$$-\frac{d^2u(x)}{dx^2} - u(x) = \sin(x) \qquad u(0)=1 \qquad \frac{du(2)}{dx} - u(2) = 4 \qquad x \in \langle 0,2 \rangle$$

wor. Dirichleta   wor. Cauchyego

wprowadzamy shift

$u = w + \bar{u}$

$$-u'' - u = \sin x$$

$$-\int_0^2 u'' v\,dx - \int_0^2 uv\,dx = \int_0^2 \sin x\, v\,dx$$

$$\cancel{\int_0^2 u'' v\,dx} = -$$

$$-u'v\Big|_0^2 + \int_0^2 u'v'\,dx - \int_0^2 uv\,dx = \int_0^2 \sin x\, v\,dx$$

$$-u'(2)v(2) + u'(0)v(0) + \int_0^2 u'v'\,dx - \int_0^2 uv\,dx = \int_0^2 \sin x\, v\,dx$$

Z warunku Dirichleta, funkcja testująca v na brzegu będzie równa 0 $(v(0)=0)$

$u'(2) = u(2) + 4$

$$-(u(2)+4)v(2) + \int_0^2 u'v'\,dx - \int_0^2 uv\,dx = \int_0^2 \sin x\, v\,dx$$

$$\underbrace{-u(2)v(2) + \int_0^2 u'v'\,dx - \int_0^2 uv\,dx}_{B(u,v)} = \underbrace{\int_0^2 \sin x\, v\,dx + 4v(2)}_{L(v)}$$

$$B(u,v) = L(v)$$

$$B(w+\bar{u}, v) = L(v)$$

$$B(w,v) + B(\bar{u},v) = L(v)$$

$$B(w,v) = L(v) - B(\bar{u},v)$$

$$B(w,v) = \bar{L}(v)$$

$$-w(2)v(2) + \int_0^2 w'v'\,dx - \int_0^2 wv\,dx = \int_0^2 v \sin x\,dx + 4v(2) + \int_0^2 \bar{u} v\,dx - \int_0^2 \bar{u}'v'\,dx + \bar{u}(2)v(2)$$