Chapter 7

Optical Receiver Operation

Content

- **Fundamental Receiver Operation**
  - Digital Signal Transmission
  - Error Sources
- **Digital Receiver Performance**
  - Probability of Error
  - Receiver Sensitivity
  - The Quantum Limit
- **Coherent Detection**
- **Analog Receiver**

Optical Receiver Operation

Digital Signal Transmission

- **A typical digital fiber transmission link is shown in Fig. 7-1. The transmitted signal is a two-level binary data stream consisting of either a '0' or a '1' in a *bit period* T_b.**

- **The simplest technique for sending binary data is *amplitude-shift keying,* wherein a voltage level is switched between *on* or *off* values.**

- **The resultant signal wave thus consists of a voltage pulse of amplitude *V* when a binary 1 occurs and a zero-voltage-level space when a binary 0 occurs.**

Digital Signal Transmission

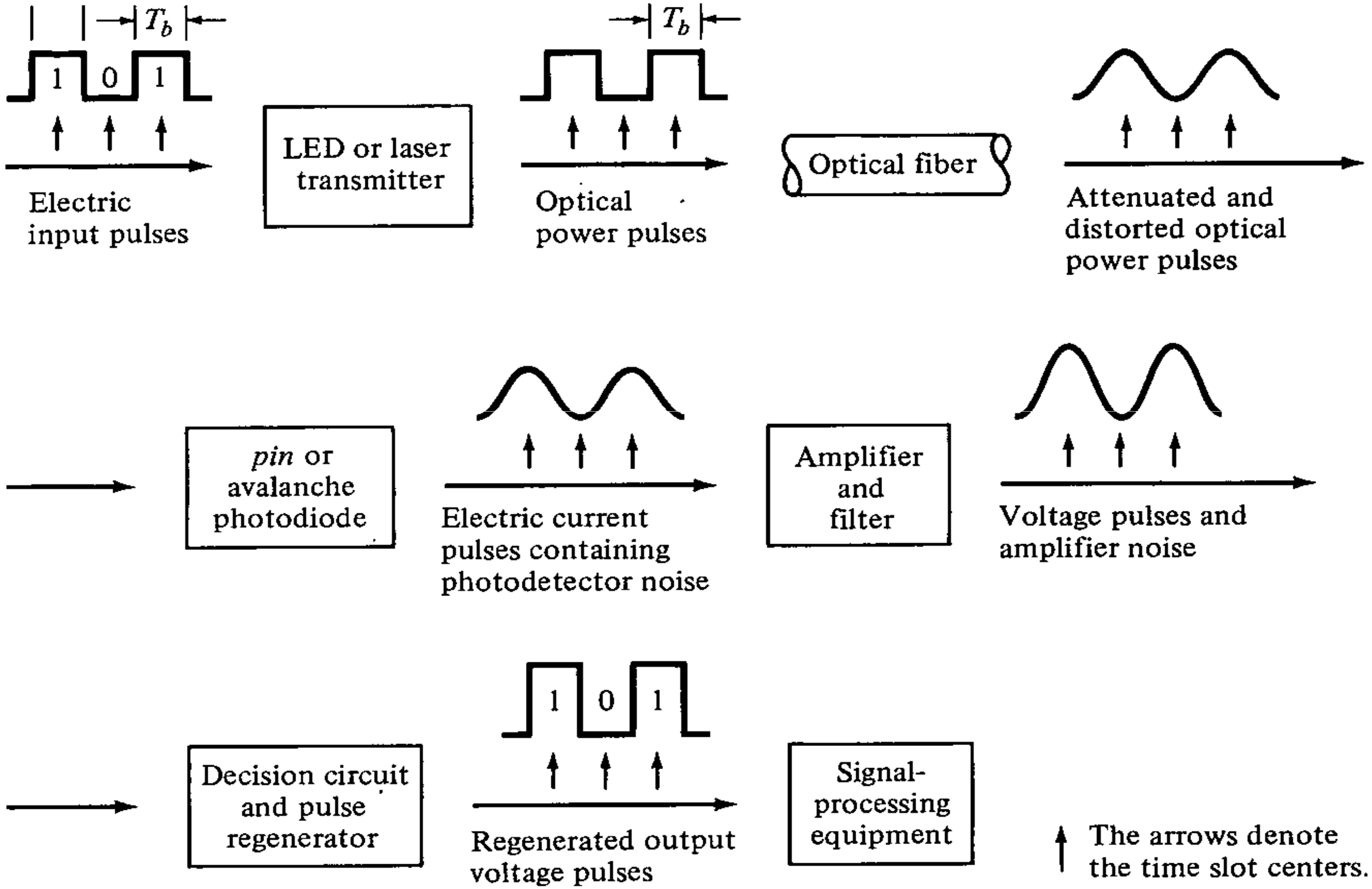


Fig. 7-1 Signal path through an optical data link.

Digital Signal Transmission (2)

- **An electric current $i(t)$ can be used to modulate directly an optical source to produce an optical output power $P(t)$.**

- **In the optical signal emerging from the transmitter, a '1' is represented by a light pulse of duration T_b, whereas a '0' is the absence of any light.**

- **The optical signal that gets coupled from the light source to the fiber becomes attenuated and distorted as it propagates along the fiber waveguide.**

Digital Signal Transmission (3)

- **Upon reaching the receiver, either a PIN or an APD converts the optical signal back to an electrical format.**

- **A decision circuit compares the amplified signal in each time slot with a *threshold level*.**

- **If the received signal level is greater than the threshold level, a ‘1’ is said to have been received.**

- **If the voltage is below the threshold level, a ‘0’ is assumed to have been received.**

Error Sources

- **Errors in the detection mechanism can arise from various noises and disturbances associates with the signal detection system.**

- **The two most common samples of the spontaneous fluctuations are shot noise and thermal noise.**

- **Shot noise arises in electronic devices because of the discrete nature of current flow in the device.**

- **Thermal noise arises from the random motion of electrons in a conductor.**

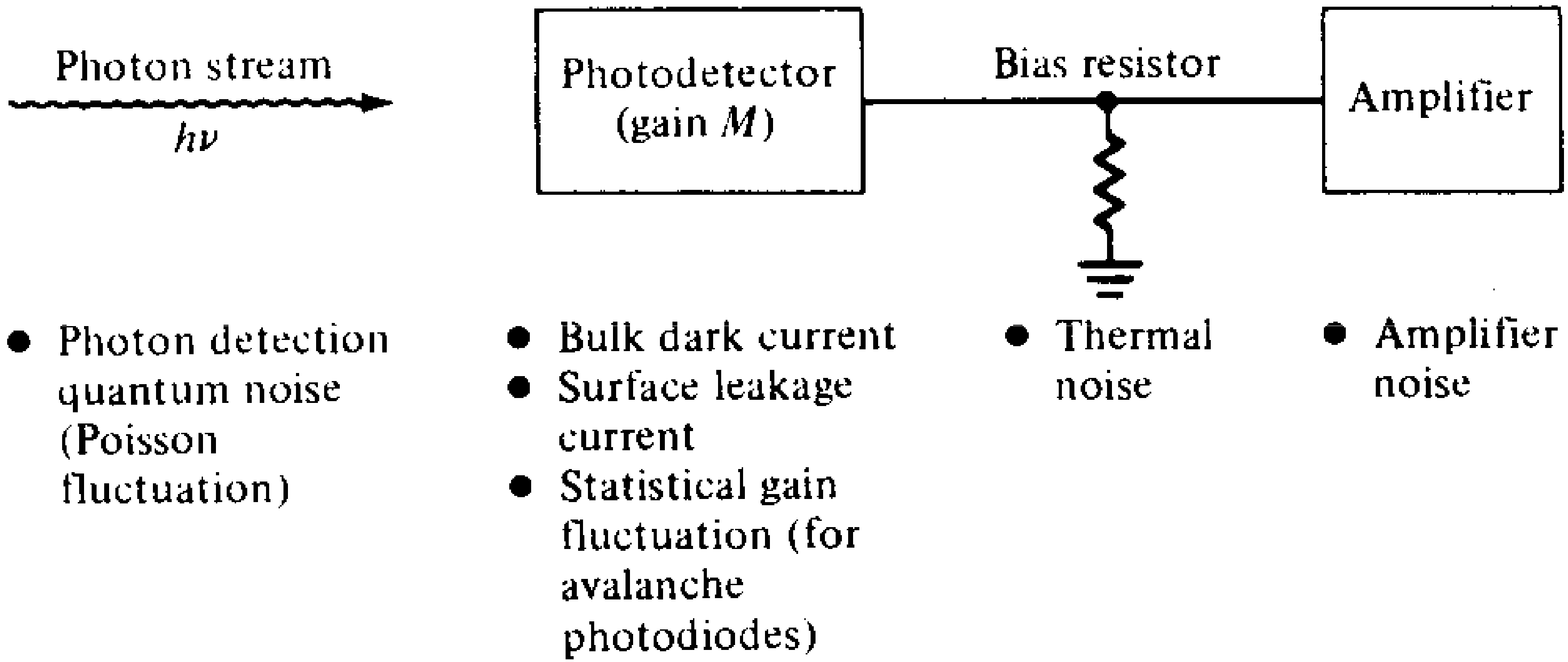
Error Sources (2)

- The random arrival rate of signal photons produces a quantum (or shot) noise at the photodetector. This noise depends on the signal level.

- This noise is of particular importance for PIN receivers that have large optical input levels and for APD receivers.

- When using an APD, an additional shot noise arises from the statistical nature of the multiplication process. This noise level increases with increasing avalanche gain M.

Error Sources (3)



Noise sources and disturbances in the optical pulse detection mechanism.

Error Sources (4)

- **Thermal noises arising from the detector load resistor and from the amplifier electronics tend to dominate in applications with low SNR when a PIN photodiode is used.**

- **When an APD is used in low-optical-signal-level applications, the optimum avalanche gain is determined by a design tradeoff between the thermal noise and the gain-dependent quantum noise.**

Error Sources (5)

- **The primary photocurrent generated by the photodiode is a time-varying Poisson process.**

- **If the detector is illuminated by an optical signal *P*(*t*), then the average number of electron-hole pairs generated in a time τ is**

$$\overline{N} = \frac{\eta}{h\nu}\int_0^{\tau} P(t)dt = \frac{\eta E}{h\nu} \qquad \textbf{(7-1)}$$

where η is the detector quantum efficiency, $h\nu$ is the photon energy, and *E* is the energy received in a time interval .

Error Sources (6)

- **The actual number of electron-hole pairs *n* that are generated fluctuates from the average according to the Poisson distribution**

$$P_r(n) = \overline{N}^{\,n} \frac{e^{-\overline{N}}}{n!} \qquad \textbf{(7-2)}$$

where $P_r(n)$ is the probability that *n* electrons are emitted in an interval τ.

Error Sources (7)

- **For a detector with a mean avalanche gain *M* and an ionization rate ratio *k*, the excess noise factor *F*(*M*) for electron injection is**

$$F(M) = kM + \left(2 - \frac{1}{M}\right)(1-k) \quad \textbf{(7-3)}$$

or

$$F(M) \cong M^{x} \quad \textbf{(7-4)}$$

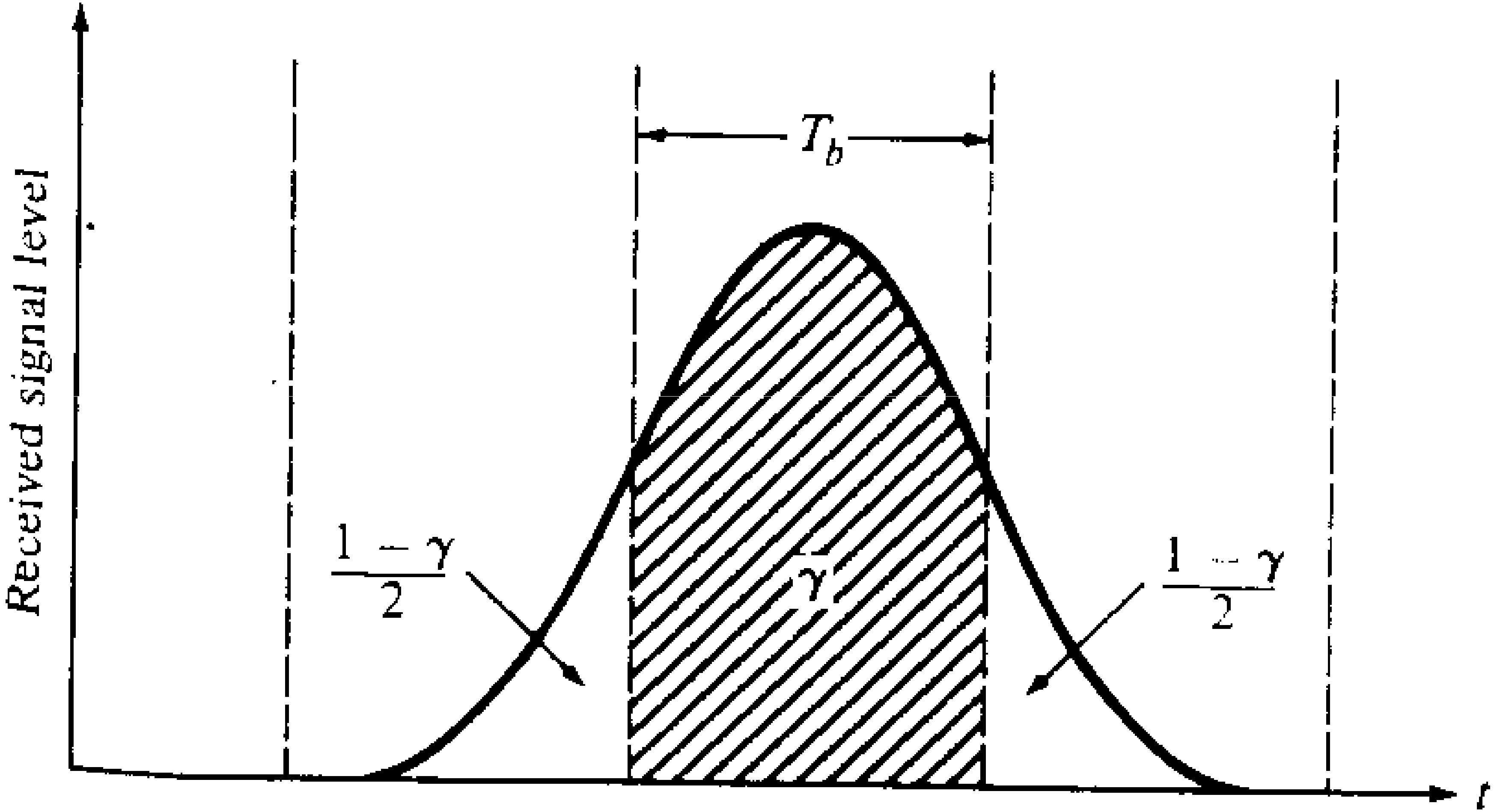
where the factor *x* ranges between 0 and 1.0 depending on the photodiode material.

Error Sources (8)

- A further error source is attributed to *intersymbol interference* (ISI), which results from pulse spreading in the optical fiber.

- The fraction of energy remaining in the appropriate time slot is designated by γ, so that 1-γ is the fraction of energy that has spread into adjacent time slots.

Error Sources (9)



Pulse spreading in an optical signal that leads to ISI.

Receiver Configuration

- A typical optical receiver is shown in Fig. 7-4.
  The three basic stages of the receiver are a photo-detector, an amplifier, and an equalizer.

- The photo-detector can be either an APD with a mean gain M or a PIN for which M=1.

- The photodiode has a quantum efficiency η and a capacitance C_d.

- The detector bias resistor has a resistance R_b which generates a thermal noise current $i_b(t)$.

Receiver Configuration (2)

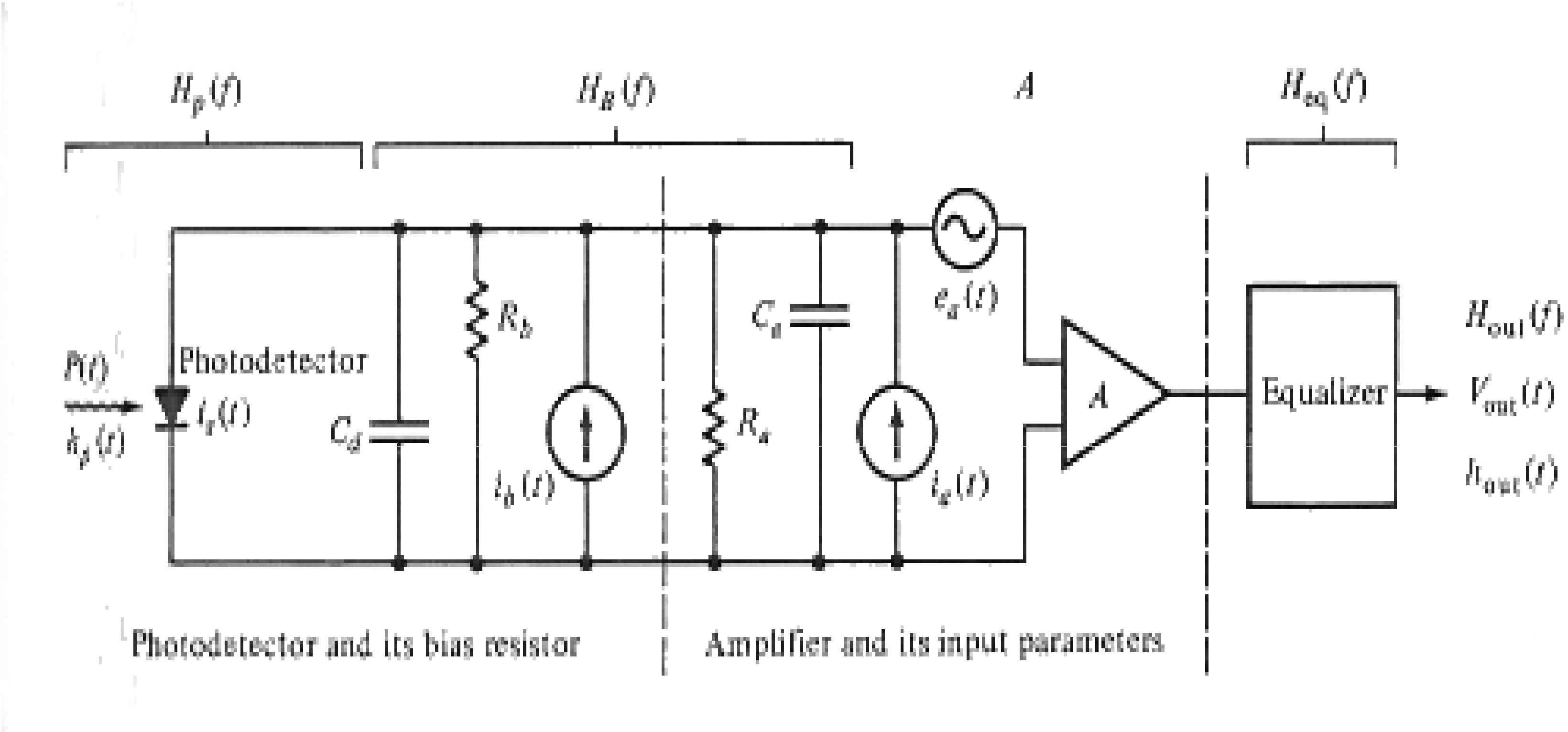


Figure 7-4. Schematic diagram of a typical optical receiver.

Receiver Configuration (3)

Amplifier Noise Sources:

- **The input noise current source $i_a(t)$ arises from the thermal noise of the amplifier input resistance R_a;**

- **The noise voltage source $e_a(t)$ represents the thermal noise of the amplifier channel.**

- **The noise sources are assumed to be Gaussian in statistics, flat in spectrum (which characterizes *white* noise), and uncorrelated (statistically independent).**

Receiver Configuration (4)

The Linear Equalizer:

- **The equalizer is normally a linear frequency-shaping filter that is used to mitigate the effects of signal distortion and intersymbol interference (ISI).**

- **The equalizer accepts the combined frequency response of the transmitter, the fiber, and the receiver, and transforms it into a signal response suitable for the following signal-processing electronics.**

Receiver Configuration (5)

- **The binary digital pulse train incident on the photo-detector can be described by**

$$P(t) = \sum_{n=-\infty}^{\infty} b_n h_p(t - nT_b)$$

- **Here, $P(t)$ is the received optical power,**

  T_b is the bit period,

  b_n is an amplitude parameter representing the n^{th} message digit,

  and $h_p(t)$ is the received pulse shape.

Receiver Configuration (6)

- Let the nonnegative photodiode input pulse $h_p(t)$ be normalized to have unit area

$$\int_{-\infty}^{\infty} h_p(t)dt = 1$$

then b_n represents the energy in the n^{th} pulse.

- The mean output current from the photodiode at time t resulting from the pulse train given previously is

$$\langle i(t) \rangle = \frac{\eta q}{h\nu} MP(t) = R_0 \sum_{n=-\infty}^{\infty} b_n h_p(t - nT_b)$$

where $R_o = \eta q / h\nu$ is the photodiode responsivity.

- The above current is then amplified and filtered to produce a mean voltage at the output of the equalizer.

Digital Receiver Performance

- **In a digital receiver the amplified and filtered signal emerging from the equalizer is compared with a threshold level once per time slot to determine whether or not a pulse is present at the photodetector in that time slot.**

- **Bit-error rate (BER) is defined as:**

$$\text{BER} = \frac{N_e}{N_t} = \frac{N_e}{Bt}$$

where B=1/T_b(bit rate). N_e,N_t : Number of errors, pulses.

- **To compute the BER at the receiver, we have to know the probability distribution of the signal at the equalizer output.**

Probability of Error (2)

The shapes of two signal pdf's are shown in Fig. 7.7.

- **These are**

$$P_1(v) = \int_{-\infty}^{v} p(y\,|\,1)\,dy \qquad \textbf{(7-6)}$$

which is the probability that the equalizer output voltage is less than *v* when a logical '1' pulse is sent, and

$$P_0(v) = \int_{v}^{\infty} p(y\,|\,0)\,dy \qquad \textbf{(7-7)}$$

which is the probability that the output voltage exceeds *v* when a logical '0' is transmitted.

Probability of Error (3)

- The different shapes of the two pdf's in Fig. 7-7 indicate that the noise power for a logical '0' is not the same as that for a logical '1'.

- The function $p(y|\mathrm{x})$ is the conditional probability that the output voltage is y, given that an x was transmitted.

Probability of Error (4)

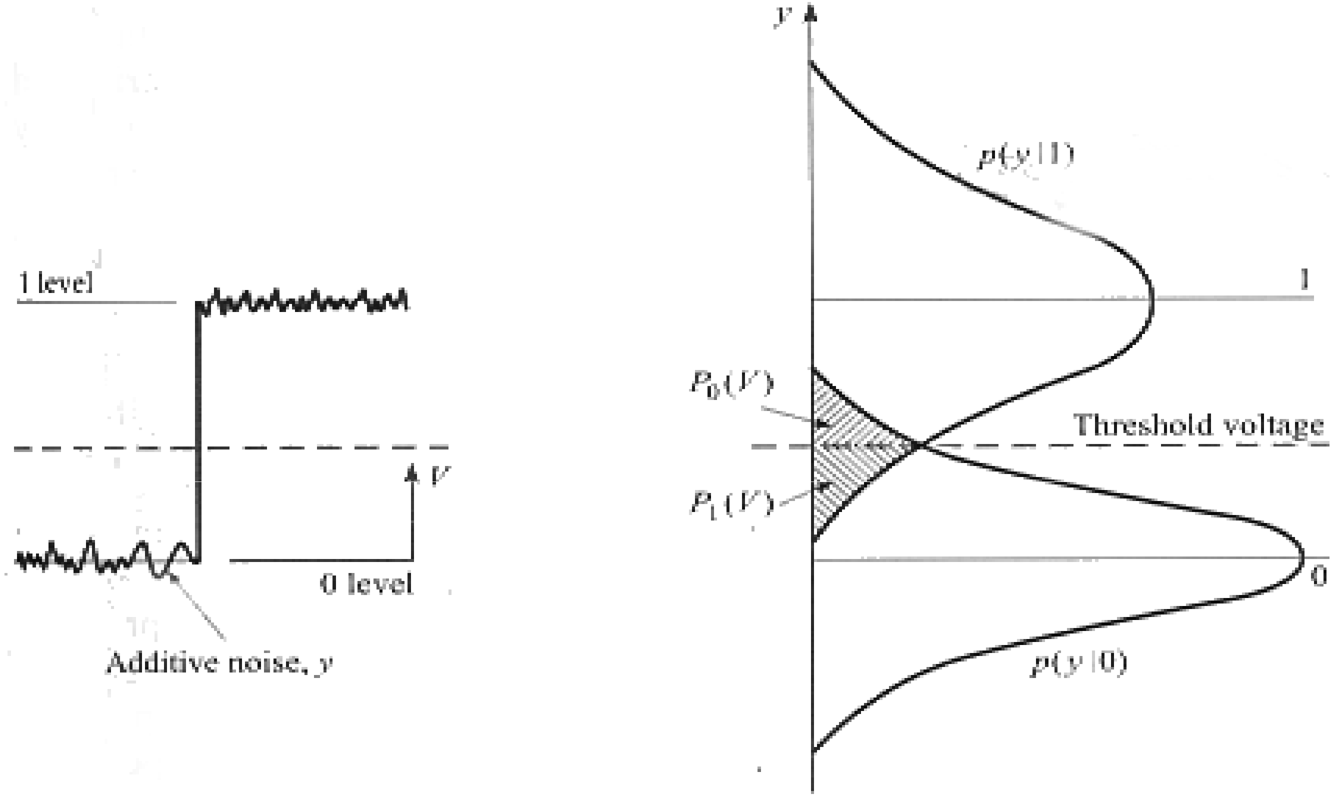


Figure 7-7. Probability distributions for received '0' and '1' signal pulses. Different widths of the two distributions are caused by various signal distortion effects.

Probability of Error (5)

- **If the threshold voltage is v_{th} then the error probability P_e is defined as**

$$P_e = aP_1(v_{th}) + bP_0(v_{th}) \qquad \textbf{(7-8)}$$

- **The weighting factors *a* and *b* are determined by the a priori distribution of the data.**

- **For unbiased data with equal probability of '1' and '0' occurrences, $a = b = 0.5$.**

- **The problem to be solved is to select the decision threshold at that point where P_e is minimum.**

Probability of Error (6)

- **To calculate the error probability we require a knowledge of the mean-square noise voltage which is superimposed on the signal voltage at the decision time.**

- **It is assumed that the equalizer output voltage $v_{out}(t)$ is a Gaussian random variable.**

- **Thus, to calculate the error probability, we need only to know the mean and standard deviation of $v_{out}(t)$.**

Probability of Error (7)

- **Assume that a signal $s(t)$ has a Gaussian pdf $f(s)$ with a mean value m. The signal sample at any s to $s+ds$ is given by**

$$f(s)ds = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(s-m)^2/2\sigma^2} ds \qquad \textbf{(7-9)}$$

where σ^2 is the noise variance, and σ the *standard deviation*.

- **The quantity measures the full width of the probability distribution at the point where the amplitude is 1/*e* of the maximum.**

Probability of Error (8)

- **As shown in Fig. 7-8, the mean and variance of the Gaussian output for a '1' pulse are b_{on} and σ_{on}^2, whereas for a '0' pulse they are b_{off} and σ_{off}^2, respectively.**

- **The probability of error $P_0(v)$ is the chance that the equalizer output voltage $v(t)$ will fall somewhere between v_{th} and ∞.**

- **Using Eqs. (7-7) and (7-9), we have**

$$P_0(v_{th}) = \int_{v_{th}}^{\infty} p(y\,|\,0)dy = \int_{v_{th}}^{\infty} f_0(v)dv$$

$$= \frac{1}{\sqrt{2\pi}\sigma_{off}} \int_{v_{th}}^{\infty} \exp\left[-\frac{(v-b_{off})^2}{2\sigma_{off}^2}\right]dv \qquad \textbf{(7-10)}$$

where the subscript 0 denotes the presence of a '0' bit.

Probability of Error (9)

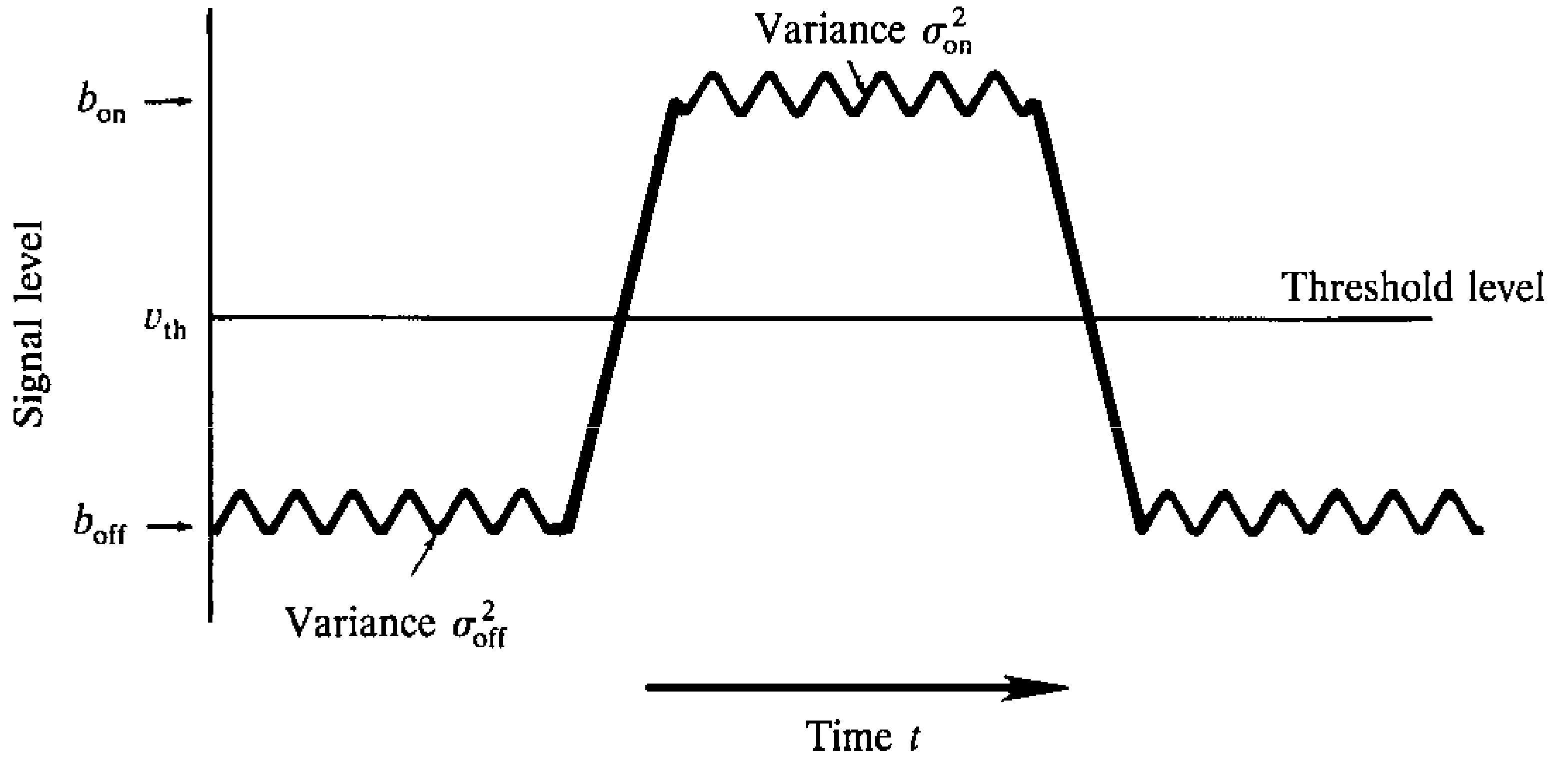


Figure 7-8. Gaussian noise statistics of a binary signal showing variances about the on and off signal levels.

Probability of Error (10)

- **Similarly, the error probability a transmitted '1' is misinterpreted as a '0' is the likelihood that the sampled signal-plus-noise pulse falls below v_{th}.**

- **From Eqs. (7-6) and (7-9), this is simply given by**

$$
\begin{aligned}
P_1(v_{th}) &= \int_{-\infty}^{v_{th}} p(y\,|\,1)\,dy = \int_{-\infty}^{v_{th}} f_1(v)\,dv \\
&= \frac{1}{\sqrt{2\pi}\sigma_{on}} \int_{-\infty}^{v_{th}} \exp\left[-\frac{(b_{on}-v)^2}{2\sigma_{on}^2}\right] dv
\end{aligned}
\qquad \textbf{(7-11)}
$$

where the subscript 1 denotes the presence of a '1' bit.

Probability of Error (11)

- **Assume that the '0' and '1' pulses are equally likely, then, using Eqs. (7-10) and (7-11), the BER or the error probability *P*$_e$ given by Eq. (7-8) becomes**

$$\begin{aligned} \mathrm{BER} = P_e(Q) &= \frac{1}{\sqrt{\pi}} \int_{Q/\sqrt{2}}^{\infty} e^{-x^2} dx \\ &= \frac{1}{2}\left[1 - \mathrm{erf}\left(\frac{Q}{\sqrt{2}}\right)\right] \approx \frac{1}{\sqrt{2\pi}} \frac{e^{-Q^2/2}}{Q} \end{aligned} \qquad \textbf{(7-12)}$$

where the parameter *Q* is defined as

$$Q = \frac{v_{th} - b_{off}}{\sigma_{off}} = \frac{b_{on} - v_{th}}{\sigma_{on}} = \frac{b_{on} - b_{off}}{\sigma_{on} + \sigma_{off}} \qquad \textbf{(7-13)}$$

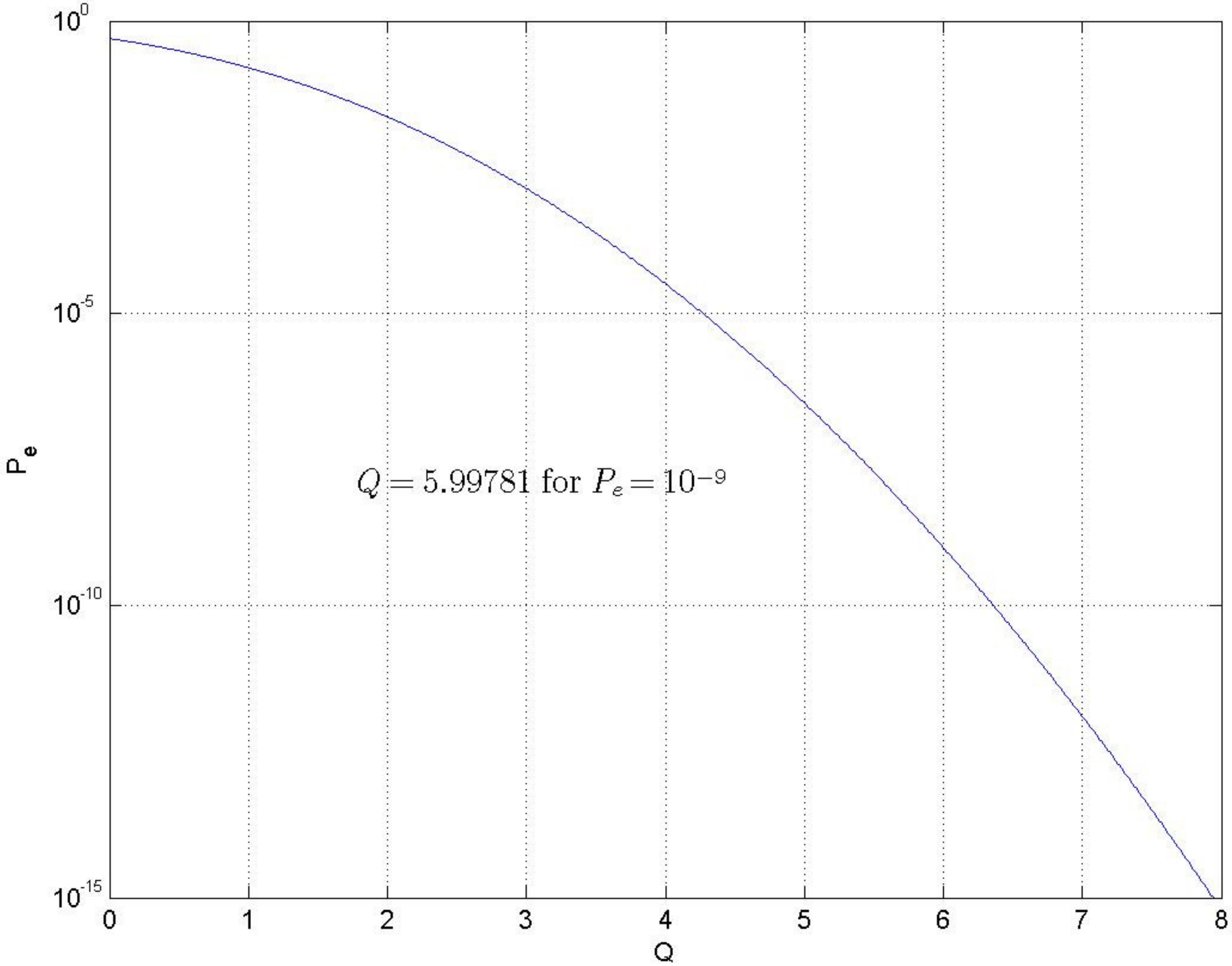
- **The approximation is obtained from the asymptotic expansion of *error function* .**

$$\mathrm{erfc}(x) = 1 - \mathrm{erf}(x) = \frac{2}{\sqrt{\pi}} \int_{x}^{\infty} e^{-y^2} dy \xrightarrow{\text{large}\, x} \frac{e^{-x^2}}{x\sqrt{\pi}}$$

Probability of Error (12)

- Figure 7-9 shows how the BER varies with Q.
- The approximation for P_e given in Eq. (7-12) and shown by the dashed line in Fig. 7-9 is accurate to 1% for Q~3 and improves as Q increases.
- A commonly quoted Q value is 6, corresponding to a BER = 10^{-9}.

Probability of Error (13)



Plot of the BER (P_e) versus the factor Q.

Calculation Examples

Example 7.1: When there is little ISI, 1-γ is small, so that $\sigma_{on}^2 \cong \sigma_{off}^2$. Then, by letting b_{off} =0,

$$Q = \frac{b_{on}}{2\sigma_{on}} = \frac{1}{2}\frac{S}{N}$$

which is one-half SNR. In this case, $v_{th} = b_{on}/2$.

Example 7.2: For an error rate of 10^{-9},

$$P_e(Q) = 10^{-9} = \frac{1}{2}\left[1 - \mathrm{erf}\left(\frac{Q}{\sqrt{2}}\right)\right] \rightarrow Q = 5.99781 \approx 6$$

The SNR then becomes 12 or 10.8 dB.

Probability of Error (14)

- **Consider the special case when $\sigma_{\text{off}} = \sigma_{\text{on}} = \sigma$ and $b_{\text{off}} = 0$, so that $b_{\text{on}} = V$.**

- **From Eq. (7-13) the threshold voltage is $v_{\text{th}} = V/2$, so that $Q = V/2\sigma$.**

- **Since σ is the *rms noise*, the ratio V/σ is the *peak- signal-to-rms-noise ratio*.**

- **In this case, Eq. (7-13) becomes**

$$P_e(\sigma_{on} = \sigma_{off}) = \frac{1}{2}\left[1 - \operatorname{erf}\left(\frac{V}{2\sqrt{2}\sigma}\right)\right] \qquad \textbf{(7-16)}$$

Probability of Error (15)

Example 7-3:

Figure 7-10 shows a plot of the BER expression from Eq. (7-16) as a function of the SNR.

(a). For a SNR of 8.5 (18.6 dB) we have $P_e = 10^{-5}$. If this is the received signal level for a standard DS1 telephone rate of 1.544 Mb/s, the BER results in a misinterpreted bit every 0.065s, which is highly unsatisfactory.

However, by increasing the signal strength so that $V/\sigma = 12.0$ (21.6 dB), the BER decreases to $P_e = 10^{-9}$. For the DS1 case, this means that a bit is misinterpreted every 650s, which is tolerable.

(b). For high-speed SONET links, say the OC-12 rate which operates at 622 Mb/s, BERs of 10^{-11} or 10^{-12} are required. This means that we need to have at least $V/\sigma = 13.0$ (22.3 dB).

Probability of Error (16)

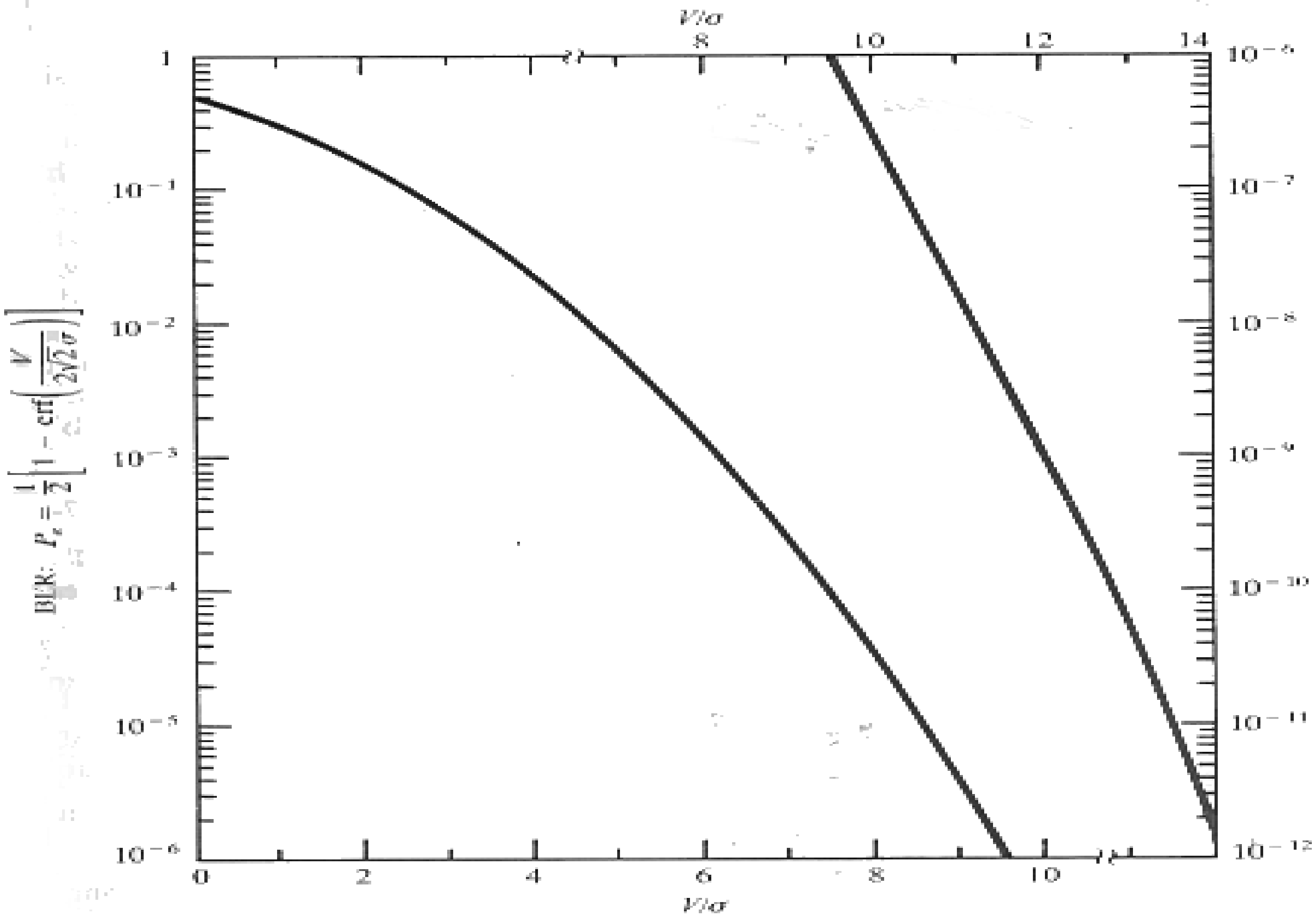


Figure 7-10. BER as a function of SNR when the standard deviations are equal ($\sigma_{on} = \sigma_{off}$) and b_{off} = 0.

The Quantum Limit

- **For an ideal photo-detector having unity quantum efficiency and producing no dark current, it is possible to find the minimum received optical power required for a specific BER performance in a digital system.**

- **This minimum received power level is known as the *quantum limit*.**

- **Assume that an optical pulse of energy E falls on the photo-detector in a time interval τ.**

- **This can be interpreted by the receiver as a '0' pulse if no electron-hole pairs are generated with the pulse present.**

The Quantum Limit (2)

- **From Eq. (7-2) the probability that n = 0 electrons are emitted in a time interval τ is**

$$P_r(0) = e^{-\overline{N}} \qquad \textbf{(7-23)}$$

where the average number of electron-hole pairs, $\underline{N}$, is given by Eq. (7-1).

- **Thus, for a given error probability $P_r(0)$, we can find the minimum energy E required at a specific wavelength λ.**

The Quantum Limit (3)

<u>Example 7-4</u>:

A digital fiber optic link operating at 850-nm requires a maximum BER of 10^{-9}.

(a). From Eq. (7-16) the probability of error is

$$P_r(0) = e^{-\overline{N}} = 10^{-9}$$

Solving for $\overline{N}$ yields $\overline{N}$ = 9ln10 = 20.7 ~ 21.

Hence, an average of 21 photons per pulse is required for this BER.

Using Eq. (7-1) and solving for *E*, we get

$$E = 20.7h\nu/\eta.$$

The Quantum Limit (4)

(b). Now let us find the minimum incident optical power P_0 that must fall on the photodetector to achieve a 10^{-9} BER at a data rate of 10 Mb/s for a simple binary-level signaling scheme.

If the detector quantum efficiency $\eta = 1$, then

$$E = P_i \tau = 20.7h\nu = 20.7hc/\lambda,$$

where $1/\tau = B/2$, B being the data rate.

Solving for P_i, we have

$$P_i = 20.7hcB/2\lambda$$

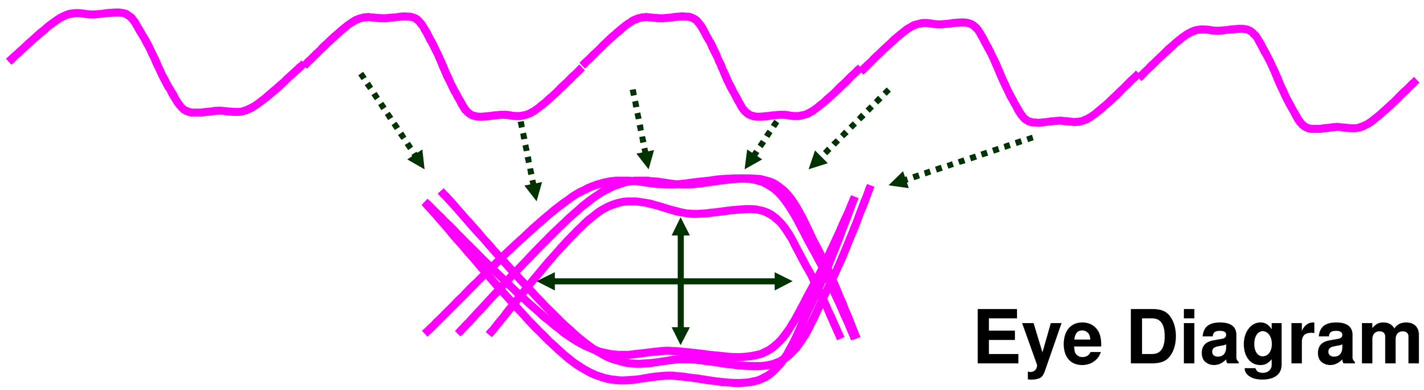
$$= \frac{20.7(6.626\times10^{-34}\text{J.s})(3\times10^{8}\text{m/s})(10\times10^{6}\text{bits/s})}{2(0.85\times10^{-6}\text{m})}$$

$$= 24.2\text{pW} = -76.2\text{ dBm}.$$

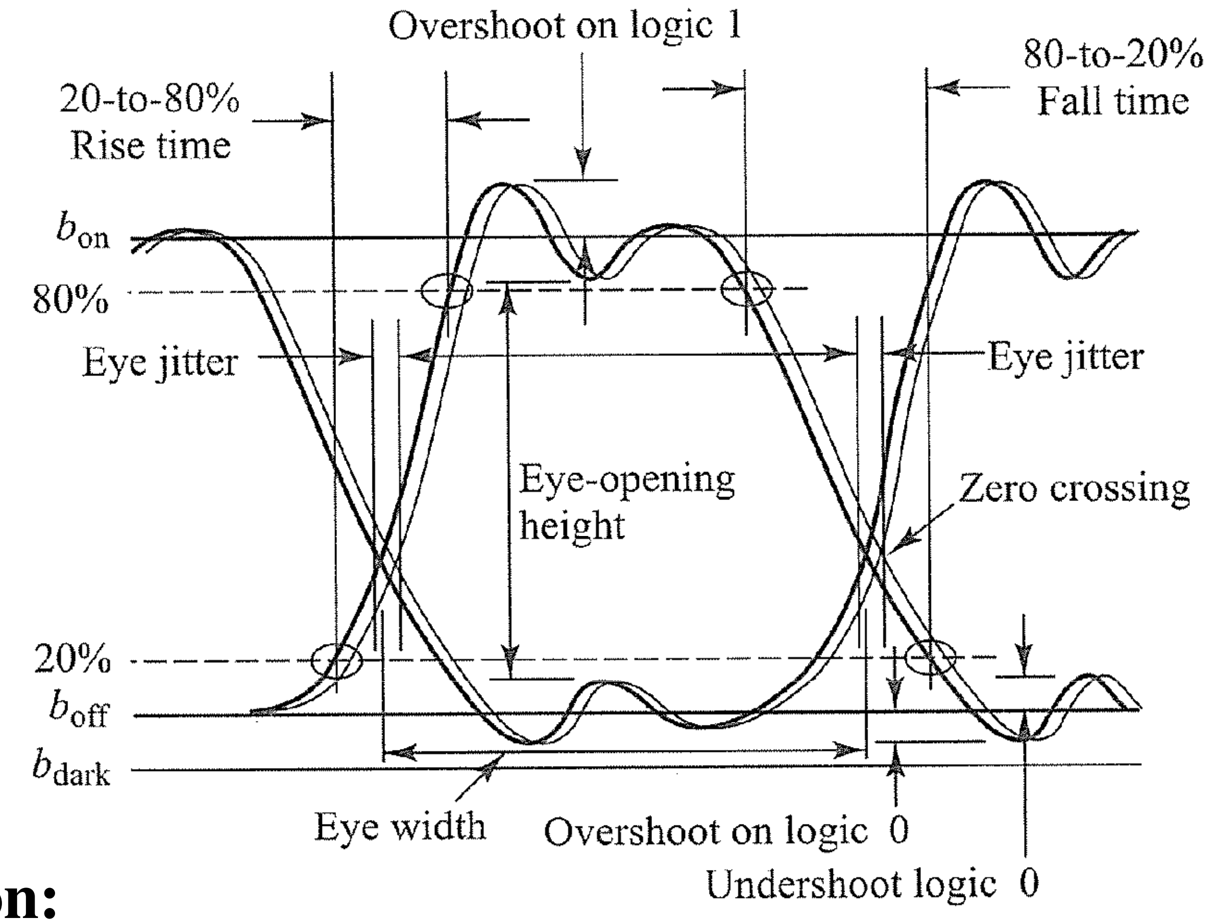
In practice, the sensitivity of most receivers is around 20 dB higher than the quantum limit because of various nonlinear distortions and noise effects in the transmission link.

Eye Diagram

- The eye diagram is a convenient way to represent what a receiver will see as well as specifying characteristics of a transmitter.
- The eye diagram maps all UI intervals on top of one and other. (UI = Unit Interval, i.e., signal duration time)
- The opening in eye diagram is measure of signal quality.
- This is the simplest type of eye diagram. The are other form which we will discuss later



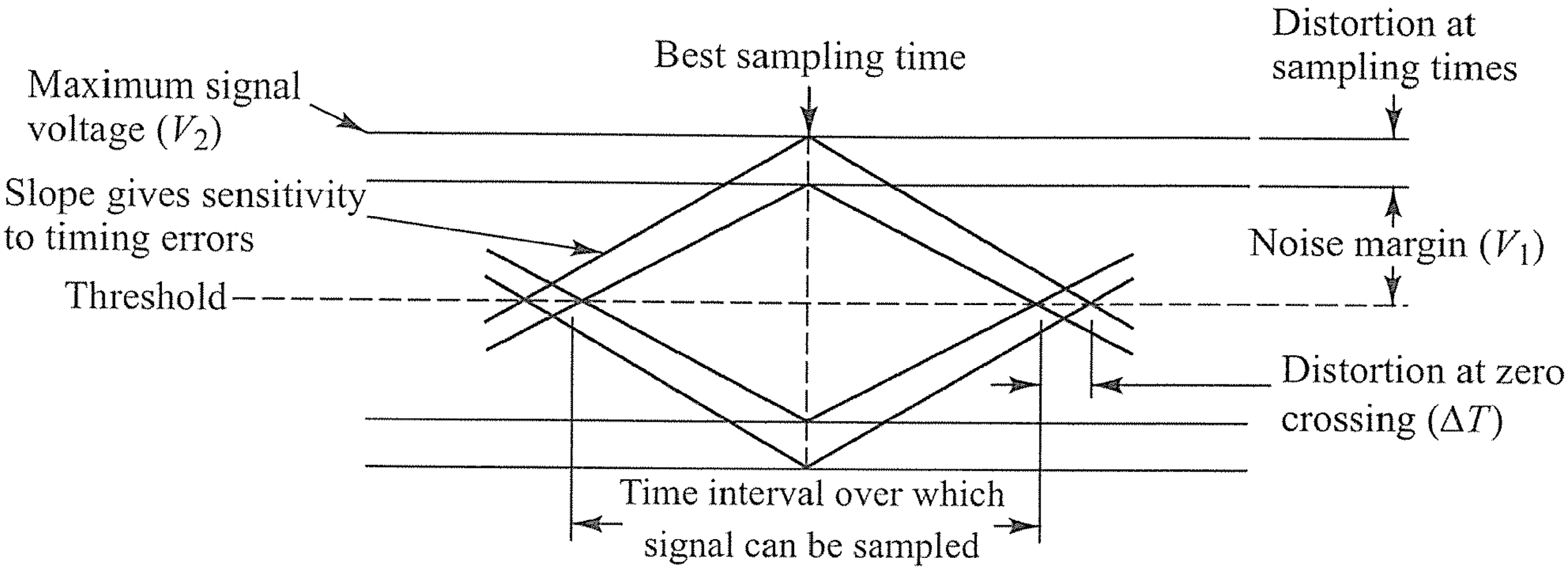
Eye Diagram



Information:

- Width of eye-opening : time interval over which the received signal can be sampled without ISI error.
- Best time to sample = when height of eye-opening is largest
- Rise time = time interval between 10% point and 90% point, can be approximated by

$$T_{10-90} = 1.25 \times T_{20-80}$$



Noise margin:

$$\text{Noise margin (percent)} = \frac{V_1}{V_2} \times 100\%$$

Timing jitter (edge jitter, phase distortion): due to noise in the receiver and pulse distortion in optical fiber.

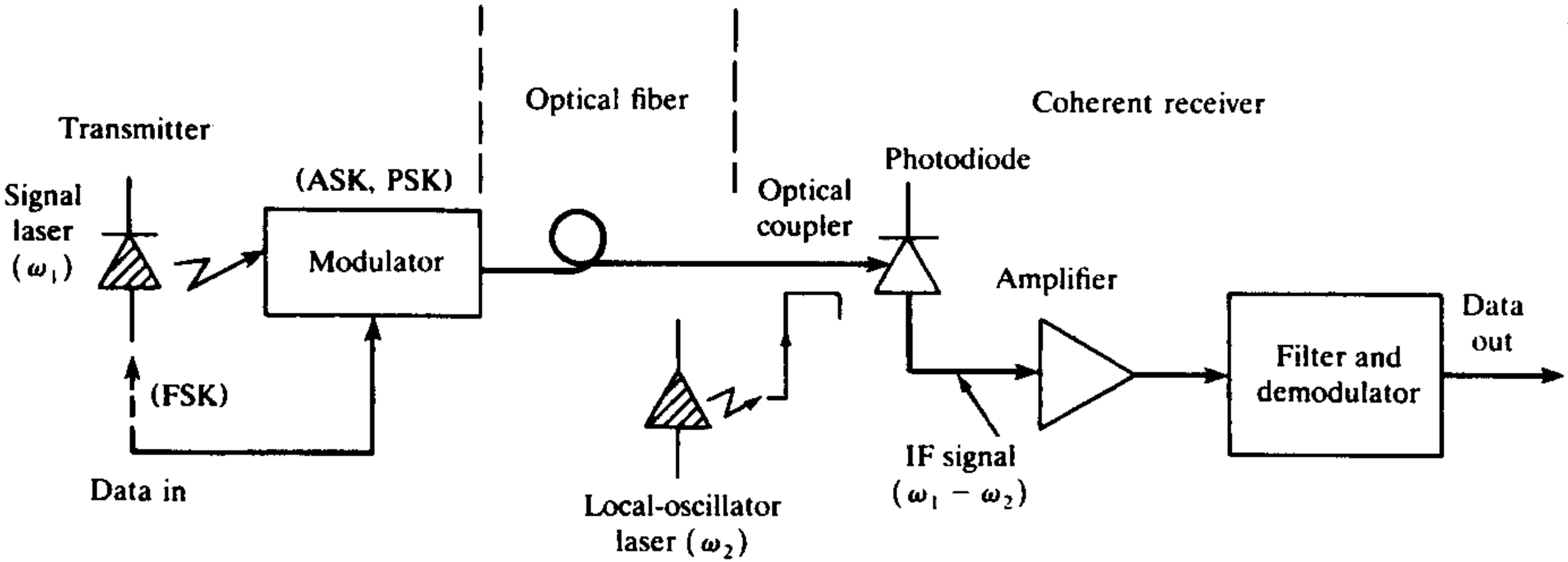
$$\text{Timing jitter (percent)} = \frac{\Delta T}{T_b} \times 100\%$$

Coherent Detection

- Primitive method : *intensity modulation with direct detection* (IM/DD)
- Since the improvement of semiconductor lasers around 70s, coherent optical communication systems become available.
- Coherent detection depends on *phase coherence* of the optical carrier.

Fundamental Concepts

- Key principle : mixing incoming signal with a locally generated continuous-wave (CW) optical field.
- Shot noise from local oscillator limits sensitivity.



Modulation Techniques

- Transmitted optical signal given by:

$$E_s = A_s \cos\left[\omega_s t + \phi_s(t)\right]$$

where A_s, ω_s, ϕ_s are amplitude, angular frequency, phase of the optical signal. Following techniques are possible:

1. *Amplitude shift keying* (ASK) or *on-off keying* (OOK) : ϕ_s is constant, and A_s have two levels for 0 or 1.
2. *Frequency shift keying* (FSK) : A_s is constant and $\phi_s(t) = \omega_1 t$, or $\omega_2 t$.
3. *Phase shift keying* (PSK) : A_s is constant and $\phi_s(t)$ differs by π.

Direct Detection

- The detected current is proportional to the intensity I_{DD} (the square of the electric field) of the optical signal, yielding

$$I_{DD} = E_s E_s^* = \frac{1}{2} A_s^2 \left[1 + \cos\left(2\omega_s t + 2\phi_s\right)\right]$$

- The double angle term gets eliminated since its frequency is beyond the response capability of the detector. Thus,

$$I_{DD} = E_s E_s^* \approx \frac{1}{2} A_s^2$$

Coherent Lightwave Systems

- Four basic demodulation formats:
  - How optical signal mixed with the local oscillator (*homodyne* or *heterodyne*)
  - How electrical signal is detected (*synchronous* or *asynchronous*)
- If the local-oscillator (LO) field is given by

$$E_{LO} = A_{LO} \cos[\omega_{LO} t + \phi_{LO}(t)]$$

- The detected current $I_{\mathrm{coh}}(t) \propto$ square of the total electric field of the signal falling on the photodetector, i.e.,

$$I_{coh}(t) = (E_s + E_{LO})^2$$

$$= \frac{1}{2} A_s^2 + \frac{1}{2} A_{LO}^2 + A_s A_{LO} \cos[(\omega_s - \omega_{LO})t + \phi(t)] \cos\theta(t)$$

where $\phi(t) = \phi_s(t) - \phi_{LO}(t)$, and

$$\cos\theta(t) = \frac{\overline{\mathbf{E}}_s \cdot \overline{\mathbf{E}}_{LO}}{\left|\overline{\mathbf{E}}_s\right|\left|\overline{\mathbf{E}}_{LO}\right|}$$

represents the polarization misalignment between the signal wave and the LO wave.

The optical power then becomes

$$P(t) = P_s + P_{LO} + 2\sqrt{P_s P_{LO}}\cos\left[\omega_{IF} t + \phi(t)\right]\cos\theta(t); \omega_{IF} = \omega_s - \omega_{LO}$$

where P_s , P_{LO} : signal and LO optical powers with $P_{LO} >> P_s$. ω_{IF} is intermediate frequency. $\phi(t)$ represents the time-varying phase difference between signal and LO.

Homodyne Detection

- $\omega_s = \omega_{LO}$, i.e., $\omega_{IF} = 0$. The power becomes

$$P(t) = P_s + P_{LO} + 2\sqrt{P_s P_{LO}} \cos\phi(t) \cos\theta(t)$$

- The first term can be ignored (since $P_{LO} >> P_s$), the second term is constant, thus the last term contains transmitted information.
- Can be used for both OOK and PSK.
- Most sensitive receiver.
- Difficult to build; needs local oscillator controlled by optical PLL.
- Restrictions on optical sources for transmitter and LO.

Heterodyne Detection

- $\omega_s \neq \omega_{LO}$ and no need for PLL.
- Ignoring P_s, the detected current contains two terms:

$$i_{dc} = \frac{\eta q}{h\nu} P_{LO}$$

$$i_{IF}(t) = \frac{2\eta q}{h\nu}\sqrt{P_s P_{LO}}\cos[\omega_{IF} t + \phi(t)]\cos\theta(t)$$

- The dc term is filtered out and the information is recovered from the amplified IF term.
- Can be used for OOK, FSK and PSK.

BER : direct-detection OOK

Assume 1 and 0 occur with equal probability, $\overline{N}$ and 0 electron-holes pairs are created during 1 and 0 pulses, and unity quantum efficiency ($\eta = 1$), the average number of photons per bit is

$$\overline{N}_p = \frac{1}{2}\overline{N} + \frac{1}{2}(0) \rightarrow \overline{N} = 2\overline{N}_p$$

The probability of error becomes:

$$P_r(0) = e^{-2\overline{N}_p}$$

Taking quantum efficiency into account, then

$$\text{BER} = P_e = \frac{1}{2}P_r(0) = \frac{1}{2}e^{-2\eta\overline{N}_p}$$

BER : OOK homodyne system

When a 0 pulse of duration T is received, the average number $\overline{N}_0$ of electron-holes pairs created is the number generated by the local-oscillator, i.e., $\overline{N}_0 = A_{LO}^2 T$

For a 1 pulse, the average number is

$$\overline{N}_1 = \left(A_s + A_{LO}\right)^2 T \approx \left(A_{LO}^2 + 2A_s A_{LO}\right)T$$

Since the LO output power is much higher than the received signal power, the voltage V seen by the receiver during a 1 pulse is

$$V = \overline{N}_1 - \overline{N}_0 = 2A_s A_{LO} T$$

and the associated rms noise is

$$\sigma \cong \sqrt{\overline{N}_1} \approx \sqrt{\overline{N}_0}$$

BER : OOK homodyne system (2)

BER becomes

$$\mathrm{BER} = P_e = \frac{1}{2}\left[1 - \mathrm{erf}\left(\frac{V}{2\sqrt{2}\sigma}\right)\right] = \frac{1}{2}\mathrm{erfc}\left(\frac{V}{2\sqrt{2}\sigma}\right) = \frac{1}{2}\mathrm{erfc}\left(\frac{A_s\sqrt{T}}{\sqrt{2}}\right)$$

For example, to achieve BER=10^{-9}, $V/\sigma = 12$ and $A_s^2 T = 36$

which is the expected number of signal photons per pulse.

Assume 0 and 1 occur with same probability, then the average number of photons per bit is half the required number per pulse. Since $\overline{N}_p = A_s^2 T / 2$ and taking quantum efficiency into account yields

$$\mathrm{BER} = \frac{1}{2}\mathrm{erfc}\left(\sqrt{\eta \overline{N}_p}\right) \xrightarrow{\eta \overline{N}_p \geq 5} \frac{e^{-\eta \overline{N}_p}}{\sqrt{\pi \eta \overline{N}_p}}$$

BER : PSK homodyne system

The average number of electron-holes pairs created during a 0 and 1 pulse are given by

$$\overline{N}_{\substack{1\\0}} = \left(A_{LO} \pm A_s\right)^2 T$$

assuming 1 and 0 pulses are in-phase and out-of-phase.

The voltage V seen by the receiver is

$$V = \overline{N}_1 - \overline{N}_0 = 4 A_s A_{LO} T$$

and the associated rms noise is $\sigma = \sqrt{A_{LO}^2 T}$

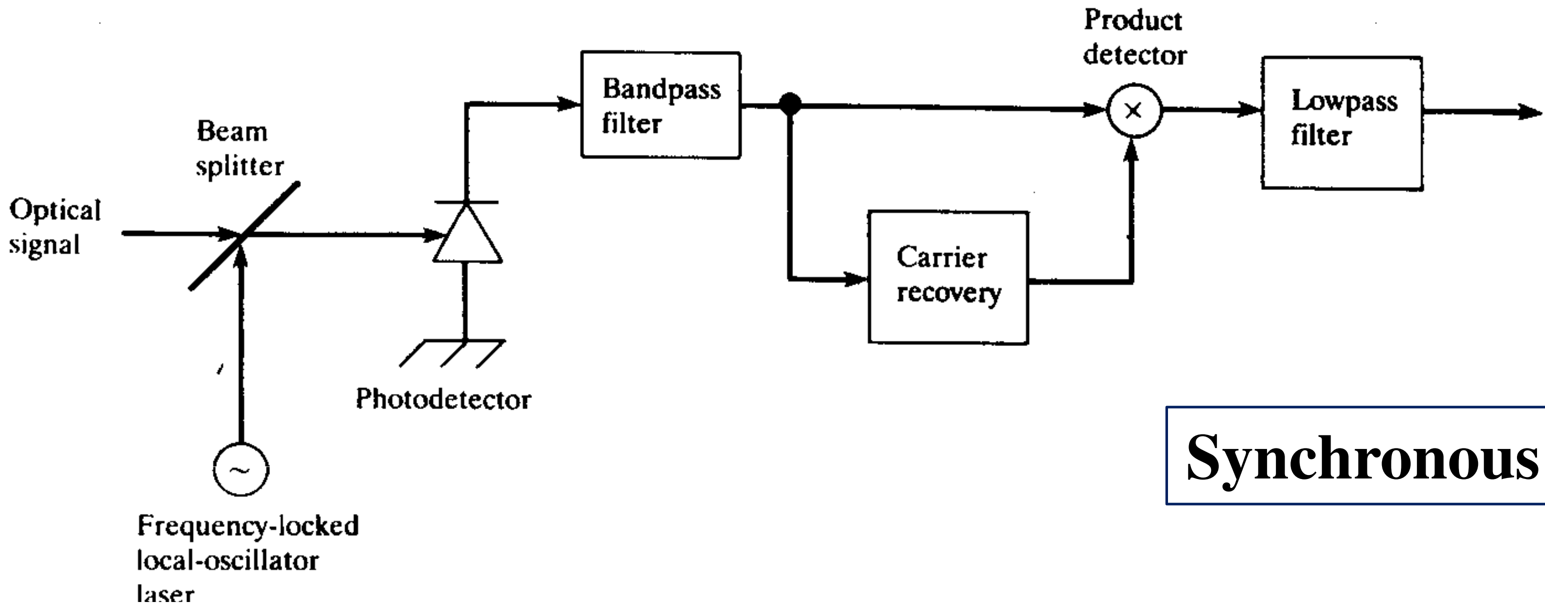
For example, to achieve BER=10^{-9}, V/σ= 12 and $A_s^2 T = 9$

Since $\overline{N}_p = A_s^2 T$

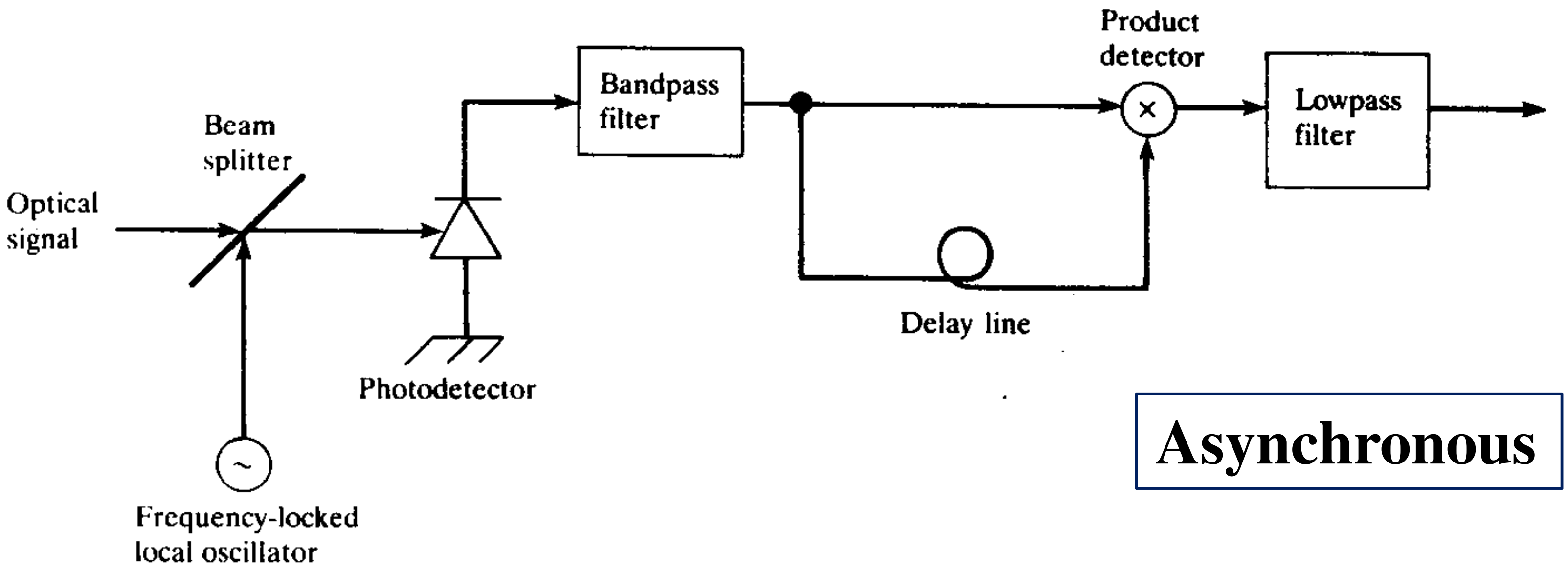
it follows that

$$\text{BER} = \frac{1}{2}\text{erfc}\left(\sqrt{2\eta \overline{N}_p}\right)$$

Heterodyne System



Synchronous



Asynchronous

BER : Summary

Modulation	Probability of error: Homodyne	Heterodyne: Synchronous detection	Heterodyne: Asynchronous detection	Direct detection
On-off keying (OOK)	$\frac{1}{2}\,\mathrm{erfc}\,(\eta \bar{N}_p)^{1/2}$	$\frac{1}{2}\,\mathrm{erfc}\,(\frac{1}{2}\eta \bar{N}_p)^{1/2}$	$\frac{1}{2}\exp(-\frac{1}{2}\eta \bar{N}_p)$	$\frac{1}{2}\exp(-2\eta \bar{N}_p)$
Phase-shift keying (PSK)	$\frac{1}{2}\,\mathrm{erfc}\,(2\eta \bar{N}_p)^{1/2}$	$\frac{1}{2}\,\mathrm{erfc}\,(\eta \bar{N}_p)^{1/2}$	$\frac{1}{2}\exp(-\eta \bar{N}_p)$	—
Frequency-shift keying (FSK)	—	$\frac{1}{2}\,\mathrm{erfc}\,(\frac{1}{2}\eta \bar{N}_p)^{1/2}$	$\frac{1}{2}\exp(-\frac{1}{2}\eta \bar{N}_p)$	—

BER $= 10^{-9}$

Modulation	Number of photons: Homodyne	Heterodyne: Synchronous detection	Heterodyne: Asynchronous detection	Direct detection
On-off keying (OOK)	18	36	40	10
Phase-shift keying (PSK)	9	18	20	—
Frequency-shift keying (FSK)	—	36	40	—

Analog Transmission System

- In photonic analog transmission system the performance of the system is mainly determined by signal-to-noise ratio at the output of the receiver.
- In case of amplitude modulation the transmitted optical power *P(t)* is in the form of:

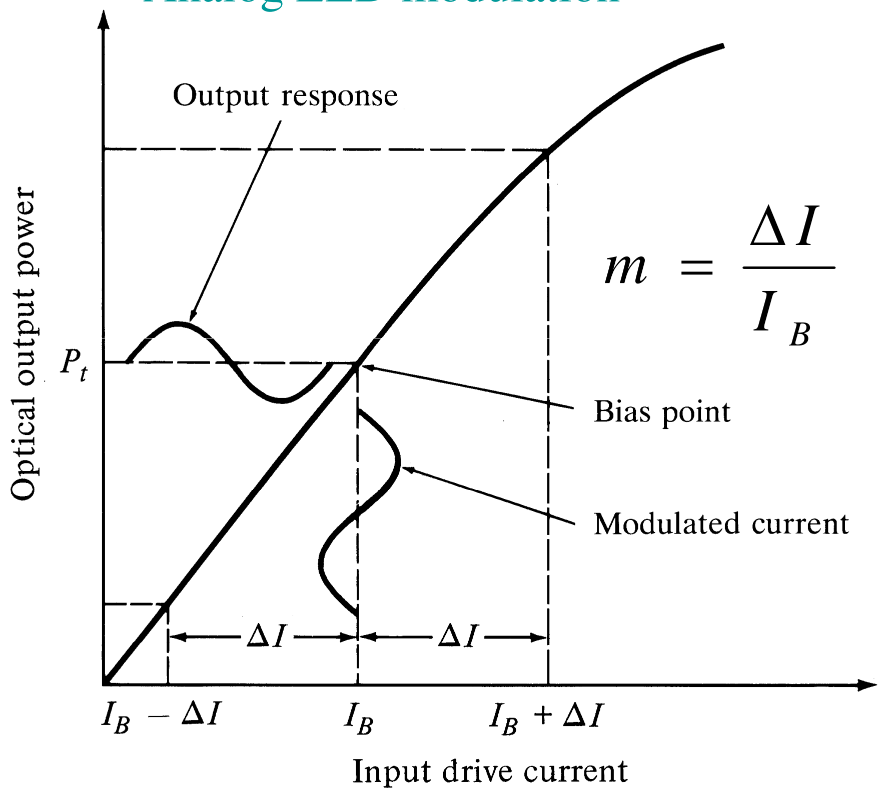
$$P(t) = P_t[1 + ms(t)]$$

where m is modulation index, and $s(t)$ is analog modulation signal.

- The photocurrent at receiver can be expressed as:

$$i_s(t) = \mathcal{R}MP_r[1+ms(t)] = I_P M[1+ms(t)]$$

Analog LED modulation



- By calculating mean square of the signal and mean square of the total noise, which consists of quantum, dark and surface leakage noise currents plus resistance thermal noise, the S/N can be written as:

$$\frac{S}{N} = \frac{\langle i_s^2 \rangle}{\langle i_N^2 \rangle} = \frac{(1/2)(\mathscr{R} M m P_r)^2}{2q(\mathscr{R} P_r + I_D) M^2 F(M) B + (4 k_B T B / R_{eq}) F_t}$$

$$= \frac{(1/2)(M m I_P)^2}{2q(I_P + I_D) M^2 F(M) B + (4 k_B T B / R_{eq}) F_t}$$

I_P : primary photocurrent $= \mathscr{R} P_r$; I_D : primary bulk dark current;

I_L : Surface - leakage current; $F(M)$: excess photodiode noise factor $\approx M^x$

B : effective noise bandwidth; R_{eq} : equivalent resistance of photodetector load and amplifier

F_t : noise figure of baseband amplifier; P_r : average received optical power

pin Photodiode *SNR*

- For *pin* photodiode, *M=1:*

Low input signal level, thermal noise dominates:

$$\frac{S}{N} \cong \frac{(1/2)(I_P m)^2}{(4k_B TB/R_{eq})F_t} = \frac{(1/2)m^2\mathcal{R}^2 P_r^{\,2}}{(4k_B TB/R_{eq})F}$$

Large signal level, shot noise dominates:

$$\frac{S}{N} \cong \frac{m^2 \mathcal{R} P_r}{4qB}$$

SNR vs. optical power for photodiodes