Assignment 5

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The link to the solution is

https://github.com/Adarsh1310/EE5609

***Abstract*—This documents solves a problem based on circles.**

1 Problem

Find the area of the region bounded by the circle $\mathbf{x}^{\mathbf{T}}\ \mathbf{x}= 4$ and $\left\|\mathbf{x}-\begin{pmatrix}2\\0\end{pmatrix}\right\| = 2.$

2 Solution

$\mathrm{x}^T\mathbf{x}+2\mathbf{u}^T\mathbf{x}+f=0$

So from above equation we can say that,

2.1 Circle 1

Taking equation of the first circle to be,

$$\mathbf{x}^T\mathbf{x}+2\mathbf{u_1}^T\mathbf{x}+f_1=0=0 \tag{2.1.1}$$

$$\mathbf{x}^T\mathbf{x}-4=0(\text{given}) \tag{2.1.2}$$

$$\mathbf{u_1}=\begin{pmatrix}0\\0\end{pmatrix} \tag{2.1.3}$$

$$f_1=-4 \tag{2.1.4}$$

2.2 Circle 2

Taking equation of the second circle to be,

$$\left\|\mathbf{x}-\begin{pmatrix}2\\0\end{pmatrix}\right\|^2=2^2(\text{given}) \tag{2.2.1}$$

$$\mathbf{x}^T\mathbf{x}+2\mathbf{u_2}^T\mathbf{x}=0 \tag{2.2.2}$$

$$\mathbf{u_2}=\begin{pmatrix}-2\\0\end{pmatrix} \tag{2.2.3}$$

$$f_2=0 \tag{2.2.4}$$

Now,Subtracting equation (2.2.2) from (2.1.2) We get,

$$\mathbf{x}^T\mathbf{x}-2\mathbf{u_2}^T\mathbf{x}+f_1-\mathbf{x}^T\mathbf{x}=0 \tag{2.2.5}$$

$$2\mathbf{u}^T\mathbf{x}=-4 \tag{2.2.6}$$

$$\begin{pmatrix}-4 & 0\end{pmatrix}\mathbf{x}=-4 \tag{2.2.7}$$

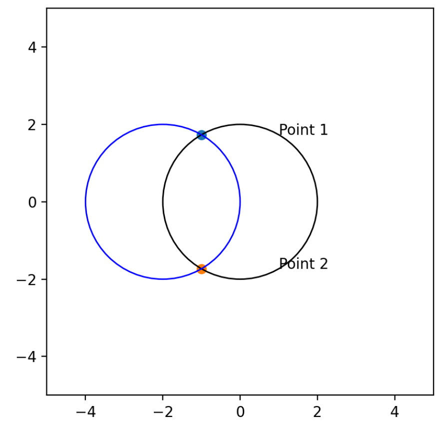

Fig. 0: Figure depicting intersection points of circle

Which can be written as:-

$$\begin{pmatrix}-1 & 0\end{pmatrix}\mathbf{x}=-1 \tag{2.2.8}$$

$$\mathbf{x}=\lambda\mathbf{m}, \text{where}\mathbf{m}=\begin{pmatrix}1\\0\end{pmatrix} \tag{2.2.9}$$

$$\mathbf{x}=\lambda\begin{pmatrix}1\\0\end{pmatrix} \tag{2.2.10}$$

Substituting (2.2.9) in (2.2.2)

$$\mathbf{x}^T\mathbf{x}+2\mathbf{u_2}^T\mathbf{x}=0 \tag{2.2.11}$$

$$(\lambda\mathbf{m})^T(\lambda\mathbf{m})+2*\mathbf{u}^T\lambda\mathbf{m}=0 \tag{2.2.12}$$

$$\tag{2.2.13}$$

Substituting the value of λ,$\mathbf{u}$ and solving

$$\lambda^2-4\lambda=0 \tag{2.2.14}$$

$$\lambda=0,4 \tag{2.2.15}$$