Assignment 3

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Abstract—**This document uses the properties of a parallelogram to prove a statement**

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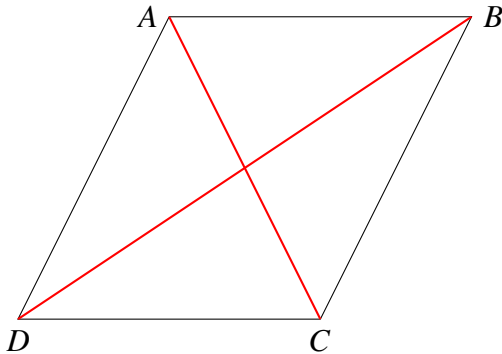
1 PROBLEM

Prove that the sum of the squares of the diagonals of parallelogram is equal to the sum of the squares of its sides.
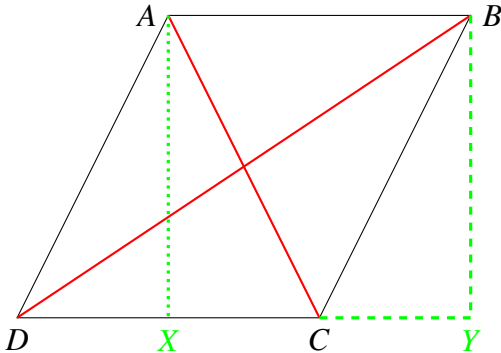
2 EXPLANATION

Given a parallelogram ABCD we have to prove that

$$AC^2 + BD^2 = AB^2 + BC^2 + CD^2 + DA^2 \quad (2.0.1)$$

In the parallelogram ABCD AB=CD and AC=BD Draw perpendiculars from A to DC and B to DC extended as shown

3 SOLUTION

From $\triangle AXD$ and $\triangle BYD$

$$AC^2 = AX^2 + CX^2 \quad (3.0.1)$$
$$BD^2 = BY^2 + DY^2 \quad (3.0.2)$$

From equation (3.0.1)

$$AC^2 = AX^2 + (CD - DX)^2 \quad (3.0.3)$$
$$AC^2 = AX^2 + CD^2 + DX^2 - 2CD.DX \quad (3.0.4)$$
$$AC^2 = (AX^2 + DX^2) + CD^2 - 2CD.DX \quad (3.0.5)$$
$$AC^2 = AD^2 + CD^2 - 2CD.DX \quad (3.0.6)$$

From equation (3.0.2)

$$BD^2 = BY^2 + (CD + CY)^2 \quad (3.0.7)$$
$$BD^2 = BY^2 + CD^2 + CY^2 + 2.CD.CY \quad (3.0.8)$$
$$BD^2 = (BY^2 + CY^2) + CD^2 + 2.CD.CY \quad (3.0.9)$$
$$BD^2 = BC^2 + CD^2 + 2.CD.CY \quad (3.0.10)$$

In $\triangle AXD$ and $\triangle BYC$

$$AX = BY \quad (3.0.11)$$
$$\angle AXD = \angle BYC \quad (3.0.12)$$
$$AD = BC \quad (3.0.13)$$

Therefore by RHS Congruency

$$DX = CY \quad (3.0.14)$$

Substituting the value of DX from (3.0.14) to equation (3.0.6)

$$AC^2 = AD^2 + CD^2 - 2CD.CY \quad (3.0.15)$$

Combining equation (3.0.15) and (3.0.10) and simplifying

$$AC^2 + BD^2 = AD^2 + CD^2 + BC^2 + CD^2 \quad (3.0.16)$$
$$\quad (3.0.17)$$

We know that CD=AB , therefore

$$AC^2 + BD^2 = AB^2 + BC^2 + CD^2 + DA^2 \quad (3.0.18)$$