Assignment 3

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***Abstract*—This document uses the properties of a parallelogram to prove a statement**

Download latex-tikz codes from

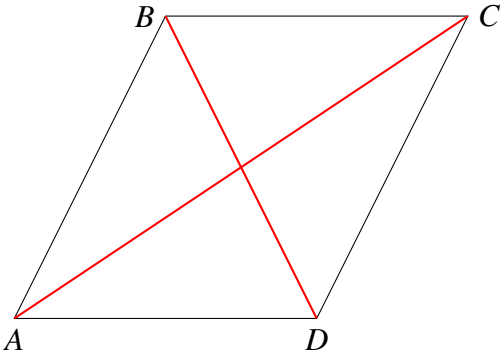
https://github.com/AddagallaSatyanarayana/AI5006/tree/master/Assignment3/assignment3.tex

1 Problem

Prove that the sum of the squares of the diagonals of parallelogram is equal to the sum of the squares of its sides.

2 Explanation

Given a parallelogram ABCD we have to prove



that

$$\left\|\mathbf{A}-\mathbf{C}\right\|^2+\|\mathbf{B}-\mathbf{D}\|^2 = \|\mathbf{A}-\mathbf{B}\|^2+\left\|\mathbf{B}-\mathbf{C}\right\|^2+\left\|\mathbf{D}-\mathbf{C}\right\|^2+\|\mathbf{A}-\mathbf{D}\|^2 \tag{2.0.1}$$

In the parallelogram $ABCD$,let

$$\mathbf{a} = \mathbf{A}-\mathbf{D} \tag{2.0.2}$$

$$\mathbf{b} = \mathbf{D}-\mathbf{C} \tag{2.0.3}$$

3 Solution

If $\mathbf{a}$ and $\mathbf{b}$ represent the sides of the parallelogram then,the diagonals are

$$\mathbf{A}-\mathbf{C} = \mathbf{a}+\mathbf{b} \tag{3.0.1}$$

$$\mathbf{B}-\mathbf{D} = \mathbf{a}-\mathbf{b} \tag{3.0.2}$$

The sum of the squares of diagonals is

$$\left\|\mathbf{A}-\mathbf{C}\right\|^2+\|\mathbf{B}-\mathbf{D}\|^2 = \|\mathbf{a}+\mathbf{b}\|^2+\|\mathbf{a}-\mathbf{b}\|^2 \tag{3.0.3}$$

$$\left\|\mathbf{A}-\mathbf{C}\right\|^2+\|\mathbf{B}-\mathbf{D}\|^2 = 2\|\mathbf{a}\|^2+2\|\mathbf{b}\|^2 \tag{3.0.4}$$

from equation (2.0.3) and (3.0.4)

$$\left\|\mathbf{A}-\mathbf{C}\right\|^2+\|\mathbf{B}-\mathbf{D}\|^2 = 2\|\mathbf{A}-\mathbf{D}\|^2+2\left\|\mathbf{D}-\mathbf{C}\right\|^2 \tag{3.0.5}$$

In the parallelogram $ABCD$

$$\|\mathbf{a}\| = \|\mathbf{A}-\mathbf{D}\| = \left\|\mathbf{B}-\mathbf{C}\right\| \tag{3.0.6}$$

$$\|\mathbf{b}\| = \|\mathbf{A}-\mathbf{B}\| = \left\|\mathbf{D}-\mathbf{C}\right\| \tag{3.0.7}$$

from equation (3.0.5) , (3.0.6) and (3.0.7)

$$\left\|\mathbf{A}-\mathbf{C}\right\|^2+\|\mathbf{B}-\mathbf{D}\|^2 = \|\mathbf{A}-\mathbf{B}\|^2+\left\|\mathbf{B}-\mathbf{C}\right\|^2+\left\|\mathbf{D}-\mathbf{C}\right\|^2+\|\mathbf{A}-\mathbf{D}\|^2 \tag{3.0.8}$$