Control Systems

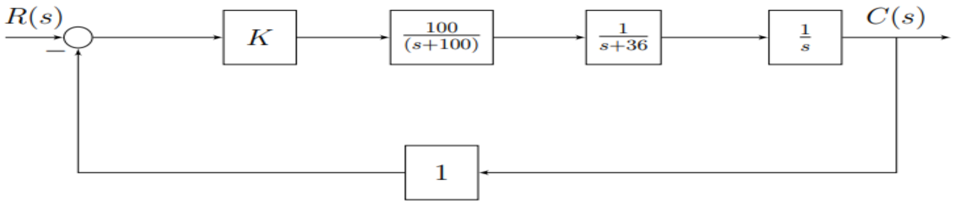
Lab Assessment #7

Date of experiment – 03/11/2023

Group Members: Adarsh Santoria (B21176), Vanshaj Nathani (B20237)

Objective:

To refine or tune system gain through the application of frequency response methodologies.



Experiment Design:

For plotting the bode plot we need to take the open loop transfer function and then proceed. After having the transfer function, we can find the initial slope, phase equation and others as follows.

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)}$$

$$G(s) = \frac{K\times 100}{s+100} \times \frac{1}{s+36} \times \frac{1}{s}$$

$$= \frac{100K}{(s+100)(s+36)s}$$

$$G(s) = \frac{100K}{100\times 36} \times \frac{1}{s(1+s/36)(1+s/100)}$$

$K = 1.$

$$G(s) = \frac{1}{36}\,\frac{1}{s(1+s/36)(1+s/100)}$$

$$\phi = -90^\circ - \tan^{-1}\left(\frac{\omega}{36}\right) - \tan^{-1}\left(\frac{\omega}{100}\right)$$

$$\text{Gain}\big|_{\omega=1} = 20\log\left|\frac{1}{36}\right| = -31.126\ dB$$

Have a pole at zero so initial slope = -20dB/dec.

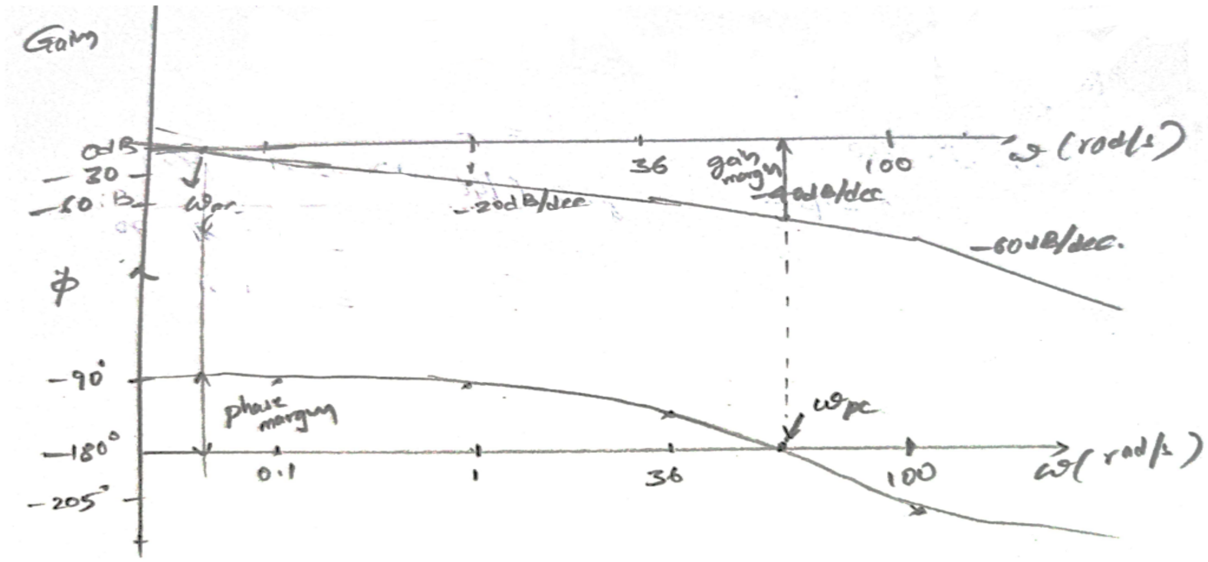
Stability:
$\omega_{pc} > \omega_{gc}$ so the system is stable.

ω	ϕ
0.1	-90.216°
1	-92.164°
36	-154.793°
100	-205.201°

Gain $\uparrow$

1. The Bode plot for given system:

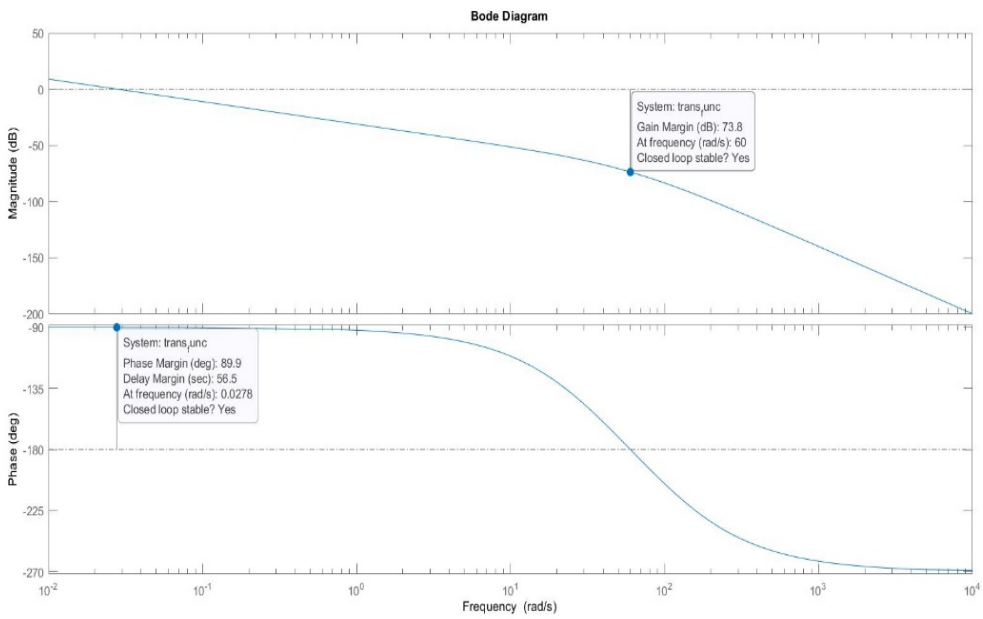
By having the required equation and data, we can plot the bode gain and phase plot as follows.



Code:

```
k = 1;
trans_func = tf(100*k, [1 136 3600 0]);
bode(trans_func);
```

Results:



Inferences:

- By the matlab plot we can verify that our bode plot is correct which we have drawn initially.
- Phase crossover frequency is around 60 rad/s and gain crossover frequency is around 0.0278rad/s.
- The phase cross over frequency is greater than the gain crossover frequency.
- By this condition we can say that the system is stable.
- We can also observe that both the phase and gain margins are positive.
- The phase margin is 89.9º and the gain margin is 73.78dB, which can be verified from the plot too.

Calculations:

a) Phase margin

$k = 1$

$$|G(j\omega)| = 1$$

$$\frac{100}{\omega\sqrt{100^2+\omega^2}\sqrt{36^2+\omega^2}} = 1$$

$$100^2 = \omega^2(100^2+\omega^2)(36^2+\omega^2)$$

$$100^2 = (\omega^2 100^2 + \omega^4)(36^2+\omega^2)$$

$$100^2 = \omega^2 3600^2 + \omega^4 100^2 + 36^2\omega^4 + \omega^6$$

$$\omega^6 + \underbrace{136\omega^4}_{(100^2+36^2)} + 3600^2\omega^2 - 100^2 = 0.$$

$$\omega^2 = -9999.99,\ -1296.0,\ 7.716\times10^{-4}$$

$$\omega_{gc} \simeq 0.0278$$

$\cancel{\phi = -180°}$

$$\phi = -90° - \tan^{-1}\left(\frac{\omega}{36}\right) - \tan^{-1}\left(\frac{\omega}{100}\right)$$

$$= -90° - \tan^{-1}\left(\frac{0.0278}{36}\right) - \tan^{-1}\left(\frac{0.0278}{100}\right)$$

$$\phi = -90.06°$$

$$PM = 180° - 90.06°$$

$$\boxed{PM = 89.939°.}$$

Phase Margin

b) Gain Margin

$k = 1$ $\quad \phi = -180°$ at ω_{pc}

$$-90° - \tan^{-1}\left(\frac{\omega}{36}\right) - \tan^{-1}\left(\frac{\omega}{100}\right) = -180°$$

$$\tan^{-1}\left(\frac{136\omega}{3600-\omega^2}\right) = 90°$$

$$\Rightarrow \omega^2 = 3600$$

$$\omega = 60 \text{ rad/sec}$$

$$|G(j\omega)| = \frac{100k}{\omega\sqrt{100^2+\omega^2}\cdot\sqrt{36^2+\omega^2}}$$

$$= \frac{100}{60\times\sqrt{100^2+60^2}\sqrt{36^2+60^2}}$$

$$= 2\times10^{-4}.$$

$$20\log_{10}(|G(j\omega)|) = -3.689 \times 70$$

$$\text{Gain Margin} = +3.689\text{dB} \times 70$$

$$= 73.78 \text{ dB}$$

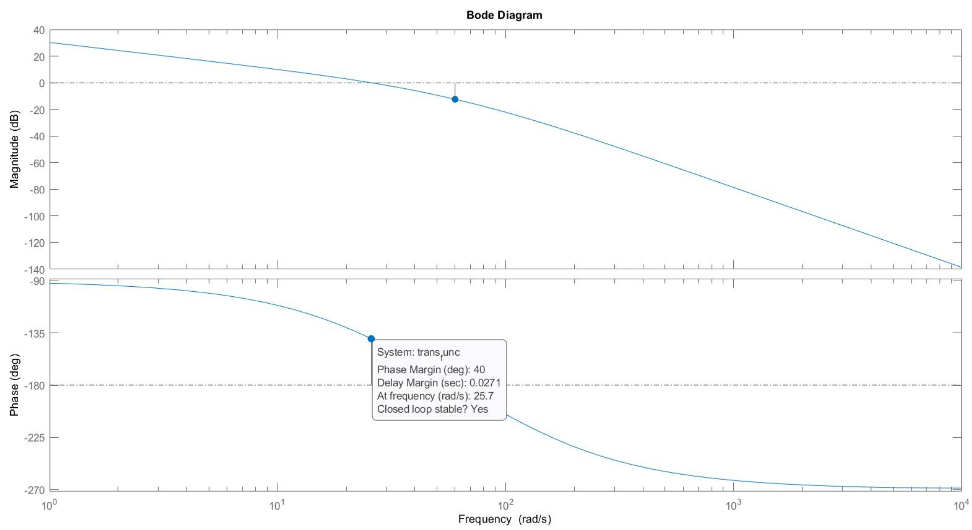
2. The 40º phase margin case:

Code:

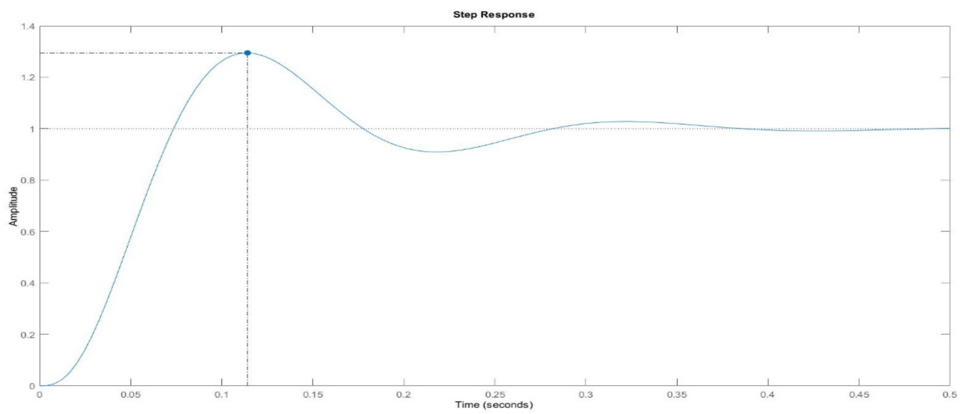
```
k = 1176.26;
trans_func = tf(100*k, [1 136 3600 0]);
bode(trans_func);

trans_func_sys = tf(100*k, [1 136 3600 100*k]);
step(trans_func_sys);
```

Results:

a) Bode Plot



b) Step Response



Inferences:

- For 40º phase margin we need the value of K to be 1176.26
- The calculations for the same are mentioned below.
- From the plot we can observe that the gain crossover frequency is 25.74rad/s.
- At this value we got the required phase margin of 40º.
- For this value of K the step response of the plot can be seen in the results too.
- First the system rises to a peak and goes down to settle but there is also a second oscillation after which the system settles.

Calculations:

b)

Phase Margin of 40°

$K : ?$

$$\phi = -180^\circ + 40^\circ = -140^\circ$$

$$-140^\circ = -90^\circ - \tan^{-1}\left(\frac{\omega}{36}\right) - \tan^{-1}\left(\frac{\omega}{100}\right)$$

$$50 = \tan^{-1}\left[\frac{\frac{\omega}{36} + \frac{\omega}{100}}{1 - \frac{\omega^2}{3600}}\right]$$

$$1.192 = \frac{136\omega}{3600} \times \frac{3600}{3600 - \omega^2}$$

$$\omega^2 + \frac{136}{1.192}\omega - 3600 = 0$$

$$\omega^2 + 114.1\omega - 3600 = 0$$

$$\omega = 25.74 \; , \; -139.86$$

$$\frac{100K}{(\omega)\sqrt{100^2+\omega^2}\sqrt{36^2+\omega^2}} = 1$$

$$100K = \omega\sqrt{100^2+\omega^2}\sqrt{36^2+\omega^2}$$

$$K = 1176.26.$$

3. K for the 9.5% overshoot condition:

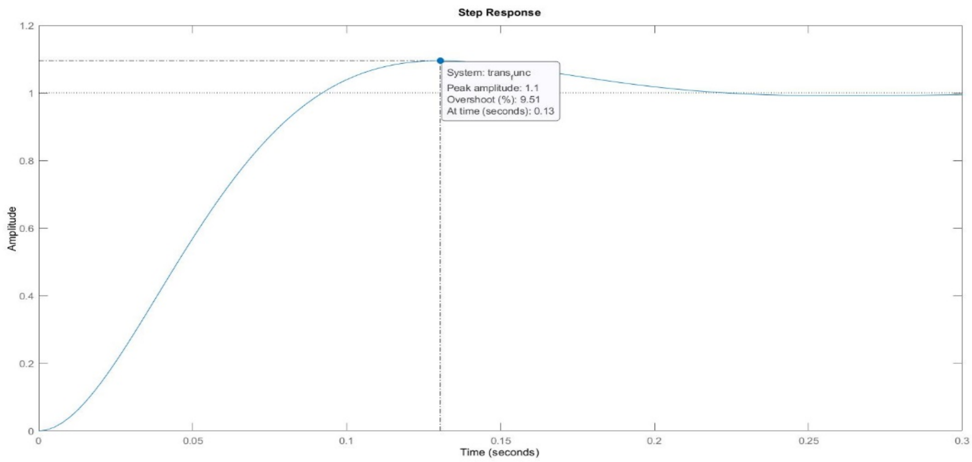
Our system is a 3rd order system (closed loop case). So we need to approximate it to the 2nd order in order to analyse and perform the calculations required. So we selected the farthest pole which is 100 and neglected it. Now the system is 2nd order. The further analysis is as follows

Code:

Q1.m Q2.m Q3.m

```
k = 901.598;
trans_func = tf(k, [1 36 k]);
step(trans_func);
```

Results:



Inferences:

- • As calculated, for the value of K as 901.598, we have the overshoot of the approximated second order system to be 9.5% as given.
- From the plot we can observe that at 0.13s we have this peak point which is 1.095.

Calculations:

c).

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)} = \frac{100k}{\cancel{s(s+36)(s+100)}} \cdot \frac{\cancel{s(s+36)(s+100)}}{s(s+36)(s+100)+100k}$$

$$= \frac{100k}{s(s+36)(s+100)+100k}.$$

ω_n

Approximating the system to 2^{nd} order. $s+100 \rightarrow 100$.
farthest

$$\frac{C(s)}{R(s)} = \frac{100k}{100(s^2+36s)+100k} = \frac{k}{s^2+36s+k}$$

$\omega_n^2 = k$

$2\zeta\omega_n = 36$

$\omega_n = \frac{36}{2\zeta}$

$= 30.026$

$\boxed{k = 901.598}$

$\% M = 9.5\%$

$e^{-\pi\zeta/\sqrt{1-\zeta^2}} \times 100 = 9.5$

$-\frac{\pi \times \zeta}{\sqrt{1-\zeta^2}} = -2.3538$

$\frac{\zeta^2}{1-\zeta^2} = 0.561$

$\zeta^2(1+0.561) = 0.561$

$\zeta^2 = \frac{0.561}{1.561}$

$\zeta = 0.5995$

Improvements and Learnings:

1. From this lab we have seen the advantage of using frequency based analysis.
2. Instead of lengthy approaches, we can simply check the crossover frequency conditions to check the stability of the system.
3. We can use a slider gain for K and tune it as required by using the Simulink models.