Team ANYmation

Astrophysics Subsystem

Instructions

- This assignment has 5 questions. The points corresponding to each question/sub-part are given in the right side margin. The assignment is for a maximum of **300 p**.
- **You are free to use the internet (obviously) and any other resources**.
- The questions are based on some interesting phenomemon in astronomy. **There are no prerequisites for this assignment**. But there is a lot of astronomy jargon floating around. You are expected to browse for those terms over the internet.
- Questions 1, 3, 4 and 5 have coding component, so, any coding knowledge would be helpful. Coding **must be done in `python`**. You will mainly be needing `Numpy` and `Matplotlib` libraries, but you are free to use any other python library. For those who are not familiar with `python`, here are some tutorials (click). Tutorials 1, 2, 4 and 7 are relevant.
- Please use `jupyter` notebooks or Google colab (`.ipynb` extension) for questions 1, 3, 4 and 5. Please explain the steps and the chosen variable names clearly. For question 5, write the answers in the colab/jupyter notebook itself. Name the files `Q<question number>.ipynb`.
- For question 2, submit a `pdf` file named `Q2.pdf`.
- We will be using Git Classroom for taking the submissions. Create a GitHub account if you do not already have one. Here is the link to the assignment.
- If you have any comments/clarifications/doubts regarding the questions, contact us on our WhatsApp group.

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1. **Cosmology** 40 p

   Hubbles' law states that a galaxy's recessional velocity v is proportional to the distance d.

$$v = H_0 d$$

Where $H_0 = 70 \text{ kms}^{-1}/\text{Mpc}$ is called as the Hubble's parameter/constant. However this law is not a strict law, especially for nearby galaxies. Nearby galaxies have a random velocity v_r over the Hubble velocity with an RMS of σ. That is

$$v = H_0 d + v_r$$

Assuming that the number density of galaxies is constant in the distance range 1 Mpc to 100 Mpc, plot the velocity (magnitude) vs distance graph for the galaxies.
Hint: v_r is drawn from a Gaussian with zero mean and SD σ. Also note that v_r can be in any random direction but the v due to Hubble's law is radially outward.

2. **Radiative Cleaning** 40 p

A newly formed star is usually surrounded by a cloud of gas and dust. The dust aggregates, forms small rocks which further aggregate and eventually form a planetary system. But a lot of dust, which doesn't interact is slowly removed from the system by various mechanisms to get to a state similar to our solar system. How is the dust removed when the main force is gravitational? Well do not forget the **radiation pressure** due to the star! If radiation pressure is the dominant force, the dust particle obiviously moves out of the system. Dust particles come of various shapes (irregular shapes) and sizes, but it is reasonable to model it as a spherical particle of **density** ρ and **radius** r. Consider a dust particle at a **distance** d from a **star** of mass M. The star has a luminosity L.

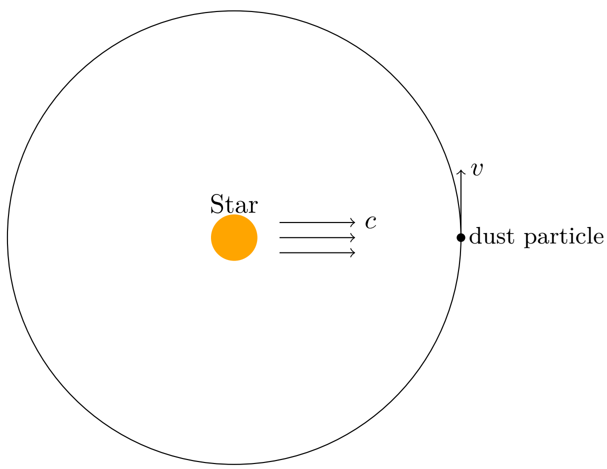
$$\text{Radiation Pressure}\,(P_r) = \frac{L}{4\pi d^2 c}$$

$$F_r = kP_r\pi r^2$$

Assume that $k = 1$.

(a) With the above assumptions find the ratio R of the gravitational force F_g to the force due to the radiation pressure F_r, on the dust particle, as a function of r, other required parameters and fundamental constants. 10 p

(b) Find is the radius r_0 of the dust particle if $R = 1$ and $\rho = 1000$ Kgm^{-3}. 5 p



Clearly if $R < 1$, the particle will eventually move out of the star system. But there are many dust particles with $R > 1$. What about them? Well, they spiral into the star due to drag caused by radiation. For simplicity, assume that the particle is in a circular orbit around the star with radius d and let the velocity of particle be v. In the reference frame of the particle, the photons from the star appear to be coming at an angle of $\arctan\left(\frac{v}{c}\right)$ with the radius vector (Ignoring relativistic effects which will be second order in $\frac{v}{c}$). So there is a tangential component of force due to radiation pressure $F_{r\text{-tangential}} \approx \frac{vF_r}{c}$.

(c) Find the torque τ due to radiative drag. 5 p

(d) Find the time taken by the particle to spiral into the star. Assume that the particle has a quasi-circular orbit at every instant. 20 p

3. **Transit Simulation** 60 p

Given a bright circle of radius r_1, uniformly emitting light, let another circle of radius r_2 pass in front of it with uniform velocity. The circles start out separate (no overlap) and end separate. The second circle is opaque and does not emit light, hence it causes a drop in total light intensity when it passes in front. Simulate this situation and plot out the light intensity (or logarithm of that, whichever plot looks better) over the whole period of the transit. Note that you can choose arbitrary units for intensity. For plotting choose some reasonable values of r_1 and r_2.

Hint: Calculating the area via integration will be complex. Instead, try to maintain an array of points over the whole larger sphere and keep track of which points are emitting light at the given timestep. Take the shadow into account by simply switching off the points within the shadow. Let the light intensity be some constant multiplied by the number of points emitting light. As your grid becomes finer, your answer will become more precise.

4. **Co-orbital Satellites** 60 p

Suppose that two small satellites of masses m_1 and m_2 are approximately co-orbital (moving on very similar orbits) around a large central body of mass M , with $m_1, m_2 \ll M$. At any instant, the orbits of the satellites may be approximated as circular Keplerian orbits with radii r_1 and r_2 respectively, although r_1 and r_2 will vary slightly over time due to the mutual gravitational interaction between the satellites.

Assume the planet is not affected by the gravity of the satellites. Simulate the motion of these satellites in the rotating reference frame centered at the planet, such that the total angular momentum of the system is zero in this frame.

Let it be given that both satellites have the same initial orbital radius and they start off on opposite sides of the earth. Take the radius where the period would be 6 hours if calculated using the radius. Let both satellites start with velocities corresponding to perfect keplerian orbits at that radius. Run the simulation for a long period and plot the trajectories in the given frame of reference in a plot. Use the python library, matplotlib. Describe the kind of motion you observe.

Hint: This might help : Euler-Richardson Integrator.

5. **Distance Estimation using Parallax** 100 p

Finding distance to an astronomical object is highly non-trivial and has multiple methods. Each method is suitable to measure distance in only a certain range. In this question, we will see how parallax is used to measure distance to stars. Note that this method works well for distances upto only a few thousand light years. **For the purpose of the question ignore proper motions of stars**

Parallax is the apparent shift of position of an object relative to distant background.

(a) Stellar parallax is dependent on which of the following factors? (Select all that apply) Explain each of the selected options in 1-2 sentences. 5 p

A. Distance to the Star

B. Size of the Star

C. Distance between observer's positions.

To measure the distance to a star, we keep track of its position in the sky over a year. The position of star changes slightly with respect to the distant stars due to the parallax caused by the variation in position of Earth in its orbit over a year. Apart from parallax there is one more effect that causes shifts in positions of stars over a year.

(b) Name the other significant effect that causes shift in positions of stars in the sky. Also describe the effect in a few sentences. 10 p

Hint: This effect is very similar to the effect in the question on **Radiative Cleaning**.

Before proceeding further please read up on the Ecliptic Coordinate System. Just read the definitions of terms used below.

Due to the parallax and the other effect (in part (b)), ecliptic coordinates of stellar objects are a function of Earth's position in its orbit. Let the ecliptic coordinates of a star S be ($\beta + \Delta\beta$, $\lambda + \Delta\lambda$).

$$\Delta\beta = \sin\beta[\Pi\cos(\lambda_\odot - \lambda) - k\sin(\lambda_\odot - \lambda)]$$

$$\Delta\lambda = \frac{1}{\cos\beta}[\Pi\sin(\lambda_\odot - \lambda) - k\cos(\lambda_\odot - \lambda)]$$

$$\lambda_\odot = \omega t$$

$$\Pi = \frac{a_\oplus}{d}$$

$$k = \frac{v}{c}$$

Description of parameters:

- β, λ - respectively are the mean ecliptic latitude and mean ecliptic longitude of S
- $\lambda_\odot$ - the ecliptic longitude of Sun
- ω - the angular velocity of Earth around Sun (assume nearly circular orbit)
- t - time measured from Spring (Vernal) Equinox, that is, March 21^{st}
- $a_\oplus$ - orbital radius of Earth about Sun
- d - distance to the star S
- c - speed of light
- v - orbital velocity of Earth about Sun

(c) What will be the shape of curve formed by plotting $\Delta\beta$ vs $\Delta\lambda$? 5 p

(d) The file `data.csv` (in Q5 folder) has measurements of ecliptic latitude $\beta(t)$, ecliptic longitude $\lambda(t)$ and time t (in days from Vernal/Spring equinox) of a Star. **Plot** $\beta(t)$ vs $\lambda(t)$ and determine the parameters : β (**mean ecliptic latitude**), λ (**mean ecliptic longitude**) and the **distance** d to the star. **Briefly explain** your approach for finding the parameters. 80 p

Note that the measurements have only been done for a part of year (because for the rest of the duration, star is not observable due to its close proximity to Sun). The data also has some measurement errors (random errors due to instrument). **Please use the following values only** (All values are given in SI units)

- $G = 6.67430 \times 10^{-11}$
- Mass of Sun $= 1.981 \times 10^{30}$
- $c = 299792458$
- $\pi = 3.141592653589793$
- 1 year = 365.25 days
- $a_\oplus = 1.496 \times 10^{11}$
- $1° = \frac{180}{\pi}$

Hint: A general 2^{nd} degree curve can be written as $Ax^2 + Bxy + (1-A)y^2 + Dx + Ey + F = 0$. `Numpy` has a function that does least square fitting.