

Assignment 8

Adarsh Sai - AI20BTECH11001

Download latex-tikz codes from

<https://github.com/Adarsh541/AI1103-prob-and-ranvar/blob/main/Assignment8/Assignment8.tex>

1 PROBLEM(GOV/STATS/2015/STATISTICS-I(1),Q.3(A))

Let X_1, X_2, \dots, X_n be independent Poisson variates with $E[X_i] = \mu_i$. Find the conditional distribution of $X_1, \dots, X_n \mid \sum_{i=1}^n X_i = y$

2 SOLUTION(GOV/STATS/2015/STATISTICS-I(1),Q.3(A))

$$F \left\{ x_1, \dots, x_n \mid \sum_{i=1}^n x_i = y \right\} = \Pr \left(X_1 \leq x_1, \dots, X_n \leq x_n \mid \sum_{i=1}^n x_i = y \right) \quad (2.0.1)$$

$$= \frac{\Pr(X_1 \leq x_1, \dots, X_n \leq x_n, \sum_{i=1}^n x_i = y)}{\Pr(\sum_{i=1}^n x_i = y)} \quad (2.0.2)$$

1) Solving numerator of (2.0.2). The only solution of $\{X_1 \leq x_1, \dots, X_n \leq x_n, \sum_{i=1}^n x_i = y\}$ is $X_1 = x_1, \dots, X_n = x_n$. Also X_1, X_2, \dots, X_n are independent. So

$$\Pr \left(X_1 \leq x_1, \dots, X_n \leq x_n, \sum_{i=1}^n x_i = y \right) = \Pr(X_1 = x_1) \Pr(X_2 = x_2) \dots \Pr(X_n = x_n) \quad (2.0.3)$$

$$= \prod_{i=1}^n \left(\frac{\mu_i^{x_i} e^{-\mu_i}}{x_i!} \right) \quad (2.0.4)$$

$$= (e^{-\sum \mu_i}) \prod_{i=1}^n \left(\frac{\mu_i^{x_i}}{x_i!} \right) \quad (2.0.5)$$

2) **Theorem:** If the random variables X_1, X_2 are independent and poisson distributed with parameters λ_1, λ_2 , then their sum $Z = X_1 + X_2$ is

also poisson distributed with parameter $\lambda_1 + \lambda_2$. **proof:** The characteristic function of a poisson random variable is given by

$$\Phi_X(\omega) = e^{-\lambda(1-e^{j\omega})} \quad (2.0.6)$$

Using convolution

$$\Phi_Z(\omega) = \Phi_{X_1}(\omega) \Phi_{X_2}(\omega) \quad (2.0.7)$$

$$= e^{-\lambda_1(1-e^{j\omega})} e^{-\lambda_2(1-e^{j\omega})} \quad (2.0.8)$$

$$= e^{-(\lambda_1 + \lambda_2)(1-e^{j\omega})} \quad (2.0.9)$$

This theorem can be extended to n variables. So

$$\Pr \left(\sum_{i=1}^n x_i = y \right) = \frac{(\sum \mu_i)^y e^{-\sum \mu_i}}{y!} \quad (2.0.10)$$

From (2.0.2), (2.0.5) and (2.0.10)

$$F \left\{ x_1, \dots, x_n \mid \sum_{i=1}^n x_i = y \right\} = \frac{y!}{(\sum \mu_i)^y} \prod_{i=1}^n \left(\frac{\mu_i^{x_i}}{x_i!} \right) \quad (2.0.11)$$