

## ANNUITIES

An annuity is a series of equal payments occurring at equal periods of time. It may also be said as "Equal payment or uniform payment series". In certain business dealings, equal payments are made at the end of equal periods of time and all such accumulated payments are allowed to earn compound interest. Periods of time may be of any length, say a year and month etc. but periods should be of equal length. Interest is expressed in yearly terms but the actual interest is paid at the end of each equal period. Hire purchase payments, instalment buying, LIC premium payments etc. are made by this method.

**Features.** These have the following common features

- (i) These involve series of payments
- (ii) All payments are of equal amount
- (iii) Payments occur at equal time intervals
- (iv) All payments are made at the end of periods
- (v) Compound interest is earned on all accumulated payments

### Kinds of Annuities

1. Capital Recovery Annuity,      2. Present Worth Annuity,
3. Sinking Fund Annuity,      4. Compound Annuity

#### 1. Capital Recovery Annuity

This is applied in case of debt payments where this initial debt or capital is recovered in uniform or equal periodical payments. It is commonly seen in deferred payments, instalment purchase etc. in all such cases the seller recovers his capital along with accumulated compensating interest not in open single lump sum payment but in "Periodical equal payments". The desired formula for this is

$$D = P \left[ \frac{i(1+i)^n}{(1+i)^n - 1} \right]$$

where  $D$  = Equal payment amount,  $P$  = Initial capital

$i$  = Compound interest rate and  $n$  = No. of interest periods

and  $\frac{i(1+i)^n}{(1+i)^n - 1}$  is called CRF (EP)

where CRF (EP) is known as Capital Recovery Factor (Equal Payments).

The values of CRF (EP), can be seen from standard interest tables. These can be expressed through hyphens as [CRF (EP)-8%-7] where 8% means interest rate is 8% per year and the money is borrowed in 7 years.

For example, a person took a loan of Rs. 10,000 from a bank for 5 years at 15% compound interest. The person wants to pay-off the amount in 5 equal annual instalments. Determine the amount of each equal payment.

#### Solution

$$\begin{aligned} D &= P \left[ \frac{i(1+i)^n}{(1+i)^n - 1} \right] = 10,000 \left[ \frac{0.15(1.15)^5}{(1.15)^5 - 1} \right] \\ &= \frac{10,000 \times 0.317}{1.01135} = \text{Rs. 2983 Ans.} \end{aligned}$$

## INTEREST, ANNUITIES AND PROFIT

### 2. Present Worth Annuity

This is applied in cases of L.I.C. premiums and all other retirement plans. These are usually known as premium annuities, income annuities and other future provisions. The desired formula is

$$PW = P \left[ \frac{(1+i)^n - 1}{i(1+i)^n} \right]$$

where  $PW$  = Present worth of entire annuity

$P$  = Periodical payment amount

and  $\frac{(1+i)^n - 1}{i(1+i)^n}$  =  $PWF$  (EP) called present worth factor equal payments.

This can be expressed in the form of hyphens as [ $PWF$  (eP) 5%-6] means interest rate is 5% on 6 years. The value of  $PWF$ -(eP) at different interest rates corresponding to different periods are found in standard interest tables. For example, a person wants that his daughter should get a fixed amount of Rs. 500 at the end of every year for the coming 7 years, so that they may put today itself in the bank total present worth capital which earning a 7% interest, compounded annuity

$$PW = P \left[ \frac{(1+i)^n - 1}{i(1+i)^n} \right] = 500 \left[ \frac{(1+0.07)^7 - 1}{0.07(1+0.07)^7} \right]$$

Here

$$= 500 \left[ \frac{(1.07)^7 - 1}{0.07(1.07)^7} \right] = 500 \left[ \frac{1.606 - 1}{0.07 \times 1.606} \right]$$

$$= \frac{500 \times 0.606}{0.07 \times 1.606} = \text{Rs. 2695.25. Ans.}$$

### 3. Sinking Fund Annuity

This is applied when a definite sum requires to be collected at a future date by setting aside at equal intervals of time equal amounts, so that these equal period payments while earning compound interest total up to the desired amount at the desired future date. Suppose a manufacturing concern wants to open a depreciation reserve fund for machinery replacement by setting apart equal depreciation amounts in the sinking fund.

The desired formula is :

$$D = S \left[ \frac{i}{(1+i)^n} \right]$$

where  $D$  = Periodical equal payment amount and  $S$  = Desired future total

$$\frac{i}{(1+i)^n - 1} = \text{Sinking fund factor or SFF (EP)}$$

**Example 3.** A manufacturing concern desires at the end of 5 years a sinking fund of Rs. 30,000. What equal amount should it deposit every year and, earning 10% interest

**Solution.** Here,

$$D = S \left[ \frac{i}{(1+i)^n - 1} \right]$$

$$= 30,000 \left[ \frac{0.10}{(1+0.1)^5 - 1} \right] = \frac{3000}{(1-10)^5 - 1} = \text{Rs. 4914. Ans.}$$

### 4. Compound Amount Annuity

In this a person deposits equal amounts at the end of a number of periods and each amount is allowed to earn compound interest per period. This is used in savings deposit schemes and cumulative time deposit schemes.



D = perpendicular pavement increments

$$\frac{(1 + r)^n - 1}{r} = \text{CAF (CFY) called compound amount factor equal payment}$$

**Example 4.** Calculate the total compound amount for a 5 years annuity paying Rs. 5000 at every year and earning interest rate of 5% per year.

$$C_A = D_A \left[ \frac{(1 + i)^n - 1}{i} \right] = \frac{500 \left[ \frac{(1 + 0.09)^5 - 1}{0.09} \right]}{0.09} = \text{Rs. } 2994.45 \text{ Ans.}$$

## PROFIT

The profit of a firm equals the total sale proceeds minus the cost of production. It is the residual part of the total sale proceeds left over after paying off all the items of expenditure in the cost of production including rent on land, wages for labour, interest on borrowed capital, salaries of management and organisation etc. But out of this profit later on certain adjustment is to be made. We have to subtract not only actual cost but also certain imputed cost in order to obtain "gross profit". Owner's capital and owner's labour is to be paid although they have not been actually paid. These cost should be added to the actual costs. In addition, certain tax obligations might have arisen during that year. These can be paid next year also, but to arrive at a correct figure of gross profits we should add those tax payments on the cost side. The value of the existing stock is also obtained and this should also be deducted to obtain the true value of "Gross Profits".

To obtain 'Net Profits', we are required to make still certain adjustments. From the gross profit, deduct depreciation charges and the cost of new investment during that period. The final position is called as 'Net Profits'. It can be summarised in the following ways:

- 1 **Residue** = Actual Receipts during a period—Actual payments made during that period
- 2 **Gross Profit** = Residue—Imputed charges of owner's labour and capital—Tax obligations—Value of balance stock
- 3 **Net Profit** = Gross Profit—Value of capital equipment added during the period—Depreciation charges

## Theories of Profits

Various theories of profit have been put forth by different economists to explain the profits. The important among these are

- 1 Risk and Uncertainty theory
- 2 Dynamic Approach to the profit theory
- 3 Residual theory of Profits

1. **Risk and Uncertainty theory.** This theory was introduced by Howley and according to him net profit is the residual income of the owner after making payments for all factors of production and is the reward for the risk taken by him. It concludes that profits are due to the risk taken by the owner. The owner has to bear the risk of losing capital, there are certain risks which can not be insured. They are known as uninsurable risks. We cannot predict when fashion will change or when new invention will come or when will war outbreak etc. There are unforeseeable changes and hence an undue risks which cannot be insured payments made for these uninsurable risks are called 'profits'.

## INTEREST, ANNUITIES AND PROFITS

2. *Dynamic theory of profits*. In a two-period model, profit in a dynamic society is the residual incomes of its rent, wages interest and salary of management. The result only in a dynamic society when the *technology* of production is known, management price (plus the cost of production) is known, no pure profit may exist. Thus if there are no such changes, no pure profit may exist. Thus if profit can exist, it may produce a new technology, competition sets in, the profit declines. Thus in a dynamic society, the theory of profits, this theory was not natural.

A **Rent theory of profits**, this theory was rather common in the 19th century. It says that *over-rent* (a profit on the form of rent) is the source of the theory for the *profit*. Marshall has criticized this theory for the form of *profit* in the form of *profit*.

- (a) Where an event is associated with a probability  $p$  for (i) land may produce zero or positive, or (ii) be positive, zero or negative

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1. Explain the various elements which are involved in interest.
2. Why does an actual rate of interest differ from a nominal rate?
3. Name the various theories of interest and explain them.
4. Discuss and explain the net, basic and gross interest.
5. Define (i) Gross profit (ii) Net profit (iii) Net interest.
6. Write short notes on: (i) Rent and wages (ii) Interest.
7. If Rs. 1000 loan earns Rs. 45 interest in six months, find the rate of interest.
8. How long will it take Rs. 10 to double if it earns 10% interest per annum?
9. What interest rate compounded quarterly will double a sum of Rs. 1000 in 10 years?
10. Find the present value of Rs. 900 due at the end of 10 years if interest is compounded semi-annually. A large concern has made a loan of Rs. 100,000 at the end of the one year and Rs. 100,000 at the end of the second year. Find the present value of the owner of the firm.
11. How much will have to be deposited at the rate of 12% per annum to receive Rs. 12,000 at the end of 10 years?
12. A loan company advertises that they will give Rs. 1000 for each Rs. 1000 repaid each year. What rate of interest is it?
13. A sum of money is accumulated by depositing Rs. 1000 at the end of each year. Interest is paid at the annual rate of 10%. Find the amount at the end of 10 years.