IC160P-Electrical Systems Around Us Lab

Frequency Response of R – C and R – L – C Circuits

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1.Objectives

To understand Frequency Response of R – C and R – L – C Circuits

2.Theory

Introduction to sinusoidal waveform

A sinusoidal waveform (say voltage) is described by the following relation (1), where, Vm is the amplitude (peak or maximum value) of the wave, ω is the angular frequency (given by (2), where, f is the frequency in Hz) and ϕ is the phase relative to a reference wave with $\phi = 0^0$.

$v = Vm \sin(\omega t + \phi) \quad (1)$

$\omega = 2\pi f \quad (2)$

If the phase ϕ is positive with respect to the reference (which has $\phi = 0^0$), the wave is said to lead the reference, if ϕ is negative, the wave lags the reference. Fig. 1 shows the reference as well as leading and lagging voltage waveforms.

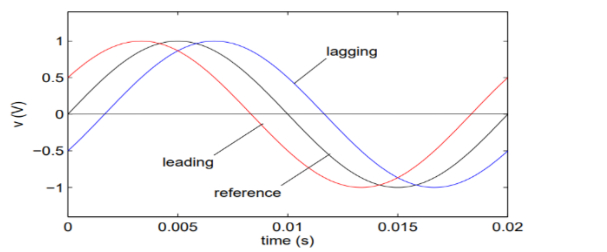


Fig. 1: Lagging and Leading Voltage Waveforms with unit magnitude

1. Average Value = $\int_0^T Vm \sin(\omega t + \varphi)\, dt = 0$

2. RMS Value $= \sqrt{\frac{1}{T}\int_0^T Vm^2 \sin^2(\omega t + \varphi)dt} = \frac{Vm}{\sqrt{2}}$

If a sinusoidal source is connected to a network comprising linear passive elements, and after the transients have died down, every voltage and current in the network is sinusoidal with the same frequency as that of the source. Such a response is termed as sinusoidal steady-state response.

In sinusoidal steady state, the impedance can be represented as a complex quantity where, the resistive parameter is the real part and the inductive and capacitive parameters represent the imaginary part with appropriate sign (positive for inductance and negative for capacitance). Thus, complex impedance associated with different circuit parameters can be given as showed in (3).

$Z_R(j\omega) = R + j0$

$Z_L(j\omega) = 0 + j\omega L$

$Z_C(j\omega) = 0 - j\,(1/\,\omega C)$

3.Procedure and Circuit Diagrams

Consider three circuits consisting of a resistance connected in series with first a capacitance, then an inductor and lastly with both respectively connected across a sinusoidal voltage source of magnitude Vm and frequency f, as shown in Fig. 2. The instantaneous value of the voltage at time t is then given by (1).

Sinusoidal steady-state response of an R – C circuit

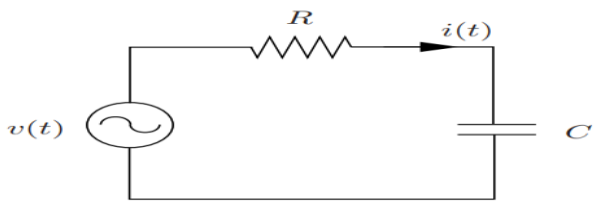
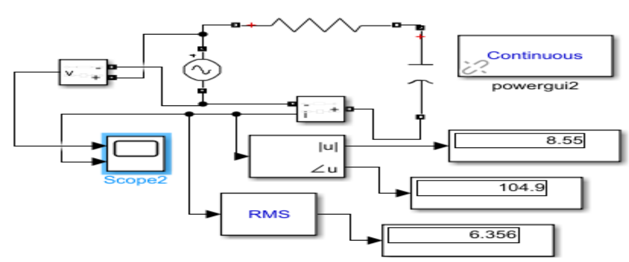


Fig. 2: A series $R - C$ circuit



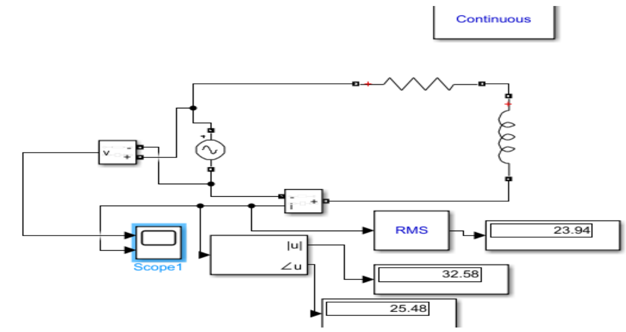
Writing a KVL around the loop gives (4), assuming i(t) = 0 before t = 0.

v = R i + (1/C) $\int_0^T$ i dt (4)

Results on solving:-

i = (V/ i|Z|) sin(ωt + ϕ), where, ϕ = tan^{-1}(1/ωCR) and Z = R – j (1/ωC) (5)

Sinusoidal steady-state response of an R – L circuit



Writing a KVL around the loop gives (6), assuming i(t) = 0 before t = 0.

v = R i + L di/ dt (6)

Results on solving:-

i = (V/ i|Z|) sin(ωt + ϕ), where, ϕ = tan^{-1}(wL/R) and Z = R +j (wL) (7)

Sinusoidal steady-state response of an R – L - C circuit

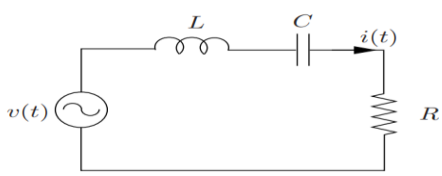


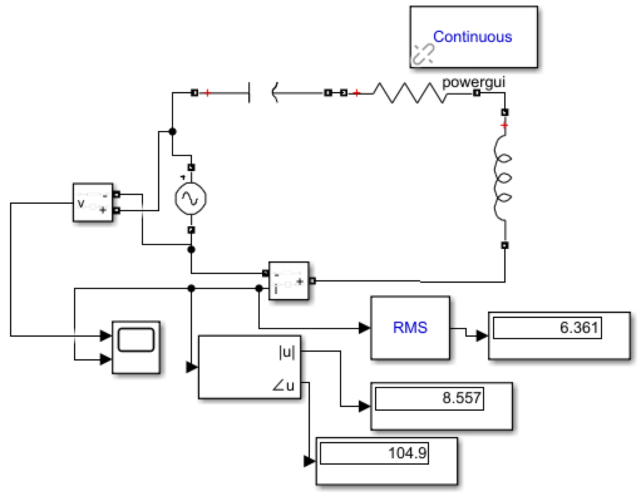
Fig. 3: A series $R - L - C$ circuit

Writing a KVL around the loop gives (8), assuming i(t) = 0 before t = 0.

v = R i + L di/ dt + (1/C) $\int_0^T$ i dt (8)

Results on solving:-

i = (V/ i|Z|) sin(ωt + ϕ), where, ϕ = tan^{-1}((wL-1/ωC)/R) and Z = R +j (wL) – j (1/ωC) (9)



3. Graphs and Analysis

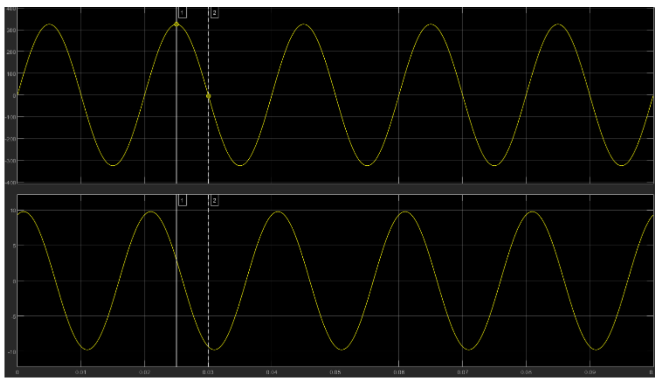
Frequency response of an R – C circuit

Consider that the output voltage is measured across the capacitor. Using voltage divider rule, the output voltage magnitude can be calculated as given by (10).

|vC| = |–j/ωC(R – j/ωC)| Vm (10)

Equation in (6) can be rearranged and written as (11), where, H(ω) is represents frequency response of the circuit, and |H(ω)| is its magnitude.

|vC|/Vm = |H(ω)| = 1/(1 + $(\omega RC)^2$) (11)



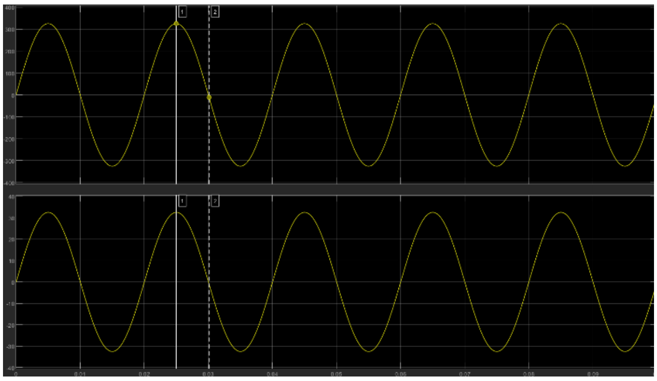
Frequency response of an R – L circuit

Consider that the output voltage is measured across the capacitor. Using voltage divider rule, the output voltage magnitude can be calculated as given by (12).

|vL| = |jwL/(R+jwL)| Vm (12)

Equation in (12) can be rearranged and written as (13), where, H(ω) is represents frequency response of the circuit, and |H(ω)| is its magnitude.

|vL|/Vm = |H(ω)| = 1/(1 + $(R/wL)^2$) (13)



Frequency response of an R − L − C circuit

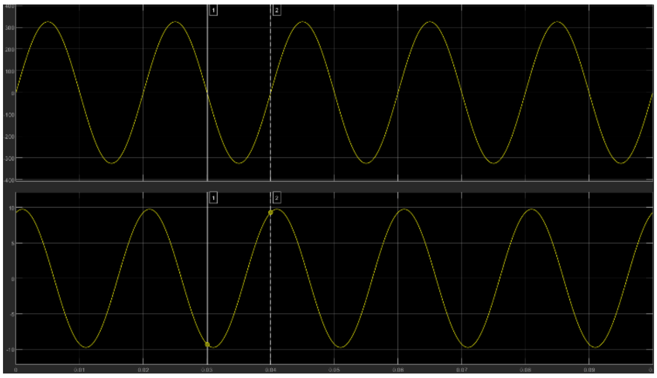
Consider a circuit consisting of a resistance connected in series with a capacitance and inductance connected across a sinusoidal voltage source of magnitude Vm and frequency f, as shown in Fig. 3.

Consider that the output voltage is measured across the resistor. Using voltage divider rule, the output voltage magnitude can be calculated as given by (14).
|vR|/ Vm = |R/(R + j(wl-1/wc))| (14)

Equation in (14) can be rearranged and written as (), where, H(ω) is represents frequency response of the circuit, and |H(ω)| is its magnitude.

|vR|/Vm = |H(ω)| = ωRC/√($(\omega RC)^2 + (\omega^2 LC - 1)^2$) (15)

From (15), the frequency at which the value of |H(ω)| is maximum can be found out, this frequency is called as frequency of resonance.



4.Conclusion

From the above experiments, we get that if (xL-xC) is positive current lags voltage , if negative current leads voltage and if zero this is the condition of resonance in phasor. Here, xL or xC can be zero which completely ignore it's effect.

5.Questions

1.What can be said about the nature of frequency response?

Nature of frequency response describes the nature of the circuit about it's magnitude and phase.

2. Replace capacitor C by an inductor L, what will be the frequency response of this new circuit?

On replacing C with L, direction of phase reverses and magnitude may change as it depends on formers.

3. If we were to use the R−C circuit discussed with voltage across capacitor as the output, what kind of filter will it act as (low-pass or high-pass)?

There will be Low-pass type of filter as voltage decreases on increasing frequency.

4. Consider that the output voltage is taken across R instead of C, calculate the frequency response. Also, answer Q.3 for this case.

Vout =Vt-VC

There will be High-pass type of filter as voltage increases on increasing frequency.

5. Find out resonance frequency from (15)

Resonant Frequency= 2π$1/\sqrt{LC}$