

Assignment 18

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The link to the solution is

<https://github.com/Adarsh1310/EE5609>

Abstract—This documents solves a problem based on Linear Transformation.

1 PROBLEM

Describe explicitly the linear transformation T from \mathbb{F}^2 into \mathbb{F}^2 such that $T(\epsilon_1) = (a, b)$, $T(\epsilon_2) = (c, d)$.

2 SOLUTION

We are given a linear transformation,

$$T : \mathbb{F}^2 \rightarrow \mathbb{F}^2 \quad (2.0.1)$$

This means that T takes a two dimensional vector and returns another vector in two dimensional space after performing some linear transformation.

$$T(\epsilon_1) = \begin{pmatrix} a \\ b \end{pmatrix} \quad (2.0.2)$$

$$T(\epsilon_2) = \begin{pmatrix} c \\ d \end{pmatrix} \quad (2.0.3)$$

Now, let's assume ϵ_1 and ϵ_2 as linearly independent. So the linear transformation T for any vector \mathbf{v} in two dimensional space will be,

$$T(\mathbf{v}) = \begin{pmatrix} T(\epsilon_1) & T(\epsilon_2) \end{pmatrix} \begin{pmatrix} k_1 \\ k_2 \end{pmatrix} \quad (2.0.4)$$

$$= \begin{pmatrix} a & c \\ b & d \end{pmatrix} \begin{pmatrix} k_1 \\ k_2 \end{pmatrix} \quad (2.0.5)$$

$$= \mathbf{A}\mathbf{v} \quad (2.0.6)$$

Now, there can be two cases here, transformation of linearly independent vector can be independent or it can be dependent. Considering the first case and (2.0.5) we can say that,

$$\text{Range}(T) = \text{columnspace of } \begin{pmatrix} a & c \\ b & d \end{pmatrix} \quad (2.0.7)$$

Now, considering the case when linear transformation will be linearly dependent,

$$\text{Range}(T) = \text{columnspace of } \begin{pmatrix} a \\ b \end{pmatrix} \quad (2.0.8)$$

Now, considering that vectors ϵ_1 and ϵ_2 itself are linearly dependent. Let $\mathbf{v} = \epsilon_1 + \epsilon_2$

$$T(\mathbf{v}) = T(\epsilon_1) + T(\epsilon_2) \quad (2.0.9)$$

$$= T(\epsilon_1) + T(k\epsilon_1) \quad (2.0.10)$$

$$= (k+1)T(\epsilon_1) \quad (2.0.11)$$

$$= (k+1) \begin{pmatrix} a \\ b \end{pmatrix} \quad (2.0.12)$$

We can see from above equation that when ϵ_1 and ϵ_2 as linearly dependent then the transformation T will be along the line only.

Vectors Independent	Vectors Dependent
$T(\mathbf{v}) = \begin{pmatrix} a & c \\ b & d \end{pmatrix} \begin{pmatrix} k_1 \\ k_2 \end{pmatrix}$	$T(\mathbf{v}) = (k+1) \begin{pmatrix} a \\ b \end{pmatrix}$
Output: On the plane	Output: On the line