

# Assignment 7

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Download latex-tikz codes from

<https://github.com/Adarsh541/AI1103-prob-and-ranvar/blob/main/Assignment7/Assignment7.tex>

From central limit theorem

$$Z = \lim_{n \rightarrow \infty} \sqrt{n} \left( \frac{\frac{S_n}{n} - \mu_s}{\sigma_s} \right) \quad (2.0.3)$$

$$Z = \lim_{n \rightarrow \infty} \sqrt{n} \left( \frac{S_n}{n} \right) \quad (2.0.4)$$

$$Z = \lim_{n \rightarrow \infty} \left( \frac{S_n}{\sqrt{n}} \right) \quad (2.0.5)$$

where Z is a standard normal variable N(0,1). Given

$$\alpha = P\{Z > z_\alpha\} \quad (2.0.6)$$

So from (2.0.5)

$$\lim_{n \rightarrow \infty} P\left\{ \frac{S_n}{\sqrt{n}} > z_\alpha \right\} = \alpha \quad (2.0.7)$$

$$\Rightarrow \lim_{n \rightarrow \infty} P\{S_n > \sqrt{n}z_\alpha\} = \alpha \quad (2.0.8)$$

## 1 PROBLEM(CSIR UGC NET EXAM(JUNE 2012)Q.112)

Let  $X_1, X_2, X_3, \dots, X_n$  be i.i.d observations from a distribution with continuous probability density function  $f$  which is symmetric around  $\theta$  i.e

$$f(x - \theta) = f(\theta - x) \quad (1.0.1)$$

for all real  $x$ . Consider the test  $H_0 : \theta = 0$  vs  $H_1 : \theta > 0$  and the sign test statistic

$$S_n = \sum_{i=1}^n \text{sign}(X_i) \quad (1.0.2)$$

where

$$\text{sign}(x) = \begin{cases} 1 & x > 0 \\ 0 & x = 0 \\ -1 & x < 0 \end{cases} \quad (1.0.3)$$

Let  $z_\alpha$  be the upper  $100(1 - \alpha)^{th}$  percentile of the standard normal distribution where  $0 < \alpha < 1$ . Which of the following is/are correct?

1) If  $\theta = 0$  then  $\lim_{n \rightarrow \infty} P\{S_n > \sqrt{n}z_\alpha\} = 1$

2) If  $\theta = 0$  then  $\lim_{n \rightarrow \infty} P\{S_n > \sqrt{n}z_\alpha\} = \alpha$

3) If  $\theta > 0$  then  $\lim_{n \rightarrow \infty} P\{S_n > \sqrt{n}z_\alpha\} = 1$

4) If  $\theta > 0$  then  $\lim_{n \rightarrow \infty} P\{S_n > \sqrt{n}z_\alpha\} = \alpha$

## 2 SOLUTION(CSIR UGC NET EXAM(JUNE 2012)Q.112)

Assume hypothesis  $H_0 : \theta = 0$  is true. Since  $X_i$  is symmetric around zero,  $\text{sign}(X_i)$  is also symmetric around zero.

$$\Rightarrow \mu_s = 0 \quad (2.0.1)$$

$$\sigma_s^2 = 1 \quad (2.0.2)$$