

Assignment 5

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Download all python codes from

https://github.com/Adarsh541/EE3900/blob/main/EE3900_As5/codes/EE3900_As5.py

Download latex-tikz codes from

https://github.com/Adarsh541/EE3900/blob/main/EE3900_As5/EE3900_As5.tex

1 PROBLEM(QUADRATIC FORMS Q.2.36)

Find the area bounded by the ellipse $\mathbf{x}^\top \begin{pmatrix} \frac{1}{a^2} & 0 \\ 0 & \frac{1}{b^2} \end{pmatrix} \mathbf{x} = 1$ and $x = ae$, where $b^2 = a^2(1 - e^2)$ and $e < 1$.

2 SOLUTION

Given ellipse is

$$\mathbf{x}^\top \begin{pmatrix} \frac{1}{a^2} & 0 \\ 0 & \frac{1}{b^2} \end{pmatrix} \mathbf{x} = 1 \quad (2.0.1)$$

On comparing with the standard form

$$\mathbf{c} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (2.0.2)$$

$$\mathbf{D} = \begin{pmatrix} \frac{1}{a^2} & 0 \\ 0 & \frac{1}{b^2} \end{pmatrix} \quad (2.0.3)$$

$$\mathbf{u}^\top \mathbf{V}^{-1} \mathbf{u} - f = 1 \quad (2.0.4)$$

$$\lambda_1 = \frac{1}{a^2} \quad (2.0.5)$$

$$\lambda_2 = \frac{1}{b^2} \quad (2.0.6)$$

Semi major and minor axes of the ellipse are

$$\text{length of semi major axis} = \sqrt{\frac{\mathbf{u}^\top \mathbf{V}^{-1} \mathbf{u} - f}{\lambda_1}} \quad (2.0.7)$$

$$= a \quad (2.0.8)$$

$$\text{length of semi minor axis} = \sqrt{\frac{\mathbf{u}^\top \mathbf{V}^{-1} \mathbf{u} - f}{\lambda_2}} \quad (2.0.9)$$

$$= b \quad (2.0.10)$$

\therefore equation of ellipse can be written as

$$\frac{x}{a^2} + \frac{y}{b^2} = 1 \quad (2.0.11)$$

$$\text{Area of ABCDA} = 2 (\text{Ar}(\text{ABDA})) \quad (2.0.12)$$

$$= 2 \left(\int_{ae}^a y dx \right) \quad (2.0.13)$$

$$= \frac{2b}{a} \left(\int_{ae}^a \sqrt{a^2 - x^2} dx \right) \quad (2.0.14)$$

$$= \frac{2b}{a} \left[\frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} \right]_{ae}^a \quad (2.0.15)$$

$$= \frac{\pi ab}{2} - b^2 e - ab \sin^{-1} e \quad (2.0.16)$$

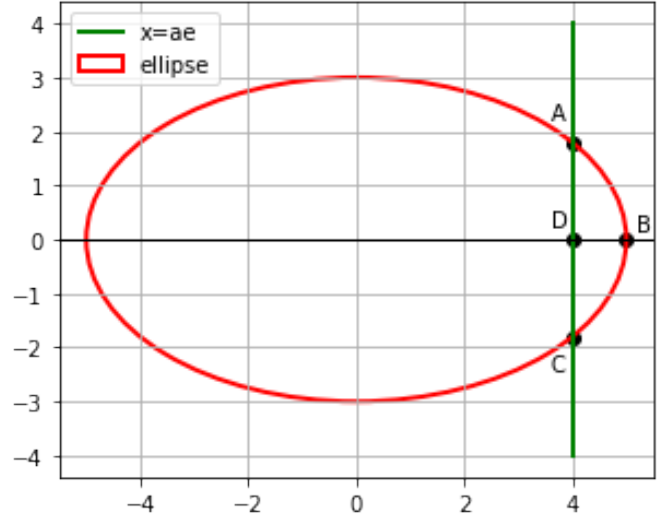


Fig. 0: Reference plot. Here $a = 5$, $b = 3$ and $e = \frac{4}{5}$