}) \text{ in } (\mathbf{??})) \tag{2.2.7}$$

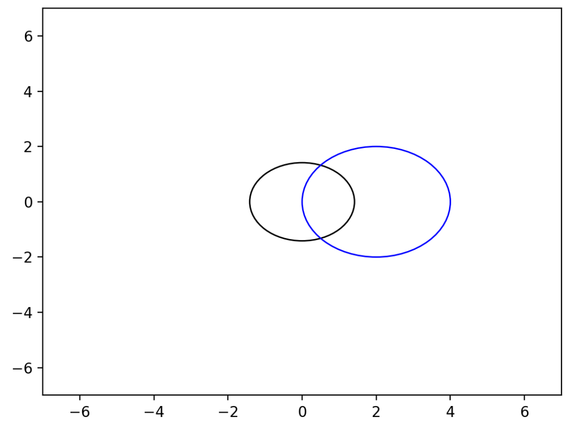

Fig. 0: Figure depicting intersection points of circle

Which can be written as:-

$$\begin{pmatrix}1 & 0\end{pmatrix}\mathbf{x} = 0 \tag{2.2.8}$$

$$\mathbf{x} = \lambda\mathbf{m}, \text{where}\mathbf{m} = \begin{pmatrix}0\\1\end{pmatrix} \tag{2.2.9}$$

$$\mathbf{x} = \lambda\begin{pmatrix}0\\1\end{pmatrix} \tag{2.2.10}$$

Substituting 2.2.9 in 2.2.14

$$\mathbf{x}^T\mathbf{x} - f_2 = 0 \tag{2.2.11}$$

$$\lambda_{\mathbf{2}}\|\mathbf{m}\|\lambda_2\|\mathbf{m}\| - f_2 = 0 \tag{2.2.12}$$

$$\lambda_2^2\|\mathbf{m}\|^2 = f_2(\because f_2 = 0) \tag{2.2.13}$$

$$\lambda_2 = 0 \tag{2.2.14}$$

Now finding direction vector between two lines

$$\tag{2.2.15}$$

Now to find the area enclosed between these circles we have to find the integral of these point w.r.t the circles.For this we need to find the area of segment and double it to find the area of the entire overlapped region. To find the area of segment we need angle segment makes.Here,r= $\sqrt{2}$ and θ=angle of segment.Now we have to find the angle.

$$\sin\theta = \frac{\mathbf{P}}{\mathbf{H}}$$
$$\theta = 90^\circ$$
$$Totalangle = 180^\circ$$

$$Area = \frac{1}{2}(\frac{180}{360}\pi - \sin(180))r^2 \tag{2.2.16}$$

(Area of Sector-Area of Triangle)

$$Area = \frac{1}{2}(\frac{\pi}{2} - \sin 180)2 \tag{2.2.17}$$
$$= 2\frac{1}{2}(\frac{\pi}{2} - \sin 180) \tag{2.2.18}$$
$$Totalarea = 2 * Area \tag{2.2.19}$$
$$= \frac{2\pi - 4}{2} \tag{2.2.20}$$