Assignment 8

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Download all python codes from

```
https://github.com/Adarsh541/AI1103−prob−and−
    ranvar/blob/main/Assignment8.1/codes/
    Assignment8.1.py
```

and latex-tikz codes from

```
https://github.com/Adarsh541/AI1103−prob−and−
    ranvar/blob/main/Assignment8.1/Assignment8
    .1.tex
```

1 Problem

Let X_1, X_2, X_3, X_4, X_5 be a random sample of size 5 from a population having standard normal distribution.If $\overline{X} = \frac{1}{5}\sum_{i=1}^{5} X_i$ and $T = \sum_{i=1}^{5}\left(X_i - \overline{X}\right)^2$ then $E[T^2\overline{X}^2]$ is equal to

1) 3
2) 3.6
3) 4.8
4) 5.2

2 Solution

2.1 Theorem

Let $\overline{X_n}$ be the sample mean of size n from a normal distribution with mean μ and variance σ^2.Then

1) $\overline{X_n} \sim N(\mu, \frac{\sigma^2}{n})$
2) $\overline{X}$ and T are independent.
3) $\frac{T}{\sigma^2} \sim \chi^2_{n-1}$

where χ^2_{n-1} is chi-square distribution with $(n-1)$ degrees of freedom and T is as defined in the question.

2.2 Useful concepts

If X,Y are independent random variables.then

$$E[XY] = E[X]E[Y] \tag{2.2.1}$$

$$E[X^2] = Var[X] + (E[X])^2 \tag{2.2.2}$$

If X is chi-square distributed with parameter k,then

$$E[X] = k \tag{2.2.3}$$

$$Var[X] = 2k \tag{2.2.4}$$

2.3 solution

For standard normal distribution $\mu = 0, \sigma^2 = 1$.So from the above theorem for $n = 5$ we have

1) T and $\overline{X}$ are independent.
2) $\overline{X} \sim N(0, 1/5)$
3) $T \sim \chi^2_4$

So from (2.2.3) and (2.2.4)

$$E[T] = 4 \tag{2.3.1}$$

$$Var[T] = 8 \tag{2.3.2}$$

Since T and $\overline{X}$ are independent

$$E[T^2\overline{X}^2] = E[T^2]E[\overline{X}^2] \tag{2.3.3}$$

from (2.2.2)

$$E[\overline{X}^2] = \frac{1}{5} \tag{2.3.4}$$

$$E[T^2] = 24 \tag{2.3.5}$$

So from (2.3.3)

$$E[T^2\overline{X}^2] = 4.8 \tag{2.3.6}$$