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Time-variant Orthogonal Matrix Feedback Delay Network Reverberator

Shreyas A. Paranjpe
Parthus Technologies
San Jose, CA, USA
paranjpe@parthus.com

ABSTRACT
We developed an artificial reverberation device based on a novel, time-variant orthogonal matrix feedback delay network topology. Our novel topology uses multiple, time-variant output taps for each delay line, and therefore simultaneously reduces the amount of delay memory required without introducing coloration, and increases the echo density. Furthermore, the system is guaranteed stable provided that certain constraints on the delay line lengths and tap weights are fulfilled. Our implementation on a 24-bit digital signal processor requires only 16384 words of delay line memory for a four channel input / four channel output reverb.

INTRODUCTION

Reverberation is the complicated set of reflections that are produced when a sound travels from a source to a listener by bouncing off many different surfaces. We spend our lives in reverberant environments -- we are surrounded by objects (such as walls, floors, ceilings, desks, windows, furniture, etc.) that reflect sound. These reflections provide important signals to our brain that describe our environment and allow us to distinguish if we are in a small bathroom or a large stadium.

An artificial reverberation device, also referred to as a reverb, transforms our acoustic environment to provide a more enjoyable listening experience. Music that is processed by a reverb transforms our ordinary living room into a world-class concert hall.

Developing artificial reverbs that simulate real and imagined environments is a complicated and nontrivial endeavor. Practical constraints such as limited delay line memory and processing power must also be considered. Bad reverbs are relatively easy to design. Unfortunately, a bad reverb is often worse than having no reverb at all. Bad reverbs can muddy, color, or distort the sound enough to diminish the listening experience and can be objectionable and annoying.

The goal is to produce a reverb that sounds good, uses as little delay line memory and processing power as possible, and is easy to design. Gardner writes in [1] that "[t]he challenge is to design an artificial reverberator which has sufficient echo density in the time domain, sufficient density of maxima in the frequency domain, and a natural colorless timbre."

The reflections generated in reverberant environments are lumped into two groups: early reflections occur within the first 60 to 80 milliseconds; late reflections occur thereafter. Artificial reverberators therefore often have one unit to generate early reflections and another to generate late reflections. Because the early reflections occur over a finite time duration, they are often simulated using finite impulse response (FIR) filters with a sparse number of taps. The late reflections, however, may persist for a much longer time. The reverberation time is one type of measurement that determines how long the late reflections persist. Concert halls typically have reverberation times of 1.5 to 3 seconds; artificial acoustic spaces may have much longer, possibly even infinite reverberation times. Because it is impractical to use such a large amount of delay line memory to directly synthesize the late reflections, the late reflections generator typically uses infinite impulse response (IIR) filters. That is, the delay lines use feedback and attenuation to recirculate and control the signal's decay.

PREVIOUSLY ATTEMPTED SOLUTIONS

Artificial reverberation has been studied for over 50 years. The following paragraphs summarize previously attempted solutions and their shortcomings. Series and/or parallel combinations of comb and allpass filters as described in [2] suffer from various problems such as flutter echo and ringing in the decay. Furthermore, choosing the appropriate delay line lengths and filter coefficients is difficult and largely empirical. Typically, incommensurate delay lengths (that is, lengths which are mutually prime) are suggested in order to reduce any obvious reflection patterns.

Feedback delay networks (FDNs) as described in [3] can be designed to be guaranteed stable, but also suffer from flutter and tonal coloration in the decay. They suggest continuously varying the delay line lengths randomly to reduce flutter and tonal coloration.

A more formalized analytical approach that reduces some of the empirical nature of reverb design, as described in [4], also relies on FDNs. Even though those designs can produce natural sounding reverberation, they still require a large amount of memory and close attention in selecting delay line lengths.

Vaananen et al. in [5] reduce the amount of memory required by combining elements of the designs proposed in [2 – 4], but thereby reduce the theoretical modal density and increase the difficulty in choosing delay line lengths in order to prevent tonal coloration.

In [6] multiple time varying feedback taps are used to help increase echo density, but at the expense of introducing pitch modulation and other distortions. Unfortunately, feedback from multiple taps can cause the reverberator to become unstable and should thus be avoided, as described in [7] and later again in [8].

Previously designed reverberators suffer from various combinations of bad sound, use of too much delay line memory, and difficult design procedures.

THE NEW REVERB ALGORITHM

Our new reverberator algorithm has a high echo density in the time domain, a sufficient density of maxima in the frequency domain, and a natural colorless timbre. Delay line memory usage is reduced because we have developed a novel delay line topology (patent pending) that increases echo density and doesn't introduce tonal coloration. Furthermore, the design process is greatly simplified because delay line lengths and other parameters are chosen based on room geometry and psychoacoustic principles.

DETAILED DESCRIPTION

Figure 1 is a high level block diagram that shows the early and late reflection generators, and a direct path that allows the input to pass directly to the output. The letters in the blocks correspond to some type of signal transformation, such as (and including combinations of) filtering, delaying, attenuating, and summing. We first describe the late reflection generator and then the early reflection generator.

Late Reflection Generator

The late reflection generator is shown in Figure 2. The late reflections are generated by recirculating the input signal through a

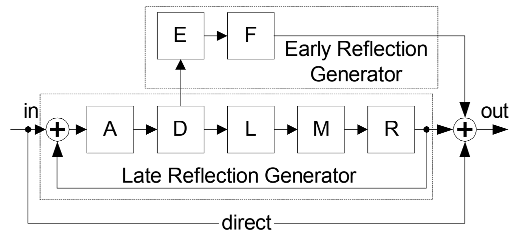

Figure 1. High level block diagram of new reverberator

lossless prototype. The lossless prototype is an energy conserving system based on a unitary system with feedback. After the input signal is injected into the lossless prototype system, it never decays as it recirculates. However, in real acoustic spaces, the late reflections eventually decay, and we model the frequency dependent absorption that results when sound propagates through air and reflects off of surfaces by inserting lowpass filters into the lossless prototype. As long as the lossless prototype is unitary, and the magnitude of the absorptive losses is less than unity, then the system is stable.

Note that a unitary system is represented by a unitary matrix, and since the product of unitary matrices results in another unitary matrix, then the series cascade of unitary systems is also another unitary system. Recall that a given matrix, U, is unitary when the matrix times the transpose of the complex conjugate of the matrix produces the identity matrix, I. Algebraically, this is described by

$$U \cdot \overline{U}^{T} = I \tag{1}$$

The overbar represents taking the complex conjugate of U, and the superscript T represents taking the transpose of U.

The late output signal , y, (that is, the late reflections) is described in terms of the input signal, x, by the following equation:

$$y = (I - R \cdot M \cdot L \cdot D \cdot A)^{-1} \cdot R \cdot M \cdot L \cdot D \cdot A \cdot x \tag{2}$$

The lossless prototype system is defined by the matrix transformations given by R M D A. As long as each of these matrices is unitary, then cascading them in series produces another unitary matrix. The frequency dependent absorption that results when sound propagates and reflects is modelled by the lossy lowpass filters represented by matrix L; the L matrix should not be unitary.

The following paragraphs describe each section's (and corresponding matrix's) functionality in the late reflection generator.

Allpass Filter

The allpass filter (A) increases echo density [5] and also adds realism to the early reflections [7].

Novel Delay Line Topology

The novel delay line topology provides the following three important features: multiple output taps, time varying output taps, and guaranteed stability. Multiple output taps increase echo density because for every one pulse that enters this topology, multiple pulses exit. This provides a more realistic room simulation because echo density increases as time progresses, as it does in the real world. Furthermore, the multiple output taps change position and amplitude; that is, the delay line lengths and coefficients are time variant.

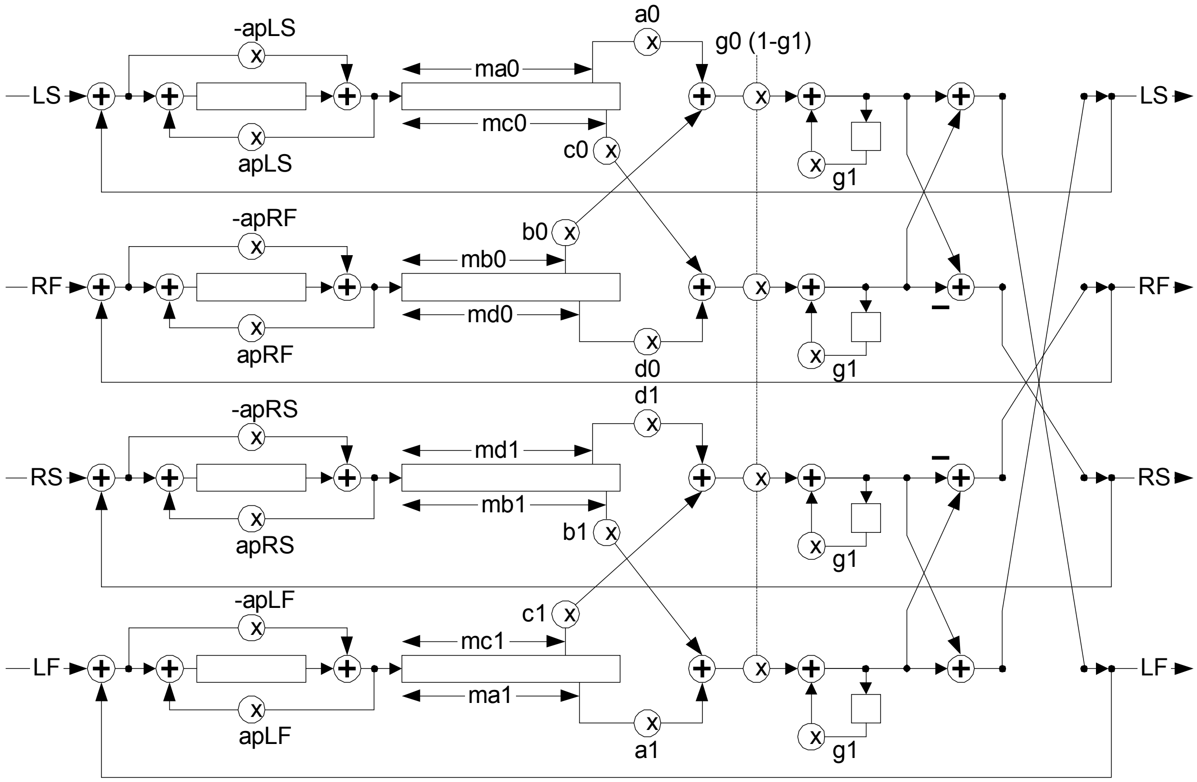

Figure 2. Late Reflection Generator

Varying the delay line lengths and coefficients changes the resonant frequencies, and thereby reduces coloration and fluttering and allows us to use less delay line memory than a time invariant reverberation algorithm. Coloration is reduced because no one set of modes persists any longer than any other neighboring set of modes. As long as there is at least one lossy element somewhere in the recirculating system, our novel delay line topology is guaranteed to be stable because it is unitary. (The lowpass filter, L, as described below, is the lossy element in our recirculating system.)

Previous solutions [3, 4] increase echo density by reducing the number of nonzero coefficients in the feedback matrix, and reduce coloration by choosing incommensurate delay line lengths. These feedback delay network (FDN) reverberators use the same delay line topology: multiple delay lines where each delay line has a single output tap. The delay line topology is represented by a matrix where the only entries are delay elements along the matrix's principal diagonal as shown by the following matrix, where z^{-m} represents a time delay of m samples:

$$D = \begin{bmatrix} z^{-ma} & 0 & 0 & 0 \\ 0 & z^{-mb} & 0 & 0 \\ 0 & 0 & z^{-mc} & 0 \\ 0 & 0 & 0 & z^{-md} \end{bmatrix} \tag{3}$$

In [7], they attempted to increase echo density by using multiple output taps from the same delay line. The following equations show why their delay line topology is not unitary, and it is therefore not surprising that they experienced unstable systems that were controllable using low feedback values.

$$D = a\cdot z^{-ma} + b\cdot z^{-mb} \qquad \overline{D}^{T} = a\cdot z^{ma} + b\cdot z^{mb} \tag{4}$$

The product simplifies to

$$D\cdot\overline{D}^{T} = a^2 + b^2 + a\cdot b\cdot\left(z^{-ma+mb} + z^{-mb+ma}\right) \tag{5}$$

Evaluating the product (5) on the unit circle produces the following result which cannot be unity for all frequencies.

$$a^2 + b^2 + 2\cdot a\cdot b\cdot\cos(\omega\cdot(ma - mb)) \tag{6}$$

Even if the approach in [7] were extended to a feedback delay network instead of just a single delay line, feeding back multiple taps to the same delay line does not allow for a unitary system.

Our novel design allows for multiple taps from the same delay line as long as certain restrictions are fulfilled. For example, for a 2x2 system that has 2 inputs, 2 delay lines, and 2 outputs, figure 3 shows the delay line topology and equation 7 the corresponding delay matrix, D (and its conjugate transpose).

$$D = \begin{bmatrix} a\cdot z^{-ma} & b\cdot z^{-mb} \\ c\cdot z^{-mc} & d\cdot z^{-md} \end{bmatrix} \qquad \overline{D}^{T} = \begin{bmatrix} a\cdot z^{ma} & c\cdot z^{mc} \\ b\cdot z^{mb} & d\cdot z^{md} \end{bmatrix} \tag{7}$$

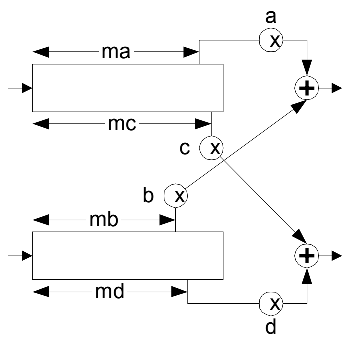

Figure 3. Novel multiple output tap delay line topology

In order for the delay line topology to be unitary, the product of the delay matrix and its conjugate transpose, as shown below in (8), must be the identity matrix.

$$\begin{bmatrix} a^2+b^2 & a\cdot c\cdot z^{-ma+mc}+b\cdot d\cdot z^{-mb+md} \\ a\cdot c\cdot z^{-mc+ma}+b\cdot d\cdot z^{-md+mb} & c^2+d^2 \end{bmatrix} \tag{8}$$

The product matrix in (8) is unitary provided the following constraints are fulfilled: the elements along the diagonal must equal unity while all other elements must cancel to zero. For the simple 2x2 case, we can easily satisfy these constraints using the following values for the tap coefficients (a, b, c, d) and delay line lengths (ma, mb, mc, md):

$$\begin{aligned} &a=\cos(\theta) && b=\sin(\theta) \\ &c=\sin(\theta) && d=-\cos(\theta) \\ &ma+md=mb+mc \end{aligned} \tag{9}$$

The delay matrix corresponding to our novel delay line topology has entries off the main diagonal (represented by $b\,z^{-mb}$ and $c\,z^{-mc}$), thus we have multiple output taps from the same delay line. By satisfying the design constraints in (9), our delay line topology is unitary, and when placed in a larger system that includes at least some lossy elements, the overall system stability is guaranteed.

The 2x2 system can easily be extended to a larger system. For example, one way of designing a 4x4 system is to use two 2x2 systems with the following topology shown in figure 4 and corresponding matrix equation 10.

$$D=\begin{bmatrix} a0\cdot z^{-ma0} & b0\cdot z^{-mb0} & 0 & 0 \\ c0\cdot z^{-mc0} & d0\cdot z^{-md0} & 0 & 0 \\ 0 & 0 & d1\cdot z^{-md1} & c1\cdot z^{-mc1} \\ 0 & 0 & b1\cdot z^{-mb1} & a1\cdot z^{-ma1} \end{bmatrix} \tag{10}$$

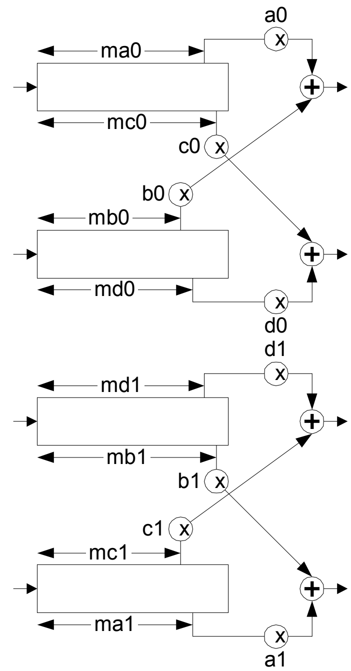

Figure 4. 4x4 multiple tap delay line

Because the simple 4x4 delay matrix is constructed of two separate 2x2 systems, we can ensure the matrix in (10) is unitary by using the following set of values in (11) as the coefficients and delay line lengths, similarly to the 2x2 case.

$$\begin{aligned} &a0=\cos(\theta) && b0=\sin(\theta) \\ &c0=\sin(\theta) && d0=-\cos(\theta) \\ &ma0+md0=mb0+mc0 \\ &a1=\cos(\phi) && b1=\sin(\phi) \\ &c1=\sin(\phi) && d1=-\cos(\phi) \\ &ma1+md1=mb1+mc1 \end{aligned} \tag{11}$$

By using cosine and sine for our coefficients (a0, b0, c0, d0; a1, b1, c1, d1), we can parameterize the coefficients as functions of time. That is, as the cosine and sine phase angle changes over time, the coefficient values also change but the unitary matrix constraints for the coefficients are still fulfilled.

The delay line length constraints indicate that for each 2x2 system, the sum of the delay line lengths for the terms on the principal diagonal must equal the sum of the delay line lengths for the terms on the anti-diagonal. The actual delay line lengths can be any positive real numbers that satisfy the above criteria, but we use room geometry and psychoacoustics to set the delay line lengths. Since the early reflections in a concert hall occur during the first 60 to 80 milliseconds and the late reflections thereafter, the delay line lengths should correspond nominally to 60 to 80 milliseconds. For practical constraints, we choose a maximum delay line length of 65 milliseconds. By choosing 65 milliseconds as the maximum delay line length for each delay line, we use the same delay lines to both directly generate the early reflections and indirectly generate the late reflections. The early reflections are generated by sparsely placing taps along the delay lines. As the input signal progresses through the delay for the first time, the taps simulate the early reflections. After reaching the end of the delay line (65 milliseconds later, after the early reflections are done), the signal is then attenuated and fed back to the input and recirculates to simulate the late reflections. The delay line lengths, however, can also change as time progresses as long as the delay line length constraints are fulfilled.

For example, in [6] Moore suggests varying the delay line lengths either completely randomly or with a periodic motion. From investigations into pitch and amplitude fluctuations causing “wow” and “flutter”, Stott and Axon [9] developed a device that modulated the position of the output tap using random noise. Our delay line topology modulates the delay line length with a random number such that the delay line length constraint is always satisfied and that the maximum delay line length of 65 milliseconds is never exceeded.

Lowpass Filter

The lowpass filter is represented by the matrix, L, and is the only lossy element for the recirculating system. The lowpass filter provides a frequency dependent gain that simulates the absorption of sound as it propagates through air and reflects off surfaces. A simple single pole, no zero lowpass filter provides adequate control of the reverb time as a function of frequency. Higher order lowpass filters can be used if more control is desired.

Because each channel's delay line length is on average identical, since we add random numbers to the nominal delay line lengths, we use one lowpass filter per channel with identical feedback and feedforward gain values. For the 4 channel case, the lowpass matrix transfer function is given in (12) . The gain *g0* controls the reverb time at DC.

$$L = g0 \cdot \begin{bmatrix} \frac{1-g1}{1-g1\cdot z^{-1}} & 0 & 0 & 0 \\ 0 & \frac{1-g1}{1-g1\cdot z^{-1}} & 0 & 0 \\ 0 & 0 & \frac{1-g1}{1-g1\cdot z^{-1}} & 0 \\ 0 & 0 & 0 & \frac{1-g1}{1-g1\cdot z^{-1}} \end{bmatrix} \quad (12)$$

Mixing and Routing

Equations (13) and (14) specify how the delayed and filtered signals are combined according to the mixing matrix, M, and then fed back to the input according to the routing matrix, R.

$$M = \frac{1}{\sqrt{2}} \cdot \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \quad (13)$$

$$R = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \quad (14)$$

The mixing and routing matrices ensure that some portion of the delayed input is always propagated in a common sense fashion. For example, in a 4 input, 4 output system, after the early reflections for the left front signal are generated, some portion of the left front signal is then sent to the right front and left surround delay lines to produce the late reflections. Even when the time varying delay matrix has output taps that are exclusively cross coupled or exclusively forward coupled, some portion of the input signal will always propagate to the nearest two adjacent speakers.

Early Reflection Generator

The early reflection generator, shown in figure 5, shares the delay line memory with the late reflection generator, thus reducing the total memory used by our reverberator.

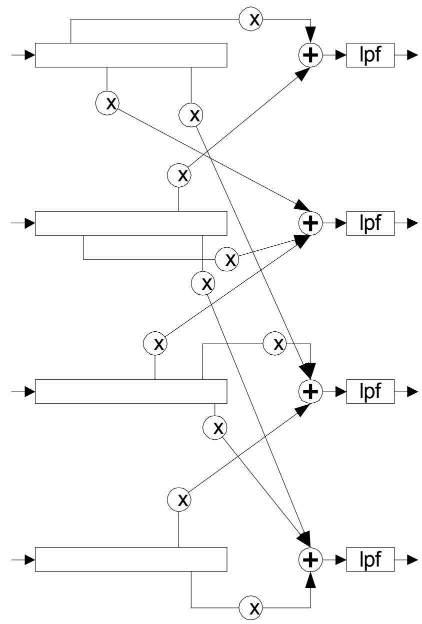

Figure 5. Early Reflection Generator

Early Reflections and Lowpass Filter

The early reflection matrix, E, describes how the input signal sent to the delay lines is delayed and combined to simulate the early reflections in a given acoustic environment. Each delay line may have multiple output taps that are summed to form a composite early reflection signal. The tap positions correspond to the delay lengths, and the tap weights correspond to the strength of the reflections. The tap positions and weights can be determined using a ray tracing model for the early reflections. The early reflections are generated as the input signal progresses through the delays for the first time. After the early reflections are generated, the input signal is combined, filtered, and fed back according to the D, L, M, and R matrices, sent through the allpass filters again (A matrix), and sent out using the same taps used for the early reflections. That is, the early reflection taps are re-used to generate the late reflections as the input signal recirculates through the entire structure.

The lowpass filters in matrix F provide additional realism by simulating sound absorption. Simple single pole, no zero filters are sufficient.

Conclusions

Our new reverb algorithm improves previously attempted solutions because it sounds good, uses little delay line memory, and is easy to design. The reverberator sounds good because it has a high echo density in the time domain, has a sufficient density of maxima in the frequency domain, and a natural colorless timbre. Delay line memory usage is reduced because we have developed a novel delay line topology that increases echo density and doesn't introduce tonal coloration. Furthermore, the design process is greatly simplified because delay line lengths and other parameters are chosen based on room geometry and psychoacoustic principles.

Also, the algorithm is scalable. A simple 2 channel input, 2 channel output (2 in, 2 out) system can easily be expanded to multichannel reverbs. For example, car audio systems can be (2 in, 4 out), video game applications can be (4 in, 4 out), and home theatre systems can be (6 in, 6 out) and beyond.

Acknowledgements

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