Assignment 4

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***Abstract*—This document uses the properties of a tangent to a circle**

Download latex-tikz codes from

https://github.com/AddagallaSatyanarayana/AI5006/tree/master/Assignment4/Assignment4.tex

1 Problem

Find the length of the tangent from the point $\begin{pmatrix}7\\4\end{pmatrix}$ to the circle

$$\mathbf{x}^T\mathbf{x} - \begin{pmatrix}4 & 6\end{pmatrix}\mathbf{x} + 12 = 0 \tag{1.0.1}$$

2 Explanation

The general equation of a second degree can be expressed as :

$$\mathbf{x}^T\mathbf{V}\mathbf{x} + 2\mathbf{u}^T\mathbf{x} + f = 0 \tag{2.0.1}$$

Let the point of intersection of the tangent and the circle be denoted by $\mathbf{p}$ as shown in figure below.

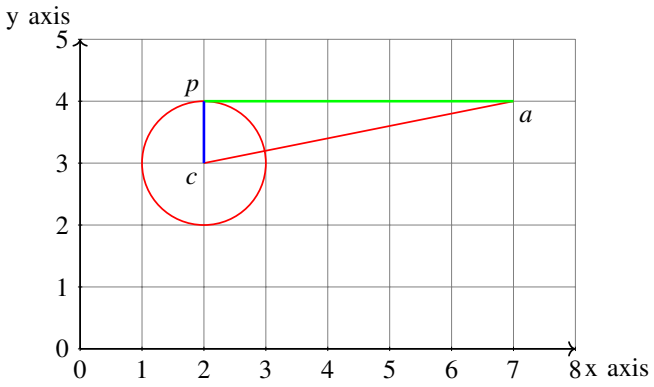

Fig. 0: Tangent to Circle

3 Solution

We know that, for a circle,

$$\mathbf{V} = \mathbf{I} \tag{3.0.1}$$

$$\mathbf{c} = -\mathbf{u} \tag{3.0.2}$$

Comparing the equation (1.0.1) and (2.0.1) we get

$$\mathbf{u} = \begin{pmatrix}-2\\-3\end{pmatrix}, f = 12 \tag{3.0.3}$$

$$\mathbf{c} = \begin{pmatrix}2\\3\end{pmatrix} \tag{3.0.4}$$

$$radius = \sqrt{\mathbf{u}^T\mathbf{u} - f} \tag{3.0.5}$$

$$radius = \sqrt{\|\mathbf{u}\|^2 - f} = 1 \tag{3.0.6}$$

let $\mathbf{a} = \begin{pmatrix}7\\4\end{pmatrix}$, then

$$\|\mathbf{a} - \mathbf{c}\| = \sqrt{26} \tag{3.0.7}$$

We know that the $\triangle$ formed by the centre of the circle to the point, the centre to the point of contact ,and the tangent to the circle from the point make a right triangle. Hence in right angle $\triangle cpa$

$$\|\mathbf{a} - \mathbf{c}\|^2 = \left\|\mathbf{a} - \mathbf{p}\right\|^2 + \|\mathbf{r}\|^2 \tag{3.0.8}$$

$$\left\|\mathbf{a} - \mathbf{p}\right\|^2 = \|\mathbf{a} - \mathbf{c}\|^2 - \|\mathbf{r}\|^2 \tag{3.0.9}$$

$$\left\|\mathbf{a} - \mathbf{p}\right\|^2 = 26 - 1 \tag{3.0.10}$$

$$\left\|\mathbf{a} - \mathbf{p}\right\| = \sqrt{25} \tag{3.0.11}$$

$$\left\|\mathbf{a} - \mathbf{p}\right\| = 5 \tag{3.0.12}$$

The length of the tangent from point $\begin{pmatrix}7\\4\end{pmatrix}$ to the circle is equal to 5.