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25/11/19

Practical No. 1.

1. Limit.

Q.1. $\lim_{x \to a} \left[\dfrac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}} \right]$

Q.2 $\lim_{y \to 0} \left[\dfrac{\sqrt{a+y} - \sqrt{a}}{y\sqrt{a+y}} \right]$

Q.3. $\lim_{x \to \frac{\pi}{6}} \left[\dfrac{\cos x - \sqrt{3}\sin x}{\pi - 6x} \right]$

Q.4 $\lim_{x \to \infty} \left[\dfrac{\sqrt{x^2+5} - \sqrt{x^2-3}}{\sqrt{x^2+3} - \sqrt{x^2+1}} \right]$

Q.5 Examine the continuity of the following function :-

(i) $F(x) = \dfrac{\sin 2x}{\sqrt{1-\cos 2x}}$, $0 < x \le \frac{\pi}{2}$
$= \dfrac{\cos x}{\pi - 2x}$, $\frac{\pi}{2} < x < \pi$ } at $x = \frac{\pi}{2}$.

(ii) $f(x) = \dfrac{x^2-9}{x-3}$, $0 < x < 3$
$= x+3$, $3 \le x < 6$
$= \cancel{\dfrac{x^2-9}{x+3}}$ } at $x = 3$, $x = 6$

Q.6 Find value of k, so that the function $f(x)$ continous at the indicated points -

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(i) $f(x) = \frac{1-\cos 4x}{x^2}, \quad x<0$
$= K, \quad x=0$ } at $x=0$

(ii) $f(x) = (\sec^2 x)^{\cot^2 x} \quad x \neq 0$
$= K$ } at $x=0$

(iii) $f(x) = \frac{\sqrt{3}-\tan x}{\pi - 3x}, \quad x \neq \frac{\pi}{3}$
$= K.$ } at $x = \frac{\pi}{3}$.

Q.7. Discuss the continuity of the following function which of these function have removable which is discontinuity? Redefine function on so as to remove the discontinuity.

(i) $f(x) = \frac{1-\cos 3x}{x \tan x}, \quad x \neq 0$
$= 9, \quad x=0$ } at $x=0$

(ii) $f(x) = \frac{(e^{3x}-1)\sin x^{\circ}}{x^2}, \quad x=0$
$= \frac{\pi}{60}, \quad x=0$ } $x=0$

Q.8 If $f(x) = \frac{e^{x^2} - \cos x}{x^2}$

for $x \neq 0$ is continious at $x=0$. Find $f(0)$.

Q.9. If $f(x) = \dfrac{\sqrt{2} - \sqrt{1-\sin x}}{\cos^2 x}$, for $x \neq \pi/2$ is continuous at $x = \dfrac{\pi}{2}$, find $f\left(\dfrac{\pi}{2}\right)$.

Solution

1.

$$\lim_{x \to a} \left[\frac{\sqrt{a+2x} - \sqrt{3}\,x}{\sqrt{3a+x} - 2\sqrt{x}} \right]$$

$$\lim_{x \to a} \left[\frac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}} \times \frac{\sqrt{3a+x} + 2\sqrt{x}}{\sqrt{3a+x} + 2\sqrt{x}} \right]$$

$$\lim_{x \to a} \left[\frac{(\sqrt{a+2x} - \sqrt{3x}) \times (\sqrt{3a+x} + 2\sqrt{x})}{3a + x - 4x} \right]$$

$$\lim_{x \to a} \left[\frac{(a+2x)(3a+x) + (a+2x)(2x) - (3x)(3a+x) - (3x)(2}{3a - 3x} \right]$$

$$\lim_{x \to a} \left[\frac{3a^2 + \cancel{ax} + \cancel{6ax} + \cancel{2x^2} + \cancel{2ax} + \cancel{4x^2} - \cancel{9ax} - 3x^2 - \cancel{6x^2}}{3a - 3x} \right]$$

$$\lim_{x \to a} \left[\frac{3a^2 - 3x^2}{3a - 3x} \right]$$

$$\lim_{x \to a} \left[\frac{\cancel{3}(a^2 - x^2)}{\cancel{3}(a - x)} \right]$$

$$\lim_{x \to a} \left[\frac{(a+x)\cancel{(a-x)}}{\cancel{(a-x)}} \right]$$

$$\therefore \left[a + a \right]$$

$$= \underline{2a}.$$

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Q.2.

$$\rightarrow \lim_{y\to 0}\left[\frac{\sqrt{y+a}-\sqrt{a}}{y(\sqrt{a+y})}\right]$$

$$\lim_{y\to 0}\left[\frac{\sqrt{y+a}-\sqrt{a}}{y(\sqrt{a+y})}\times\frac{(\sqrt{y+a}+\sqrt{a})}{\sqrt{y+a}+\sqrt{a}}\right]$$

$$\lim_{y\to 0}\left[\frac{y+\cancel{a}-\cancel{a}}{y\sqrt{a+y}(\sqrt{y+a}+\sqrt{a})}\right]$$

$$\lim_{y\to 0}\left[\frac{\cancel{y}}{\cancel{y}\sqrt{a+y}(\sqrt{y+a}+\sqrt{a})}\right]$$

$$\lim_{y\to 0}\left[\frac{1}{\sqrt{a+y}(\sqrt{y+a}+\sqrt{a})}\right]$$

$$=\frac{1}{\sqrt{a}(\sqrt{a}+\sqrt{a})}$$

$$=\frac{1}{a+a}=\frac{1}{2a}$$

Q.3

$$\rightarrow \lim_{x\to \pi/6}\left[\frac{\cos x-\sqrt{3}\sin x}{\pi-6x}\right]$$

$x \to \pi/6 + h$, $x - h = \frac{\pi}{6}$, $x \to \frac{\pi}{6}$, $h \to 0$

$$\therefore \lim_{h\to 0}\left[\frac{\cos(\frac{\pi}{6}-h)-\sqrt{3}\sin(\pi/6-h)}{\pi-6(\pi/6-h)}\right]$$

$$\lim_{h\to 0}\left[\frac{\cos\pi/6\cosh+\sin\pi/6\sin h-\sqrt{3}(\sin\frac{\pi}{6}\cos h-\sin h\cos\ldots}{6h}\right.$$

$$\lim_{h\to 0}\left[\frac{\frac{1}{2}\cos h+\frac{\sqrt{3}}{2}\sin h-\sqrt{3}(\frac{\sqrt{3}}{2}\cos h-\sin h\frac{1}{2})}{6h}\right]$$

$$\lim_{h\to 0}\left[\frac{1}{2}\times\frac{\cos h}{6h}+\frac{\sqrt{3}}{2}\times\frac{\sin h}{6h}-\frac{3}{2}\frac{\cos h}{6h}+\frac{\sqrt{3}}{2}\frac{\sin h}{6h}\right]$$

$$=\left[\frac{1}{12}+\frac{\sqrt{3}}{12}-\frac{3}{12}+\frac{\sqrt{3}}{12}\right]$$

$$=\left[\frac{1+\sqrt{3}-3+\sqrt{3}}{12}\right]$$

$$=\left[\frac{2\sqrt{3}-2}{12}\right]$$

$$=\frac{\sqrt{3}-1}{6}$$

Q.4

$$\rightarrow \lim_{x\to\infty}\left[\frac{\sqrt{x^2+5}-\sqrt{x^2-3}}{\sqrt{x^2+3}-\sqrt{x^2+1}}\right]$$

$$\lim_{x\to\infty}\left[\frac{\sqrt{x^2+5}-\sqrt{x^2-3}}{\sqrt{x^2+3}-\sqrt{x^2+1}}\times\frac{\sqrt{x^2+5}+\sqrt{x^2-3}}{\sqrt{x^2+5}+\sqrt{x^2-3}}\right]$$

$$\lim_{x\to\infty}\left[\frac{(x^2+5)-(x^2-3)}{(\sqrt{x^2+3}-\sqrt{x^2+1})(\sqrt{x^2+5}+\sqrt{x^2-3})}\right]$$

$$\lim_{x\to\infty}\left[\frac{2}{(x^2+3)(x^2+5)+(x^2+3)(x^2-3)-(x^2+1)(x^2+5)-(x^2+1)(x^2-3)}\right]$$

$$\lim_{x\to\infty}\left[\frac{2}{\cancel{x^4}+\cancel{5x^2}+3x^2+15+\cancel{x^4}-\cancel{3x^2}+\cancel{3x^2}-9-\cancel{x^4}-\cancel{5x^2}-\cancel{x^2}-5-\cancel{x^4}+3x^2+\cancel{x^2}-3}\right]$$

$$\lim_{x\to\infty}\left[\frac{2}{6x^2-2}\right]$$

RS

Q.8 If $f(x) = \dfrac{e^{x^2} - \cos x}{x^2}$ for $x \neq 0$, is continous at $x = 0$. find $f(0)$.

$\rightarrow$ $f(x) = \dfrac{e^{x^2} - \cos x}{x^2}$

$\because$ f is continous at $x = 0$,

$\therefore \lim_{x \to 0} f(x) = f(0)$

$\lim_{x \to 0} \dfrac{e^{x^2} - \cos x}{x^2} = f(0)$

$\lim_{x \to 0} \dfrac{e^{x^2} - 1 + 1 - \cos x}{x^2} = f(0)$

$\lim_{x \to 0} \left(\dfrac{e^{x^2} - 1}{x^2}\right) + \dfrac{2\sin^2 x/2}{x^2} = f(0)$

$\lim_{x \to 0} \left(\dfrac{e^{x^2} - 1}{x^2}\right) + 2 \times \dfrac{1}{4}\dfrac{\sin^2 x/2}{(x/2)^2} = f(0)$

$= \log e + \dfrac{1}{2}(1) = f(0)$

$= 1 + \dfrac{1}{2} = f(0)$

$= \dfrac{3}{2} = f(0)$.

$\therefore f(0) = \dfrac{3}{2}$ at $x = 0$.

Q-9. If $f(x) = \dfrac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x}$, for $x \neq \frac{\pi}{2}$ is continous at $x = \pi/2$, find $f(\frac{\pi}{2})$.

$\rightarrow$ $f(x) = \dfrac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x}$

f is continous at $\frac{\pi}{2}$,

$\therefore \lim\limits_{x \to \frac{\pi}{2}} \left[\dfrac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x}\right]$

$= \lim\limits_{x \to \frac{\pi}{2}} \left[\dfrac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x} \times \dfrac{\sqrt{2} + \sqrt{1+\sin x}}{\sqrt{2} + \sqrt{1+\sin x}}\right]$

$= \lim\limits_{x \to \frac{\pi}{2}} \left[\dfrac{(2) - (1+\sin x)}{\cos^2 x\,(\sqrt{2} + \sqrt{1+\sin x})}\right]$

$= \lim\limits_{x \to \frac{\pi}{2}} \left[\dfrac{(1-\sin x)}{(1-\sin^2 x)\,(\sqrt{2} + \sqrt{1+\sin x}}\right]$

$= \lim\limits_{x \to \frac{\pi}{2}} \left[\dfrac{\cancel{(1-\sin x)}}{\cancel{(1-\sin x)}(1+\sin x)\,(\sqrt{2} + \sqrt{1+\sin x}}\right]$

$= \lim\limits_{x \to \frac{\pi}{2}} \dfrac{1}{(1+\sin x)\,(\sqrt{2} + \sqrt{1+\sin x}}]$

$= \dfrac{1}{1\,(\sqrt{2}+1)}$

$=$

$$4\lim_{x\to\infty}\left[\frac{\cancel{x}}{\cancel{x}(3x^2-1)}\right]$$

$$\lim_{x\to\infty}\left[\frac{1}{3x^2}-\frac{1}{1}\right]$$

$$= \underline{\underline{\cancel{0}4}}$$

Q.5.

$$\rightarrow (i)\ f(x)=\left.\begin{array}{ll}\dfrac{\sin 2x}{\sqrt{1-\cos 2x}} & 0<x\le\dfrac{\pi}{2}\\[2ex] =\dfrac{\cos x}{\pi-2x} & \dfrac{\pi}{2}<x<\pi\end{array}\right\}\ x=\frac{\pi}{2}$$

① $$F(\pi/2)=\frac{\sin 2(\pi/2)}{\sqrt{1-\cos 2(\pi/2)}}$$

$$=\frac{\sin\pi}{\sqrt{1-\cos\pi}}=\frac{0}{\sqrt{2}}=0.$$

② $$\lim_{x\to\pi/2^+} f(x)=\frac{\cos x}{\pi-2x}$$

put $x-\frac{\pi}{2}=h\ \therefore x=\frac{\pi}{2}+h,\ h\to 0$

$$=\lim_{h\to 0}\frac{\cos(\pi/2+h)}{\pi-2(\pi/2+h)}$$

$$=\lim_{h\to 0^+}\frac{\cancel{-}\sin h}{\cancel{-}2h}$$

$$\frac{1}{2}=RHS.$$

$$f(x)=\lim_{x\to 0^+}\frac{\sin 2x}{\sqrt{1-\cos 2x}}$$

$$=\lim_{x\to 0^-}\frac{2\cos x\cancel{\sin x}}{\sqrt{2}\cancel{\sin x}}$$

$$=\cancel{\lim_{x\to 0}}\frac{2}{\sqrt{2}}\cos(0)$$

$$=\sqrt{2}\ne RHS$$

$\because$ LHS $\neq$ RHS

$\therefore$ F is not continous at $x = \frac{\pi}{2}$.

(ii) $f(x) = \frac{x^2-9}{x-3}$, $0 < x < 3$

$= x+3$, $3 \leq x < 6$

$= \frac{x^2-9}{x+3}$, $6 \leq x < 9$

at $x = 3$ & $x = 6$.

① for $x = 3$

$f(3) = 3+3 = 6$ —(i)

f is define.

② $\lim\limits_{x \to 3^+} f(x) = (x+3)$

$= 3+3 = 9$

3 $f(x) = \lim\limits_{x \to 3^-} \frac{x^2-9}{x-3}$

$\lim\limits_{x \to 3^-} \frac{\cancel{(x-3)}(x+3)}{\cancel{(x-3)}}$

$\lim\limits_{x \to 3} x+3$

$= 3+3 = 6$

$\therefore \lim\limits_{x \to 3^-} f(x) = \lim\limits_{x \to 3^+} = f(3)$ —(ii)

from (i) & (ii)

$\therefore$ function is continous at $x = 3$

For, $x = 6$.

① $\lim\limits_{x \to 6} \frac{x^2-9}{x+3} = \lim\limits_{x \to 6} \frac{\cancel{(x+3)}(x-3)}{\cancel{(x+3)}} = 6-3 = 3$.

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② i) $f(x) = \lim_{x \to 6^+} \cancel{f = \lim} \frac{x^2 - 9}{x+3} = 6$

ii) $f(x) = \lim_{x \to 6^-} (x^{2}+3) = 6+3 = 9.$

$\therefore$ Above f is not continous at $x=6$.

Q.6 Find the value of K, So that the function f(x) is continous at the indicated point

(i)

$\rightarrow$ $f(x) = \frac{1-\cos 4x}{x^2}$, $x \neq 0$
$= K$, $x = 0$ $\}$ at $x=0$.

$\because$ f is continous at $x=0$

$\lim_{x \to 0} f(x) = f(0)$

$\lim_{x \to 0} \frac{1-\cos 4x}{x^2} = K$

$\lim_{x \to 0} \frac{2\sin^2 2x}{x^2} = K$

$\lim_{x \to 0} 2 \times 4 \left(\frac{\sin 2x}{2x}\right)^2 = K$

$= 8 (1)^2 = K$

$\therefore$ $\underline{K = 8}$.

(ii) $f(x) = \frac{\sqrt{3} - \tan x}{\pi - 3x}$, $x \neq \pi/3$
$= K$, $x = \pi/3$ $\}$ at $x = \pi/3$

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$\therefore$ f is continous at $x = \pi/3$

$$\lim_{x\to\frac{\pi}{3}} f(x) = f(\pi/3)$$

$$\lim_{x\to\frac{\pi}{2}} \frac{\sqrt{3}-\tan x}{\pi - 3x} = K$$

Put, $x = \frac{\pi}{3} + h$

$$\therefore \lim_{h\to 0} \frac{\sqrt{3} - \tan\left(\frac{\pi}{3}+h\right)}{\pi - 3\left(\frac{\pi}{3}+h\right)} = K$$

$$\lim_{h\to 0} \left[\frac{\sqrt{3} - \left(\frac{\tan\frac{\pi}{3} + \tan h}{1-\tan\frac{\pi}{3}\cdot\tan h}\right)}{\cancel{\pi} - \cancel{\pi} - 3h}\right] = K$$

$$\lim_{h\to 0} \left[\frac{\cancel{\sqrt{3}} - \sqrt{3}\tan h - \tan\frac{\pi}{3} - \tan h}{-3h\left(1-\tan\frac{\pi}{3}\tan h\right)}\right] = K$$

$$\lim_{h\to 0} \left[\frac{-4\tan h}{-3h\left(1-\sqrt{3}\tan h\right)}\right] = K$$

$$\lim_{h\to 0} \left[\frac{4}{3}\left(\frac{\tan h}{h}\right) \times \frac{1}{\left(1-\sqrt{3}\tan h\right)}\right] = K$$

$$= \left[\frac{4}{3}(1) \times \frac{1}{1-\sqrt{3}\tan(0)}\right] = K$$

$$= \frac{4}{3} \times 1 = \frac{4}{3} = K$$

ii) $f(x) = (\sec^2 x)^{\cot^2 x}$, $x \neq 0$
$= K$, $x = 0$ } at $x = 0$

f is continous at $x = 0$.

$$\lim_{x\to 0} f(x) = f(0)$$

$$\lim_{x\to 0} (\sec^2 x)^{\cot^2 x} = K$$

$$\lim_{x\to 0} \left(1-\tan^2 x\right)^{-\left(\frac{-1}{\tan^2 x}\right)} = K.$$

$$= (\log e)^{-1} = K$$

$$\frac{1}{\log e} = \underline{\underline{1 = K}}$$

Q.7.

(i) $f(x) = \frac{1-\cos 3x}{x.\tan x}$, $x \neq 0$ } at $x = 0$
$= 9$, $x = 0$

$$\lim_{x\to 0} \frac{2\sin^2 \frac{3}{2}x}{x\tan x}$$

$$\lim_{x\to 0} 2 \frac{\sin^2 \frac{3}{2}x \times \frac{9}{4}}{\frac{3x}{2} \times x \times \frac{9}{4}} \times \frac{1}{\frac{\tan x}{x}}$$

$$\lim_{x\to 0} \cancel{2} \times \frac{9}{\cancel{4}2} \times \frac{1}{\frac{\tan x}{x}}$$

$$\cancel{\lim_{x\to 0}} \quad \frac{9}{2} \times \frac{1}{(1)}$$

$$\frac{9}{2} //$$

$\therefore$ f is not continous at $x = 0$

Redefine function

$f(x) = \frac{1-\cos 3x}{x\tan x}$, $x \neq 0$
$= \frac{9}{2}$, $x = 0$

Now, $\lim_{x\to 0} f(x) = f(0)$

f has removable discontinuity at $x = 0$

(ii) $f(x) = \frac{(e^{3x}-1)\sin x^\circ}{x^2}$, $x \neq 0$
$= \frac{\pi}{60}$, $x = 0$ } at $x = 0$

$\lim_{x \to 0} \frac{(e^{3x}-1)\sin x^\circ}{x^2}$

$\lim_{x \to 0} \left(3 \times \frac{e^{3x}-3}{3x}\right)\left(\frac{\sin x^\circ}{x} \times \frac{\pi}{180}\right)$

$\cancel{\lim_{x \to 0}} = 3 \log e \,(1) \times \frac{\pi}{180}$

$= 3 \times \frac{\pi}{180} = \frac{\pi}{60} = f(0)$

$\therefore$ f is continous at $x = 0$.

AK
2/12/19

PRACTICAL-02

Topic :- Derivative

Q.1. Show that the following function defined from R to R are differentiable :-

1) $\cot x$.

$\rightarrow$ $f(x) = \cot x$

$$Df(a) = \lim_{x\to a} \frac{f(x) - f(a)}{x - a}$$

$$= \lim_{x\to a} \frac{\cot x - \cot a}{x - a}$$

$$= \lim_{x\to a} \frac{\frac{1}{\tan x} - \frac{1}{\tan a}}{x - a}$$

$$= \lim_{x\to a} \frac{\tan a - \tan x}{(x-a)\tan a \tan x}$$

put $x - a = h$

$x = h + a$

as $x \to a$ $h \to 0$

$$Df(a) = \lim_{h\to 0} \frac{\tan a - \tan(a+h)}{(a+h-a)\tan a \cdot \tan(a+h)}$$

$$= \lim_{h\to 0} \frac{\tan a - \tan(a+h)}{h \tan h \cdot \tan(a+h)}$$

$\because$ formula: $\tan(A-B) = \frac{\tan A - \tan B}{1+\tan A \tan B}$

$$= \lim_{h\to 0} \frac{\tan(a-a-h) - (1+\tan a \tan(a+h))}{h \times \tan(a+h) \tan a}$$

$$= \lim_{h\to 0} \frac{-\tan h}{h} \times \frac{1+\tan a \tan(a+h)}{\tan(a+h)\tan a}$$

$$= -1 \times \frac{1+\tan^2 a}{\tan^2 a}$$

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$= \frac{\sec^2 a}{\tan^2 a} = \frac{1}{\cancel{\cos^2 a}} \times \frac{\cancel{\cos^2 a}}{\sin^2 a}$

$= -\operatorname{cosec}^2 a$

$\therefore Df(a) = -\operatorname{cosec}^2 a$

$\therefore$ f is Differentiable $\forall a \in R$.

2) $\operatorname{cosec} x$

$\rightarrow \quad f(x) = \operatorname{cosec} x$

$$Df(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$= \lim_{x \to a} \frac{\operatorname{cosec} x - \operatorname{cosec} a}{x - a}$$

$$= \lim_{x \to a} \frac{\frac{1}{\sin x} - \frac{1}{\sin a}}{x - a}$$

$$= \lim_{x \to a} \frac{\sin a - \sin x}{(x-a)\sin x \cdot \sin a}$$

Put $x - a = h$, $x = h + a$,
$h \to 0$, $x \to a$

$$\therefore Df(h) = \lim_{h \to 0} \frac{\sin a - \sin(a+h)}{(\cancel{a} + h - \cancel{a})\sin(a+h) \cdot \sin a}$$

$$= \lim_{h \to 0} \frac{\sin a - \sin(a+h)}{h \times \sin(a+h)\sin a}$$

formula : $\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right) \cdot \sin\left(\frac{C-D}{2}\right)$.

$$= \lim_{h \to 0} \frac{2\cos\left(\frac{a+a+h}{2}\right) \cdot \sin\left(\frac{\cancel{a} - \cancel{a} - h}{2}\right)}{h \times \sin(a+h)\sin a}$$

$$= \lim_{h \to 0} -\frac{\sin\frac{h}{2}}{h/2} \times \frac{1}{2} \times 2\cos\left(\frac{2a+h}{2}\right)$$

$$= \frac{-\frac{1}{2} \times 2\cos\left(\frac{2a+0}{2}\right)}{\sin a \cdot \sin a}$$

$=$

$$-\frac{\cos a}{\sin^2 a} = -\cot a \operatorname{cosec} a.$$

Ae

Sec x.

$f(x) = \sec x$

$$Df(x) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$= \lim_{x \to a} \frac{\sec x - \sec a}{x - a}$$

$$= \lim_{x \to a} \frac{\frac{1}{\cos x} - \frac{1}{\cos a}}{x - a}$$

$$= \lim_{x \to a} \frac{\cos a - \cos x}{(x-a)\cos x \cos a}$$

Put $x - a = h$, $x = a+h$,

$x \to a$, $h \to 0$,

$$Df(h) = \lim_{h \to 0} \frac{\cos a - \cos(a+h)}{(\cancel{x}a+h-a)\cos(a+h)\cos a}$$

$$= \lim_{h \to 0} \frac{\cos a - \cos(a+h)}{h \times \cos(a+h) \cdot \cos a}.$$

formula: $\cos C - \cos D = -2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$

$$= \lim_{h \to 0} \frac{-2\sin\left(\frac{a+a+h}{2}\right) \cdot \sin\left(\frac{a-a-h}{2}\right)}{h \times \cos a \cos(a+h)}$$

$$= \lim_{h \to 0} -\frac{\sin\frac{h}{2}}{h/2} \times \frac{1}{2} \times \frac{-2\sin\left(\frac{2a+h}{2}\right)}{\cos a \cos(a+h)}$$

$$= \frac{-\frac{1}{\cancel{2}} \times (-\cancel{2}) \sin\left(\frac{2a+0}{2}\right)}{\cos a \cdot \cos a}$$

$$= \frac{\sin a}{\cos^2 a} = \tan a \cdot \sec a.$$

Q.2 If $f(x) = 4x+1$, $x \leq 2$
$= x^2+5$, $x > 0$, at $x=2$, then find function is differentiable or not.

Solⁿ:- LHD :-

$$Df(2^-) = \lim_{x \to 2^-} \frac{f(x) - f(2)}{x-2}$$

$$= \lim_{x \to 2^-} \frac{4x+1-(4\times\cancel{2}+2)}{x\cancel{+}2}$$

$$= \lim_{x \to 2^-} \frac{4x+1-9}{x-2}$$

$$= \lim_{x \to 2^-} \frac{4x-8}{x-2}$$

$$= \lim_{x \to 2^-} \frac{4\cancel{(x-2)}}{\cancel{x-2}} = 4.$$

$Df(\cancel{-}2^-) = 4$

RHD :-

$$Df(2^+) = \lim_{x \to 2^+} \frac{x^2+5-9}{x-2}$$

$$= \lim_{x \to 2^+} \frac{x^2-4}{x-2}$$

$$= \lim_{x \to 2^+} \frac{(x+2)\cancel{(x-2)}}{\cancel{(x-2)}}$$

$= 2+2 = 4$

$Df(2^+) = 4$

$\therefore$ RHD = LHD.

f is Differentiable at $x=2$.

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Q.3 If $f(x) = 4x+7$, $x < 3$
$= x^2+3x+1$, $x \geq 3$ at $x=3$ then find f is differentiable or not?

Solns-

RHD:

$$Df(3^+) = \lim_{x \to 3^+} \frac{f(x) - f(a)}{x-a}$$

$$\lim_{x \to 3^+} \frac{x^2+3x+1-(3^2+3\times3+1)}{x-3}$$

$$\lim_{x \to 3^+} \frac{x^2+3x+1-(19)}{x-3}$$

$$\lim_{x \to 3^+} \frac{x^2+3x-18}{(x-3)}$$

$$\lim_{x \to 3^+} \frac{(x+6)\cancel{(x-3)}}{\cancel{(x-3)}}$$

$\therefore$ $3+6 = 9$.

$Df(3^+) = 9$

LHD:

$$Df(3^-) = \lim_{x \to 3^-} \frac{f(x) - f(3)}{x-3}$$

$$= \lim_{x \to 3^-} \frac{4x+7-\cancel{(4\times3+7)}\ 19}{x-3}$$

$$\lim_{x \to 3^-} \frac{4x-12}{(x-3)}$$

$$\lim_{x \to 3^-} \frac{4\cancel{(x-3)}}{\cancel{(x-3)}} = 4.$$

$Df(3^+) = 4$

RHD $\neq$ LHD

f is not differentiable at $x=3$.

Q.4. If $f(x) = 8x - 5 \quad , x \leq 2$

$= 3x^2 - 4x + 3 \quad , x > 2$ at $x = 2$ 36 then find f is differentiable or not.

Soln:-

$f(x) = 8 \times 2 - 5 = 16 - 5 = 11$

R.H.D:

$$Df(2^+) = \lim_{x \to 2^+} \frac{f(x) - f(2)}{x - 2}$$

$$= \lim_{x \to 2^+} \frac{3x^2 - 4x + 7 - 11}{x - 2}$$

$$= \lim_{x \to 2^+} \frac{3x^2 - 4x - 4}{x - 2}$$

$$= \lim_{x \to 2^+} \frac{(3x + 2)\cancel{(x - 2)}}{\cancel{(x - 2)}}$$

$$= \lim_{x \to 2^+} 3x + 2 = 3(2) + 2 = 8.$$

$Df(2^+) = 8.$

LHD:

$$Df(2^-) = \lim_{x \to 2^-} \frac{f(x) - f(2)}{x - 2}$$

$$= \lim_{x \to 2^-} \frac{8x - 5 - 11}{x - 2}$$

$$\lim_{x \to 2^-} \frac{8x - 16}{x - 2}$$

$$= \lim_{x \to 2^-} \frac{8\cancel{(x - 2)}}{\cancel{(x - 2)}} = 8$$

$= \cancel{\lim}$ $\therefore Df(3^-) = 8$

$\therefore$ LHD = RHD

$\therefore$ f is differentiable at $x = 3$.

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PRACTICAL-03

Topic - Application Of Derivative.

Q.1. Find the intervals in which function is increasing or decreasing.

(i) $f(x) = x^3 - 5x - 11$

$\rightarrow$ $F(x) = x^3 - 5x - 11$

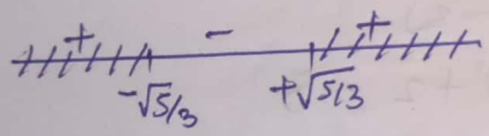
$f'(x) = 3x^2 - 5$

$\because$ f is increasing iff $f'(x) > 0$.

$3x^2 - 5 > 0$

$3(x^2 - 5/3) > 0$

$(x - \sqrt{5/3})(x + \sqrt{5/3}) > 0$



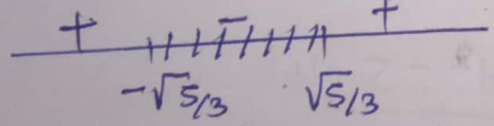
$x \in (-\infty, -\sqrt{5/3}) \cup (\sqrt{5/3}, \infty)$.

and f is decreasing iff $f'(x) < 0$

$3x^2 - 5 < 0$

$\therefore (x^2 - 5/3) < 0$

$(x - \sqrt{5/3})(x + \sqrt{5/3}) < 0$



$x \in (-\sqrt{5/3}, \sqrt{5/3})$

(ii) $f(x) = x^2 - 4x$.

$\rightarrow$ $f(x) = x^2 - 4x$

$\therefore f'(x) = 2x - 4$

$\because$ f is decreasing iff $f'(x) < 0$

$2x - 4 < 0$

$2(x - 2) < 0$

$\therefore x - 2 < 0$

$\therefore x \in (-\infty, 2)$

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$\because$ f is increasing iff $f'(x) > 0$,

$2x - 4 > 0$

$2(x-2) > 0$

$\therefore x - 2 > 0$

$\therefore x \in (2, \infty)$.

(iii) $F(x) = 2x^3 + x^2 - 20x + 4$.

$\rightarrow$ $F(x) = 2x^3 + x^2 - 20x + 4$

$\therefore F'(x) = 6x^2 + 2x - 20$

$\because$ F is increasing, iff $F(x) > 0$,

$\therefore 6x^2 + 2x^2 - 20 > 0$

$2(3x^2 + x - 10) > 0$

$\therefore 3x^2 + 6x - 5x - 10 > 0$

$3x(x+2) - 5(x+2) > 0$

$(x+2)(3x-5) > 0$

$-2 \qquad 5/3$ $\quad x \in (-\infty, -2) \cup (5/3, \infty)$

$\because$ F is decreasing iff $F(x) < 0$.

$\therefore 6x^2 + 2x - 20 < 0$

$2(3x^2 + x - 10) < 0$

$3x^2 + x - 10 < 0$

$3x^2 + 6x - 5x - 10 < 0$

$3x(x+2) - 5(x+2) < 0$

$(3x-5)(x+2) < 0$

$-2 \qquad 5/3$

$x \in (-2, 5/3)$

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(iv) $f(x) = x^3 - 27x + 5$

$\rightarrow f(x) = x^3 - 27\cancel{x+5}$

$\therefore f'(x) = 3x^2 - 27$

$\because f$ is increasing, iff $f(x) > 0$,

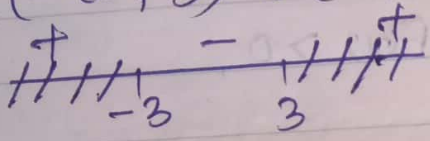
$3x^2 - 27 > 0$

$3(x^2 - 9) > 0$

$x^2 - 9 > 0$

$(x+3)(x-3) > 0$

$\therefore x \in (-\infty, -3) \cup (3, \infty)$



8

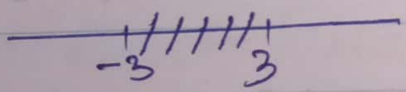
$\because f$ is decreasing, iff $f(x) < 0$

$\therefore 3x^2 - 27 < 0$

$\therefore 3(x^2 - 9) < 0$

$\therefore (x-3)(x+3) < 0$

$\therefore x \in (-3, 3)$



(v) $f(x) = 2x^3 - 9x^2 - 24x + 69$

$\rightarrow f(x) = 2x^3 - 9x^2 - 24x + 69$

$f'(x) = 6x^2 - 18x - 24$

$\because f$ is increasing, iff $f'(x) > 0$

$\therefore 6x^2 - 18x - 24 > 0$

$6(x^2 - 3x - 4) > 0$

$x^2 - 3x - 4 > 0$

$x^2 + x + 4x - 4 > 0$

$x(x+1) - 4(x+1) > 0$

$(x-4)(x+1) > 0$

$+ \quad -1 \quad - \quad 4 \quad +$

$x \in (-\infty, -1) \cup (4, \infty)$

$\therefore$ f is decreasing, iff $f'(x) < 0$,

$\therefore 6x^2 - 18x - 24 < 0$

$6(x^2 - 3x - 4) < 0$

$x^2 - 3x - 4 < 0$

$x^2 + x - 4x - 4 < 0$

$x(x+1) - 4(x+1) < 0$

$(x-4)(x+1) < 0$

$+ \quad -1 \quad - \quad 4 \quad +$

Q.2. Find the intervals in which function is concave upwards.

(i) $y = 3x^2 - 2x^3$

$\rightarrow y = 3x^2 - 2x^3$

$\therefore f(x) = 3x^2 - 2x^3$

$\therefore f'(x) = 6x - 6x^2$

$\therefore f''(x) = 6 - 12x$

f is concave upward iff $f''(x) > 0$

$(6 - 12x) > 0$

$12\left(\frac{1}{2} - x\right) > 0$

$\frac{1}{2} - x > 0$

$\therefore x > \frac{1}{2}$

$\therefore f''(x) > 0$

$\therefore x \in (1/2, \infty)$.

(ii) $y = x^4 - 6x^3 + 12x^2 + 5x + 7$

$\rightarrow f'(x) = 4x^3 - 18x^2 + 24x + 5$

$f''(x) = 12x^2 - 36x + 24$

f is concave upward iff $f''(x) > 0$

$12x^2-36x+24>0$

$12(x^2-3x+2)>0$

$\therefore x^2-3x+2>0$

$x^2-2x-x+2>0$

$x(x-2)-1(x-2)>0$

$\therefore (x-1)(x-2)>0$

$\therefore f''(x)>0$

$\therefore x \in (-\infty,1)\cup(2,\infty)$.

$+ \quad - \quad +$

$1 \quad 2$

(iii) $y=x^3-27x+5$

$\rightarrow f'(x)=3x^2-27$

$f''(x)=6x$

f is concave upward, iff $f''(x)>0$

$6x>0$

$x>0$

$\therefore f''(x)>0$

$\therefore x\in(0,\infty)$

(iv) $y=69-24x-9x^2-2x^3$.

$\rightarrow f(x)=69-24x-9x^2-2x^3$

$f'(x)=6x^2-18x-24$

$f''(x)=12x-18$.

f is concave upward iff $f''(x)>0$

$12x-18>0$

$12(x-18/12)>0$

$(x-3/2)>0$

$\therefore f''(x)>0$

$\therefore x\in(3/2,\infty)$

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(v) $y = 2x^3 + x^2 - 20x + 4$.

→ $f(x) = 2x^3 + x^2 - 20x + 4$

$f'(x) = 6x^2 + 2x - 20$.

$f''(x) = 12x + 2$

∴ f is concave upward, iff $f''(x) > 0$

$12x + 2 > 0$

$12(x + 1/6) > 0$

$x + 1/6 > 0$

∴ ~~f~~ $f''(x) \neq 0$

∴ $x \in \{-1/6 \cdot \cancel{\infty}\}$.

∴ There exist ~~no~~ interval.

$x \in (-1/6, \infty)$

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P.E.

PRACTICAL-04.

Topic :- Application of Derivative & Newton's Method.

Q.1. Find Maximum & Minimum value of following function :-

(i) $f(x) = x^2 + \frac{16}{x^2}$.

$\rightarrow$ $f(x) = x^2 + \frac{16}{x^2}$

$f'(x) = 2x - 32/x^3$.

Now, consider,

$f'(x) = 0$

$\therefore 2x - 32/x^3 = 0$

$2x^4 - 32 = 0$

$2x^4 = 32$

$\therefore x^4 = 16$

$x = \pm 2$

$f''(x) = 2 + 96/x^4$

$f''(2) = 2 + 96/2^4$

$= 2 + 6 = 8 > 0$.

$\therefore$ f has minimum value at $x = 2$.

~~Now, x=2~~

$f(2) = 2^2 + 16/2^2$

$= 4 + 16/4$

$= \cancel{16}\, 8$.

$f''(-2) = 2 + 96/-2^4$

$= 2 + 6 = 8 > 0$

$\therefore$ f has minimum value at $x = -2$

$\therefore$ Function reaches minimum value at $x = 2$, and $x = -2$.

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(ii) $f(x) = 3 - 5x^3 + 3x^5$.

→ $f'(x) = -15x^2 + 15x^4$

consider,

$f'(x) = 0$

$-15x^2 + 15x^4 = 0$

$x^4 = x^2$

$x^2 = 1$

$x = \pm 1$

$\therefore f''(x) = -30x + 60x^3$

$f(1) = -30 + 60$

$= 30 > 0$

$\therefore f$ has minimum value at $x = 1$.

$\therefore f(1) = 3 - 5(1)^3 + 3(1)^5$

$= \cancel{3} - 5 + \cancel{3}$

$= 6 - 5 \cancel{= 0} = 1$

$\therefore f''(-1) = -30 - 60$

$= -30$

$\therefore f$ has maximum value at $x = -1$.

$\therefore f(-1) = 3 - 5(-1)^3 + 3(-1)^5$

$= 3 + 5 - 3$

$= 5$

$\therefore f$ has the maximum value 5 at $x = -1$ & has the minimum value 1 at $x = 1$.

(iii) $f(x) = x^3 - 3x^2 + 1$.

→ $f'(x) = 3x^2 - 6x$

consider,

$f'(x) = 0$

$\therefore 3x^2 - 6x = 0$

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$\therefore 3x(x-2)=0$

$x-2=0$

$x=2$ or $x=0$.

$f''(x)=6x-6$

$\therefore f''(0)=6(0)-6$

$=-6<0$

$\therefore$ f has maximum value at $x=0$.

$\therefore f(0)=(0)^3-3(0)^2+1$

$=1$

$f''(2)=6(2)-6$

$f''(2)=12-6=6>0$

$\therefore$ f has minimum value at $x=2$

$f(2)=(2)^3-3(2)^2+1$

$=8-12+1$

$=-3$

$\therefore$ f has maximum value 1 at $x=0$ &

f has minimum value -3 at $x=2$.

(iv) $f(x)=2x^3-3x^2-12x+1$.

$\rightarrow$ $f'(x)=6x^2-6x-12$.

~~consider,~~ consider,

$f'(x)=0$

$6x^2-6x-12=0$

$6(x^2-x-2)=0$

$x^2-2x+x-2=0$

$(x-2)(x+1)=0$

$x=2$ OR $x=-1$.

$\therefore f''(x)=12x-6$.

$f''(2)=12(2)-6$

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$f''(x) = 18 > 0$

$\therefore$ f has minimum value at $x = 2$.

$f(2) = 2(2)^3 - 3(2)^2 - 12(2) + 1$

$= 16 - 12 - 24 + 1 = -19$.

$f''(-1) = 12(-1) - 6$

$= -12 - 6 = -18 < 0$

f has maximum value at $x = -1$.

$\therefore f(-1) = 2(-1)^3 - 3(-1)^2 - 12(-1) + 1$

$= -2 - 3 + 12 + 1$

$= 8$.

$\therefore$ f has maximum value 8 at $x = -1$ &

f has minimum value -19 at $x = 2$.

Q.2. find the root of following equation by Newton Method [Take 4 iteration only] correct upto 4 decimal.

(i) $f(x) = x^3 - 3x^2 - 55x + 9.5$ (take $x_0 = 0$).

$\rightarrow f'(x) = 3x^2 - 6x - 55$

By Newton's Method,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$\therefore x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$x_1 = 0 + \frac{9.5}{55}$$

$x_1 = 0.1727$

$\therefore f(x_1) = (0.1727)^3 - 3(0.1727)^2 - 55(0.1727) + 9.5$

$= 0.0051 - 0.0895 - 9.4985 + 9.5$

$= -0.0829$.

$f'(x_1) = 3(0.1727)^2 - 6(0.1727) - 55$

$= 0.0895 - 1.0362 - 55$

$= -55.9467$

$\therefore x_2 = x_1 - \dfrac{f(x_1)}{f'(x_1)}$

$= 0.1727 + \dfrac{0.0829}{55.9467}$

$= 0.1712$

$f(x_2) = (0.1712)^3 - 3(0.1712)^2 - 55(0.1712) + 9.5$

$= 0.0050 - 0.0879 - 9.416 + 9.5$

$= 0.0011$

$f'(x_2) = 3(0.1712)^2 - 6(0.1712) - 55$

$= 0.0879 - 1.0272 - 55$

$= -55.9393$

$x_3 = x_2 - \dfrac{f(x_2)}{f'(x_2)}$

$= 0.1712 + \dfrac{0.0011}{55.9893}$

$= 0.1712$

The root of the equation is 0.1712.

(ii) $f(x) = x^3 - 4x - 9$

$\rightarrow f'(x) = 3x^2 - 4$.

$f(2) = 2^3 - 4(2) - 9$

$= 8 - 8 - 9$

$= -9$

$f(3) = 3^3 - 4(3) - 9$

$= 27 - 12 - 9$

$= 6$

Let $x_0 = 3$ be the initial approximation,

$\therefore$ By Newton's Method,

$x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$.

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$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 3 - \frac{6}{23}$$

$$= 2.7392.$$

$f(x_1) = (2.7392)^3 - 4(2.7392) - 9$

$= 20.5528 - 10.9568 - 9$

$= 0.596$

$f''(x_1) = 3(2.7392)^2 - 4$

$= 22.5096 - 4$

$= 18.5096.$

$f(x_2) = (2.7071)^3 - 4(2.7071) - 9$

$= 19.8386 - 10.8284 - 9$

$= 0.0102.$

$f'(x_2) = 3(2.7071)^2 - 4.$

$= 21.9851 - 4$

$= 17.9851.$

$\therefore x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$

$x_3 = 2.7071 - \frac{0.0102}{17.9851}$

$= 2.7071 - 0.0056$

$= 2.7015.$

$f(x_3) = (2.7015)^3 - 4(2.7015) - 9$

$= 19.7158 - 10.806 - 9$

$= -0.0901$

$f'(x_3) = 3(2.7015)^2 - 4$

$= 21.8943 - 4$

$= 17.8943.$

$$x_4 = 2.7015 + \frac{0.0901}{17.8943}$$

$= 2.7015 + 0.0050$

$= ~~2.7065~~\ 2.7065$

(iii) $f(x) = x^3 - 1.8x^2 - 10x + 17$ $[1,2]$.

→ $f'(x) = 3x^2 - 3.6x - 10$

$f(1) = (1)^3 - 1.8(1)^2 - 10(1) + 17$

$= 1 - 1.8 - 10 + 17$

$= 6.2$

$f(2) = (2)^3 - 1.8(2)^2 - 10(2) + 17$

$= 8 - 7.2 + 20 + 17$

$= -2.2$

Let $x_0 = 2$, be initial approximation,
by Newton's method,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_1 = x_0 - \frac{f(x_2)}{f'(x_2)}$$

$$= 2 - \frac{2.2}{5.2}$$

$= 1.577$

$f(x_1) = (1.577)^3 - 1.8(1.577)^2 - 10(1.577) + 17$

$= 3.9219 - 4.4764 - 15.77 + 17$

$= 0.6755$

$f'(x_1) = 3(1.597)^2 - 3.6(1.577) - 10$

$= 7.4608 - 5.6772 - 10$

$= -8.2164$.

$$\therefore x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$= 1.577 + \frac{0.6755}{8.2164}$

$= 1.577 + 0.0822$

$= 1.6592.$

$f(x_2 = (1.6592)^3 - 1.8(1.6592)^2 - 10(1.6592) + 17$

$= 4.5677 - 0.9653 - 16.592 + 17$

$= 0.0204$

$f'(x_2) = 3(1.6592)^2 - 3.6(1.6592) - 10.$

$= 8.2588 - 5.97312 - 10$

$= -7.7143$

$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$

$= 1.6592 + \frac{0.0204}{7.7143}$

$= 1.6592 + 0.0026$

$= 1.6618.$

$\cancel{f}(x_3) = (1.6618)^3 - 1.8(1.6618)^2 - 10(1.6618) + 17$

$= 4.5892 - 4.9708 - 16.618 + 17$

$= 0.0004$

$f'(x_3) = 3(1.6618)^2 - 3.6(1.6618) \cancel{+} -10$

$= 8.2844 - 5.98$

$= -7.6977.$

$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$

$= 1.6618 + \frac{0.0004}{7.6977}$

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$= 1.6618$

$\therefore$ The root of equation 1.6618.

PRACTICAL-05

Topic :- Integration

Q.1. Solve the following Integration:-

(i) $\int \frac{dx}{\sqrt{x^2+2x-3}}$

$\rightarrow$ ~~$I = \int (x^2+2x-3)^{-1/2} dx$~~

~~$I = \left[\frac{(x^2+2x-3)^{1/2}}{1/2}\right] + C$~~

~~$I = \pm 2\sqrt{x^2}$~~

$I = \int \frac{-1}{4(x+3)} + \frac{1}{4(x-1)} dx$

$\because \int f(x) \pm g(x)\, dx = \int f(x)\, dx \pm$

$= \int \frac{1}{\sqrt{x^2+2x-3}} dx$

$= \int \frac{1}{\sqrt{x^2+2x+1-4}}$

$\because a^2+b^2+2ab = (a+b)^2$

$= \int \frac{1}{\sqrt{(x+1)^2-4}} dx$

Substitution,

put $x+1 = t$

$dx = \frac{1}{t} \times dt$ where $t = 1$ $t = x+1$

$\int \frac{1}{\sqrt{t^2-4}} dt$

using,

$\because \int \frac{1}{\sqrt{x^2-a^2}} dx = \ln(|x+\sqrt{x^2-a^2}|)$

$= \ln(|t+\sqrt{t^2-4}|)$

$t = x+1$

$= \ln(|x+1+\sqrt{(x+1)^2-4}|)$

$= \ln(|x+1+\sqrt{x^2+2x+3}|)$

$= \ln(|x+1+\sqrt{x^2+2x-3}|)+C$

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i) $\int (4e^{3x}+1)\,dx$.

$\Rightarrow = \int (4e^{3x}\,dx + 1)\,dx$

$= 4\int e^{3x}\,dx + \int 1\,dx \quad \because \int e^{ax}\,dx = \frac{1}{a} e^{ax}$.

$= \frac{4 e^{3x}}{3} + x$.

$= \frac{4e^{3x}}{3} + x + c$.

ii) $\int 2x^2 - 3\sin(x) + 5\sqrt{x}\,dx$.

$\Rightarrow = \int 2x^2 - 3\sin(x) + 5x^{1/2}\,dx \quad \because \sqrt[n]{a^m} = a^{m/n}$

$= \int 2x^2\,dx - \int 3\sin(x)\,dx + \int 5x^{1/2}\,dx$

$= \frac{2x^3}{3} - 3\cos x + \frac{10x\sqrt{x}}{3} + c$

$= \frac{2x^3 + 10x\sqrt{x}}{3} + 3\cos x + c$.

) $\int \frac{x^3+3x+4}{\sqrt{x}}\,dx$.

$\int \frac{x^3+3x+4}{x^{1/2}}\,dx$

$\because$ Split the denominator.

$= \int \frac{x^3}{x^{1/2}} + \frac{3x}{x^{1/2}} + \frac{4}{x^{1/2}}\,dx$

$= \int x^{5/2}\,dx + \int 3x^{1/2}\,dx + \int \frac{4}{x^{1/2}}\,dx$.

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$= \frac{x^{5/2+1}}{5/2+1}$

$= \frac{2x^3\sqrt{x}}{7} + 2x\sqrt{x} + 8\sqrt{x} + C$

(v) $\int t^7 \times \sin(2t^4)\, dt$.

$\rightarrow$ put $u = 2t^4$

$du = 2 \times 4t^3$

$= \int t^7 \times \sin(2t^4) \times \frac{1}{2\times 4t^3}\, du$

$= \int t^4 \sin(2t^4 \times \frac{1}{2\times 4}\, du$

$= \int t^4 \sin(2t^4) \times \frac{1}{8}\, du.$

$= \frac{t^4 \times \sin(2t^4)}{8}\, du$

Substitute t^4 with $\frac{u}{2}$.

$= \int \frac{u/2 \times \sin(2u)}{8}\, du.$

$= \int \frac{u \times \sin(u)}{2} / 8\, du.$

$= \frac{1}{16} \int u \times \sin u\, du$

$\therefore \int u\,dv = uv - \int v\,du$

where $u = u$

$dv = \sin u \times du$

$du = 1\,du \quad v = -\cos(u).$

$= \frac{1}{16}(4 \times (-\cos u)) + \int \cos(u)\, du.$

$\because \int \cos x\, dx = \sin(x)$

$= \frac{1}{16} \times (4 \times (-\cos(u)) + \sin 4(x))$

Return the substitution $u = 2+^4$.

$= 1/16 \times (2+^4 \times (-\cos(2+^4)) + \sin(2+^4))$

$= -\frac{+^4 \times \cos(2+^4)}{8} + \frac{\sin(2+^4)}{16} + C.$

(vi) $\int \sqrt{x}\,(x^2-1)\, dx.$

$\rightarrow = \int \sqrt{x}\, x^2 - \sqrt{x}\, dx$

$= \int x^{1/2} \cancel{\times} x^2 - x^{1/2}\, dx$

$= \int x^{5/2} - x^{1/2}\, dx$

$= \int x^{5/2} dx - \int x^{1/2} dx$

$= I_1 = \frac{x^{5/2+1}}{5/2+1} = \frac{x^{7/2}}{7/2} = \frac{2x^{7/2}}{7} = \frac{2\sqrt{x^7}}{7} = \frac{2x^3\sqrt{x}}{7}$

$= I_2 = \frac{x^{1/2+1}}{1/2+1} = \frac{x^{3/2}}{3/2} = \frac{2x^{3/2}}{3/2} = \frac{2\sqrt{x^3}}{3}$

$= \frac{2x^3\sqrt{x}}{7} + \frac{2\sqrt{x^3}}{3} + C.$

(viii) $\int \frac{\cos x}{\sqrt[3]{\sin^2 x}}\, dx.$

$\rightarrow = \int \frac{\cos x}{(\sin x)^{2/3}}\, dx$ L

$\because$ put $t = \sin x$

$dt = \cos x\, dx \therefore \frac{dt}{\cos x} = dx$

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$$= \int \frac{\cancel{\cos x}}{(\sin x)^{2/3}} \times \frac{dt}{\cancel{\cos x}}$$

$$= \int \frac{1}{t^{2/3}}\, dt$$

$$I = \int \frac{1}{t^{2/3}}\, dt = \frac{-1}{(2/3-1)\, t^{2/3-1}} = \frac{-1}{(2/3-1)\, t^{2/3-1}}$$

$$= \frac{-1}{-1/3\, t^{2/3-1}} = \frac{1}{1/3\, t^{-1/3}} = \frac{t^{1/3}}{1/3} = 3\, t^{1/3}$$

$$= 3\sqrt[3]{t}$$

Resubstitution $t = \sin x$.

$$= 3\sqrt[3]{\sin x} + c.$$

(X) $\int \frac{x^2 - 2x}{x^3 - 3x^2 + 1}\, dt.$

$\rightarrow$ Put $x^3 - 3x^2 + 1 = dt$

$$I = \int \frac{x^2 - 2x}{x^3 - 3x^2 + 1} \times \frac{1}{3x^2 - 3\cancel{x} \cdot 2x}\, dt$$

$$= \int \frac{x^2 - 2x}{x^3 - 3x^2 + 1} \times \frac{1}{3x^2 - 6x}\, dt$$

$$= \int \frac{x^2 - 2x}{x^3 - 3x^2 + 1} \times \frac{1}{3(x^2 - 2x)}\, dt$$

$$= \int \frac{1}{x^3 - 3x^2 + 1} \times 1/3\, dt$$

$$= \int \frac{1}{3(x^3 - 3x^2 + 1)}\, dt = \int \frac{1}{3t}\, dt$$

$$= 1/3 \int 1/t\, dt. \quad \because \int \frac{1}{x}\, dx = \log(x).$$

$$= \frac{1}{3} \times \log|t| + c$$

Resubstituting,

$$\frac{1}{3} \times \log(|x^3 - 3x^2 + 1|) + c.$$

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(viii) $\int \frac{1}{x^3} \sin\left(\frac{1}{x^2}\right) dx$

$\rightarrow$ $I = \int \frac{1}{x^3} \sin\left(\frac{1}{x^2}\right) dx$

let $\frac{1}{x^2} = t$

$x^{-2} = t \quad = \frac{-2}{x^3} dx = dt$

$I = \frac{-1}{2} \int \frac{-2}{x^3} \sin\left(\frac{1}{x^2}\right) dx$

$= \frac{-1}{2} \int \sin t \cdot$

$= \frac{-1}{2} (-\cos t) + c$

$= \frac{1}{2} \cos t + c$

Resubstitution $t = 1/x^2$

$I = \frac{1}{2} \cos\left(\frac{1}{x^2}\right) + c.$

(ix) $\int e^{\cos^2 x} \cdot \sin 2x \, dx$

$\rightarrow$ $I = \int e^{\cos^2 x} \sin 2x \, dx$

let $\cos^2 x = t$

$-2\cos x \cdot \sin x \, dx = dt$

$-2 \sin 2x \, dx = dt$

$I = \int -\sin 2x \cdot e^{\cos^2 x} dx$

$= -\int e^t + c$

Resubstituting $t = \cos^2 x$

$I = -e^{\cos 2x} + c.$

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PRACTICAL-06.

Topic :- Application of integration & Normal integration.

Q.1) Find the length of the following curve :-

1) $x = t - \sin t$ $y = 1 - \cos t$ to $[0, 2\pi]$, t belong to $[0, 2\pi]$

$\rightarrow L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

$x = t - \sin t$

$\frac{dx}{dt} = 1 - \cos t$

$y = 1 - \cos t$

$\frac{dy}{dt} = 0 - \cancel{\cos}(-\sin t)$

$\frac{dy}{dt} = \sin t$

$L = \int_0^{2\pi} = \sin t$

$L = \int_0^{2\pi} \sqrt{(1 - \cos t \cancel{+ \sin t})^2 + (\sin t)^2}\, dt$

$L = \int_0^{2\pi} \sqrt{1 - 2\cos t + 1}\, dt$

$= \int_0^{2\pi} \sqrt{2 - 2\cos t \cancel{dt} + 1}\, dt$

$\cancel{= \int_0^{2\pi} 2 \left| \sin \frac{1}{2} \right| dt} = \int_0^{2\pi} \sqrt{1 - 2\cos t + 1}\, dt$

$= \int_0^{2\pi} \sqrt{2 - 2\cos t}\, dt$

$= \int_0^{2\pi} 2 \left| \sin \frac{t}{2} \right| dt \quad \because \sin^2 \frac{t}{2} = \frac{1 - \cos t}{2}$

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PRACTICAL-06.

Topic :- Application of integration & Normal integration. 46

Q1) Find the length of the following curve :-

1) $x = t - \sin t$ $y = 1 - \cos t$ to $[0, 2\pi]$, t belong to $[0, 2\pi]$

$\rightarrow L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

$x = t - \sin t$

$\frac{dx}{dt} = \cancel{t} - \cos t$

$y = 1 - \cos t$

$\frac{dy}{dt} = 0 - \cancel{\cos}(-\sin t)$

$\frac{dy}{dt} = \sin t$

$L = \int_0^{2\pi} = \sin t$

$L = \int_0^{2\pi} \sqrt{(\cancel{t} - \cos t \cancel{+ \sin t})^2 + (\sin t)^2}\, dt$

$L = \int_0^{2\pi} \sqrt{1 - 2\cos t + 1}\, dt$

$= \int_0^{2\pi} \sqrt{2 - 2\cos t \cancel{dt} + 1}\, dt$

$\cancel{= \int_0^{2\pi} 2\left|\sin \frac{1}{2}\right| dt} = \int_0^{2\pi} \sqrt{1 - 2\cos t + 1}\, dt$

$= \int_0^{2\pi} \sqrt{2 - 2\cos t}\, dt$

$= \int_0^{2\pi} 2\left|\sin \frac{1}{2}\right| dt$ $\therefore \sin^2 \frac{t}{2} - \cancel{1} - \frac{\cos t}{2}$

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$$= \int_0^{2\pi} 2\pi \sin\frac{t}{2}\, dt$$

$$= \left(-4\cos\left(\frac{t}{2}\right)\right)_0^{2\pi} = (-4\cos\pi) - (-4\cos 0)$$

$$= \underline{4+4} = \underline{\underline{8}}$$

2) $y = \sqrt{4-x^2}$ $\quad x \in [-2, 2]$

Sol^n:- $L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

$$\frac{dy}{dx} = 2\int_0^2 \sqrt{1 + \left(\frac{-x}{\sqrt{4-x^2}}\right)^2}\, dx$$

$$= 2\int_0^2 \sqrt{1 + \frac{x^2}{4-x^2}}\, dx$$

$$= 4\int_0^2 \frac{1}{\sqrt{4-x^2}}\, dx$$

$$= 4\left(\sin^{-1}\left(\frac{x}{2}\right)\right)_0^2$$

$$= 2\pi.$$

3) $y = x^{3/2}$ $\quad$ log $[0, 4)$

Sol^n:- $f'(x) = \frac{3}{2} x^{1/2}$

$$[f'(x)]^2 = \frac{9}{4} x$$

$$l = \int_a^b \sqrt{1 + [f'(x)]^2}\, dx$$

$$= \int_0^x \sqrt{1 + \frac{9}{4}x}\, dx$$

put $u = 1 + \frac{9}{4}x$, $dx = \frac{9}{4} dx$

$$L = \int_1^{1+\frac{9}{4}x} \frac{4}{9}\sqrt{u}\, du = \left[\frac{4}{9}\cdot\frac{2}{3}(u^{3/2})\right]^{1+\frac{9}{4}x}$$

$$= \frac{8}{27}\left[\left(1+\frac{9}{4}x\right)-1\right]$$

4) $x = 3\sin t$, $y = 3\cos t$.

Sol^ng $\frac{dx}{dt} = 3\cos t$

$\frac{dy}{dt} = -3\sin t$

$$L = \int_0^{2\pi} \sqrt{\left(\frac{dy}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2}\, dt$$

$$= \int_0^{2\pi} \sqrt{(3\cos t)^2 + (-3\sin t)^2}\, dt$$

$$= \int_0^{2\pi} \sqrt{9\sin^2 t + 9\cos^2 t}\, dt$$

$$= \int_0^{2\pi} \sqrt{9\sin^2 t + 9\cos^2 t}\, dt$$

$$= \int_0^{2\pi} 3\sqrt{x}\, dt$$

$$= 3\int_0^{2\pi} x\, dt = 3[x]_0^{2\pi}$$

$$= 3[2\pi - 0]$$

$L = 6\pi$ units

5) $x = \frac{1}{6}y^3 + \frac{1}{2y}$ on $y = [1, 2]$.

Sol^ng. $\frac{dx}{dy} = \frac{y^2}{2} - \frac{1}{2y^2}$

$$\frac{dx}{dy} = \frac{y^4-1}{2y^2}$$

$$L = \int_1^2 \sqrt{1+\left(\frac{dx}{dy}\right)^2}\, dy.$$

$$= \int_1^2 \sqrt{\frac{(y^4+1)^2}{(2y)^2}}\, dy.$$

$$= \int_1^2 \frac{y^4+1}{2y^2}\, dy.$$

$$= \frac{1}{2}\int_1^2 y^2\, dy + \frac{1}{2}\int_1^2 y^{-2}\, dy.$$

$$= \frac{1}{2}\left[\frac{y^3}{3} - \frac{y^{-2}}{1}\right]_1^2$$

$$= \frac{1}{2}\left[\frac{8}{3} - \frac{1}{2} - \frac{1}{3} + 1\right]$$

$$= \frac{1}{2}\left[\frac{7}{3} - \frac{1}{2}\right] = \frac{17}{12} \text{ units.}$$

SQ.2

1) $\int_0^2 e^{x^2}\, dx$ with $n = 4$.

Solⁿ: $\int_0^2 e^{x^2}\, dx = 16.4526$

In each case the width of the Sub interv be $\Delta x = \frac{2-0}{4} = \frac{1}{2}$,

and So the Sub intervals will be [0, 0.5] [0.5, 1] [1, 1.5] [1.5, 2]

by Simpson rule

$$\int_0^2 e^{x^2} dx = \frac{1/2}{3} [y_0 + 4y_1 + 2y_2 + 4y_3 + y_4]$$

$$\approx \frac{1/2}{3} (e^{0^2} + 4e^{(0.5)^2} + 2e^{(1)^2} + 4e^{(1.5)^2} + e^{(2)^2})$$

$$\approx 17.3536.$$

2) $\int_1^4 x^2 dx \quad n = 4.$

Sol^n: $\Delta x = \frac{4-0}{4} = 1$

$$\int_a^b f(x)dx = \frac{\Delta x}{3} [y_0 + 4y_1 + 2y_2 + 4y_3 + y_4]$$

$$= \frac{1}{3} [y(0) + 4(1)^2 + 2(2)^2 + 4(3)^2 + 4^2]$$

$$= \frac{1}{3} \{0^2 + 4(1)^2 + 2(2)^2 + 4(3)^2 + 4^2]$$

$$= 64/3.$$

3) $\int_0^{\pi/3} \sqrt{\sin x}\, dx \quad n = 6.$

Sol^n:- $\Delta x = \frac{b-a}{n}$

$$\Delta x = \frac{b-a}{n} = \frac{\pi/3 - 0}{6} = \frac{\pi}{18}.$$

x	0	$\pi/18$	$2\pi/18$	$3\pi/18$	$4\pi/18$	$5\pi/18$
y	0	0.41	0.584	0.707	0.801	0.87
	y_0	y_1	y_2	y_3	y_4	y_5

$$\int_0^{\pi/3} \sqrt{\sin x}\, dx \approx \frac{\Delta x}{3} (y_0 + 4(y_1 + y_3 + y_5)$$

$$= \frac{\pi/18}{3} (0 + 4(0.4167 + 0.707 + 0.875) + 0.93$$

$$= 2(0.584 + 0.801) + 0.930$$

$$\approx \underline{0.681}$$

PRACTICAL:- 7.

Differential Equation

Q.1. Solve the following differential equations:-

1) $x\frac{dy}{dx} + \frac{1}{x}y = \frac{e^y}{x}$.

Sol^{n}- $P(x) = \frac{1}{x}$ $\quad Q(x) = \frac{e^y}{x}$

$IF\ e^{\int 1/x}$

x

$y = (IF) = \int Q(x)(IF)\,dx + c$

$= \int \frac{e^y}{x} x \, dx + c$

$= \int e^x \, dx + c$

$xy = e^y + c$

2) $e^x\frac{dy}{dx} + 2e^{xy} = 1$.

Sol^{n}- $\frac{dy}{dx} + 2\frac{e^x}{e^x} = \frac{1}{e^x}$

$\frac{dy}{dx} + 2y = \frac{1}{e^x}$

$\frac{dy}{dx} + 2y = e^{-x}$

$P(x) = 2 \quad Q(x) = e^{-x}$

$\int P(x) = dx$

$IF = e^{\int 2dx}$

$= e^{2x}$

$y(IF) = \int Q(x)(IF)\,dx + c$.

$= \int e^x dx + c$

$= y \cdot e^{2x}$

$= e^x + c$.

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3) $x \frac{dy}{dx} = \frac{\cos x}{x} - 2y.$

Sol^n:- $x \frac{dy}{dx} = \frac{\cos x}{x} - 2y$

$$\frac{dy}{dx} + \frac{2y}{x} = \frac{\cos x}{x^2}$$

$$P(x) = 2(x) = Q(x) = \frac{\cos x}{x^2}$$

$$IF = e^{\int P(x)dx}$$
$$= e^{\int 2x\, dx}$$

$$y(IF) = \int Q(x)(IF)dx + C$$
$$= \int \frac{\cos x}{x^2} \cdot x^2 dx + C$$
$$= \int \cos x + C$$
$$\therefore x^2 y = \sin x. + C.$$

4) $x \cdot \frac{dy}{dx} + 3y = \frac{\sin x}{x^2}$

Sol^n $\frac{dy}{dx} + \frac{3y}{x} = \frac{\sin x}{x^3}$

$$P(x) = 3/x \qquad Q(x) = \sin x/x^3.$$
$$\cancel{P(x) = \int 3/x\, dx}$$
$$= x^3$$
$$If = e^{\int P(x)dx}$$
$$= x^3$$

$$y(IF) = \int Q(x)(IF)\, dx + C$$
$$= \int \frac{\sin x}{x^2} \cdot x^3 dx + C$$
$$= \int \sin x + C$$
$$x^3 y = -\cos x + C$$

5) $e^{2x}\frac{dy}{dx} + 2e^{2x}y = 2x$

Sol^n: $\frac{dy}{dx} + 2y = \frac{2x}{e^{2x}}$

$P(x) = 2$

$Q(x) = \frac{2x}{e^{2x}} = 2xe^{-2x}$

$I(F) = e^{\int P(x)\,dx}$

$= e^{\int 2\,dx}$

$= e^{2x}$

$y(IF) = \int Q(x)\,(IF)\,dx + c$

$= \int 2x\,e^{-2x}\,e^{2x} + c$

$ye^{2x} = \int 2x + c = x^2 + c.$

6) $\sec^2 x \cdot \tan y\,dx + \sec^2 y \cdot \tan x\,dy = 0.$

Sol^n: $\sec^2 x \cdot \tan y\,dx = -\sec^2 y \tan x\,dy.$

$\frac{\sec^2 x\,dx}{\tan x} = -\int \frac{\sec^2 x}{\tan y}\,dy.$

$\int \frac{\sec^2 y\,dx}{\tan x} = -\int \frac{\sec^2 x}{\tan y}\,dy$

$\int \frac{\sec^2 x}{\tan x}\,dx = -\int \frac{\sec^2 x}{\tan y}\,dy.$

$\log|\tan x| = -\log|\tan y| + c$

$\log|\tan x + \tan y| = c$

$\tan x + \tan y = e^c.$

7) $\frac{dy}{dx} = \sin^2(x - y + 1)$

Sol^n:- put $x - y + 1 = v$

$x - y + 1 = v$

$1 - \frac{dy}{dx} = \frac{dv}{dx}$

$\frac{1 - dy}{dx} = \frac{dy}{dx}$

$1 - \frac{dy}{dx} = 1 - \sin^2 v$

$\frac{dv}{dx} = \cos^2 v$

$\frac{dv}{\cos^2 v} = dx$

$\int \sec^2 v\, dv = \int dx$

$\tan v = x + c \quad \tan(x - y + 1) = \underline{\underline{x + c}}$

8) $\frac{dy}{dx} = \frac{2x + 3y - 1}{6x + 9y + 6}$.

Sol^n: put $2x + 3y = v$

$2 + 3\frac{dy}{dx} = \frac{dv}{dx}$

$\frac{dy}{dx} - \frac{1}{3}\left(\frac{dv}{dx} - 2\right)$

$= \frac{1}{3}\left(\frac{dv}{dx} - 2\right) = \frac{1}{3}\left(\frac{v-1}{v+2}\right)$

$\frac{dv}{dx} = \frac{v-1}{v+2} + 2$

$\frac{dv}{dx} = \frac{v - 1 + 2v + 4}{v + 2}$

$= \frac{3v + 3}{v + 2}$

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$$= \int \frac{v+2}{v+1} dv + \int \frac{1}{v+1} dv = \int 3\, dx .$$

$v \log|v| - 3x + c$

$2x + 3y + \log|2x+3y+1| = 3x + c$

$3y = x - \log|2x+3y+1| + c .$

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PRACTICAL:- 08.

Topic :- Euler's Method

Q.1. $\frac{dy}{dx} = y + e^x - 2$ $y(0) = 2$ $h = 0.5$ find $y(2) = ?$

Sol^n:- $f(x) = y + e^x - 2$ $x_0 = 0$ $y(0) = 2$ $h = 0.5$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	2	1	2.5
1	0.5	2.5	2.1487	3.5743
2	1	3.5743	4.2925	5.7205
3	1.5	5.7205	8.2021	9.8215
4	2	9.8215		

$\therefore y(2) = 9.8215$.

Q.2. $\frac{dy}{dx} = 1 + y^2$, $y(0) = 1$ $h = 0.2$ find $y(1) = ?$

Sol^n:- $y_0 = 0$, $y_0 = 0$, $h = 0.2$.

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	0	1	0.2
1	0.2	0.2	1.04	0.408
2	0.4	0.408	1.1664	0.641
3	0.6	0.6412	1.4111	0.923
4	0.8	0.9234	1.8856	1.293
5	1	1.2939		

$\therefore y(1) = 1.2939$.

Q.3. $\frac{dy}{dx} = \sqrt{\frac{2}{y}}$ $y(0) = 1$, $h = 0.2$, find $y(1) = ?$

Sol^n:- $x_0 = 0$, $y_0(0) = 1$ $h = (0.2)$.

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n	x_0	y_0	$f(x_n, y_n)$	y_{n+1}
0	0	1	0	1
1	0.2	1	0.4472	1.0894
2	0.4	1.0894	0.6089	1.2105
3	0.6	1.2105	0.7040	1.3519
4	0.8	1.3513	0.7696	1.5051
5	1	1.5051		

$y(1) = 1.5051$

Q.4 (1) $\frac{dy}{dx} = 3x^2+1$ $y(1)=2$ find $y(2)$, $h=0.5$

Sol^n: $y_0 = 2$ $x_0 = 1$ $h = 0.1$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	2	4	4
1	1.5	4	7.25	7.875
2	2	7.875		

$y(2) = 7.875$

(2) $y_0 = 2$, $x_0 = 1$ $h = 0.25$

Sol^n:

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	2	4	3
1	1.25	3	5.6875	4.4218
2	1.5	4.4218	59.6569	19.3360
3	1.75	19.3360	1122.6426	299.9960
4	2	299.9966		

$y(2) = 299.9960$

Q.5 $\frac{dy}{dx} = \sqrt{xy} + 2$ $y(1) = 1$ $h = 0.2$.

Sol^n $x_0 = 1$ $y_0 = 1$ $h = 0.2$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	1	3	3.6
1	1.2	3.6		

$\therefore y(\cancel{1}) = 3.6$

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PRACTICAL :- 09

AIM :- Limit and partial order

Q1.

1) $\lim\limits_{(x,y)\to(-4,-1)} \dfrac{x^3-3y+y^2-1}{xy+5}$

$\to \lim\limits_{(x,y)\to(-4,-1)} \dfrac{x^3-3y+y^2-1}{xy+5}$

At $(-4,-1)$, Denominator $\neq 0$

$\therefore$ By applying limit,

$= \dfrac{(-4)^3-3(-1)+(-1)^2-1}{-4(-1)+5}$

$= \dfrac{-64+3+1-1}{4+5} = \dfrac{-61}{4}$

2) $\lim\limits_{(x,y)\to(2,0)} \dfrac{(y+1)(x^2+y^2-4x)}{x+3y}.$

$\to \lim\limits_{(x,y)\to(2,0)} \dfrac{(y+1)(x^2+y^2-4x)}{x+3y}.$

At $(2,0)$, Denominator $\neq 0$

$\therefore$ By applying limit,

$= \dfrac{(0+1)((2)^2+0-4(2))}{2+0}$

$= \cancel{(0+2)}\ \dfrac{1(4+0-8)}{2}$

$= \dfrac{-\cancel{4}}{\cancel{2}} -2$

$= -2$

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3) $\lim_{(x,y,z)\to(1,1,1)} \dfrac{x^2 - y^2 - z^2}{x^3 - x^2yz}$

$\rightarrow \lim_{(x,y,z)\to(1,1,1)} \dfrac{x^2 - y^2 - z^2}{x^3 - x^2yz}$

At $(1,1,1)$, Denominator $= 0$,

$\therefore \lim_{(x,y,z)\to(1,1,1)} \dfrac{x^2 - y^2 - z^2}{x^3 - x^2yz}$

$\lim_{(x,y,z)\to(1,1,1)} \dfrac{\cancel{(x-yz)}(x+yz)}{x^2\cancel{(x-yz)}}$

$\therefore \lim_{(x,y,z)\to(1,1,1)} \dfrac{x+yz}{x^2}$

On Applying limit,

$= \dfrac{1+1(1)}{(1)^2} = \underline{\underline{2}}$.

Q.2

1) $f(x,y) = xye^{x^2+y^2}$.

$\rightarrow \therefore f_x = \dfrac{\delta}{\delta x}(f(x,y))$

$= \dfrac{\delta}{\delta x}(xye^{x^2+y^2})$

$= ye^{x^2+y^2}(2x)$

$\therefore f_x = 2xye^{x^2+y^2}$

$f_y = \dfrac{\delta}{\delta y}(f(x,y))$

$= \dfrac{\delta}{\delta y}(xye^{x^2+y^2})$

$= \cancel{\dfrac{\delta}{\delta y}} \cancel{(} \; xe^{x^2+y^2}(2y)$

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$\therefore f_y = 2yxe^{x^2+y^2}$

2) $f(x,y) = e^x \cos y$

$\rightarrow f_x = \frac{\delta}{\delta x}(f(x,y))$

$= \frac{\delta}{\delta x}(e^x \cos y)$

$\therefore f_x = e^x \cos y$

$f_y = \frac{\delta}{\delta y}(e^x \cos y)$

$= \frac{\delta}{\delta y}(e^x(\cos y))$

$\therefore f_y = -e^x \sin y$

3) $f(x,y) = x^3y^2 - 3x^2y + y^3 + 1$

$\rightarrow f_x = \frac{\delta}{\delta x}(f(x,y))$

$= \frac{\delta}{\delta x}(x^3y^2 - 3x^2y + y^3 + 1)$

$\therefore f_x = 3x^2y^2 - 6xy$

$f_y = \frac{\delta}{\delta y}(f(x,y))$

$= \frac{\delta}{\delta y}(x^3y^2 - 3x^2y + y^3 + 1)$

$= 2x^3y - 3x^2 + 3y^2.$

Q.3

1) $f(x,y) = \frac{2x}{1+y^2}$

$\rightarrow f_x = \frac{\delta}{\delta x} \cancel{\frac{2x}{1+y^2}} (f(x,y))$

$= \frac{\delta}{\delta x}\left(\frac{2x}{1+y^2}\right)$

$= \frac{(1+y^2)\frac{\delta}{\delta x}(2x) - 2x\frac{\delta}{\delta x}(1+y^2)}{(1+y^2)^2}$

$= \frac{2+2y^2-0}{(1+y^2)^2}$

$= \frac{2(1+y^2)}{(1+y^2)(1+y^2)} = \frac{2}{1+y^2}$

At $(0,0)$,

$\frac{2}{1+0} = \underline{2}$

Now,

$f_y = \frac{\delta}{\delta y}\left(\frac{2x}{1+y^2}\right)$

$= \frac{1+y^2 \frac{\delta}{\delta x} 2x - 2x \frac{\delta}{\delta x}(1+y^2)}{(1+y^2)^2}$

$= \frac{1+y^2(0) - 2x(2y)}{(1+y^2)^2} = \frac{-4xy}{(1+y^2)^2}$

At $(0,0)$,

$= \frac{-4(0)(0)}{(1+0)^2}$

$= \underline{0}$

Q.4

1) $f(x,y) = \dfrac{y^2 - xy}{x^2}$

$\rightarrow f_x = \dfrac{x^2 \frac{\delta}{\delta x}(y^2 - xy) - (y^2 - x.y)\frac{\delta}{\delta x}(x^2)}{(x^2)^2}$

$= \dfrac{x^2(-y) - (y^2 - xy)(2x)}{x^4}$

$f_y = \dfrac{2y - x}{x^2}$

$f_{xx} = \dfrac{\delta}{\delta x}\left(\dfrac{-x^2y - 2x(y^2 - xy)}{x^4}\right)$

$= \dfrac{x^4\left(\frac{\delta}{\delta x}(-x^2y - 2xy^2 + 2x^2y)\right) - (-x^2y - 2xy + 2x^2y)\frac{\delta}{\delta}}{(x^4)^2}$

$= \dfrac{x^4(-2xy - 2y^2 + 4xy) - 4x^3(-x^3y - 2xy + 2x^2y}{x^6}$ — (1)

$f_{yy} = \dfrac{\delta}{\delta y}\left(\dfrac{2y - x}{x^2}\right)$

$= \dfrac{2 - 0}{x^2} = \dfrac{2}{x^2}$ — (2)

$f_{xy} = \dfrac{\delta}{\delta xy}\left(\dfrac{-x^2y - 2xy^2 + 2x^2y}{x^4}\right)$ — (3)

$= \dfrac{-x^2 - 4xy + 2x^2}{x^4}$

$f_{yx} = \dfrac{\delta}{\delta x}\left(\dfrac{2y - x}{x^2}\right)$

$= \dfrac{x^2\frac{\delta}{\delta x}(2y - x) - (2y - x)\frac{\delta}{\delta x}(x^2)}{(x^2)^2}$

$= \dfrac{-x^2 - 4xy + 2x^2}{x^4}$

from (3) & (4);

$f_{xy} - f_{yx}$

2) $f(x,y) = x^3 + 3x^2y^2 - \log(x^2+1)$

$\rightarrow f(x) = \frac{\delta}{\delta x}(x^3 + 3x^2y - \log(x^2+1))$

$= 3x^2 + 6xy^2 - \frac{2x}{x^2+1}$

$fy = \frac{\delta}{\delta y}(x^3 + 3x^2y^2 - \log(x^2+1))$

$= 0 + 6x^2y - 0$

$= 6x^2y$

$fxx = 6x + 6y^2 - \left(\frac{x^2+1 \frac{\delta}{\delta x}(2x) - 2x\frac{\delta}{\delta x}(x^2+1)}{(x^2+1)^2}\right)$

$= 6x + 6y^2 - \left(\frac{2(x^2+1) - 4x^2}{(x^2+1)^2}\right)$ — 1

$fyy = \frac{\delta}{\delta xy}(6x^2y)$

$= 6x^2$ — 2

$fxy = \frac{\delta}{\delta y}\left(3x^2 + 6xy^2 - \frac{2x}{x^2+1}\right)$

$= 0 + 12xy - 0. = 12xy.$ — 3

$fyx = \frac{\delta}{\delta x}(6x^2y)$

$= 12xy$ — 4

from 3 & 4,

$\therefore fxy = fyx$

3) $f(x,y) = \sin(xy) + e^{x+y}$

$\rightarrow f_x = y\cos(xy) + e^{x+y}(1)$

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$= y\cos(xy) + e^{x+y}$ $\qquad$ $f_y = x\cos(xy) + e^{x+y}(1)$

$= x\cos(xy) + e^{x+y}$

$$f_{xx} = \frac{\delta}{\delta x}\left(y\cos(xy) + e^{x+y}\right)$$

$= -y(\sin(xy)\cdot(y)) + e^{x+y}(1)$.

$= -y^2\sin(xy) + e^{x+y}$

$$f_{yy} = \frac{\delta}{\delta y}\left(x\cos(xy) + e^{x+y}\right)$$

$= -x\sin(xy)(x) + e^{x+y}(1)$

$= -x^2\sin(xy) + e^{(x+y)}$.

$$f_{xy} = \frac{\delta}{\delta y}\left(y\cos(xy) + e^{x+y}\right)$$

$= -y^2\sin(xy) + \cos(xy) + e^{(x+y)}$ — (3)

$$f_{yx} = \frac{\delta}{\delta x}\left(x\cos(xy) + e^{x+y}\right)$$

$= -x^2\sin(xy) + \cos(xy) + e^{x+y}$ — (4)

$\therefore$ From (3) & (4),

$f_{xy} \neq f_{yx}$.

Q.5

(i) $f(x,y) = \sqrt{x^2+y^2}$ at $(1,1)$

$\rightarrow$ $f(1,1) = \sqrt{(1)^2+(1)^2} = \sqrt{2}$

$$f_x = \frac{1}{2\sqrt{x^2+y^2}}(2x)$$

$$= \frac{x}{\sqrt{x^2+y^2}}$$

f_x at $(1,1) = \frac{1}{\sqrt{2}}$

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$L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$

$= \sqrt{2} + \frac{1}{\sqrt{2}}(x-1) + \frac{1}{\sqrt{2}}(y-1)$

$= \sqrt{2} + \frac{1}{\sqrt{2}}(x-1+y-1)$

$= \sqrt{2} + \frac{1}{\sqrt{2}}(x+y-2)$

$= \sqrt{2} + \frac{1}{\sqrt{2}}x + \frac{1}{\sqrt{2}}y - \frac{2}{\sqrt{2}}$

$= \frac{x+y}{\sqrt{2}}$

2) $f(x,y) = 1-x+y\sin x$ at $(\pi/2, 0)$

$\rightarrow f(\pi/2, 0) = 1-\pi/2+0 = 1-\frac{\pi}{2}$

$f_x = 0-1+y\cos x$

$f_x = 0-1+y\cos x$ $\quad$ $f_y = 0-0+\sin x$

f_x at $(\pi/2, 0) = -1+0 = -1$ $\quad$ f_y at $(\pi/2, 0) = \sin\frac{\pi}{2} = 1$.

$L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$

$= 1-\frac{\pi}{2} + (-1)(x-\frac{\pi}{2}) + 1(y-0)$

$= 1-\frac{\pi}{2} - x + \frac{\pi}{2} + y$

$= 1-x+y$.

3) $f(x,y) = \log x + \log y$ at $(1,1)$.

$\rightarrow f(1,1) = \log(1) + \log(1) = 0$

$f_x = \frac{1}{x} \cancel{+0}$ $\quad$ $f_y = 0 + \frac{1}{y}$

f_x at $(1,1) = 1$ $\quad$ f_y at $(1,1)$

$L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$

$= 0 + 1(x-1) + 1(y-1)$

$= x-1+y-1$

$= x+y-2$.

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PRACTICAL : 10

Q1. Find the directional deviation of the following function at given point & in the direction of given vector.

1) $f(x,y) = x+2y-3$ $\quad a = (1,-1)$, $u = 3i - j$.

→ Here, $u = 3i - j$ is not a unit vector.

$|\bar{u}| = \sqrt{(3)^2 + (-1)^2} = \sqrt{9+1} = \sqrt{10}$.

unit vector along u is $\frac{u}{|u|} = \frac{1}{\sqrt{10}}(3,-1)$

$\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

$f(a+hu) = f(1,-1) + h\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

$f(a) = f(1,-1) = (1) + 2(-1) - 3 = 1-2-3 = -4$

$f(a+hu) = f(1,-1) + h\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

$= f = \left(1 + \frac{3}{\sqrt{10}}\right), \left(-1 - \frac{h}{\sqrt{10}}\right)$

$f(a+hu) = \left(1 + \frac{3}{\sqrt{10}}\right), \left(-1 - \frac{h}{\sqrt{10}}\right)$

$= 1 + \frac{3}{\sqrt{10}} - 2 - \frac{2h}{\sqrt{10}}$.

$f(a+hu) = \cancel{(-1+3)}\, 4 + \frac{h}{\sqrt{10}}$.

$D_u f(a) = \lim_{h \to 0} \frac{f(a+hu) - f(a)}{h}$

$= \lim_{h \to 0} \frac{-4 + h/\sqrt{10} + 4}{h}$

$D_u f(a) = \frac{1}{\sqrt{10}}$

2) $f(x) = y^2 - 4x + 1$ $\quad a = (3,4)$. $u = i + 5j$.

→ Here, $u = i + 5j$ is not a unit vector.

$|\bar{u}| = \sqrt{(1)^2 + (5)^2} = \sqrt{26}$

unit vector u is $\frac{u}{|u|} = \frac{1}{\sqrt{26}}(1, 5)$.

$$\left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)$$

$$f(a) = f(3,4) + h\left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)$$

$$= f\left(3 + \frac{h}{\sqrt{26}}, 4 + \frac{5h}{\sqrt{26}}\right)$$

$$f_{x,y}(a+hu) = \left(4 + \frac{5h}{\sqrt{26}}\right)^2 - 4\left(3 + \frac{h}{\sqrt{26}}\right) + 1$$

$$= 16 + \frac{25h^2}{26} - \frac{40h}{\sqrt{26}} - 12 - \frac{4h}{\sqrt{26}} + 1.$$

$$= \frac{25h^2}{26} + \frac{40h}{\sqrt{26}} - \frac{4h}{\sqrt{26}} + 5$$

$$= \frac{25h^2}{26} + \frac{40h - 4h}{\sqrt{26}} + 5$$

$$D_u f(a) = \lim_{h \to 0} \frac{\frac{25h^2}{26} + \frac{36h}{\sqrt{26}} + 5 - 5}{h\left(\frac{25h}{26} + \frac{36}{\sqrt{26}}\right)}$$

$$D_u f(a) = \frac{25h}{26} + \frac{36}{\sqrt{36}} //$$

3) $2x + 3y \cdot a = (1,2)$; $u = (3i + 4j)$

→ Here, $u = 3i + 4j$ is not a unit vector

$|\bar{u}| = \sqrt{(3)^2 + (4)^2} = \sqrt{25} = 5.$

Unit vector along u is $\frac{u}{|\bar{u}|} = \frac{1}{5}(3,4)$

$$= \left(\frac{3}{5}, \frac{4}{5}\right).$$

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$f(a) = f(1,2) = 2(1) + 3(2) = 8$

$f(a+hu) = f(1,2) + h\left(\frac{3}{5}, \frac{4}{5}\right)$

$f\left(1+\frac{3h}{5}, 2+\frac{4}{5}\right)$.

$f(a+hu) = 2\left(1+\frac{3h}{5}\right) + 3\left(2+\frac{4h}{5}\right)$

$= 2 + \frac{6h}{5} + 6 + \frac{12h}{5}$

$= \frac{18h}{5} + 8$

$D_u f(a) = \lim_{h \to 0} \frac{\frac{18h}{5} + 8 - 8}{h} = \frac{18h}{5}$ //

Q.2. Find gradiant vector for the following f at ~~your~~ given point :-

1) $f(x,y) = x^y + y^x$ $a = (1,1)$

$f_x = y \cdot x^{y-1} + y^x \log y$

$f_y = x^y \log x + x y^{x-1}$

$\nabla f(x,y) = (f_x, f_y)$

$= (y x^{y-1} + y^x \log y, \; x^y \log x + x y^{x-1})$

$f(1,1) = (1+0+1+0$

$= (1,1)$

2) $f(x,y) = (\tan^{-1} x) \cdot y^2$ $a = (1,-1)$

$f_x = \frac{1}{1+x^2} \cdot y^2$

$f_y = 2y \tan^{-1} x$

$\nabla f(x,y) = (f_x, f_y)$

$= \left(\frac{y^2}{1+x^2}, 2y \tan^{-1} x\right)$

$f(1,-1) = \left(\frac{1}{2}, \tan^{-1}(1)(-2)\right)$

$= \frac{1}{2}, \frac{\pi}{4}(-2)$

$= \left(\frac{1}{2}, -\frac{\pi}{2}\right)$

3) $f(x,y,z) = xyz - e^{x+y+z}$, $a = (1,-1,0)$

→ $f_x = yz - e^{x+y+z}$

$f_y = xz - e^{x+y+z}$

$f_z = xy - e^{x+y+z}$

$\nabla f(x,y,z) = (f_x, f_y, f_z)$

$= yz - e^{x+y+z}, xz - e^{x+y+z}, xy - e^{x+y+z}$

$f(x,-1,0) = (e-1)(0) - e^{(1+(-1)+0)}, (1)(0) - e^{1+0}$

$= 0 - e^2\ 0 - e^0, -1e^0$

$= (-1, -1, 2)$

Q.3 Find the equation of Tangent & normal to each of the following

$x^2 \cos y + e^{xy} = 2$ ~~and~~ at $(1,0)$.

$f_x = \cos y\ 2x + e^{xy} y$

$f_x = -x^2 \sin y + e^{xy+y}$

$dy = \cos y\ 2x + e^{xy} y$

$\frac{dy}{dx}$. $(x_0, y_0) = (1,0)$ $x_0 = 1$, $y_0 = 0$

eq of tangent

~~each~~ $f_x(x - x_0) + f_y(y - y_0) = 0$

$fy(x-yo) = \cos 0 \; 2(1) + e^0 = 0.$

$= 1(2) + 0$

$= 2.$

$fy(xy\,yo) = (1)^2(-\sin 0 = 0) + e^0 = 1$

$= 0 + 1 + 1$

$= 0 1 1$

$2(x-1) \cdots 1(y-0) = 0$

$2(x - \cancel{1}) + (y-0) = 0$

$2x - 2 - 2y = N0$

eq of Normal $\; ax + by + c = 0$

$= \cancel{b}x + ay + a = 0$

$1(1) + 2(y) + a = 0$

$1 + 2y + a = 0$

$= 1 + 2(0) + a = 0$

$a x = 0 \quad a = 1$

2) $x^2 + y^2 - 2x + 36 + 2 = 0.$ at $(2, -2)$

$\to fx = 2x + 0 - 2 + 10 \cancel{6}$

$= 2x - 2$

$dy = 0 + 2y - 0 + 3 + 0 = 2y + 3 \;,\; x_0 = 2,\; y_0 = -2.$

$fx(x_0 y_0) = 2\cancel{2}(1) 2 = 2..$

$fy(x_0, y_0) = 2(-2) + 3 = -1$

eqn. of tangent

$fx(x - x_0) + fy(y - y_0) = 0$

$2(x-2) + (-1(y+2)) = 0$

$2x - 2 - y - 2 = 0$

$2x - y - 4 = 0 \to$ it is required eqn. $\to 0$

eqn of Normal.

$= ax + by + c = 0$

$bx + ay + d = 0$

$= -1(x) + 2y + d = 0$

$= -x + 2y + d = 0$ at $(2,2)$

$= -2 - 4 + d = 0 \quad = -6 + d \Rightarrow$

$\therefore d = -6$

Q.4 Find the eqn of tangent and Normal line following surface

1) $x^2 - 2yz + 3y + xz = 7$ at $(2, 1, 0)$.

$\rightarrow$ $f_x = 2x - 0 + 0 + z$

$f_x = 2x + z$

$f_y = 0 - 2z + 0 + y$

$= -2y + 10 + 1$

$(x_0\ y_0\ z_0) : (2, 1, 0)$.

$x_0 = 2,\ y_0 = 1,\ z = 0$

$f_x(x_0 \cdot y_0\ z_0) = (2,1,0)$

$f_x(x_0\ y_0\ z_0) = 2(2) + 0 = 4$

$f_y(x_0, y_0, x_0) = 2(0) + 3 = 3$

$f_z = -2(1) + 2 = 0$

equation of tangent,

$f_x(x_0 - x_0) + f_y(y_0 - y_0) + 0(z - 0)$

$= 4x - 8 + 3y - 3 = 0$

$4x + 3y - 11 = 0$ This is required ~~Tangent~~ equation of tangent

Eqn of Normal at $(4, 3, 4)$

$$\frac{x - x_4}{dx} = \frac{y - y_0}{dy} = \frac{z - z_0}{dz}$$

$$= \frac{x - 2}{4} = \frac{y - 1}{33} = \frac{z + 4}{6}$$ //

09

$3yzx = -4$

2) $3y^2x - x - y + z = -4$

$\rightarrow$ $3xy^2 - x - y + z + 4 = 0$

$f_x = 3yz - 1 - 0 + 0 + 0$

$= 3yz - 1$

$f_y = 3xz - 0 - 1 + 0 + 0 + 0$.

$= 3xz - 1$

$f_z = 3xy - 0 - 0 + 1 + 0$

$= 3yx + 1$

$x_0, y_0, z_0) = (1, -1, 2)$

$(x_0\ y_0\ z_0) = (1, -1, 2)$ ∴ $x_0 = 1, y_0 = -1, z_0 = 2$

$f_x(x_0, y_0, z_0) = 3(-1)(2) - 1 = -7$

$f_y(x_0, y_0, z_0) = 3(+1)(2) - 1 = 5$

$f_z(x_0, y_0, z_0) = 3(1)(-1) + 1 = -2$.

eq^n of tangent,

$-7(x-1) + 5(y+1) - 2(z-2) = 0$

$-7x + 7 + 5y + 5 - 2z + 4 = 0$

$-7x + 5y - 2z + 16 = 0$ — This is required eq^n of tangent

Eq^n of normal at $(-7, 5, 2)$

$$\frac{x - x_0}{f_x} = \frac{y - y_0}{f_y} = \frac{z - z_0}{-2}$$

Q.5 Find the local maxima and minima for the following function.

1) $f(x, y) = 3x^2 + y^2 - 3xy + 6x - 4y$

$\rightarrow$ $f_x = 6x + 0 - 3y + 6 = 0$

$= 6x - 3y + 6$

$f_y = 0 + 2y - 3x + 0 - 4$

$= 2y - 3x - 4.$

$f_x = 0$

$6x - 3y + 6 = 0$

$3(2x - y + 2) = 0$

$2x - y + 2 = 0$

$2x - y = -2$ —— ①

$f_y = 0$

$2y - 3x - 4 = 0$

$2y - 3x = 4$ —— ②

~~$f_x = 0$~~

~~$6x - 3y + 6 = 0$~~

~~$3(2x - y + 2) = 0$~~

~~$2x - y + 2 = 0$~~

~~$2x - y = -2$ — ①~~

Multiply eq^n 1 with 2,

$4x - 2y = -4$

$2y - 3x = 4$

$x = 0$

Substitution value of x in eq^n ①

$2(0) - y = -2$

$+y = +2 \quad = y = 2$

Critical point on $(0, 2)$.

$r = f_{xx} = 6$

$t = f_{yy} = 2$

$s = f_{xy} = -3$

Here; $r > 0$

$= rt = s$

$= 6(2) - (-3)^2$

$= 12 - 9$

$= 3 > 0$

$\therefore f$ has maximum at $(0, 2)$

$3x^2 + y^2 - 3xy + 6x - 4y$ at $(0, 2)$

$3(0)^2 + 2^2 - 3(0)(2) + 6(0) - 4(2)$

$0 + 4 - 0 + 0 - 8$

$= \underline{-4}$

2) $f(x,y) = 2x^4 + 3x^2 y - y^2$

$\longrightarrow$ $f_x = 8x^3 + 6xy$

$f_y = 3x^2 - 2y$

$f_x = 0$

$\therefore 8x^3 + 6xy = 0$

$2x(4x^2 + 3y) = 0$

$4x^2 + 3y = 0$ — ①

$f_y = 0$

$3x^2 - 2y = 0$ — ②

Multiply eq^n ① with 3

② with 4

$12x^2 + 9y = 0$

$-12x^2 - 8y = 0$

$\overline{\qquad 17y = 0}$

$\therefore y = 0$

Substitute value of y in eq^n ①.

$4x^2 + 3(0) = 0$

$4x^2 = 0$

$x = 0$

Critical point is $(0, 0)$

$r = f_{xx} = 24x^2 + 6x$

$t =$ [illegible]

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$s = f_{xy} = 6x - 0 = 6x = 6(0) = 0$

r at $(0,0)$.

$= 24(0) + 6(0) = 0$

$\therefore r = 0$

$rt - s^2 = 0(-2) - s^2$

$= 0 - 0 = 0.$

$r = 0$ & $rt - s^2 = 0$

(nothing to say)

3) $f(x,y) = x^2 - y^2 + 2x + 8y - 70$

→ $f_x = 2x + 2$

$f_y = -2y + 8$

$f_x = 0 \therefore 2x + 2 = 0$

$x = -\frac{2}{2} = 1 \quad \therefore x = -1$

$f_y = 0$

$-2 + 8 = 0$

$y = \frac{-8}{-2} = 4$

$\therefore \underline{\underline{y = 4}}$,

Critical point is $(-1, 4)$.

$r = f_{xx} = 2$

$t = f_{yy} = -2$

$s = f_{xy} = 0$

$r > 0$

$rt - s^2 = 2(-2) - (0)^2$

$= -4 - 0$

$= -4 < 0$

$f(x,y)$ at $(-1, 4)$

(5)

$f(-1,4) = (-1) - (4)^2 + 2(-1) + 8(4) - 70$

$= 1 + 16 - 2 + 32 - 70$

$= 17 + 30 - 70$

$= 37 - 70$

$= \underline{\underline{33}}$

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