

Assignment 4

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Abstract—This document uses the properties of a tangent to a circle

Download latex-tikz codes from

<https://github.com/AddagallaSatyanarayana/AI5006/tree/master/Assignment4/Assignment4.tex>

1 PROBLEM

Find the length of the tangent from the point $\begin{pmatrix} 7 \\ 4 \end{pmatrix}$ to the circle

$$\mathbf{x}^T \mathbf{x} - (4 \ 6) \mathbf{x} + 12 = 0 \quad (1.0.1)$$

2 EXPLANATION

The general equation of a second degree can be expressed as :

$$\mathbf{x}^T \mathbf{V} \mathbf{x} + 2\mathbf{u}^T \mathbf{x} + f = 0 \quad (2.0.1)$$

Let the point of intersection of the tangent and the circle be denoted by \mathbf{p} as shown in figure below.

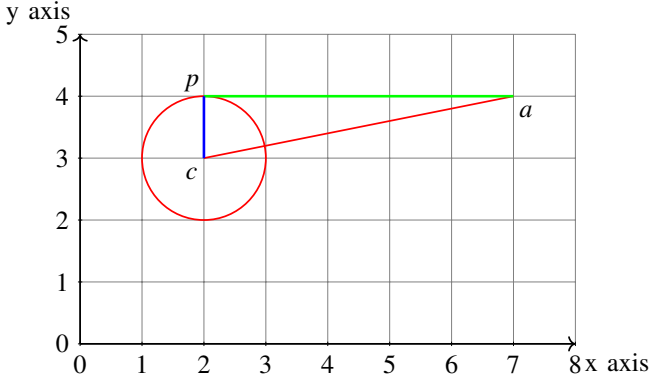


Fig. 0: Tangent to Circle

3 SOLUTION

We know that, for a circle,

$$\mathbf{V} = \mathbf{I} \quad (3.0.1)$$

$$\mathbf{c} = -\mathbf{u} \quad (3.0.2)$$

Comparing the equation (1.0.1) and (2.0.1) we get

$$\mathbf{u} = \begin{pmatrix} -2 \\ -3 \end{pmatrix}, f = 12 \quad (3.0.3)$$

$$\mathbf{c} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \quad (3.0.4)$$

$$radius = \sqrt{\mathbf{u}^T \mathbf{u} - f} \quad (3.0.5)$$

$$radius = \sqrt{\|\mathbf{u}\|^2 - f} = 1 \quad (3.0.6)$$

let $\mathbf{a} = \begin{pmatrix} 7 \\ 4 \end{pmatrix}$, then

$$\|\mathbf{a} - \mathbf{c}\| = \sqrt{26} \quad (3.0.7)$$

We know that the \triangle formed by the centre of the circle to the point, the radius from the centre to the point of contact, and the tangent to the circle from the point make a right triangle. Hence in right angle $\triangle cpa$

$$\|\mathbf{a} - \mathbf{c}\|^2 = \|\mathbf{a} - \mathbf{p}\|^2 + \|\mathbf{r}\|^2 \quad (3.0.8)$$

$$\|\mathbf{a} - \mathbf{p}\|^2 = \|\mathbf{a} - \mathbf{c}\|^2 - \|\mathbf{r}\|^2 \quad (3.0.9)$$

$$\|\mathbf{a} - \mathbf{p}\|^2 = 26 - 1 \quad (3.0.10)$$

$$\|\mathbf{a} - \mathbf{p}\| = \sqrt{25} \quad (3.0.11)$$

$$\|\mathbf{a} - \mathbf{p}\| = 5 \quad (3.0.12)$$

The length of the tangent from point $\begin{pmatrix} 7 \\ 4 \end{pmatrix}$ to the circle is equal to 5.