Assignment 11

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The link to the solution is

https://github.com/Adarsh1310/EE5609

Abstract—This documents show a method to perform row exchange using elementary row operations.

1 PROBLEM

Prove that the interchange of two rows of a matrix can be accomplished by a finite sequence of elementary row operations of the other two types.

2 SOLUTION

Let us assume an elementary matrix $\mathbf{M}$

$$\mathbf{M} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (2.0.1)$$

Let's exchange row $\mathbf{a}_1$ and $\mathbf{a}_2$.

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (2.0.2)$$

Now, to prove that same matrix can be obtained by row operations of other two types, we will first add row 1 and row 2.

$$\begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (2.0.3)$$

Now, subtract row 1 from row 2.

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (2.0.4)$$

Now adding row 2 to row 1.

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (2.0.5)$$

Now, multiplying row 2 by -1.

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (2.0.6)$$

Hence, (2.0.2) was obtained from other two operations.