Control Systems

Lab Assessment #9

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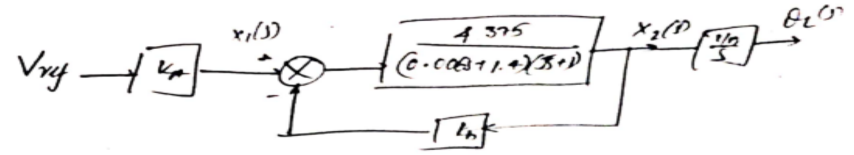
Objective:

To use MATLAB to design the gain of a controller via root locus method.

Pre-Lab:

1.

a) Open loop T.F



$\theta_L = X_2(s) \cdot \frac{-1}{200s}$

$X_2(s) = \left(X_1(s) - X_2(s)\, 0.00567^{K_b}\right) \frac{K_m}{(Ls+R)(Js+D)}$

$J = J_m + J_L/n^2$

$= 8.465 \times 10^{-3}$

$X_2(s)\left[1 + \frac{K_b K_m}{(Ls+R)(Js+D)}\right] = \frac{X_1(s) K_m}{(Ls+R)(Js+D)}$

$X_1(s) = K_a V_{ref}$

$X_2(s) = \frac{K_m K_a V_{ref} \,/\, (Ls+R)(Js+D)}{1 + \frac{K_b K_m}{(Ls+R)(Js+D)}}$

$X_2(s) = \frac{K_m K_a V_{ref}}{(Ls+R)(Js+D) + K_b K_m}$

θ_L

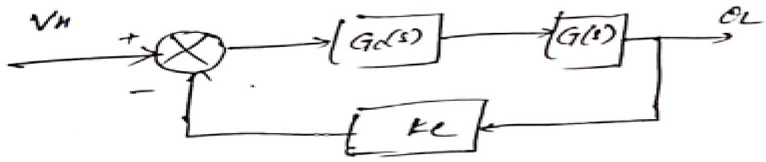
$\theta_L = \frac{K_m K_a V_{ref}}{200s\left[(Ls+R)(Js+D) + K_b K_m\right]}$

$= \frac{52.5}{200s\left[5.061 \times 10^{-5} s^2 + s\left(8.55 \times 10^{-7} + 0.01851\right) + 1.995 \times 10^{-4} + 0.0379\right]}$

$\frac{\theta_L}{V_{ref}} = \frac{52.5}{200s\left[5.061 \times 10^{-5} s^2 + 0.01151 s + 0.0381\right]}$

2.

b). Closed loop System.



3.

c). closed loop transfer function

$$\theta_L(s) = (V_{in} - \theta_L k_e)\, G_c(s)\, G(s)$$

$$\theta_L = \frac{V_{in}\, G_c(s) G(s)}{1 + k_e G_c(s) G(s)}$$

$$TF: \quad \frac{\theta_L}{V_{in}} = \frac{G(s)\, G_c(s)}{k_e G_c(s) G(s) + 1}$$

a) Root locus Derivation.

$$k_p = 4 k_D$$
$$k_e = 1$$
$$G_c = k_D s + k_p$$

and $G(s) = \dfrac{4866}{s^3 + 233.5 s^2 + 7163 s}$

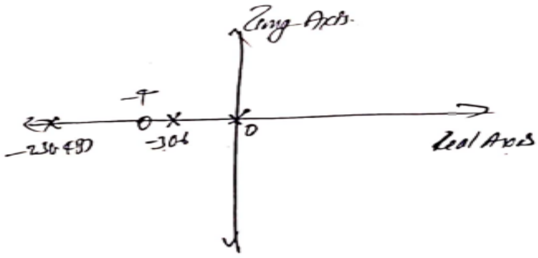
For new closed loop,

$$TF: \quad \frac{\theta_L}{V_{in}} = \frac{G(s) G_c(s)}{1 + G_c(s) G(s)} = \frac{4866\,(k_D(s+4))}{s^3 + 233.5 s^2 + 7163 s + 4866 k_D(s+4)}$$

Characteristic Eqn,

$$1 + G_c(s) G(s) = 0.$$

$$1 + \frac{4866\,[k_D(s+4)]}{s^3 + 233.5 s^2 + 7163 s} = 0$$



poles: 0, −230.433, −3.0637

zeros: −4.

Asymptotes angles: $\frac{180(2k+1)}{3-1} = 90°, 270°$

Point of intersection $= \frac{-(0+230.433+3.067)-4}{2} = -118.75.$

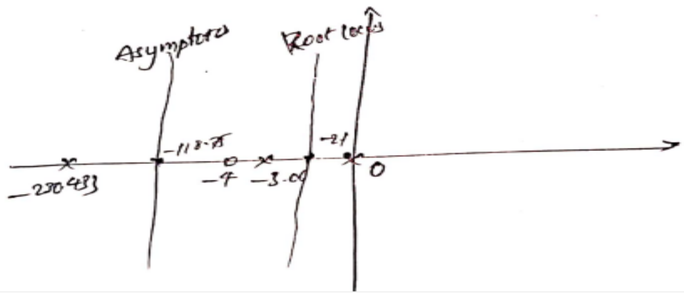
Breakaway point.

$$k_D = \frac{-s^3 + 233.5s^2 + 706.25}{4866(s+4)}$$

$$\frac{dk_D}{ds} = 0.$$

$$\Rightarrow -2s^3 + 2455s^2 + 186305 + 28272 = 0$$

$s = 130.01, -5.66, -2.1$ B/w poles. → break-away point



Experiment Design:

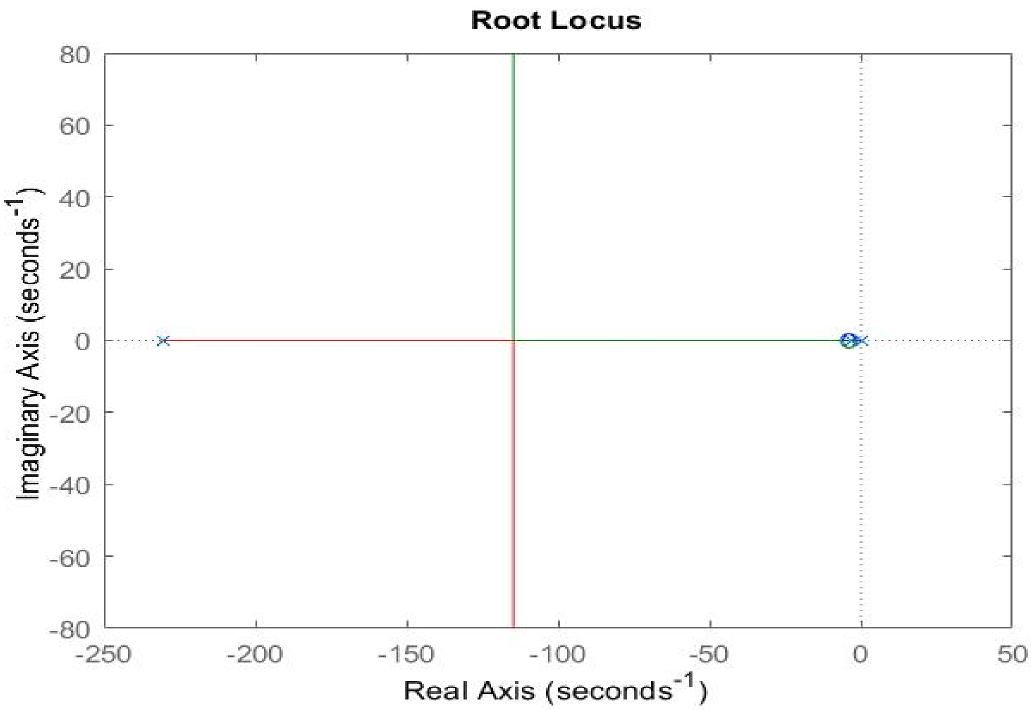
1. The Rlocus:

Code:

```

syms s
syms k
system = tf([4866, 19464],[1,233.5,706.8,0]);
rlocus(system)
```

Results:



Inferences:

- The root locus plotted in the matlab is same as the derived root locus.
- Locus starts from poles and end at zeros.
- Here we have 3 poles and 1 zero.
- So, the locus from one pole end at zero and locus from other poles end at infinity.

2. Gain at (-115,50) i.e. at point -115+50i:

Calculations:

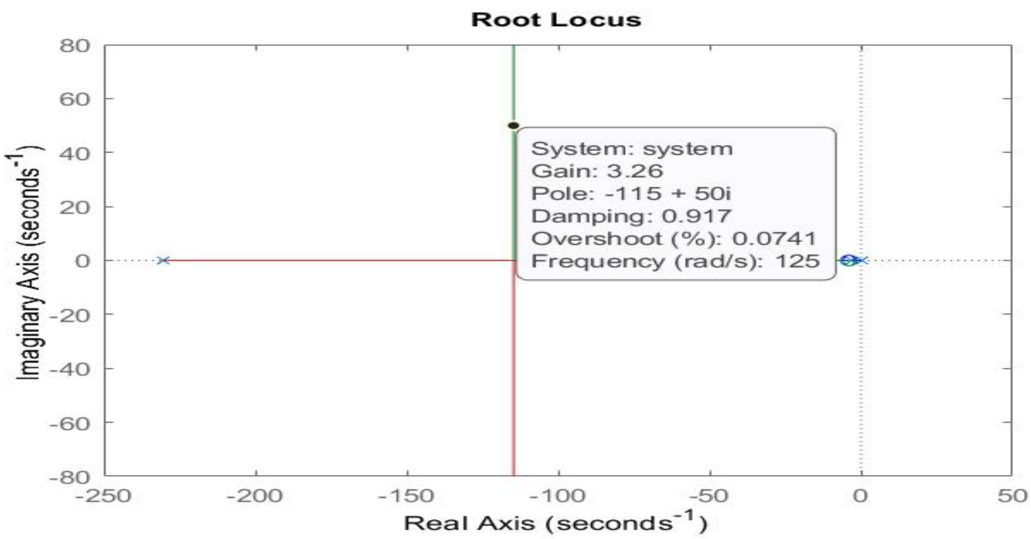
b) Gain at $(-115+50i)$

$$k_D = \left| \frac{-(s^3+233.5s^2+406.85)}{4866(s+4)} \right|_{s(-115+50i)}$$

$$k_D = \frac{-\left[(-115+50i)^3 + 233.5(-115+50i)^2 + 7068(-115.50i)\right]}{4866\left[(-115+50i)+4\right]}$$

$$k_D = 3.26$$

Results:



Inferences:

- From the plot we can observe that at the point given, the gain 3.26
- From the calculation too we got the gain as 3.26.
- Damping ratio is 0.917
- Natural frequency is 125 rad/s

3. Second order approximation:

Our system is a 3rd order system (closed loop case). So we need to approximate it to the 2nd order in order to analyse and perform the calculations required. So we selected the farthest pole which is 100 and neglected it. Now the system is 2nd order. The further analysis is as follows

Calculations:

c)

$$\frac{\theta_L}{V_{in}} = \frac{k_D(s+4) \times 4866}{s^3 + 233.5s^2 + 4866(s+4)k_I)}$$

we have, $k_D = 3.26$

$$\frac{\theta_L}{V_{in}} = \frac{15863.16(s+4)}{s^3 + 233.5s^2 + 16270.8s + 6228t.8}$$

so,

$$s^3 + 233.5s^2 + 16270.8s + 6228t.8 = (s+p)(s^2 + 2\zeta\omega_n s + p\omega_n^2)$$

$$233.5 = p + \cancel{\ldots}, \quad 16270.8 = 2\zeta\omega_n p + \omega_n^2 \quad 6228t.8 = p\omega_n^2$$

$p = 4.6,$

$\omega_n = 125$

$\zeta = 0.9.$

Inferences:

- From the calculations we got the damping ratio as 0.917.
- Also we got the natural frequency as 125 rad/s.
- We can observe that the calculated values are also approximately the same.

4. Step Response:

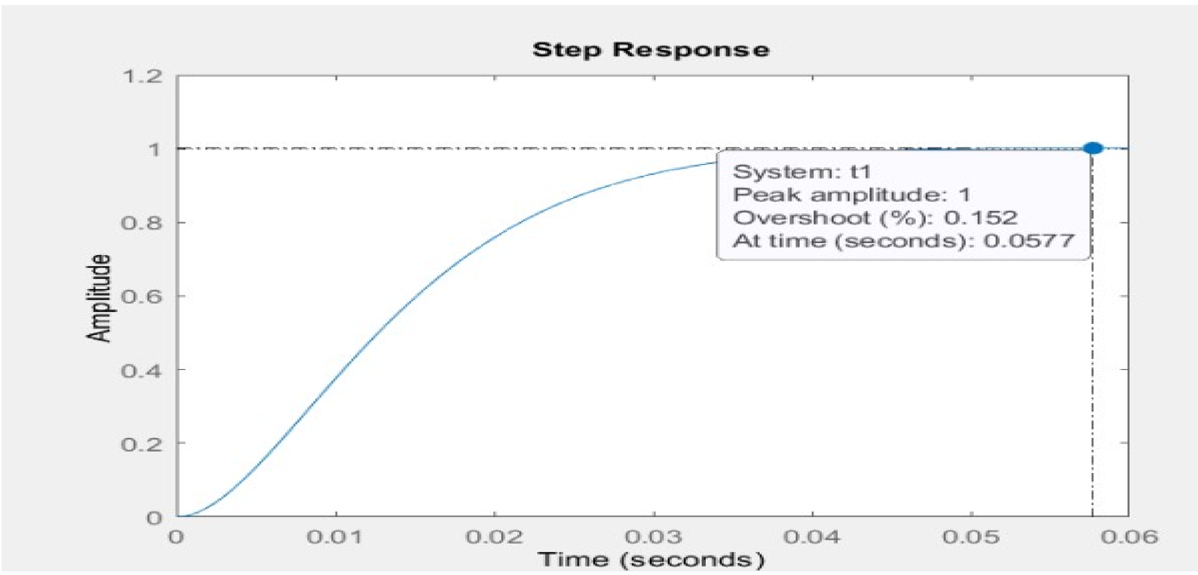
Our system is a 3rd order system (closed loop case). So we need to approximate it to the 2nd order in order to analyse and perform the calculations required. So we selected the farthest pole which is 100 and neglected it. Now the system is 2nd order. The further analysis is as follows

Code:

```
k = 3.26;
syms  s;
trans = tf([k*4866, 19464*k],[1,233.5,706.8+k*4866,19464*k]);
omega = 125;
zeta = 0.9;
trans_1 = tf([omega*omega],[1,2*zeta*omega, omega*omega]);

step(trans_1)
```

Results:



Calculations:

d) Peak overshoot $= e^{-\zeta\pi/\sqrt{1-\zeta^2}}$

$$= e^{-0.9\pi/\sqrt{1-0.9^2}}$$

$$= 0.15\%$$

Inferences:

- From the above plot, we can observe that the peak overshoot of the approximated systems is 0.152%
- By the calculations we got it as 0.15% which is approximately the same as we got.

Improvements and Learnings:

1. Root locus method is very useful to locate all the points which satisfy the given transfer function.
2. Using this method, we can design the gain of the controller.
3. We can also improve our program to give the new parameters which are within the trade-off to get the required gain params.