Assignment 19

Adarsh Srivastava

The link to the solution is

https://github.com/Adarsh1310/EE5609

***Abstract*—This documents solves a problem based on Linear Transformation.**

1 Problem

Let T be the linear transformation from $\mathbb{R}^3$ into $\mathbb{R}^2$ defined by,

$$T\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_1 + x_2 \\ 2x_3 - x_1 \end{pmatrix}$$

If β is the standard ordered basis for $\mathbb{R}^3$ and $\beta^{'}$ is the standard ordered basis for $\mathbb{R}^2$, what is the matrix of T relative to the pair β,$\beta^{'}$

2 Solution

We know that,

$$[T\alpha]_\beta = \mathbf{A}[\alpha]_{\beta'} \tag{2.0.1}$$

where **A** is called the matrix of T relative to ordered basis β,$\beta^{'}$ Using the ordered basis,

$$\beta = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{2.0.2}$$

$$\beta^{'} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{2.0.3}$$

Let,

$$\mathbf{V}_j = [T\alpha_j]_\beta \tag{2.0.4}$$

$$\mathbf{V} = \begin{pmatrix} \mathbf{V}_1 & \mathbf{V}_2 & \mathbf{V}_3 \end{pmatrix} \tag{2.0.5}$$

Using the given transformation,

$$T\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{2.0.6}$$

$$T\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{2.0.7}$$

$$T\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix} \tag{2.0.8}$$

So,T(β) can be written as,

$$\mathbf{V} = \begin{pmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \end{pmatrix} \tag{2.0.9}$$

Now, since the transformation has to be found relative to the pair β, $\beta^{'}$ we can rewrite the equation in the question as,

$$\left(\begin{array}{cc|ccc} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & -1 & 0 & 2 \end{array}\right) \tag{2.0.10}$$

but from here we can see that $\beta^{'}$ is already an identity matrix and hence no row reduction is required. So by using (2.0.1) we can say that,

$$[T\alpha]_\beta = \begin{pmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \end{pmatrix}[\alpha]_{\beta'} \tag{2.0.11}$$