Gate Assignment 2

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Download all python codes from

```
https://github.com/Adarsh541/EE3900/blob/main/
    Gate2/codes/Gate2.py
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Download latex-tikz codes from

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https://github.com/Adarsh541/EE3900/blob/main/
    Gate2/Gate2.tex
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1 Problem(Gate EC 2010 Q.41)

A continuous time LTI system is described by

$$\frac{d^2y(t)}{dt^2} + 4\frac{dy(t)}{dt} + 3y(t) = 2\frac{dx(t)}{dt} + 4x(t) \quad (1.0.1)$$

Assuming zero initial conditions, the response $y(t)$ of the above system for the input $x(t) = e^{-2t}u(t)$ is given by

1) $\left(e^{t} - e^{3t}\right) u(t)$
2) $\left(e^{-t} - e^{-3t}\right) u(t)$
3) $\left(e^{-t} + e^{-3t}\right) u(t)$
4) $\left(e^{t} + e^{3t}\right) u(t)$

2 Solution

Lemma 2.1 (Table of Laplace Transforms)**.**

Time Function $f(t) = \mathcal{L}^{-1}\{F(s)\}$	***Laplace transform*** *of f(t)* $F(s) = \mathcal{L}\{f(t)\}$
$u(t)$	$\frac{1}{s},\ s > 0$
$g'(t)$	$sG(s) - g(0)$
$g''(t)$	$s^2G(s) - sg(0) - g'(0)$
$e^{-at}u(t)$	$\frac{1}{s+a},\ s + a > 0$

Lemma 2.2. *Linearity of Laplace Transform*

$$\mathcal{L}\{af(t) + bg(t)\} = a\mathcal{L}\{f(t)\} + b\mathcal{L}\{g(t)\} \quad (2.0.1)$$

From Lemma-2.1 Laplace transform of $x(t) = e^{-2t}u(t)$ is given by

$$X(s) = \frac{1}{s+2} \quad (2.0.2)$$

Since initial conditions are zero. Laplace Transform of (1.0.1) gives

$$s^2Y(s) + 4sY(s) + 3Y(s) = 2sX(s) + 4X(s) \quad (2.0.3)$$

$$Y(s) = \frac{2(s+2)}{s^2+4s+3}X(s) \quad (2.0.4)$$

$$= \frac{1}{s+3}X(s) + \frac{1}{s+1}X(s) \quad (2.0.5)$$

$$= \frac{1}{s+1} - \frac{1}{s+3} \quad (2.0.6)$$

From Lemma-2.1. Inverse Laplace transform of $Y(s)$ is given by

$$y(t) = e^{-t}u(t) - e^{-3t}u(t) \quad (2.0.7)$$

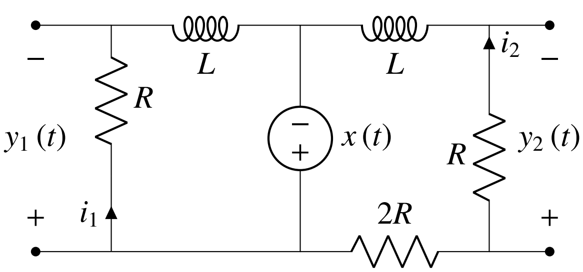
$$= \left(e^{-t} - e^{-3t}\right)u(t) \quad (2.0.8)$$

$\therefore$ The required option is B.

Corollary 2.2.1. *An RLC circuit that satisfies* (1.0.1). *Assume* $\frac{R}{L} = 1$.

$$\textit{Input} : x(t) \quad (2.0.9)$$

$$\textit{Output} : y(t) = y_1(t) + y_2(t) \quad (2.0.10)$$



Using KVL laws

$$x(t) - L\frac{di_2}{dt} - 3i_2R = 0 \quad (2.0.11)$$

$$y_2(t) = i_2R \quad (2.0.12)$$

$$x(t) - L\frac{di_1}{dt} - i_1R = 0 \quad (2.0.13)$$

$$y_1(t) = i_1R \quad (2.0.14)$$

Eliminating $y_1(t), y_2(t), i_1, i_2$ *from equations* (2.0.10) − (2.0.14) *we get* (1.0.1).

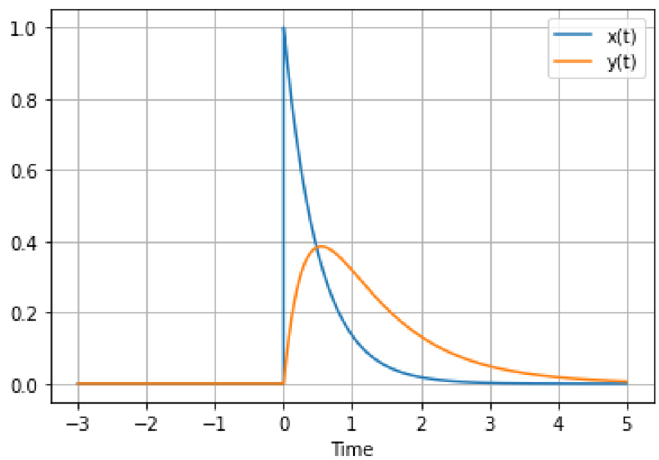


Fig. 4: Plot of input and output responses in time domain.