IC252 - Lab 5

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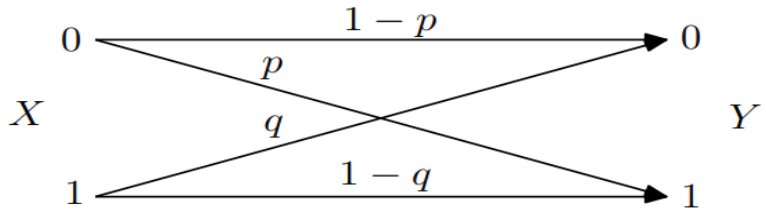
Questions with code and terminal displayed :-

Preamble: In practice, we use channels (e.g., mobile communication via a wireless channel) to communicate information. But in the physical world, the channels are usually not “perfect”. That is, due to the noise in the channels, a transmitted message may be received as some other message. In this lab session, we shall study a model for such channel and understand marginal, conditional and joint distributions and their interrelation via the model.

Problem statement: The input messages to a binary channel are chosen from the set {0, 1} with probability p(X = 0) and p(X = 1). Output of the channel is a stream of messages from the set {0, 1} with probability P(Y = 0) and P(Y = 1). In a binary channel, an input message 0 (1) is altered to 1 (0) with probability p (q). Hence,

$P(Y = 0|X = 0) = 1 - p$, (1) $P(Y = 1|X = 0) = p$, (2) $P(Y = 1|X = 1) = 1 - q$, (3) $P(Y = 0|X = 1) = q$. (4)

A binary channel is depicted in the following figure.



Let p = .25, q = .35. Let the length of the input stream be 10000, in which the messages are drawn according to the uniform distribution, i.e., P(X = 0) = .5 and P(X = 1) = .5.

1. (a) Find via simulation the distribution of Y and plot it. (b) Verify that the distribution $P(Y = y)$, $y \in \{0, 1\}$ obtained by simulation approximately matches the values analytically given by $P(Y = 0) = P(Y = 0|X = 0)P(X = 0) + P(Y = 0|X = 1)P(X = 1)$, $P(Y = 1) = P(Y = 1|X = 0)P(X = 0) + P(Y = 1|X = 1)P(X = 1)$.

2. (a) Find via simulation the joint distribution for (X, Y) and plot it. (b) Verify that the conditional distribution $P(Y = y|X = x)$, $x, y \in \{0, 1\}$ obtained by simulation approximately matches the conditional probabilities (1)-(4) via the equality $P(Y = y|X = x) = P(X = x, Y = y)\ P(X = x)$.

Hint: To obtain an input sequence of size 10000, you need to use a random number generator which generates 0 with probability .5 and 1 with probability .5. To implement the channel, you again need to use two random number generators as follows. If X = 0 then Generator 1 should generate (i.e., output Y) the number 0 with probability .75 and 1 with probability .25. If X = 1 then Generator 2 should generate (i.e., output Y) the number 0 with probability .35 and 1 with probability .65.

Spyder

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C:\Users\Adars\.spyder-py3

C:\Users\Adars\.spyder-py3\untitled0.py

temp.py | untitled0.py* | untitled1.py*

```
'''a'''
import random as r                  #importing modules
import matplotlib.pyplot as plt
I=[]
N=10000                             #N is number of simulations
for i in range (N):
    I.append(r.randint(0,1))        #randomly taking values for X
print("P(X=0)=",I.count(0)/N)
print("P(X=1)=",I.count(1)/N)
O=[]
x=int(0.25*100)*[1]+int(0.75*100)*[0] #setting probability for 0 & 1 in Y
y=int(0.65*100)*[1]+int(0.35*100)*[0]
for i in I:
   if(i==0):
       O.append(r.choice(x))
   elif(i==1):
       O.append(r.choice(y))
y0=O.count(0)/N
y1=O.count(1)/N
print("P(Y=0)=",y0)
print("P(Y=1)=",y1)
yvalues = ['0','1']            #plotting of graph
prob= [y0,y1]
plt.bar(yvalues,prob)
plt.xlabel("Values of Y")
plt.ylabel("Probability")
plt.title("Distribution of Y")
plt.show()
'''b'''
a=0
b=0
c=0
d=0
for i in range(10000):          #finding requiured outputs
    if(I[i]==O[i]==0):
        a=a+1
    elif(I[i]==O[i]==1):
        b=b+1
    elif(I[i]==0) and (O[i]==1):
        c=c+1
```

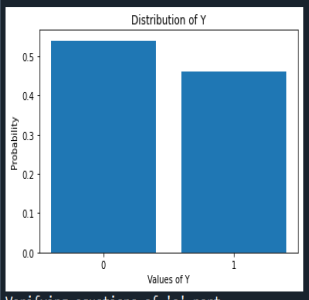
Source Editor Object

Built-in mutable sequence.

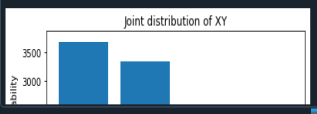
Help | Variable Explorer | Plots | Files

Console 1/A

```
In [34]: runfile('C:/Users/Adars/.spyder-py3/untitled0.py', wdir='C:/Users/Adars/.spyder-py3')
P(X=0)= 0.495
P(X=1)= 0.505
P(Y=0)= 0.5401
P(Y=1)= 0.4599
```




```
Verifying equations of 'a' part
P(Y=0 0.5401
P(Y=1 0.4599
P(Y=0|X=0)= 0.7448484848484849
P(Y=1|X=1)= 0.6605940594059406
P(Y=1|X=0)= 0.2551515151515513
P(Y=0|X=1)= 0.33940594059405943
```



IPython Console | History

LSP Python: ready | custom (Python 3.7.9) | Line 34, Col 59 | UTF-8 | CRLF

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ENG IN

Spyder

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C:\Users\Adars\.spyder-py3

C:\Users\Adars\.spyder-py3\untitled0.py

temp.py | untitled0.py* | untitled1.py*

```
22  yvalues = ['0','1']            #plotting of graph
23  prob= [y0,y1]
24  plt.bar(yvalues,prob)
25  plt.xlabel("Values of Y")
26  plt.ylabel("Probability")
27  plt.title("Distribution of Y")
28  plt.show()
29  '''b'''
30  a=0
31  b=0
32  c=0
33  d=0
34  for i in range(10000):          #finding requiured outputs
35      if(I[i]==O[i]==0):
36          a=a+1
37      elif(I[i]==O[i]==1):
38          b=b+1
39      elif(I[i]==0) and (O[i]==1):
40          c=c+1
41      elif(I[i]==1) and (O[i]==0):
42          d=d+1
43  y0x0=a/I.count(0)
44  y1x1=b/I.count(1)
45  y1x0=c/I.count(0)
46  y0x1=d/I.count(1)
47  print("Verifying equations of 'a' part")
48  print("P(Y=0",y0x0*I.count(0)/N+y0x1*I.count(1)/N)
49  print("P(Y=1",y1x0*I.count(0)/N+y1x1*I.count(1)/N)
50  print("P(Y=0|X=0)=",y0x0)
51  print("P(Y=1|X=1)=",y1x1)
52  print("P(Y=1|X=0)=",y1x0)
53  print("P(Y=0|X=1)=",y0x1)            #plotting of graph
54  yval = ['Y=0,X=0' ,'Y=1,X=1' ,'Y=1,X=0' ,'Y=0,X=1']
55  prob= [a,b,c,d]
56  plt.bar(yval,prob)
57  plt.xlabel("XY")
58  plt.ylabel("Outcomes")
59  plt.title("Joint distribution of XY")
60
```

Source Editor Object

Built-in mutable sequence.

Help

Console 1/A

```
Verifying equations of 'a' part
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P(Y=1 0.4599
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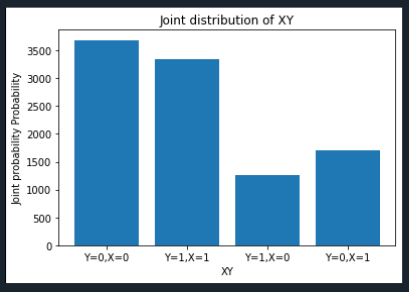

In [35]:

LSP Pyth

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