Assignment 8

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Download all python codes from

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https://github.com/Adarsh541/AI1103−prob−and−ranvar/blob/main/Assignment8/codes/Assignment8.py
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and latex-tikz codes from

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https://github.com/Adarsh541/AI1103−prob−and−ranvar/blob/main/Assignment8/Assignment8.tex
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1 Problem(gov/stats/2015/statistics-I(1),Q.3(a))

Let $X_1, X_2, ..., X_n$ be independent Poisson variates with $E[X_i] = \mu_i$.Find the conditional distribution of $X_1, ..., X_n \Big| \sum_{i=1}^{n} X_i = y$

2 Solution(gov/stats/2015/statistics-I(1),Q.3(a))

$$F\left\{x_1, ..., x_n \middle| \sum_{i=1}^{n} x_i = y\right\} = \Pr\left(X_1 \le x_1, ..., X_n \le x_n \middle| \sum_{i=1}^{n} x_i = y\right) \tag{2.0.1}$$

$$= \frac{\Pr\left(X_1 \le x_1, ..., X_n \le x_n, \sum_{i=1}^{n} x_i = y\right)}{\Pr\left(\sum_{i=1}^{n} x_i = y\right)} \tag{2.0.2}$$

1) Solving numerator of (2.0.2).The only solution of $\{X_1 \le x_1, ..., X_n \le x_n, \sum_{i=1}^{n} x_i = y\}$ is $X_1 = x_1, ..., X_n = x_n$.Also $X_1, X_2, ..., X_n$ are independent.So

$$\Pr\left(X_1 \le x_1, ..., X_n \le x_n, \sum_{i=1}^{n} x_i = y\right) = \Pr(X_1 = x_1)\Pr(X_2 = x_2)...\Pr(X_n = x_n) \tag{2.0.3}$$

$$= \prod_{i=1}^{n}\left(\frac{\mu_i^{x_i} e^{-\mu_i}}{x_i!}\right) \tag{2.0.4}$$

$$= \left(e^{-\sum \mu_i}\right)\prod_{i=1}^{n}\left(\frac{\mu_i^{x_i}}{x_i!}\right) \tag{2.0.5}$$

2) **Theorem**:If the random variables X_1, X_2 are independent and poisson distributed with parameters λ_1, λ_2,then their sum $Z = X_1 + X_2$ is also poisson distributed with parameter $\lambda_1 + \lambda_2$.**proof**:The characteristic function of a poisson random variable is given by

$$\Phi_X(\omega) = e^{-\lambda(1-e^{j\omega})} \tag{2.0.6}$$

Using convolution

$$\Phi_Z(\omega) = \Phi_{X_1}(\omega)\Phi_{X_2}(\omega) \tag{2.0.7}$$

$$= e^{-\lambda_1(1-e^{j\omega})}e^{-\lambda_2(1-e^{j\omega})} \tag{2.0.8}$$

$$= e^{-(\lambda_1+\lambda_2)(1-e^{j\omega})} \tag{2.0.9}$$

This theorem can be extended to n variables.So

$$\Pr\left(\sum_{i=1}^{n} x_i = y\right) = \frac{\left(\sum \mu_i\right)^y e^{\sum \mu_i}}{y!} \tag{2.0.10}$$

From (2.0.2),(2.0.5) and (2.0.10)

$$F\left\{x_1, ..., x_n \middle| \sum_{i=1}^{n} x_i = y\right\} = \frac{y!}{\left(\sum \mu_i\right)^y}\prod_{i=1}^{n}\left(\frac{\mu_i^{x_i}}{x_i!}\right) \tag{2.0.11}$$