

# Assignment 20

Adarsh Srivastava

The link to the solution is

<https://github.com/Adarsh1310/EE5609>

Consider the following mapping,

$$g = T^t f \quad (2.0.7)$$

**Abstract**—This documents solves a problem based on Linear Transformation.

Now,

$$g(\mathbf{x}) = T^t f(\mathbf{x}) \quad (2.0.8)$$

Using (2.0.3) in (2.0.8),

$$= f(T(\mathbf{x})) \quad (2.0.9)$$

$$= \mathbf{a}^T \mathbf{A} \mathbf{x} \quad (2.0.10)$$

$$= \mathbf{a}^T \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \mathbf{x} \quad (2.0.11)$$

$$= a(x_1 - x_2) + b(x_1 + x_2) \quad (2.0.12)$$

## 1 PROBLEM

Let  $\mathbb{F}$  be a field and let  $f$  be the linear functional on  $\mathbb{F}^2$  defined by,

$$f(x_1, x_2) = ax_1 + bx_2$$

For the linear operator  $T(x_1, x_2) = (x_1 - x_2, x_1 + x_2)$  Let,  $g = T^t y$  and find  $g(x_1, x_2)$

## 2 SOLUTION

The linear functional  $f$  on  $\mathbb{F}^2$  is defined by,

$$f(\mathbf{x}) = \mathbf{a}^T \mathbf{x} \quad \forall \mathbf{x} \in \mathbb{F}^2 \quad (2.0.1)$$

where,

$$\mathbf{a} = \begin{pmatrix} a \\ b \end{pmatrix} \quad (2.0.2)$$

We use the following theorem,

Let  $\mathbb{V}$  and  $\mathbb{W}$  be vector spaces, over the field  $F$ . For each linear transformation  $T : \mathbb{V} \rightarrow \mathbb{W}$ , there is a unique linear transformation  $T^t : \mathbb{W}^* \rightarrow \mathbb{V}^*$  such that,

$$(T^t g)(\alpha) = g(T\alpha) \quad (2.0.3)$$

$\forall (\mathbf{x}) \in \mathbb{F}^2$  the given linear operator  $T$  defined as,

$$T(\mathbf{x}) = \mathbf{A} \mathbf{x} \quad (2.0.4)$$

Where,

$$\mathbf{A} = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \quad (2.0.5)$$

Hence,

$$\mathbf{A} \mathbf{x} = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \mathbf{x} \quad (2.0.6)$$