Assignment 6

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***Abstract*—This document solves a question based on triangle.**

All the codes for the figure in this document can be found at

https://github.com/Adarsh1310/EE5609/tree/master/Assignment_6

1 Problem

$\triangle ABC$ is an isosceles triangle in which altitudes **BE** and **CF** are drawn to equal sides **AC** and **AB** respectively. Show that these altitudes are equal.

2 Result used

FB=EC

3 Solution

$$\mathbf{m}_{AC}^T\mathbf{m}_{BE} = 0 \tag{3.0.1}$$

$$(\mathbf{A}-\mathbf{C})^T(\mathbf{B}-\mathbf{E}) = 0 \tag{3.0.2}$$

$$\implies (\mathbf{A}-\mathbf{F}+\mathbf{F}-\mathbf{C}+\mathbf{C}-\mathbf{B})^T(\mathbf{C}-\mathbf{F}) = 0 \tag{3.0.3}$$

$$\implies (\mathbf{A}-\mathbf{F})^T(\mathbf{C}-\mathbf{F}) + ||\mathbf{C}-\mathbf{F}|| + (\mathbf{C}-\mathbf{B})(\mathbf{C}-\mathbf{F}) = 0 \tag{3.0.4}$$

$$\implies ||\mathbf{C}-\mathbf{F}|| + (\mathbf{C}-\mathbf{B})(\mathbf{C}-\mathbf{F}) = 0 \tag{3.0.5}$$

Let $\mathbf{m}_{AB}$ and $\mathbf{m}_{CF}$ be direction vector of side **AB** and altitude **CF** respectively.

$$\mathbf{m}_{AB} = \mathbf{A}-\mathbf{B} \tag{3.0.6}$$

$$\mathbf{m}_{CF} = \mathbf{C}-\mathbf{F} \tag{3.0.7}$$

Here, $\mathbf{CF} \perp \mathbf{AB}$ because **CF** is the altitude to side **AB**.So,

$$\mathbf{m}_{AB}^T\mathbf{m}_{CF} = 0 \tag{3.0.8}$$

$$(\mathbf{A}-\mathbf{B})^T(\mathbf{C}-\mathbf{F}) = 0 \tag{3.0.9}$$

$$\implies (\mathbf{A}-\mathbf{E}+\mathbf{E}-\mathbf{B}+\mathbf{B}-\mathbf{C})^T(\mathbf{B}-\mathbf{E}) = 0 \tag{3.0.10}$$

$$\implies (\mathbf{A}-\mathbf{E})^T(\mathbf{B}-\mathbf{E}) + ||\mathbf{B}-\mathbf{E}|| + (\mathbf{B}-\mathbf{C})^T(\mathbf{B}-\mathbf{E}) = 0 \tag{3.0.11}$$

$$\implies ||\mathbf{B}-\mathbf{E}|| + (\mathbf{B}-\mathbf{C})^T(\mathbf{B}-\mathbf{E}) = 0 \tag{3.0.12}$$

Comparing equation (3.0.5) and (3.0.12)

$$||\mathbf{C}-\mathbf{F}|| + (\mathbf{C}-\mathbf{B})^T\mathbf{C}-\mathbf{F} = ||\mathbf{B}-\mathbf{E}|| + (\mathbf{B}-\mathbf{C})^T\mathbf{B}-\mathbf{E} \tag{3.0.13}$$

$$||\mathbf{C}-\mathbf{F}|| + ||\mathbf{C}-\mathbf{B}||\,||\mathbf{C}-\mathbf{F}||\cos\theta = ||\mathbf{B}-\mathbf{E}|| + ||\mathbf{B}-\mathbf{C}||\,||(\mathbf{B}-\mathbf{E})||\cos\theta \tag{3.0.14}$$

$$||\mathbf{C}-\mathbf{F}||\,(1 + ||\mathbf{C}-\mathbf{B}||\,||\mathbf{C}-\mathbf{F}||\cos\theta) = ||\mathbf{B}-\mathbf{E}||\,(1 + ||\mathbf{B}-\mathbf{C}||\,||\mathbf{B}-\mathbf{E}||\cos\theta) \tag{3.0.15}$$

$$||\mathbf{C}-\mathbf{F}|| = ||\mathbf{B}-\mathbf{C}|| \tag{3.0.16}$$

Hence, the altitudes drawn to equal sides of isosceles triangle is equal.