① Conditional Probability

a) $P(A) - P(A \cap B) + P(B) - P(A \cap B)$

$= 0.3 - 0.2 + 0.4 - 0.2$

$= \underline{0.3}$

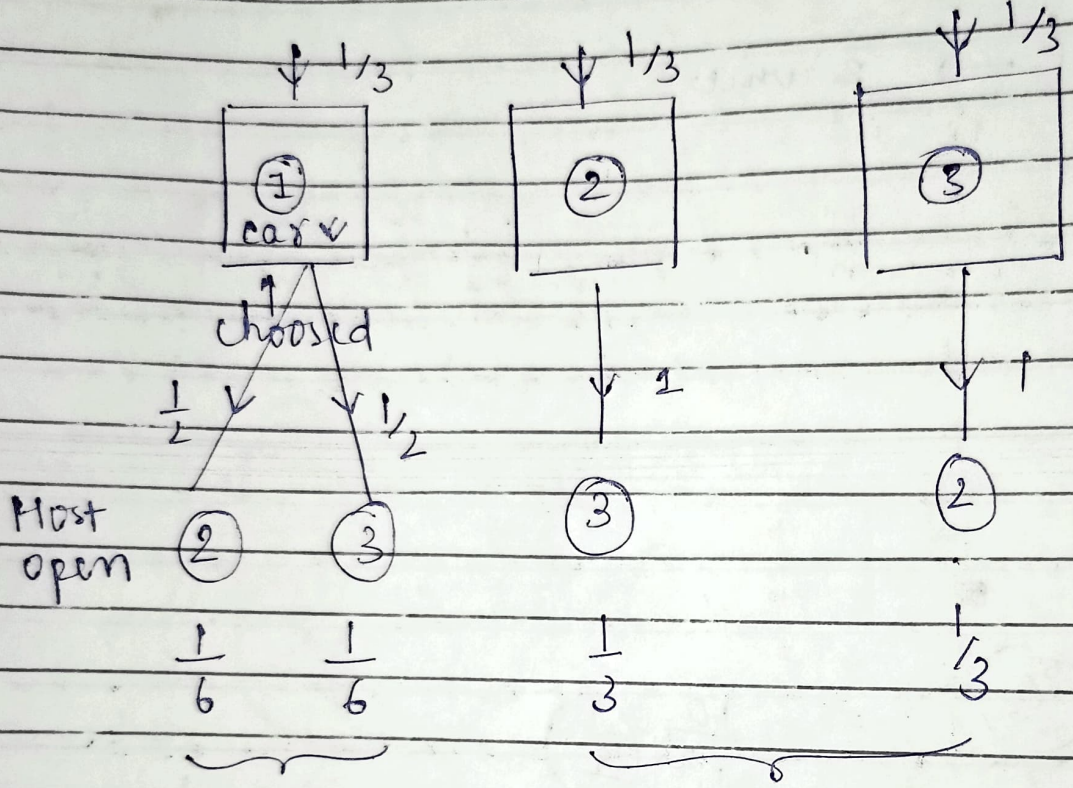
b) $P(A) + P(B) - P(A \cap B)$

$= \underline{0.5}$

c) $1 - (P(A \cup B))$

$= 1 - 0.5 = \underline{0.5}$

② Monty Hall problem



if contestent stay → win

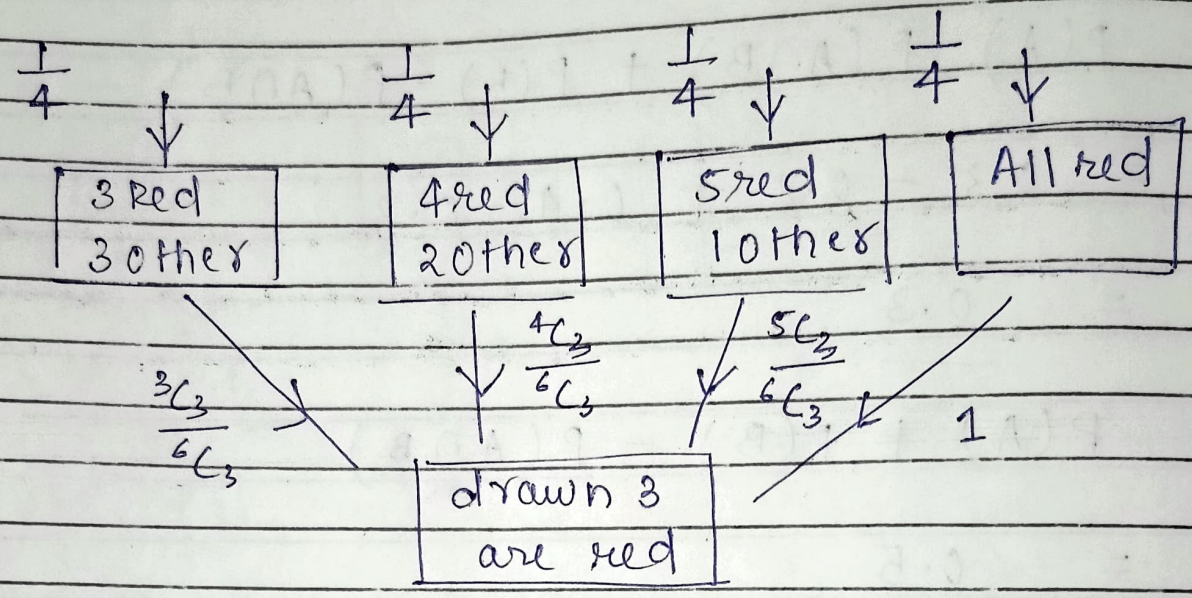
$P = \frac{1}{6} + \frac{1}{6} = \frac{1}{3}$

if contestent switch → win

$P = \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$

⇒ Switch is a better option.

(3) Bayes Theorem



$$P(\text{drawn 3 are red}) = \frac{1}{4} \times \frac{{}^3C_3}{{}^6C_3} + \frac{1}{4} \times \frac{{}^4C_3}{{}^6C_3} + \frac{1}{4} \times \frac{{}^5C_3}{{}^6C_3} + \frac{1}{4}$$

$$= \frac{1}{4} \times \frac{7}{4}$$

$$P(\text{bag have all red balls}) = \frac{\frac{1}{4} \times 1}{\frac{1}{4} \times \frac{7}{4}}$$

$$= \frac{4}{7} \quad \text{Ans}$$

④ Random Variables

a) $P(X < 0.5)$

$= P(0.2) + P(0.4) = 0.1 + 0.2$

$= \underline{0.3}$

b) $P(0.25 < X < 0.75)$

$= P(0.4) + P(0.5) = \underline{0.4}$

c) $P\left(\frac{0.2}{X < 0.6}\right)$

$P(X < 0.6) = .5$

$P((0.2) \cap (X < 0.6)) = \frac{0.1}{0.5} = \frac{1}{5}$

$= \underline{0.2}$

(5) Random variables

Since distribution f^n is right cont.

$$\frac{4c^2 - 9c + 6}{4} = 1$$

$$\Rightarrow \quad c = 2, \frac{1}{4} \qquad \Rightarrow \quad c = \frac{1}{4} \quad \text{Ans}$$

2 ↑ reject.

as $\frac{7}{6} - 2 < 0$

with $c = \frac{1}{4}$ corresponding pdf:

x	0	1	2	3
$f(x)$	$2/3$	$3/12$	$1/12$	0

$$P(1 < X < 2) = 0$$

$$P(2 \leq X < 3) = f(2) = \frac{1}{12}$$

$$P(1 \leq X \leq 2) = \frac{3}{12} + \frac{1}{12} = \frac{1}{3}$$

$$P(0 < X \leq 1) = \frac{3}{12} = \frac{1}{4}$$

$$P(X > 3) = 0$$

(6) Expectation calculation

a) $$E(X) = \int_{-\infty}^{\infty} x \cdot p(x)\, dx$$

$$= \int_{0}^{1} x \times 1\, dx = \frac{1}{2} \quad \text{Ans.}$$

b) $$\sigma^2 = E(X^2) - (E(X))^2$$

$$= \int_{0}^{1} x^2\, dx - \frac{1}{4} = \frac{1}{12} \quad \text{Ans}$$

c) $$E(X^2 + Y^2) = 1$$

$$\Rightarrow \frac{1}{3} + E(Y^2) = 1 \Rightarrow$$

$$E(Y^2) = 2/3$$

$$\sigma^2 = E(Y^2) - (E(Y))^2$$

$$\Rightarrow E(Y) = \sqrt{\frac{2}{3} - \frac{5}{9}} = \frac{1}{3} \quad \text{Ans}$$

d) $$E(X+Y)$$

$$= E(X) + E(Y) = \frac{1}{2} + \frac{1}{3}$$

$$= \frac{5}{6} \quad \text{Ans}$$