Assignment 20

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The link to the solution is

https://github.com/Adarsh1310/EE5609

***Abstract*—This documents solves a problem based on Linear Transformation.**

1 Problem

Let $\mathbb{F}$ be a field and let f be the linear functional on $\mathbb{F}^2$ defined by,

$$f(x_1, x_2) = ax_1 + bx_2$$

For the linear operator $T(x_1, x_2) = (x_1 - x_2, x_1 + x_2)$ Let, $g = T^t y$ and find $g(x_1, x_2)$

2 Solution

The linear functional f on $\mathbb{F}^2$ is defined by,

$$f(\mathbf{x}) = \mathbf{a}^{\mathbf{T}}\mathbf{x} \quad \forall \mathbf{x} \in \mathbb{F}^2 \tag{2.0.1}$$

where,

$$\mathbf{a} = \begin{pmatrix} a \\ b \end{pmatrix} \tag{2.0.2}$$

We use the following theorem,
Let $\mathbb{V}$ and $\mathbb{W}$ be vector spaces, over the field F. For each linear transformation $T : \mathbb{V} \to \mathbb{W}$, there is a unique linear transformation $T^t : \mathbb{W}^* \to \mathbb{V}^*$ such that,

$$(T^t g)(\alpha) = g(T\alpha) \tag{2.0.3}$$

$\forall(\mathbf{x}) \in \mathbb{F}^2$ the given linear operator T defined as,

$$T(\mathbf{x}) = \mathbf{A}\mathbf{x} \tag{2.0.4}$$

Where,

$$\mathbf{A} = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \tag{2.0.5}$$

Hence,

$$\mathbf{A}\mathbf{x} = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}\mathbf{x} \tag{2.0.6}$$

Consider the following mapping,

$$g = T^t f \tag{2.0.7}$$

Now,

$$g(\mathbf{x}) = T^t f(\mathbf{x}) \tag{2.0.8}$$

Using (2.0.3) in (2.0.8),

$$= f(T(\mathbf{x})) \tag{2.0.9}$$

$$= \mathbf{a}^{\mathbf{T}}\mathbf{A}\mathbf{x} \tag{2.0.10}$$

Substituting (2.0.5),

$$= \mathbf{a}^{\mathbf{T}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}\mathbf{x} \tag{2.0.11}$$

$$= a(x_1 - x_2) + b(x_1 + x_2) \tag{2.0.12}$$