

# Assignment 18

Adarsh Srivastava

The link to the solution is

<https://github.com/Adarsh1310/EE5609>

**Abstract**—This documents solves a problem based on Linear Transformation.

## 1 PROBLEM

Describe explicitly the linear transformation  $T$  from  $\mathbb{F}^2$  into  $\mathbb{F}^2$  such that  $T(\epsilon_1) = (a, b)$ ,  $T(\epsilon_2) = (c, d)$ .

## 2 SOLUTION

We are given a linear transformation,

$$T : \mathbb{F}^2 \rightarrow \mathbb{F}^2 \quad (2.0.1)$$

This means that  $T$  takes a two dimensional vector and returns another vector in two dimensional space after performing some linear transformation.

$$T(\epsilon_1) = \begin{pmatrix} a \\ b \end{pmatrix} \quad (2.0.2)$$

$$T(\epsilon_2) = \begin{pmatrix} c \\ d \end{pmatrix} \quad (2.0.3)$$

Now, let's assume  $\epsilon_1$  and  $\epsilon_2$  as linearly independent. So the linear transformation  $T$  for any vector  $\mathbf{v}$  in two dimensional space will be,

$$T(\mathbf{v}) = \begin{pmatrix} T(\epsilon_1) & T(\epsilon_2) \end{pmatrix} \begin{pmatrix} k_1 \\ k_2 \end{pmatrix} \quad (2.0.4)$$

$$= \begin{pmatrix} a & c \\ b & d \end{pmatrix} \begin{pmatrix} k_1 \\ k_2 \end{pmatrix} \quad (2.0.5)$$

$$= \mathbf{A}\mathbf{v} \quad (2.0.6)$$

Now, there can be two cases here, transformation of linearly independent vector can be independent or it can be dependent. Considering the first case and (2.0.5) we can say that,

$$\text{Range}(T) = \text{columnspace of } \begin{pmatrix} a & c \\ b & d \end{pmatrix} \quad (2.0.7)$$

Now, considering the case when linear transformation will be linearly dependent,

$$\text{Range}(T) = \text{columnspace of } \begin{pmatrix} a \\ b \end{pmatrix} \quad (2.0.8)$$

Now, considering that vectors  $\epsilon_1$  and  $\epsilon_2$  itself are linearly dependent. Let  $\mathbf{v} = \epsilon_1 + \epsilon_2$

$$T(\mathbf{v}) = T(\epsilon_1) + T(\epsilon_2) \quad (2.0.9)$$

$$= T(\epsilon_1) + T(k\epsilon_1) \quad (2.0.10)$$

$$= (k+1)T(\epsilon_1) \quad (2.0.11)$$

$$= (k+1) \begin{pmatrix} a \\ b \end{pmatrix} \quad (2.0.12)$$

We can see from above equation that when  $\epsilon_1$  and  $\epsilon_2$  as linearly dependent then the transformation  $T$  will be along the line only.

Vectors Independent	Vectors Dependent
$T(\mathbf{v}) = \begin{pmatrix} a & c \\ b & d \end{pmatrix} \begin{pmatrix} k_1 \\ k_2 \end{pmatrix}$	$T(\mathbf{v}) = (k+1) \begin{pmatrix} a \\ b \end{pmatrix}$
Output: On the plane	Output: On the line