Assignment 5

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Download latex-tikz codes from

https://github.com/Adarsh541/AI1103−prob−and−ranvar/blob/main/Assignment5/Assignment5.tex

1 Problem(GATE 2020(ST) Q16)

The characteristic function of a random variable X is given by

$$\phi_X(t) = \begin{cases} \frac{(sin(t))(cos(t))}{t} & t \neq 0 \\ 1 & t = 0 \end{cases} \tag{1.0.1}$$

Then $P\left(|X| \leq \frac{3}{2}\right) =$

2 Solution(GATE 2020(ST) Q16)

If ϕ_X is characteristic function of distribution function F_X then

$$\frac{F(x+h) - F(x-h)}{2h} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left(\frac{sin(ht)}{ht} \right) e^{-ixt} \phi_X(t)\, dt \tag{2.0.1}$$

If characteristic function ϕ_X is integrable, then F_X is absolutely continuous.Since the given characteristic function is integrable,F_X is absolutely continuous.

$$P\left(|X| \leq \frac{3}{2}\right) = P\left(-\frac{3}{2} \leq X \leq \frac{3}{2}\right) \tag{2.0.2}$$

$$= F\left(\frac{3}{2}\right) - F\left(-\frac{3}{2}^{-}\right) \tag{2.0.3}$$

$$= F\left(\frac{3}{2}\right) - F\left(-\frac{3}{2}\right) \tag{2.0.4}$$

Substituting $x = 0$ and $h = \frac{3}{2}$ in (2.0.1) we get

$$\frac{F\left(\frac{3}{2}\right) - F\left(-\frac{3}{2}\right)}{3} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left(\frac{sin\left(\frac{3t}{2}\right)}{\frac{3t}{2}} \right) \frac{(sin(t))(cos(t))}{t} dt \tag{2.0.5}$$

$$F\left(\frac{3}{2}\right) - F\left(-\frac{3}{2}\right) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{sin\left(\frac{3t}{2}\right) sin(2t)}{t^2} dt \tag{2.0.6}$$

Using wolfram alpha to solve the above integral we get

$$P\left(|X| \leq \frac{3}{2}\right) = 0.75 \tag{2.0.7}$$