Assignment 15

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The link to the solution is

https://github.com/Adarsh1310/EE5609

***Abstract*—This documents solves a problem based on vector subspaces.**

1 Problem

Let $\mathbb{W}$ be the set of all $(x_1, x_2, x_3, x_4, x_5)$ in $\mathbf{R}^5$ which satisfy

$$2x_1 - x_2 + \frac{4}{3}x_3 - x_4 = 0$$
$$x_1 + \frac{2}{3}x_3 - x_5 = 0$$
$$9x_1 - 3x_2 + 6x_3 - 3x_4 - 3x_5 = 0$$

Find a finite set of vectors which spans W.

2 Solution

The above vectors can be written as,

$$\alpha_1 = (2, -1, \frac{4}{3}, -1, 0) \tag{2.0.1}$$
$$\alpha_2 = (1, 0, \frac{2}{3}, 0, -1) \tag{2.0.2}$$
$$\alpha_3 = (9, -3, 6, -3, -3) \tag{2.0.3}$$

Vector is in the subspace W of $\mathbf{R}^5$ spanned by α_1, α_2 and α_3 if and only if there exist scalars c_1,c_2 in $\mathbf{R}$. We can see that α_3 is a linear combination of α_1 and α_2.So,

$$\alpha = c_1\alpha_1 + c_2\alpha_2 \tag{2.0.4}$$

W consists all vector of the form,

$$\alpha = (2c_1 + c_2, -1c_1, \frac{4}{3}c_1 + \frac{2}{3}c_2, -1c_1, -c_2) \tag{2.0.5}$$

where c_1,c_2 are scalar constant. Alternatively it can be written as

$$\alpha = (x_1, x_2, x_3, x_4, x_5) \tag{2.0.6}$$

with x_i in $\mathbf{R}$

$$2x_1 - x_2 + \frac{4}{3}x_3 - x_4 = 0 \tag{2.0.7}$$
$$x_1 + \frac{2}{3}x_3 - x_5 = 0 \tag{2.0.8}$$

which can be written as,

$$x_1 = -\frac{2}{3}x_3 + x_5 \tag{2.0.9}$$
$$x_2 = -x_4 + 2x_5 \tag{2.0.10}$$

Hence,

$$\mathbb{W} = (-\frac{2}{3}x_3 + x_5, x_4 + 2x_5, x_3, x_4, x_5) \tag{2.0.11}$$

$$= \left(-\frac{2}{3}, 0, 1, 0, 0\right) x_3 + (0, -1, 0, 1, 0)\, x_4 + (1, 2, 0, 0, 1)x_5 \tag{2.0.12}$$

So,($-\frac{2}{3}$,0,1,0,0),(0,-1,0,1,0) and (1,2,0,0,1) will span $\mathbf{W}$.