GATE 2005 EC Q.23

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Question

A continuous time LTI system is described by

$$\frac{d^2y(t)}{dt^2} + 4\frac{dy(t)}{dt} + 3y(t) = 2\frac{dx(t)}{dt} + 4x(t) \tag{1}$$

Assuming zero initial conditions, the response $y(t)$ of the above system for the input $x(t) = e^{-2t}u(t)$ is given by

1. $\left(e^{t} - e^{3t}\right) u(t)$
2. $\left(e^{-t} - e^{-3t}\right) u(t)$
3. $\left(e^{-t} + e^{-3t}\right) u(t)$
4. $\left(e^{t} + e^{3t}\right) u(t)$

Laplace Transform

Laplace transform

$$X(s) = \int_{-\infty}^{\infty} x(t)e^{-st}dt \tag{2}$$

Time Function $f(t) = \mathcal{L}^{-1}\{F(s)\}$	**Laplace transform** of f(t) $F(s) = \mathcal{L}\{f(t)\}$
$u(t)$	$\frac{1}{s}$, $s > 0$
$g'(t)$	$sG(s) - g(0)$
$g''(t)$	$s^2G(s) - sg(0) - g'(0)$
$e^{-at}u(t)$	$\frac{1}{s+a}$, $s + a > 0$

Linearity of Laplace Transform

$$\mathcal{L}\{af(t) + bg(t)\} = a\mathcal{L}\{f(t)\} + b\mathcal{L}\{g(t)\} \tag{3}$$

Solution

Laplace transform of $x\left(t\right) = e^{-2t}u\left(t\right)$ is given by

$$X\left(s\right) = \frac{1}{s+2} \tag{4}$$

Since initial conditions are zero. Laplace Transform of (1) gives

$$s^2Y\left(s\right) + 4sY\left(s\right) + 3Y\left(s\right) = 2sX\left(s\right) + 4X\left(s\right) \tag{5}$$

$$Y\left(s\right) = \frac{2\left(s+2\right)}{s^2+4s+3}X\left(s\right) \tag{6}$$

$$= \frac{1}{s+1} - \frac{1}{s+3} \tag{7}$$

Solution

Inverse Laplace transform of $Y(s)$ is given by

$$y(t) = e^{-t}u(t) - e^{-3t}u(t) \tag{8}$$

$$= \left(e^{-t} - e^{-3t}\right)u(t) \tag{9}$$

$\therefore$ The required option is B.