

Assignment 22

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The link to the solution is

<https://github.com/Adarsh1310/EE5609>

Abstract—This documents solves a problem based on Lagrange Interpolation

1 PROBLEM

Let \mathbb{F} be the field of real numbers,

$$\mathbf{A} = \begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$p = (x - 2)(x - 3)(x - 1)$$

- 1) Show that $p(\mathbf{A}) = 0$
- 2) Let P_1, P_2, P_3 be the Lagrange polynomials for $t_1 = 2, t_2 = 3, t_3 = 1$. Compute $E_i = P_i(\mathbf{A})$, $i=1,2,3$

2 SOLUTION

- 1) Since \mathbf{A} is a diagonal matrix, It's characteristic polynomial is,

$$\det(\mathbf{A} - x\mathbf{I}) = 0 \quad (2.0.1)$$

$$f(x) = (x - 2)^2(x - 3)(x - 1) = 0 \quad (2.0.2)$$

From, (2.0.2) and using Cayley Hamilton Theorem,

$$(\mathbf{A} - 2)^2(\mathbf{A} - 3)(\mathbf{A} - 1) = 0 \quad (2.0.3)$$

Now Since $f(\mathbf{A}) = 0$ and also we can also see that $(x-2)(x-3)(x-1)$ is a minimal polynomial, Hence $p(\mathbf{A})=0$.

- 2) Using Lagrange Interpolation,

$$P_1(x) = \frac{(x - 3)(x - 1)}{(2 - 3)(2 - 1)} \quad (2.0.4)$$

$$= -(x - 3)(x - 1) \quad (2.0.5)$$

$$P_2(x) = \frac{(x - 2)(x - 1)}{(3 - 2)(3 - 1)} \quad (2.0.6)$$

$$= \frac{(x - 2)(x - 1)}{2} \quad (2.0.7)$$

$$P_3(x) = \frac{(x - 2)(x - 3)}{(1 - 2)(1 - 3)} \quad (2.0.8)$$

$$= \frac{(x - 2)(x - 3)}{2} \quad (2.0.9)$$

Now,Substituting the value of \mathbf{A} ,

$$P_1(\mathbf{A}) = - \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (2.0.10)$$

$$= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (2.0.11)$$

$$P_2(\mathbf{A}) = \frac{1}{2} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (2.0.12)$$

$$= \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (2.0.13)$$

$$P_3(\mathbf{A}) = \frac{1}{2} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2 \end{pmatrix} \quad (2.0.14)$$

$$= \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (2.0.15)$$