QUIZ2

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Download all python codes from

https://github.com/Adarsh541/EE3900/blob/main/quiz2/codes/quiz2.py

Download latex-tikz codes from

https://github.com/Adarsh541/EE3900/blob/main/quiz2/quiz2.tex

1 Problem(Q3.16(a,b))

When the input to an LTI system is

$$x[n] = \left(\frac{1}{3}\right)^n u[n] + (2)^n u[-n-1], \tag{1.0.1}$$

the corresponding output is

$$y[n] = s\left(\frac{1}{3}\right)^n u[n] - 5\left(\frac{2}{3}\right)^n u[n]. \tag{1.0.2}$$

1) Find the system function $H(z)$ of the system. Plot the pole(s) and zero(s) of $H(z)$ and indicate the region of convergence.
2) Find the impulse response $h[n]$ of the system.

2 Solution

Lemma 2.1.

$$a^n u[n] \overset{\mathcal{Z}}{\rightleftharpoons} \frac{z}{z-a}, \ ROC: |z| > |a| \tag{2.0.1}$$

$$-a^n u[-n-1] \overset{\mathcal{Z}}{\rightleftharpoons} \frac{z}{z-a}, \ ROC: |z| < |a| \tag{2.0.2}$$

$$\delta[n] \overset{\mathcal{Z}}{\rightleftharpoons} 1, \ ROC: Allz \tag{2.0.3}$$

Lemma 2.2.

$$x[n-n_0] \overset{\mathcal{Z}}{\rightleftharpoons} z^{-n_0} X(z) \tag{2.0.4}$$

1) Since Z-transform obeys linearity and from Lemma-2.1

$$X(z) = \frac{z}{z-\frac{1}{3}} - \frac{z}{z-2}, \ \text{ROC1: } \frac{1}{3} < |z| < 2 \tag{2.0.5}$$

$$Y(z) = 5\left(\frac{z}{z-\frac{1}{3}} - \frac{z}{z-\frac{2}{3}}\right), \ \text{ROC2: } |z| > \frac{2}{3} \tag{2.0.6}$$

$$H(z) = \frac{Y(z)}{X(z)} \tag{2.0.7}$$

$$= \frac{z-2}{z-\frac{2}{3}} \tag{2.0.8}$$

$$= 1 - \frac{4}{3}z^{-1}\left(\frac{z}{z-\frac{2}{3}}\right) \tag{2.0.9}$$

From (2.0.8)

$$Zero: z = 2 \tag{2.0.10}$$

$$Pole: z = \frac{2}{3} \tag{2.0.11}$$

$$ROC: ROC1 \cap ROC2 \tag{2.0.12}$$

$$: \frac{2}{3} < |z| < 2 \tag{2.0.13}$$

2) From Lemma-2.1, 2.2 and (2.0.9)

$$h[n] = \delta[n] - \frac{4}{3}\left(\frac{2}{3}\right)^{n-1} u[n-1] \tag{2.0.14}$$

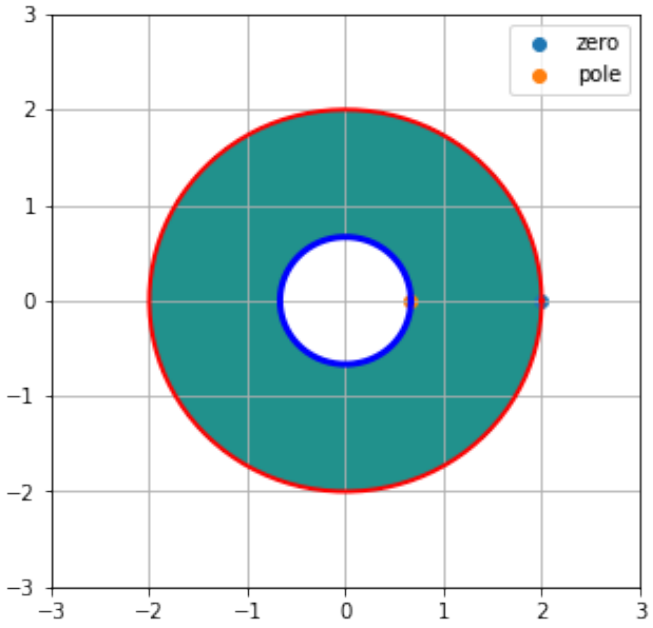

Fig. 1: Green region is the ROC.