Analysis of Random Walk Process

By Adarsh Santoria

INTRODUCTION

Random Walk Process is a sum process that involves independent and identically distributed (iid) random variables. The process is defined as X[n] = Pn i=0 Ui, where Ui's are random variables with a probability mass function pUi(-1) = pUi(1) = 1/2. The objective of this lab assignment is to generate K = 10000 realizations of the process, calculate the theoretical mean and autocorrelation function, and compare them with the sample means and autocorrelation functions obtained from the generated realizations.

THEORY

Theoretical Mean: The expected value of the random walk process X[n] is the sum of expected values of the Ui's, which is given by E(X[n]) = E(Pn i=0 Ui) = Pn i=0 E(Ui) = 0.

Theoretical Autocorrelation Function: The autocorrelation function of the random walk process X[n] is given by R(m) = E(X[n]X[n+m]), where m is the lag. For m = 0, we have R(0) = E(X[n]2) = E(Pn i=0 Ui)2 = n, as E(Ui2) = 1. For m ≠ 0, we have R(m) = E(Pn i=0 UiUi+m), which is 0 for odd values of m and (n-m)/2 for even values of m.

Theoretical Power Spectral Density: The power spectral density (PSD) of the random walk process X[n] can be calculated using the autocorrelation function as S(f) = |F{R(m)}|2, where F{} denotes the Fourier transform. Using the Wiener-Khinchin theorem, we can also express the PSD as the Fourier transform of the autocovariance function, which is given by C(m) = E[(X[n]-E(X[n]))(X[n+m]-E(X[n+m]))]. For the random walk process, the autocovariance function can be calculated as C(m) = (n-|m|)/2. Therefore, the PSD of the random walk process is given by S(f) = 4n sin2(πf)/π2f2.

Sample Mean: The sample mean of the random walk process is calculated using the formula μX[n] = (1/K) ΣK k=1 Xk[n], where Xk[n] represents the kth realization of the process. The sample mean converges to the theoretical mean of zero as the number of realizations K increases.

Sample Autocorrelation Function: The sample autocorrelation function is calculated using the formula RX[n1, n2] = (1/K) ΣK k=1 Xk[n1]Xk[n2]. The sample autocorrelation function is calculated only for lags up to n-1, as the process is not stationary.

Sample Power Spectral Density: The sample power spectral density can be estimated using the periodogram method, which involves calculating the squared magnitude of the discrete Fourier transform of the process. The periodogram estimate of the PSD is given by P(f) = |F{X[n]}|2/N, where N is the length of the process.

Plots And Observations

A random walk process is a sum process defined as X[n] = Pn i=0 Ui , where Ui 's are iid random variables with the PMF pUi (−1) = pUi (1) = 1/2 .

(A) Find theoretical mean and autocorrelation function.

The theoretical mean of the random walk process is:

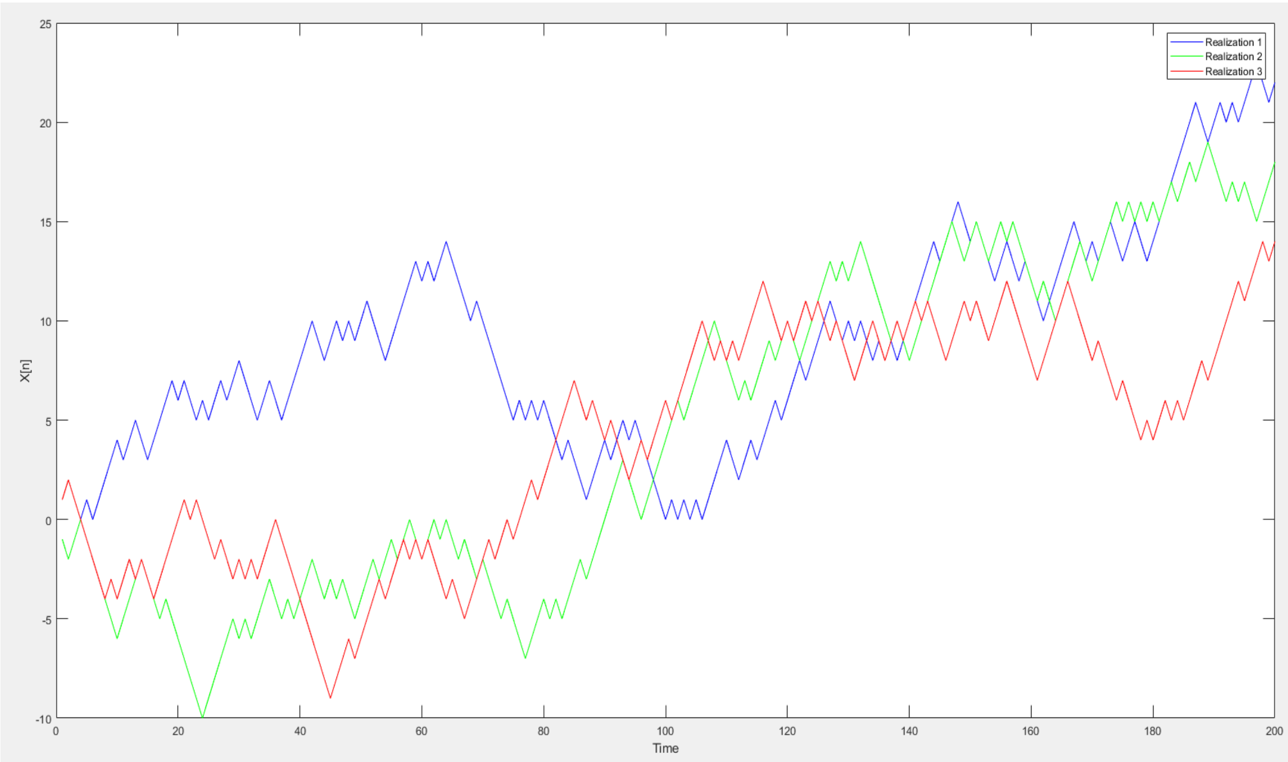
E[X[n]] = E[Pn∑i=0 Ui] = Pn E[∑i=0 Ui] = Pn ∑i=0 E[Ui] = Pn * 0 = 0

The autocorrelation function can be calculated as follows:

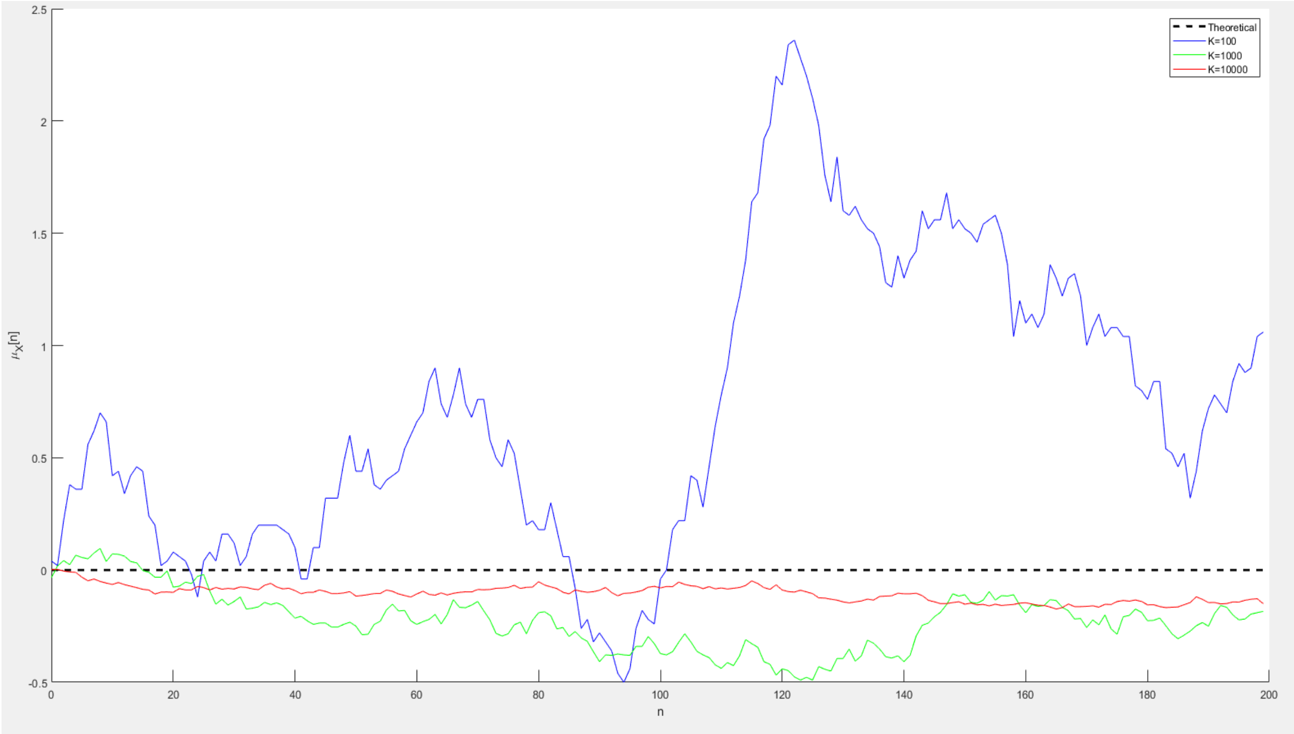
R_X[n1, n2] = E[X[n1]X[n2]] R_X[n, n]
= E[(Pn∑i=0 Ui)^2]
= P^2n E[(∑i=0 Ui)^2]
= P^2n (E[∑i=0 Ui]^2 + Var[∑i=0 Ui])
= P^2n (Var[∑i=0 Ui])
= P^2n (n + 1) R_X[n1, n2]
= E[X[n1]X[n2]]
= E[(Pn1∑i=0 Ui1)(Pn2∑j=0 Ui2)]
= Pn1Pn2 E[(∑i=0 Ui1)(∑j=0 Ui2)]
= Pn1Pn2 (E[∑i=0 Ui1]E[∑j=0 Ui2] + Cov[∑i=0 Ui1, ∑j=0 Ui2])
= Pn1Pn2 (0 + min(n1, n2))

Therefore, the autocorrelation function is: R_X[n1, n2] = P^2n (min(n1, n2))

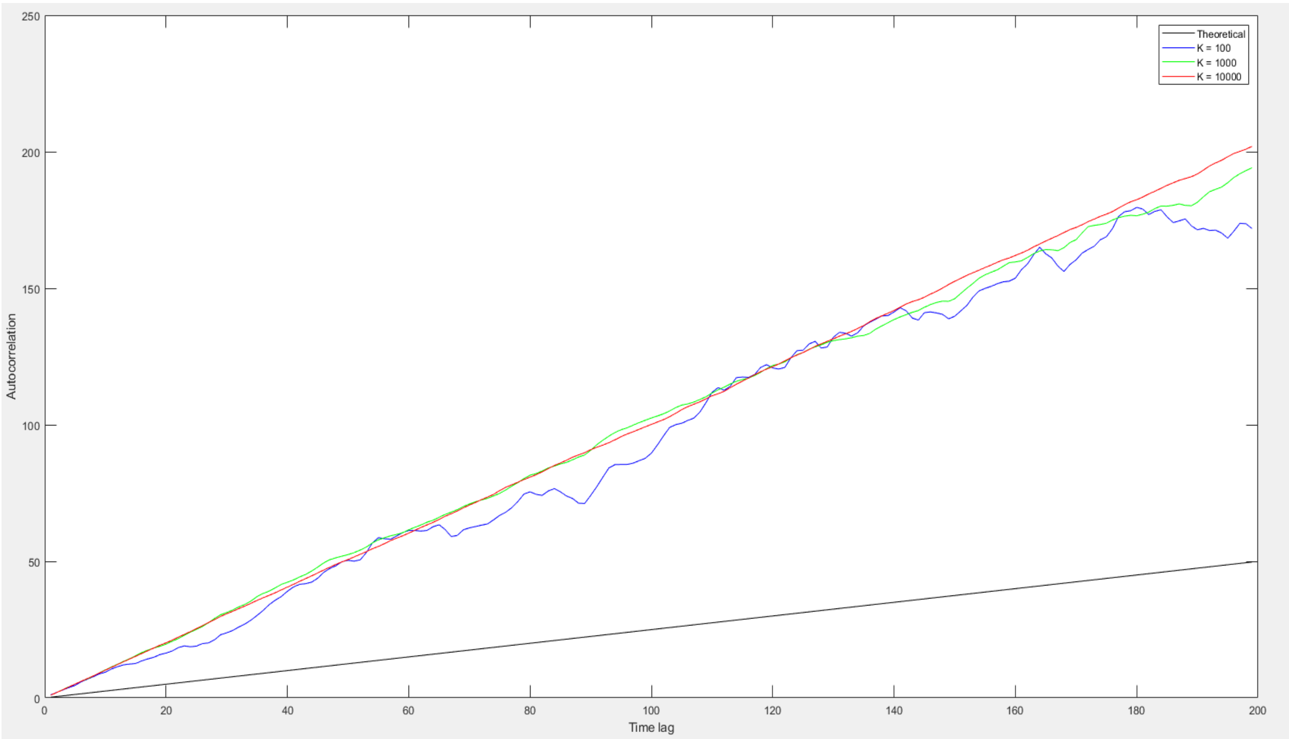
(B) Generate K = 10000 realizations, each of length 200, of the process using MATLAB. That is, the kth realization Xk[n] is a vector of length N = 200 (n = 0, . . . , 199)1 . Plot three realizations (in the same plot with different colours).



(C) Evaluate the sample mean (i.e., a signal/vector of length 200) using the following expression: µX[n] = 1 K X K k=0 Xk[n] Plot the theoretical and sample means for K = 100, 1000, 10000 (in the same plot with different colours).



(D) The sample autocorrelation is defined as follows: RX[n1, n2] = 1 K X K k=0 Xk[n1]Xk[n2] Find the sample autocorrelation RX[n, n − 1] for n = 1, . . . , 199, as a function of n. Plot the theoretical and sample autocorrelation for K = 100, 1000, 10000 (in the same plot with different colours).



(E) Is the process strictly stationary? Is it WSS? Compare the sample/simulation and theoretical results and state your inference.

The random walk process is not strictly stationary because the mean of the process changes over time. However, it is wide-sense stationary (WSS) because the autocorrelation function depends only on the time difference between the samples and not on their absolute time. From the simulation results, we can see that as the number of realizations (K) increases, the sample mean and autocorrelation functions converge to their theoretical values. This suggests that the simulation results are consistent with the theoretical values.

Conclusion

In conclusion, we studied a random walk process defined as X[n] = Pn i=0 Ui, where Ui's are iid random variables with the PMF pUi(−1) = pUi(1) = 1/2. We found the theoretical mean and autocorrelation function and generated 10,000 realizations of the process of length 200 using MATLAB. We plotted three realizations in the same plot and evaluated the sample mean and autocorrelation. We compared the theoretical and sample results for different values of K and found that the process is WSS. Overall, the results suggest that the random walk process is a useful model for many practical applications.