

Assignment 7

Adarsh Sai - AI20BTECH11001

Download all python codes from

<https://github.com/Adarsh541/AI1103-prob-and-ranvar/blob/%main/Assignment6/codes/Assignment6.py>

Download latex-tikz codes from

<https://github.com/Adarsh541/AI1103-prob-and-ranvar/blob/main/Assignment7/Assignment7.tex>

1 PROBLEM(CSIR UGC NET EXAM(JUNE 2012)Q.112)

Let $X_1, X_2, X_3, \dots, X_n$ be i.i.d observations from a distribution with continuous probability density function f which is symmetric around θ i.e

$$f(x - \theta) = f(\theta - x) \quad (1.0.1)$$

for all real x . Consider the test $H_0 : \theta = 0$ vs $H_1 : \theta > 0$ and the sign test statistic

$$S_n = \sum_{i=1}^n \text{sign}(X_i) \quad (1.0.2)$$

where

$$\text{sign}(x) = \begin{cases} 1 & x > 0 \\ 0 & x = 0 \\ -1 & x < 0 \end{cases} \quad (1.0.3)$$

Let z_α be the upper $100(1 - \alpha)^{th}$ percentile of the standard normal distribution where $0 < \alpha < 1$. Which of the following is/are correct?

- 1) If $\theta = 0$ then $\lim_{n \rightarrow \infty} P\{S_n > \sqrt{n}z_\alpha\} = 1$
- 2) If $\theta = 0$ then $\lim_{n \rightarrow \infty} P\{S_n > \sqrt{n}z_\alpha\} = \alpha$
- 3) If $\theta > 0$ then $\lim_{n \rightarrow \infty} P\{S_n > \sqrt{n}z_\alpha\} = 1$
- 4) If $\theta > 0$ then $\lim_{n \rightarrow \infty} P\{S_n > \sqrt{n}z_\alpha\} = \alpha$

2 SOLUTION(CSIR UGC NET EXAM(JUNE 2012)Q.112)

Assume hypothesis $H_0 : \theta = 0$ is true..Since X is symmetric around zero.

$$f_X(x) = f_X(-x) \quad (2.0.1)$$

$$\int_0^\infty f_X(x)dx = \int_0^\infty f_X(-x)dx \quad (2.0.2)$$

$$\int_0^\infty f_X(x)dx = \int_{-\infty}^0 f_X(x)dx \quad (2.0.3)$$

$$\Rightarrow \Pr(X \geq 0) = \Pr(X \leq 0) = \frac{1}{2} \quad (2.0.4)$$

Let Y be a random variable such that

$$Y = \text{sign}(X) \quad (2.0.5)$$

From (2.0.4) we have

$$\Pr(Y = -1) = \Pr(Y = 1) = \frac{1}{2} \quad (2.0.6)$$

So $Y = \text{sign}(X)$ is also symmetric around zero.

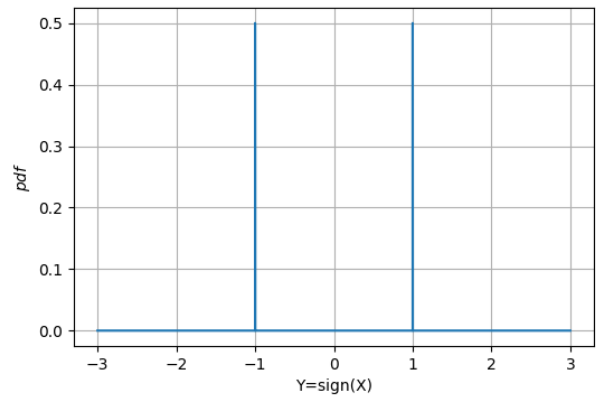


Fig. 4: pdf of $Y = \text{sign}(X)$

$$\Rightarrow \mu_Y = 0 \quad (2.0.7)$$

and variance is

$$\sigma_y^2 = (-1)^2 \left(\frac{1}{2}\right) + (1)^2 \left(\frac{1}{2}\right) \quad (2.0.8)$$

$$= 1 \quad (2.0.9)$$

From central limit theorem

$$Z = \lim_{n \rightarrow \infty} \sqrt{n} \left(\frac{\frac{S_n}{n} - \mu_y}{\sigma_y} \right) \quad (2.0.10)$$

$$= \lim_{n \rightarrow \infty} \sqrt{n} \left(\frac{S_n}{n} \right) \quad (2.0.11)$$

$$= \lim_{n \rightarrow \infty} \left(\frac{S_n}{\sqrt{n}} \right) \quad (2.0.12)$$

where Z is a standard normal variable N(0,1). Given

$$\alpha = P\{Z > z_\alpha\} \quad (2.0.13)$$

So from (2.0.12)

$$\lim_{n \rightarrow \infty} P\left\{ \frac{S_n}{\sqrt{n}} > z_\alpha \right\} = \alpha \quad (2.0.14)$$

$$\implies \lim_{n \rightarrow \infty} P\{S_n > \sqrt{n}z_\alpha\} = \alpha \quad (2.0.15)$$