# Assignment 8

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The link to the solution is

https://github.com/Adarsh1310/EE5609

***Abstract*—This documents solves a QR decomposition problem.**

## 1 Problem

Find QR decomposition of $\begin{pmatrix} -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{pmatrix}$

## 2 Solution

Let $\alpha$ and $\beta$ be transpose of column vectors of the given matrix.

$$\alpha = \begin{pmatrix} -\frac{1}{2} \\ \frac{1}{2} \end{pmatrix} \tag{2.0.1}$$

$$\beta = \begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \end{pmatrix} \tag{2.0.2}$$

We can express these as

$$\alpha = \mathrm{k}_1 \mathbf{u}_1 \tag{2.0.3}$$

$$\beta = \mathrm{r}_1 \mathbf{u}_1 + k_2 \mathbf{u}_2 \tag{2.0.4}$$

where

$$k_1 = \|\alpha\|, \mathbf{u}_1 = \frac{\alpha}{k_1} \tag{2.0.5}$$

$$r_1 = \frac{\mathbf{u}_1^T \beta}{\|\mathbf{u}_1\|^2} \tag{2.0.6}$$

$$\mathbf{u}_2 = \frac{\beta - r_1 \mathbf{u}_1}{\|\beta - r_1 \mathbf{u}_1\|} \tag{2.0.7}$$

$$k_2 = \mathbf{u}_2^T \beta \tag{2.0.8}$$

From (2.0.3) and (2.0.4),

$$\begin{pmatrix} \alpha & \beta \end{pmatrix} = \begin{pmatrix} \mathbf{u}_1 & \mathbf{u}_2 \end{pmatrix} \begin{pmatrix} k_1 & r_1 \\ 0 & k_2 \end{pmatrix} \tag{2.0.9}$$

$$\begin{pmatrix} \alpha & \beta \end{pmatrix} = \mathbf{QR} \tag{2.0.10}$$

From above we can see that $\mathbf{R}$ is an upper triangular matrix and $\mathbf{Q}^T\mathbf{Q}$ =I

$$k_1 = \sqrt{\frac{1}{2}}, \mathbf{u}_1 = \sqrt{2}\begin{pmatrix} -\frac{1}{2} \\ \frac{1}{2} \end{pmatrix}, \tag{2.0.11}$$

$$r_1 = -\sqrt{\frac{1}{2}} \tag{2.0.12}$$

$$\mathbf{u}_2 = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{2.0.13}$$

$$k_2 = 0 \tag{2.0.14}$$

Thus obtained QR decomposition is

$$\begin{pmatrix} -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{pmatrix} = \begin{pmatrix} -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & 0 \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} \end{pmatrix} \tag{2.0.15}$$