Gate Assignment 2

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Download all python codes from

https://github.com/Adarsh541/EE3900/blob/main/Gate2/codes/Gate2.py

Download latex-tikz codes from

https://github.com/Adarsh541/EE3900/blob/main/Gate2/Gate2.tex

1 Problem(Gate EC 2010 Q.41)

A continuous time LTI system is described by

$$\frac{d^2y(t)}{dt^2} + 4\frac{dy(t)}{dt} + 3y(t) = 2\frac{dx(t)}{dt} + 4x(t) \quad (1.0.1)$$

Assuming zero initial conditions, the response $y(t)$ of the above system for the input $x(t) = e^{-2t}u(t)$ is given by

1) $\left(e^{t} - e^{3t}\right)u(t)$
2) $\left(e^{-t} - e^{-3t}\right)u(t)$
3) $\left(e^{-t} + e^{-3t}\right)u(t)$
4) $\left(e^{t} + e^{3t}\right)u(t)$

2 Solution

Lemma 2.1 (Table of Laplace Transforms)**.**

Time Function $f(t) = \mathcal{L}^{-1}\{F(s)\}$	***Laplace transform*** *of f(t)* $F(s) = \mathcal{L}\{f(t)\}$
$u(t)$	$\frac{1}{s},\ s > 0$
$g'(t)$	$sG(s) - g(0)$
$g''(t)$	$s^2G(s) - sg(0) - g'(0)$
$e^{-at}u(t)$	$\frac{1}{s+a},\ s + a > 0$

Lemma 2.2. *Linearity of Laplace Transform*

$$\mathcal{L}\{af(t) + bg(t)\} = a\mathcal{L}\{f(t)\} + b\mathcal{L}\{g(t)\} \quad (2.0.1)$$

From Lemma-2.1 Laplace transform of $x(t) = e^{-2t}u(t)$ is given by

$$X(s) = \frac{1}{s+2} \quad (2.0.2)$$

Since initial conditions are zero. Laplace Transform of (1.0.1) gives

$$s^2Y(s) + 4sY(s) + 3Y(s) = 2sX(s) + 4X(s) \quad (2.0.3)$$

$$Y(s) = \frac{2(s+2)}{s^2+4s+3}X(s) \quad (2.0.4)$$

$$= \frac{1}{s+1} - \frac{1}{s+3} \quad (2.0.5)$$

From Lemma-2.1. Inverse Laplace transform of $Y(s)$ is given by

$$y(t) = e^{-t}u(t) - e^{-3t}u(t) \quad (2.0.6)$$

$$= \left(e^{-t} - e^{-3t}\right)u(t) \quad (2.0.7)$$

$\therefore$ The required option is B.

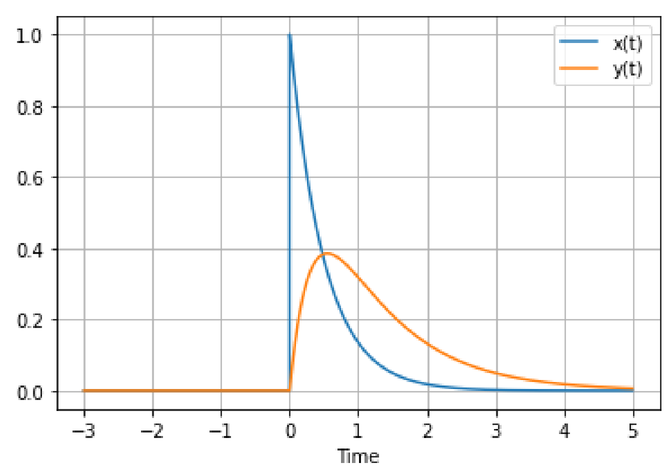

Fig. 4: Plot of input and output responses in time domain.