Assignment 16

Adarsh Srivastava

The link to the solution is

https://github.com/Adarsh1310/EE5609

***Abstract*—This documents solves a problem based on basis and dimensions.**

1 Problem

Let **V** be a vector space over a subfield **F** of complex numbers. Suppose α, β and γ are linearly independent vectors in **V**. Prove that (α+β),(β+γ) and (γ+α) are linearly independent.

2 Solution

Let α, β and γ be three n$\times$ 1 dimensional vectors. We need to prove that,

$$\begin{pmatrix} \alpha+\beta & \beta+\gamma & \gamma+\alpha \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0 \tag{2.0.1}$$

will only have a trivial solution.The above equation can be written as

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}^T \begin{pmatrix} \alpha+\beta & \beta+\gamma & \gamma+\alpha \end{pmatrix}^T = 0 \tag{2.0.2}$$

$$\begin{pmatrix} x_1 & x_2 & x_3 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} \alpha^T \\ \beta^T \\ \gamma^T \end{pmatrix} = 0 \tag{2.0.3}$$

$$\begin{pmatrix} x_1+x_2 & x_1+x_3 & x_2+x_3 \end{pmatrix} \begin{pmatrix} \alpha^T \\ \beta^T \\ \gamma^T \end{pmatrix} = 0 \tag{2.0.4}$$

Since, α, β and γ are independent Hence (2.0.4) will have a trivial solution and so, (α+β),(β+γ) and (γ+α) are linearly independent.