Assignment 2

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***Abstract*—This document explains proof by method of induction.**

Download the latex-tikz code from

https://github.com/AddagallaSatyanarayana/AI5006/tree/master/Assignment2/Assignment2.tex

1 Problem

If A= $\begin{pmatrix}1 & 1 & 1\\1 & 1 & 1\\1 & 1 & 1\end{pmatrix}$, prove that $A^n = \begin{pmatrix}3^{n-1} & 3^{n-1} & 3^{n-1}\\3^{n-1} & 3^{n-1} & 3^{n-1}\\3^{n-1} & 3^{n-1} & 3^{n-1}\end{pmatrix}, n \in \mathbb{N}$

2 Explanation

The above problem can be proven by the method of induction

$$A^2 = AA = \begin{pmatrix}3 & 3 & 3\\3 & 3 & 3\\3 & 3 & 3\end{pmatrix} = \begin{pmatrix}3^1 & 3^1 & 3^1\\3^1 & 3^1 & 3^1\\3^1 & 3^1 & 3^1\end{pmatrix} \quad (2.0.1)$$

$$A^3 = A^2A = \begin{pmatrix}3^2 & 3^2 & 3^2\\3^2 & 3^2 & 3^2\\3^2 & 3^2 & 3^2\end{pmatrix} \quad (2.0.2)$$

let this be true for n=k,then

$$A^k = \begin{pmatrix}3^{k-1} & 3^{k-1} & 3^{k-1}\\3^{k-1} & 3^{k-1} & 3^{k-1}\\3^{k-1} & 3^{k-1} & 3^{k-1}\end{pmatrix} \quad (2.0.3)$$

$$A^{k+1} = A^kA = \begin{pmatrix}3^{k-1} & 3^{k-1} & 3^{k-1}\\3^{k-1} & 3^{k-1} & 3^{k-1}\\3^{k-1} & 3^{k-1} & 3^{k-1}\end{pmatrix}\begin{pmatrix}1 & 1 & 1\\1 & 1 & 1\\1 & 1 & 1\end{pmatrix} \quad (2.0.4)$$

$$A^{k+1} = \begin{pmatrix}3.3^{k-1} & 3.3^{k-1} & 3.3^{k-1}\\3.3^{k-1} & 3.3^{k-1} & 3.3^{k-1}\\3.3^{k-1} & 3.3^{k-1} & 3.3^{k-1}\end{pmatrix} \quad (2.0.5)$$

$$A^{k+1} = \begin{pmatrix}3^k & 3^k & 3^k\\3^k & 3^k & 3^k\\3^k & 3^k & 3^k\end{pmatrix} \quad (2.0.6)$$

therefore it holds for all n$\in \mathbb{N}$

$$A^n = \begin{pmatrix}3^{n-1} & 3^{n-1} & 3^{n-1}\\3^{n-1} & 3^{n-1} & 3^{n-1}\\3^{n-1} & 3^{n-1} & 3^{n-1}\end{pmatrix}, n \in \mathbb{N} \quad (2.0.7)$$