

1 Claim -Is that the push and pop operations are of  $O(1)$  or of constant time

Push-> The implemented code is-:

```
void Stack_B::push(int data)
{
    if (size > capacity - 1)
    {
        int *temp = new int[capacity * 2];
        for (int i = 0; i < size; i++)
        {
            temp[i] = stk[i];
        }
        delete[] stk;
        stk = temp;
        stk[size] = data;
        capacity *= 2;
        size++;
    }
    else
    {
        stk[size] = data;
        size++;
    }
}
```

1. When **size** is less than **capacity**: In this case, I am simply adding an element to the existing array, which takes constant time since it involves only accessing the memory and changing the value. This can be represented as  $O(1)$  since it's a fixed number of operations regardless of the size of the stack.
2. When **size** is greater than or equal to **capacity**: In this case
  - Copy the old array to the new array takes  $O(N)$  time, where  $N$  is the number of elements in the old array.
  - Insert the new element takes  $O(1)$  time.

The total time complexity for the push operation in this case can be represented as  $O(N + 1)$ , which simplifies to  $O(N)$  since the constant term is dropped in big O notation.

Now, let's look at the amortized analysis for the push operation:

If we consider a sequence of  $N$  push operations, the total cost of copying elements will be approximately  $N$ .

Total cost =  $N$  (for copying) +  $N$  (for inserting) =  $2N$

Amortized cost per push operation = Total cost /  $N$  =  $2N / N = 2$

Since the amortized cost per push operation is a constant (2), we can conclude that the push operation's amortized time complexity is indeed  $O(1)$ .

POP->The implemented code is---:

```
int Stack_B::pop()
{
    if (size <= 0)
    {
        throw runtime_error("Empty Stack");
    }
    else if (size < capacity / 4 and capacity > 1024)
    {
        int *temp = new int[capacity / 4];
        for (int i = 0; i < size; i++)
        {
            temp[i] = stk[i];
        }
        delete[] stk;
        stk = temp;
        capacity /= 4;
        size--;
        return stk[size];
    }
    else
    {
        size--;
        return stk[size];
    }
}
```

If  $\text{size} > \text{capacity}/4$  or  $\text{size} < 1024$  then it's obvious  $O(1)$  as I am only decreasing the value of size. If  $\text{size} < \text{capacity}/4$  and  $\text{size} > 1024$  which is less frequent but let's assume  $N$  elements in old array where  $N = \text{older capacity of array}$ . Now after  $3N/4$  pop() the  $\text{size} < \text{capacity}/4$ . Now the  $(3N/4)+1$ th pop()

the steps= $(3N/4)$ for $(3N/4)$  pops)+ $O(N/4)$  (for copying N elements)+1(for size changing in the new array)

$$\text{Steps}=(3N/4)+(N/4)+1=N+1$$

$$\text{Operations}=(3N/4)+1$$

Average time complexity= $(N+1)/((3N/4)+1)=1+N/(3N+4)$  for large N the whole simplifies to 2

Hence time is constant or  $O(1)$ .