# Fractions

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# 1 Introduction

## 1.1 What are fractions?

Fractions are a way of representing numbers in a rational form. Every rational number can be representing as a pair of integers in a fraction. Here are some examples:

$$\frac{1}{2} \text{ One Half}$$

$$\frac{3}{4} \text{ Three Quarters}$$

$$\frac{4}{3} \text{ Four Thirds}$$

## 1.2 Why Fractions?

Imagine you have a pizza that is $1ft^2$, and we split it into 3 slices. What is the exact size of an indivdual slice? Using the formula $\frac{Area}{Parts}$. We find that size per slice is approximately

$$\frac{1}{3} \approx 0.33333333 = \textit{Slice Area}$$

Now lets find how much pizza we have of we have 3 slices if the same size using the formula $\textit{Slice Area} \cdot Parts$.

$$.33333333 \cdot 3 = 0.9999999$$

Hold on, it seems like a small slice of our pizza has gone missing! If we split our $1ft^2$ pizza into 3 slices, and then reassembled it back together, we should end up with a pizza that is still $1ft^2$ right? The issues lies in our first calculation where we used an approximation($\approx$) to find the area of a slice meaning that we found an answer that is very close but not exact. How about we instead kept the size of a slice as a fraction and write it as $\frac{1}{3}$, and then apply it to out seconds formula?

$$\frac{1}{3} \cdot 3 = 1$$

We see that now we have a the orginal area of our pizza when we combined the area of all three slices represented as a fraction. Not only is answer correct, but we can also garuntee it to be accurate since it does not use any approximation.

# 2 Manipulating Fractions

## 2.1 Simplifying fractions

Sometimes it's earier to work with smaller numbers which is why we simplify fractions so that their numerator and denominator are the smallest they can get. Take this fraction as an example: $\frac{24}{40}$. This fraction can be hard to work and visualize with. By simplyfying this fraction we can rewrite it as $\frac{3}{5}$. To do this we must find a number that both the numerator and denominator can be divided by. Let's try 2.

$$\frac{24 \div 2}{40 \div 2} = \frac{12}{20}$$

Note that both of these fractions are the same, we are just rewriting them. We can cintue dividing both the numerator and denominator by some number:

$$\frac{12 \div 4}{20 \div 4} = \frac{3}{5}$$

Our fraction rewritten now as $\frac{3}{5}$ is the same as $\frac{24}{40}$.

$$\frac{24}{40} = \frac{3}{5}$$

## 2.2 Rewriting fractions

Every number can be written as itself over 1:

$$a = \frac{a}{1}$$

A number over itself will always equal to 1:

$$\frac{a}{a} = 1$$

A fraction can numerator and denomination can be split up:

$$\frac{a}{b} = \frac{a}{1} \cdot \frac{1}{b} = a \cdot \frac{1}{b}$$

And be split up further more by splittung up it's factors

$$\frac{ab}{cd} = a \cdot b \cdot \frac{1}{c} \cdot \frac{1}{d}$$

$$\frac{ab}{c} = \frac{a}{c} \cdot \frac{b}{c}$$

NOTE: These rules of rewriting fractions only apply to multiplication and division ONLY!

$$\frac{a+b}{c} \neq \frac{a}{c} + \frac{b}{c}$$

but, you CAN do this:

$$\frac{a+b}{c} = \frac{1}{c} \cdot (a+b)$$

## 2.3 Multiplying fractions

Multiplying two fractions are done by the numerators and denominators:

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d} = \frac{ac}{bd}$$

$$\frac{3}{7} \cdot \frac{5}{2} = \frac{3 \cdot 5}{7 \cdot 2} = \frac{15}{14}$$

## 2.4 Dividing fractions

Dividing fractions can be done through the Keep Change Flip method:

**Step 1: Keep the first fraction**

$$\frac{a}{b} \div \frac{c}{d}$$

**Step 2: Change the sign to multiplication**

$$\frac{a}{b} \cdot \frac{c}{d}$$

**Step 3: Flip the second fraction**

$$\frac{a}{b} \cdot \frac{d}{c}$$

**And now we only need to multiply like before**

$$\frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$$

Here is an example:

**Keep**

$$\frac{3}{7} \div \frac{2}{5}$$

**Change**

$$\frac{3}{7} \cdot \frac{2}{5}$$

**Flip**

$$\frac{3}{7} \cdot \frac{5}{2}$$

**Now multiply**

$$\frac{3}{7} \cdot \frac{5}{2} = \frac{15}{14}$$