Assignment 7

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Download latex-tikz codes from

https://github.com/Adarsh541/AI1103−prob−and−ranvar/blob/main/Assignment7/Assignment7.tex

1 Problem(CSIR UGC NET EXAM(June 2012)Q.112)

Let $X_1, X_2, X_3, ..., X_n$ be i.i.d observations from a distribution with continuous probability density function f which is symmetric around θ i.e

$$f(x-\theta) = f(\theta - x) \tag{1.0.1}$$

for all real x.Consider the test $H_0 : \theta = 0$ vs $H_1 : \theta > 0$ and the sign test statistic

$$S_n = \sum_{i=1}^{n} sign(X_i) \tag{1.0.2}$$

where

$$sign(x) = \begin{cases} 1 & x > 0 \\ 0 & x = 0 \\ -1 & x < 0 \end{cases} \tag{1.0.3}$$

Let z_α be the upper $100(1-\alpha)^{th}$ percentile of the standard normal distribution where $0 < \alpha < 1$. Which of the following is/are correct?

1) If $\theta = 0$ then $\lim_{n\to\infty} P\left\{S_n > \sqrt{n} z_\alpha\right\} = 1$
2) If $\theta = 0$ then $\lim_{n\to\infty} P\left\{S_n > \sqrt{n} z_\alpha\right\} = \alpha$
3) If $\theta > 0$ then $\lim_{n\to\infty} P\left\{S_n > \sqrt{n} z_\alpha\right\} = 1$
4) If $\theta > 0$ then $\lim_{n\to\infty} P\left\{S_n > \sqrt{n} z_\alpha\right\} = \alpha$

2 Solution(CSIR UGC NET EXAM(June 2012)Q.112)

Assume hypothesis $H_0 : \theta = 0$ is true.Let $\Pr(X=0) = k$.Since X is symmetric around zero.

$$\implies \Pr(X<0) = \Pr(X>0) = \frac{1-k}{2} \tag{2.0.1}$$

$$\implies \Pr(sign(X) = -1) = \Pr(sign(X) = 1) = \frac{1-k}{2} \tag{2.0.2}$$

So sign(X) is also symmetric around zero.

$$\implies \mu_s = 0 \tag{2.0.3}$$

and variance is

$$\sigma_s^2 = (-1)\frac{1-k}{2} + 0(k) + (1)\frac{1-k}{2} \tag{2.0.4}$$

$$= 1-k \tag{2.0.5}$$

From central limit theorem

$$Z = \lim_{n\to\infty} \sqrt{n}\left(\frac{\frac{S_n}{n} - \mu_s}{\sigma_s}\right) \tag{2.0.6}$$

$$= \lim_{n\to\infty} \sqrt{n}\left(\frac{S_n}{n(1-k)}\right) \tag{2.0.7}$$

$$= \lim_{n\to\infty} \left(\frac{S_n}{\sqrt{n}(1-k)}\right) \tag{2.0.8}$$

where Z is a standard normal variable N(0,1).Given

$$\alpha = P\{Z > z_\alpha\} \tag{2.0.9}$$

So from (2.0.8)

$$\lim_{n\to\infty} P\left\{\frac{S_n}{\sqrt{n}(1-k)} > z_\alpha\right\} = \alpha \tag{2.0.10}$$