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A Weighted Buddy Method for Dynamic Storage Allocation

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An extension of the buddy method, called the weighted buddy method, for dynamic storage allocation is presented. The weighted buddy method allows block sizes of 2^k and $3 \cdot 2^k$, whereas the original buddy method allowed only block sizes of 2^k. This extension is achieved at an additional cost of only two bits per block.

Simulation results are presented which compare this method with the buddy method. These results indicate that, for a uniform request distribution, the buddy system has less total memory fragmentation than the weighted buddy algorithm. However, the total fragmentation is smaller for the weighted buddy method when the requests are for exponentially distributed block sizes.

Key Words and Phrases: weighted buddy algorithm, buddy system, memory allocation, dynamic storage allocation

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1. Introduction

The buddy system [2, 3] is an algorithm for dynamic storage allocation. It provides storage in block sizes which are powers of 2, from a pool of available memory space of size 2^m. An available space list keeps track of available blocks. When a block of size 2^k is requested, the available space list is examined. If a block of the proper size is available, it is used. Otherwise, an iterative request for a block of size 2^{k+1} is made. This block, when obtained, is split into two blocks, called *buddies*, both of size 2^k. One block is placed on the available space list, and the other is used to fill the request. When a block is released, its buddy is examined. If both buddies are free, they are combined to recreate the block from which they were created. If the buddy of this resulting larger block is free, then it is also combined with its buddy. This process repeats until no further combination is possible. The resulting block is placed on the available space list.

Two aspects of the algorithm are important: its execution speed and its effectiveness in storage utilization. The running time of the buddy system is determined by the number of blocks which are split and combined. The effectiveness of the algorithm in managing memory can be determined by measuring two types of losses in storage utilization: *internal memory fragmentation* and *external memory fragmentation* [5]. Internal fragmentation is the result of allocating only blocks of predetermined sizes, so that a request for memory must be rounded up to the next larger block size.

External fragmentation is the result of breaking down memory into separate blocks which cannot be combined into a desired larger block. External fragmentation can occur in the buddy system because two empty blocks of size 2^k cannot be used to fill requests for blocks of size 2^{k+1} unless they are buddies. This situation is illustrated in Figure 1. Knuth [3] has shown by simulation that external fragmentation is not significant for the buddy method. Purdom and Stigler [4] have shown similar results using a stochastic model of the buddy system. Internal fragmentation, however, is a major problem. Knuth has shown that there may be a loss of between one-fourth and one-third of memory due to internal fragmentation.

The weighted buddy system decreases the amount of internal fragmentation by allowing more block sizes. Blocks may be of sizes 2^k, $0 \leq k \leq m$, and $3 \cdot 2^k$, $0 \leq k \leq m - 2$. With this scheme, allowed block sizes are 1, 2, 3, 4, 6, 8, 12,. . . . In the weighted buddy system there are nearly twice as many block sizes as are available in a standard buddy system. The reduction in internal fragmentation is achieved at a cost of two extra bits of "overhead" in each block and a slight increase in running time, relative to the buddy method.

A related algorithm for dynamic storage allocation has been considered by Hirschberg [1]. That algorithm

Fig. 1. External fragmentation in the buddy system: □ available blocks; ● blocks reserved by the user; or ○ blocks split into buddies.

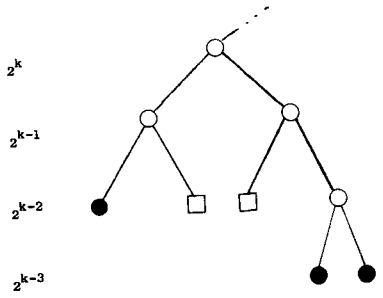


Fig. 2. Block splitting in the weighted buddy system. Each block is labeled by its size and address form (x represents either a 0 or a 1). Arcs are labeled with the TYPE field of the sub-blocks.

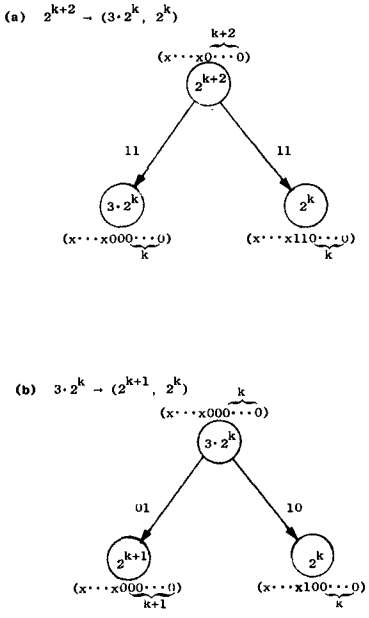
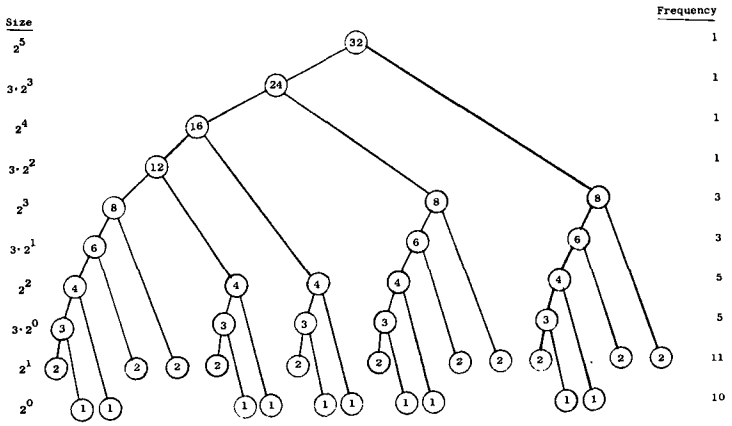


Fig. 3. Tree structure for the weighted buddy system. Original memory size is $2^5 = 32$. Frequency column gives the maximum frequency of each block size.



allocates memory in block sizes which are Fibonacci numbers.

Section 2 of this paper discusses the weighted buddy method. Section 3 describes results of simulating the algorithm.

2. The Weighted Buddy Method

The weighted buddy method is similar to the original buddy method in that large blocks are split iteratively to provide the desired smaller blocks; and when blocks are released, they are combined with their buddy if the buddy is available, or, failing this, are attached to an available space list. Other similarities are: (1) the ease of calculating the address of the buddy of a block, given the block's address; and (2) the allocation of blocks from an available space list.

The differences between the two methods lie in how the blocks are split, the mechanism of address calculation, and other more minor variations arising from a wider choice of block sizes.

Splitting Blocks

As in the buddy method, the total memory consists of 2^m words addressed for convenience from 0 through $2^m - 1$. This memory forms the first block. The blocks are split in two different ways depending on the size of the block to be split. This is illustrated in Figure 2. A block of size 2^{k+2} is split into two blocks of sizes $3\cdot 2^k$ and 2^k. Blocks of size $3\cdot 2^k$ are split into sizes 2^{k+1} and 2^k. Blocks split from the same parent block are called buddies. Since the buddies of a pair have different sizes, they are called *weighted buddies*. We shall simply call them buddies where there is no ambiguity. Note that, in any split, the larger of the two resulting buddies always has a size which is the next smaller block size of the block sizes generated by the weighted buddy system.

Example. Figure 3 gives an example of a tree for the weighted buddy method when all possible splits have been made. In this example, we have an original memory block of 32 words.

The Available Space List

An available space list (ASL) keeps track of all available blocks of storage. The ASL consists of $2m$ doubly linked table locations, AVAIL($2m$), . . ., AVAIL(1), serving as the heads and tails of linked lists of available storage of sizes 2^m, $3\cdot 2^{m-2}$, 2^{m-1}, . . ., 3, 2, 1, respectively. Available blocks have two link fields, forward and backward links, which link the blocks to the appropriate element of the ASL. Initially, there is only one block of size 2^m attached to the ASL at the top. As requests for blocks arrive, this large block is split. In general, any element of the ASL may have several blocks of the same size attached to it as a doubly linked list.

New available blocks are attached to the end of this linked list, and blocks desired for satisfying requests are

Table I. Simulation Results Comparing Fragmentation of the Buddy and Weighted Buddy Algorithms.

Fragmentation	Uniform (S1)		Exponential (S2)	
	Buddy	Weighted Buddy	Buddy	Weighted Buddy
Internal	26%	12%	28%	14%
External	1%	22%	1%	8%
Total	27%	34%	29%	22%

removed from the front of the appropriate list as in a queue. A queue is used instead of a stack because of the simulation results which are discussed later. When a block is released and its buddy is available, then the buddy is removed from the ASL for recombination and the recombined block is placed on the ASL. The ASL uses a doubly linked list to facilitate the block removal for recombination.

When a block of size 2^k is desired but not available, the following method is used to determine how a larger block is to be repeatedly split until a block of size 2^k is generated.

Given a request for an unavailable block of size 2^k, we search up the ASL until we reach the first available block of size greater than 2^k. This block is removed from the ASL, and split into two blocks. We continue splitting the smallest subblock greater than or equal to the requested size until the requested block size is matched. The same procedure holds if the request is for a block of size $3 \cdot 2^k$.

Address Calculation

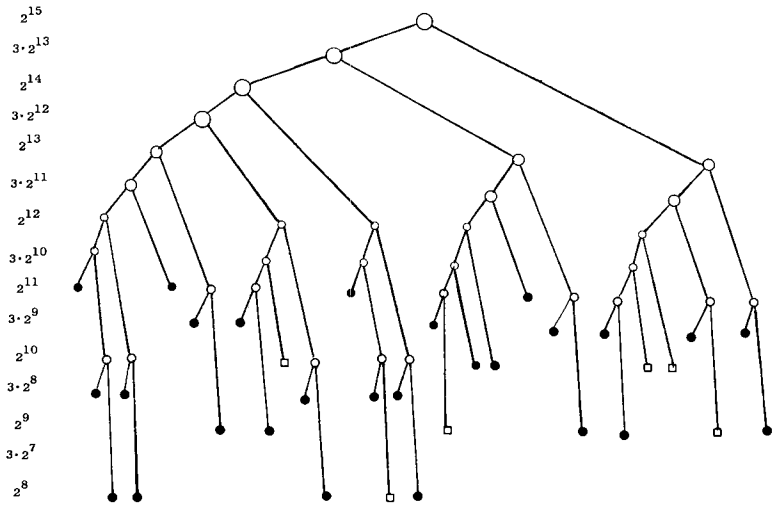
Owing to the manner in which blocks are split, the form of the address for the buddies (i.e. the address of its first word) has the following property.

THEOREM. *The address of a block of size 2^k is a multiple of 2^k (i.e. the address in binary notation has at least k zeros on the right). The address of a block of size $3 \cdot 2^k$ is a multiple of 2^{k+2}.*

PROOF. We prove this result by induction on k with $k = m, m - 1, \ldots, 0$. For $k = m$, we assume without loss of generality that the largest block of size 2^m has a starting address of zero [3]. As illustrated in Figure 2(a), this block is split into buddies, and the $3 \cdot 2^{m-2}$ sized block will have a starting address of zero, while the 2^{m-2} sized block will have a starting address of $3 \cdot 2^{m-2}$.

We next assume the form to be true for all blocks of size 2^{k+2}. As illustrated in Figure 2(a), it is also true for all blocks of size $3 \cdot 2^k$ and 2^k. This is because the address for the $3 \cdot 2^k$ size block is the same as that of the parent 2^{k+2} block, and the address for the 2^k size block comes from adding the value $3 \cdot 2^k$ in binary to the address of the block of size $3 \cdot 2^k$. As illustrated in Figure 2(b), the resulting $3 \cdot 2^k$ sized block is used as the new parent block for splitting. The form is now similarly preserved for all blocks of size 2^{k+1} and 2^k owing to the address form for the $3 \cdot 2^k$ sized block. Therefore, the form holds for all blocks of size 2^k and $3 \cdot 2^k$. □

Fig. 4. Memory map for a 2^{15} size subblock at overflow for the weighted buddy system.



A corollary of this theorem is that the address of any block of size 2^k, which is the "right-handed" split from a 2^{k+2} sized block (Figure 2a), has a binary form of $x \cdots x110 \cdots 0$ with exactly k zeros on the right. The 2^k sized block which is a right-handed split from a $3 \cdot 2^k$ sized block (Figure 2(b)) has a binary form of $x \cdots x100 \cdots 0$ with exactly $k + 1$ zeros on the right.

To distinguish between these different kinds of splits, a two-bit TYPE field (not in the original buddy method) is used in every block:

$$\begin{aligned} \text{TYPE(P)} &= 11 \text{ if the block with address P is split from a } 2^k \text{ size block,} \\ &= 01 \text{ if the block with address P is the left split from a } 3 \cdot 2^k \text{ block,} \\ &= 10 \text{ if the block with address P is the right split from a } 3 \cdot 2^k \text{ block.} \end{aligned}$$

We now show how to find the address of the buddy of a block, given the size and address of the block. Let $WB_k(x)$ be the address of the (weighted) buddy of a block of size $c \cdot 2^k$ and address x, where c is 1 or 3. (The address, x, and the type field associated with the block at x, denoted by TYPE(x), contain sufficient information to determine c. Therefore, c is omitted from the notation, $WB_k(x)$.) We define $WB_k(x)$ by:

$$\begin{aligned} WB_k(x) &= x + 3 \cdot 2^k \text{ if } x \bmod 2^{k+2} = 0 \text{ and } \text{TYPE}(x) = 11, \\ &= x - 3 \cdot 2^k \text{ if } x \bmod 2^{k+2} = 3 \cdot 2^k \text{ and } \text{TYPE}(x) = 11, \\ &= x + 2^k \text{ if } \text{TYPE}(x) = 01, \\ &= x - 2^{k+1} \text{ if } \text{TYPE}(x) = 10. \end{aligned}$$

The type field has been selected so that the address of the buddy may be calculated quickly without testing by performing a selective complement (exclusive or) between the address, x, and its type field shifted k places to the left.

Overhead

Besides the overhead of maintaining the ASL, a one-bit TAG field is needed to indicate the available/reserved status of the block. Also, a KVAL field is needed to indicate the value of k. If the block is of size $c \cdot 2^k$, only k need be remembered. These fields are also needed for the standard buddy system, so the only additional space overhead for the weighted buddy system is the two-bit TYPE field.

Frequency

We denote the maximum possible frequency of occurrence of each block size in the weighted buddy method by $F(i)$, where $i = 1, 2, \ldots, 2m$ is an index for each distinct size. This frequency may be found by solving the following recurrence relations.

$$F(i) = F(i+1), \quad \text{for } i \text{ odd}, 3 \leq i \leq 2m-1,$$

$$F(i) = F(i+1) + F(i+3) + F(i+4), \quad \text{for } i \text{ even}, 2 \leq i \leq 2m-4,$$

$$F(1) = F(3) + F(4),$$

$$F(2m) = F(2m-2) = 1.$$

For the block size of 2^k, $k = 1, 2, \ldots, m$, and the block size of $3 \cdot 2^{k-2}$, $k = 2, 3, \ldots, m$, the solution of the above equations gives a frequency of occurrence equal to $\frac{1}{3}(2^{m-k+1} + (-1)^{m-k})$.

3. Simulation Method and Results

The basic simulation program to compare the buddy and the weighted buddy methods of dynamic storage follows the outline given in Knuth [3]. We repeat the method below.

Let TIME be initially zero and all of memory initially be available.

P1. Advance TIME by 1.

P2. Free all blocks in the system that are scheduled to be released at the current value of TIME.

P3. Calculate two quantities: S (a random size), and T (a random "lifetime") based on some probability distributions.

P4. Reserve a new block of size S, to be released at TIME + T. Return to step P1.

Detailed statistics on parameters such as internal fragmentation and other steady state characteristics were printed whenever TIME was a multiple of 200. The steady state was reached for values of TIME larger than about 2000. Two block size distributions for S were used in the experiments:

S1. An integer chosen uniformly between 100 and 2000.

S2. An integer chosen according to a (truncated) exponential distribution between 100 and 2000 with a mean of 1000.

The time distribution was a random integer chosen uniformly between 1 and 100 for both block size distributions. Since both S1 and S2 limit block sizes requested to between 100 and 2000, the two buddy algorithms were limited to allocating blocks of size between 128 and 2048. The total memory size assumed available was 2^{17}.

The memory wasted for each allocated block is the difference between the size of the allocated block and the size of the request. The sum of this memory waste over all allocated blocks is the internal fragmentation.

Memory overflow occurs when a request for memory cannot be satisfied because only smaller blocks are available. When overflow occurs, the ratio of the amount of unallocated memory to the total memory size is the external fragmentation. To measure external fragmentation, memory overflow was activated by ceasing to release blocks after a given time (2000 for our simulations).

When a steady state is reached, the average number of blocks which must be split into buddies to satisfy a request and the average number of blocks which are recombined with their buddies when a block is released are equal. The average number of splits and recombinations for the weighted buddy method was 0.66 in our simulation as compared to 0.20 for the buddy method. Since the running time of these algorithms is proportional to the number of splits and recombinations needed, the weighted buddy method will increase the running time overhead by approximately a factor of three. It should be remembered that this increase in execution time is for only the request and release mechanisms, which will probably take only a small portion of the time for the overall computing task.

Table I gives the statistics for storage fragmentation for both the uniform and exponential block size distributions. For uniformly distributed block requests, the weighted buddy method achieves a significant savings in internal fragmentation but at a greater expense in external fragmentation. However, the increase in total fragmentation is only about 7 percent. For exponentially distributed block requests, though, the weighted buddy method is better than the buddy method by about 7 percent overall. This is a result of a large decrease in internal fragmentation which offsets the minor increase in external fragmentation.

Figure 4 shows a partial memory map of one large subblock which existed during overflow with uniformly distributed block size requests. Overflow occurred when a request for a block of size 2^{11} arrived. However, there were still at least 15 of each of the block sizes, 2^9 and 2^{10}, available. These were not being recombined in the

manner, $(2^9, 2^{10}) \rightarrow 3 \cdot 2^9$, $(2^9, 3 \cdot 2^9) \rightarrow 2^{11}$, because they were not buddies. So that we could probe possible remedies to this problem, the variations listed in the following section were tested on parts of the weighted buddy method, in an attempt to reduce external fragmentation. These variations were not tested for exponentially distributed block requests since external fragmentation is small here.

Variations on the Weighted Buddy Method

The first variation was on methods of continuing the splitting after finding a larger block to split. Instead of "continue splitting the smallest subblock greater than or equal to the desired size" as described in Section 2, we changed to the method discussed in V1 below. The second variation was to change from a "queue" method (for inserting and removing equal sized blocks from the circular list attached to the ASL) to two other methods, as described in V2 and V3 below. The results of these variations were either worse or statistically insignificant.

V1. Instead of using the smallest buddy, which is sufficient to satisfy our request when a larger block needs to be repeatively split, we used "always split the *larger* buddy" after finding a block to split. This is equivalent to going down the ASL step by step from some larger block until we reach the block size requested. This variation increased the external fragmentation by 3 percent and the number of splits and recombinations by about 12 percent for the uniform distribution defined in S1. Internal fragmentation remained unchanged, of course. The main disadvantage of this method is that it splits larger blocks when splitting a smaller block would suffice.

V2. Instead of a queue mechanism for adding and removing blocks from the ASL, a stack mechanism was tried, inserting and removing blocks only at the front of the list. This also increased external fragmentation by 3 percent, with other factors unchanged.

V3. Instead of a queue or stack for choosing a block to satisfy a request, the block with the largest sized buddy was chosen. This also increased external fragmentation slightly.

4. Conclusions

The weighted buddy method presented in this paper is an alternative to the buddy method for dynamic storage allocation. The weighted buddy method allows the generation and use of nearly twice as many block sizes as the buddy method, and consequently it approximately halves internal fragmentation. This is achieved at an additional cost of only two bits per block and a possibly longer execution time. However, our simulation results show that external fragmentation may increase. Therefore, the trade-off between internal and external fragmentation, as well as the expected distribution of block sizes of the requesting processes, should be considered in a choice between these two methods of dynamic storage allocation. If the distribution of block sizes is skewed towards small block sizes, the weighted buddy method may be a better allocation algorithm.

This research also indicates that, although it would be possible to consider other methods of partitioning a memory block into subblocks, such new algorithms would probably result in a still greater external fragmentation and a higher execution time overhead than those used in the buddy system. It would be necessary to determine efficient means of defining the buddy of a block such as is achieved in both the buddy and weighted buddy methods by use of a partitioning which results in a suitable address form.

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