Assignment 5

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Download all python codes from

https://github.com/Adarsh541/EE3900/blob/main/EE3900_As5/codes/EE3900_As5.py

Download latex-tikz codes from

https://github.com/Adarsh541/EE3900/blob/main/EE3900_As5/EE3900_As5.tex

1 Problem(Quadratic forms Q.2.36)

Find the area bounded by the ellipse $\mathbf{x}^\top \begin{pmatrix} \frac{1}{a^2} & 0 \\ 0 & \frac{1}{b^2} \end{pmatrix} \mathbf{x} = 1$ and $x = ae$, where $b^2 = a^2\left(1 - e^2\right)$ and $e < 1$.

2 Solution

Given ellipse is

$$\mathbf{x}^\top \begin{pmatrix} \frac{1}{a^2} & 0 \\ 0 & \frac{1}{b^2} \end{pmatrix} \mathbf{x} = 1 \tag{2.0.1}$$

On comparing with the standard form

$$\mathbf{c} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{2.0.2}$$

$$\mathbf{D} = \begin{pmatrix} \frac{1}{a^2} & 0 \\ 0 & \frac{1}{b^2} \end{pmatrix} \tag{2.0.3}$$

$$\mathbf{u}^\top \mathbf{V}^{-1} \mathbf{u} - f = 1 \tag{2.0.4}$$

$$\lambda_1 = \frac{1}{a^2} \tag{2.0.5}$$

$$\lambda_2 = \frac{1}{b^2} \tag{2.0.6}$$

Semi major and minor axes of the ellipse are

$$\text{length of semi major axis} = \sqrt{\frac{\mathbf{u}^\top \mathbf{V}^{-1} \mathbf{u} - f}{\lambda_1}} \tag{2.0.7}$$

$$= a \tag{2.0.8}$$

$$\text{length of semi minor axis} = \sqrt{\frac{\mathbf{u}^\top \mathbf{V}^{-1} \mathbf{u} - f}{\lambda_2}} \tag{2.0.9}$$

$$= b \tag{2.0.10}$$

$\therefore$ equation of ellipse can be written as

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \tag{2.0.11}$$

$$\text{Area of ABCDA} = 2\left(Ar\left(ABDA\right)\right) \tag{2.0.12}$$

$$= 2\left(\int_{ae}^{a} y dx\right) \tag{2.0.13}$$

$$= \frac{2b}{a}\left(\int_{ae}^{a} \sqrt{a^2 - x^2} dx\right) \tag{2.0.14}$$

$$= \frac{2b}{a}\left[\frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_{ae}^{a} \tag{2.0.15}$$

$$= \frac{\pi ab}{2} - b^2 e - ab\sin^{-1} e \tag{2.0.16}$$

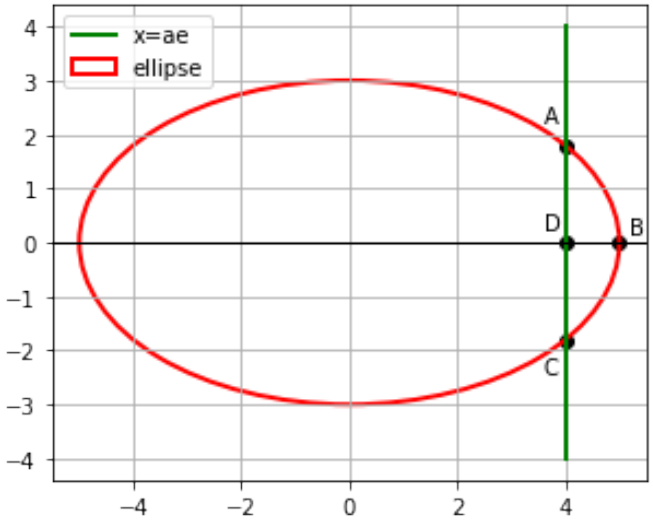

Fig. 0: Reference plot. Here $a = 5$, $b = 3$ and $e = \frac{4}{5}$