Control Systems

Lab Assessment #3

Date of experiment – 02/09/2023

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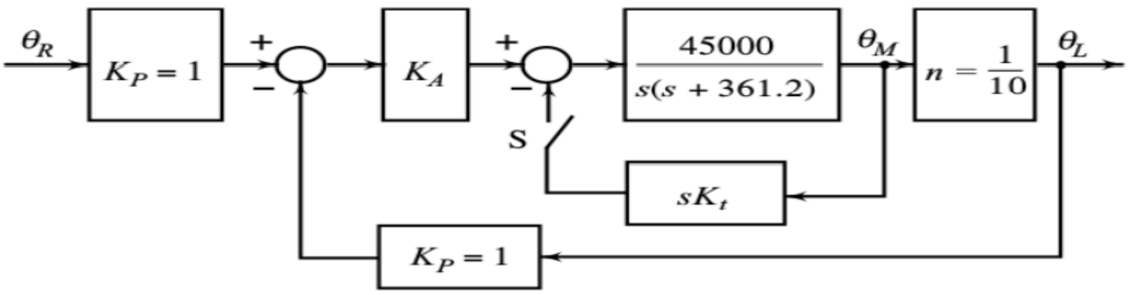
Objective:

To observe and analyse the response and behaviour of position control System, and fine tune the parameters of the system to get the desired behaviour.
The system is as follows:



Experiment Design:

1. Tuning the KA for the given conditions:

By analysing the zeta and tuning,

1) we have,

$$\zeta = \frac{180.6}{\sqrt{4500K_A}}$$

(a) over damped.

For over damped $\zeta > 1$

$$\frac{180.6}{\sqrt{4500K_A}} > 1$$

$$\frac{\sqrt{4500K_A}}{180.6} < 1$$

$$K_A < \frac{180.6^2}{4500}$$

$$K_A < 7.248$$

(b) Critically damped.

$\zeta = 1$

$\therefore K_A = 7.248$

(c) Under damped

$$0 < \zeta < 1$$

$$0 < \frac{180.6}{\sqrt{4500K_A}} < 1$$

$$\infty > \frac{\sqrt{4500K_A}}{180.6} > 1$$

$$4500K_A > 180.6^2$$

$$K_A > \frac{180.6^2}{4500}$$

$$K_A > 7.248$$

- Under Damped:

  KA > 7.248
- Critically Damped:

  KA = 7.248
- Over Damped:

  KA < 7.248

2. Finding the given metrices:

Let us take KA as 20, which is greater than 7.248 i.e. the system is in Under Damped mode.

2)

Under Damped

Let.

$KA = 20$

$$\zeta = \frac{180.6}{\sqrt{4500 \times 20}} = 0.602$$

$$\omega_n = \sqrt{4500 \times 20} = 300 \text{ rad/s}$$

$$\omega_d = \omega_n \sqrt{1-\zeta^2} = 300 \times \sqrt{1-0.602^2}$$

$$= 239.55 \text{ rad/s}$$

$$t_r = \frac{\pi - \beta}{\omega_d} = \frac{\pi - \tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right)}{239.55} = \frac{\pi - 0.925}{239.55} = 0.00925 \text{ s.}$$

$$\%M = e^{-\pi\zeta/\sqrt{1-\zeta^2}} \times 100$$

$$= 9.35\%$$

$$t_p = \frac{\pi}{\omega_d} = 0.013 \text{ s}$$

$$2\% \; t_s = \frac{4}{\omega_n \zeta} = 0.022 \text{ s}$$

Code:

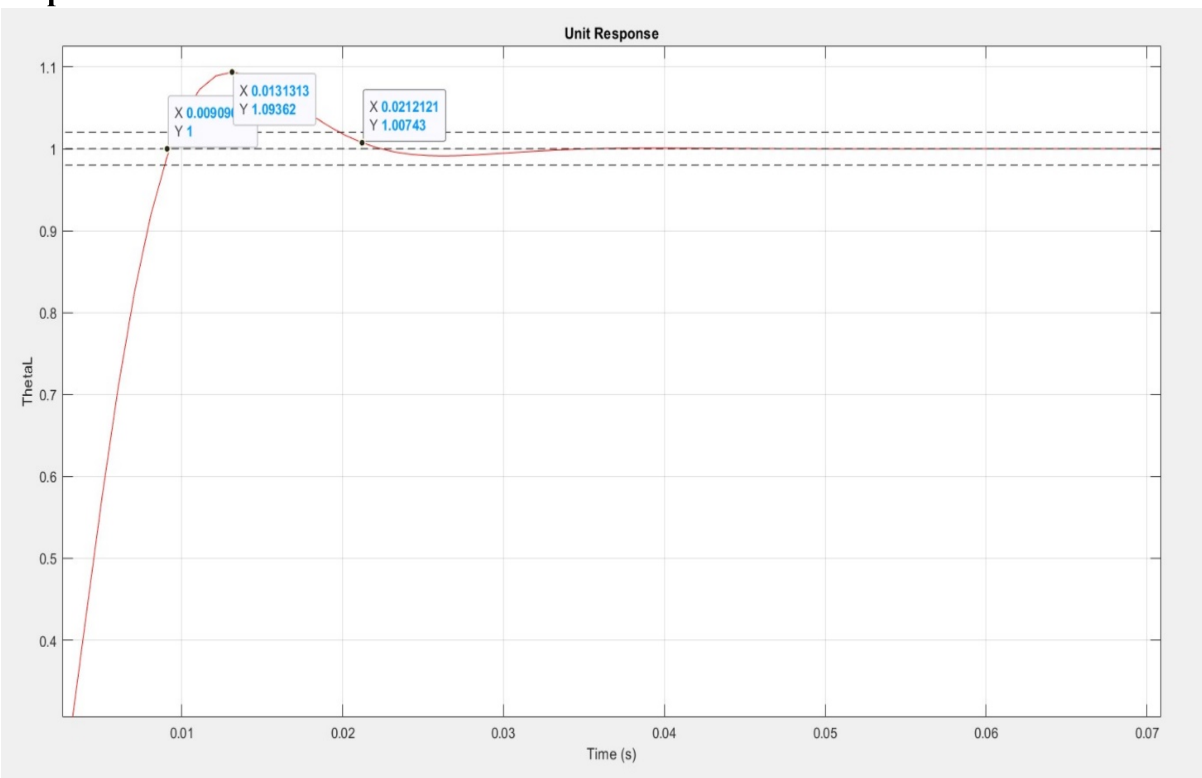
```
syms s;
k = 20*4500;
trans_func = k/(s*s+s*361.2+k);

respt = ilaplace(trans_func/s);
t = linspace(0,0.1,100);
% Plotting the response
plot(t, subs(respt,t),"Color",'red');
hold on;
plot(t,ones(1,100), '--', "Color",'black');
plot(t,ones(1,100)-.02, '--', "Color",'black');
plot(t,ones(1,100)+.02, '--', "Color",'black');
grid on;
xlabel("Time (s)")
ylabel("ThetaL")
title("Unit Response")

% Getting the metrices
stepinfo(tf([k],[1 361.2 k]))
```

Observations:

- We can see that the values we calculated are approximately the same as what we got. The error might be due to the no of samples in the time scale we considered.

- The settling time which we calculated is 2%, whereas the min and max are given by the "stepinfo" function.

Response:



Stepinfo:

```
TransientTime: 0.0198
 SettlingTime: 0.0198
  SettlingMin: 0.9046
  SettlingMax: 1.0936
    Overshoot: 9.3569
   Undershoot: 0
         Peak: 1.0936
     PeakTime: 0.0130
```

3. Plotting all the three cases:

Code:

```
ka = [20 7.248 5];
color = ["red" "green" "blue"];

% Plotting the Responses
for i=1:3
    syms s;
    k = ka(i)*4500;
    trans_func = k/(s*s+s*361.2+k);

    respt = ilaplace(trans_func/s);
    t = linspace(0,0.1,100);
    plot(t, subs(respt,t),"Color",color(i));
    hold on;
end
plot(t,ones(1,100), '--', "Color",'black');
plot(t,ones(1,100)-.02, '--', "Color",'black');
plot(t,ones(1,100)+.02, '--', "Color",'black');
grid on;
xlabel("Time (s)")
ylabel("ThetaL")
title("Unit Response")
legend(["Under Damped", "Critically Damped", "Over Damped"]);

% Plotting the poles
figure;
for i=1:3
    k = ka(i)*4500;
    pzplot(tf([k],[1 361.2 k]),color(i));
    hold on;
end
grid on;
xlabel("sigma")
ylabel("jwd")
title("Poles")
legend(["Under Damped", "Critically Damped", "Over Damped"]);
```

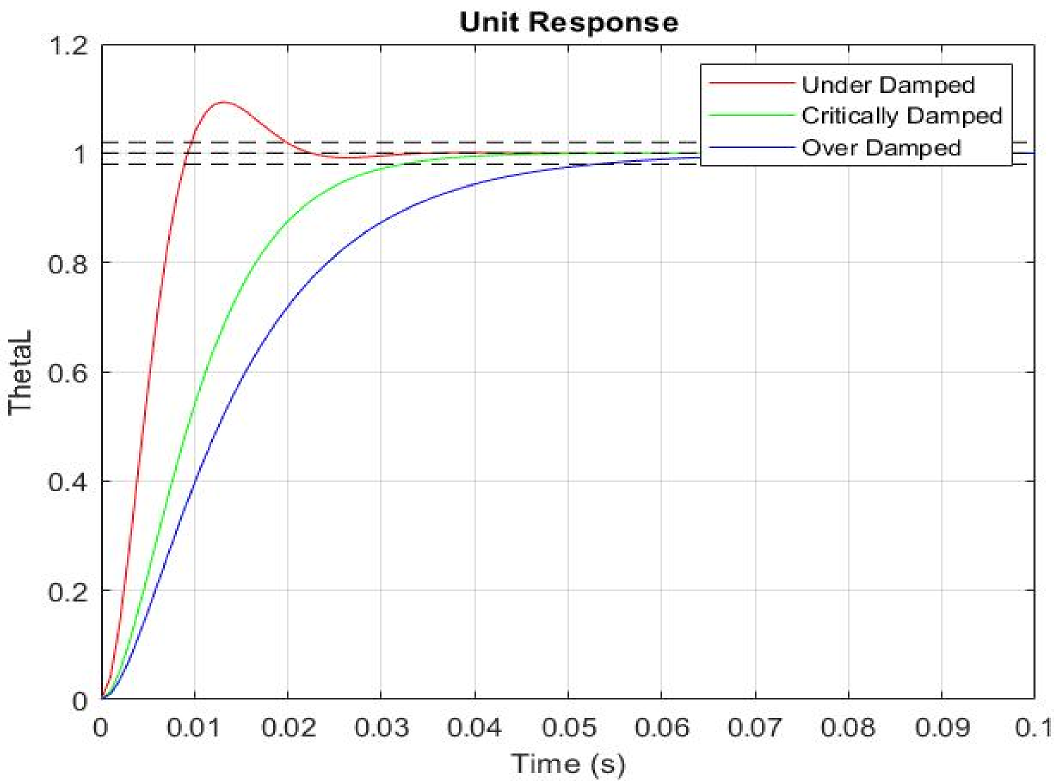
Observations:

- We can observe that the oscillations occur in the under-damped case.

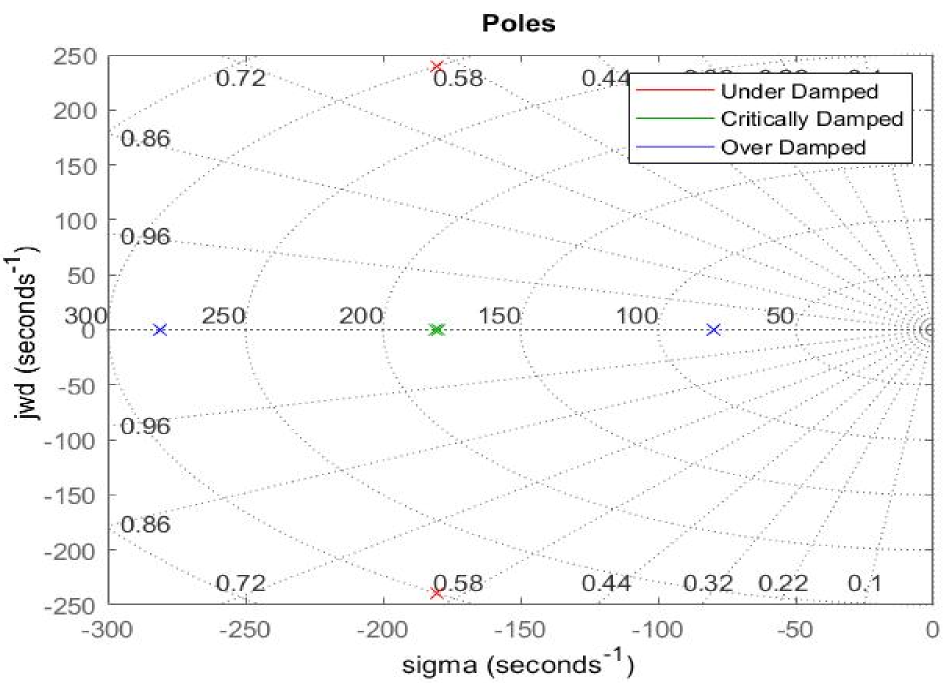
- Critically damped is the case where the system settled faster without any oscillations.

- The unit response of overdamped is similar to that of a critically damped case but the duration required to reach is more.

Response:

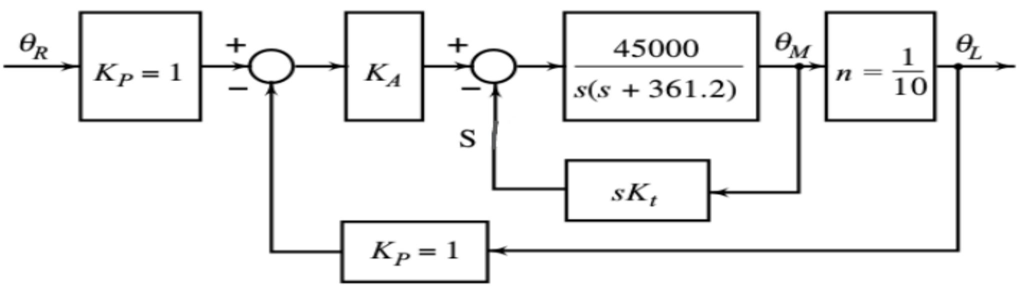


Poles:



- We have the poles of the under-damped case in the 2nd and 3rd quadrant.
- The poles of under-damped condition share the same X coordinate and opposite Y coordinate.
- The poles of Critically damped and Overdamped lie on the X axis.

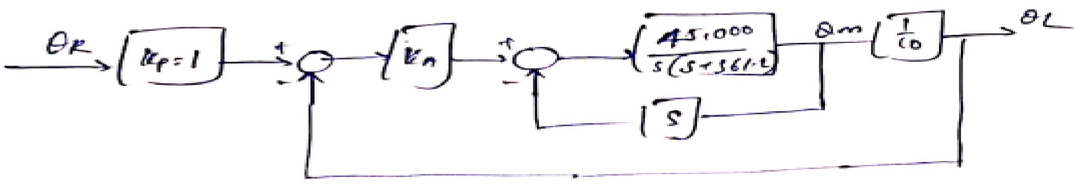
4. Attaching the Tachogenerator feedback (closing the switch):



The below procedure depicts the optimization and solving of the given block diagram (switch closed system step by step by eliminating the blocks.

The transfer function can be obtained as follows:

Assume $k_t = 1$



$$\frac{\frac{45{,}000}{s(s+361.2)}}{1+\frac{45000}{s(s+361.2)}\times s} = \frac{45000}{s(s+361.2)}\times\frac{s+361.2}{(361.2+s)+45000}$$

$$= \frac{45000}{s(s+45361.2)}$$

$$k_A \times \frac{4500\cancel{0}}{s(s+45361.2)}\times\frac{1}{10} = \frac{k_A \cdot 4500}{s(s+45361.2)}$$

$$\frac{\frac{k_A\cdot 4500}{s(s+45361.2)}}{1+\frac{k_A\cdot 4500}{s(s+45361.2)}} = \frac{4500\,k_A}{s(s+45361.2)+k_A 4500}$$

$$H(s) = \frac{4500\,k_A}{s^2+s\cdot(45361.2)+4500k_A} = \frac{\theta_L(s)}{\theta_R(s)}$$

Now the zeta (damping ratio) can be calculated as:

$$\omega_n^2 = 4500 K_A$$

$$2\omega_n \zeta = 45361.2$$

$$\zeta = \frac{45361.2}{2\omega_n} = \frac{22680.6}{\sqrt{4500 K_A}}.$$

The value of Zeta (in terms of KA) which we obtained here is different than what we got in the previous case. So, the tuning done in 1 doesn't work here.

Improvements:

1. Use a looping method in code for choosing the better case while fine tuning the parameters.