Assignment 1

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***Abstract*—This document explains how to find a line perpendicular to 2 lines and passing through a point.**

Download the python code from

https://github.com/AddagallaSatyanarayana/AI5006/tree/master/Assignment1

and latex-tikz codes from

https://github.com/AddagallaSatyanarayana/AI5006/tree/master/Assignment1/Assignment1.tex

1 Problem

Find the vector equation of the line passing through the point $\begin{pmatrix} 1 \\ 2 \\ -4 \end{pmatrix}$ and perpendicular to the two lines $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$

2 Explanation

Equation of a line passing through the point $\mathbf{a}$ and parallel to the line $\mathbf{n}$ is given by:

$$\mathbf{x} = \mathbf{a} + \lambda \mathbf{n} \tag{2.0.1}$$

where λ is some constant. Since the line passes through $\begin{pmatrix} 1 \\ 2 \\ -4 \end{pmatrix}$;$\mathbf{a} = \begin{pmatrix} 1 \\ 2 \\ -4 \end{pmatrix}$

$$\mathbf{x} = \begin{pmatrix} 8 \\ -19 \\ 10 \end{pmatrix} + \lambda_1 \begin{pmatrix} 3 \\ -16 \\ 7 \end{pmatrix} \tag{2.0.2}$$

$$\mathbf{x} = \begin{pmatrix} 15 \\ 29 \\ 5 \end{pmatrix} + \lambda_2 \begin{pmatrix} 3 \\ 8 \\ -5 \end{pmatrix} \tag{2.0.3}$$

3 Solution

Let $\mathbf{n}$ be the normal vector to both lines. If $\mathbf{m_1}$ and $\mathbf{m_2}$ are the direction vectors of the lines,then

$$\mathbf{m_1}^T \mathbf{n} = 0 \tag{3.0.1}$$

$$\mathbf{m_2}^T \mathbf{n} = 0 \tag{3.0.2}$$

Let the matrix $\mathbf{M}$ be

$$\mathbf{M} = \begin{pmatrix} \mathbf{m_1}^T \\ \mathbf{m_2}^T \end{pmatrix} \tag{3.0.3}$$

$$\mathbf{m_1} = \begin{pmatrix} 3 \\ -16 \\ 7 \end{pmatrix}, \mathbf{m_2} = \begin{pmatrix} 3 \\ 8 \\ -5 \end{pmatrix}, \mathbf{n} = \begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix} \tag{3.0.4}$$

$$\mathbf{Mn} = 0 \tag{3.0.5}$$

The matrix form is

$$\begin{pmatrix} 3 & -16 & 7 \\ 3 & 8 & -5 \end{pmatrix} \tag{3.0.6}$$

$$\begin{pmatrix} 3 & -16 & 7 \\ 3 & 8 & -5 \end{pmatrix} \xleftrightarrow{R_2 = R_1 - R_2} \begin{pmatrix} 3 & -16 & 7 \\ 0 & -24 & 12 \end{pmatrix} \tag{3.0.7}$$

$$\begin{pmatrix} 3 & -16 & 7 \\ 0 & -24 & 12 \end{pmatrix} \xleftrightarrow{R_2 = \frac{R_2}{2}} \begin{pmatrix} 3 & -16 & 7 \\ 0 & -2 & 1 \end{pmatrix} \tag{3.0.8}$$

$$\begin{pmatrix} 3 & -16 & 7 \\ 0 & -2 & 1 \end{pmatrix} \xleftrightarrow{R_1 = R_1 - 8R_2} \begin{pmatrix} 3 & 0 & -1 \\ 0 & -2 & 1 \end{pmatrix} \tag{3.0.9}$$

We have 2 equations and 3 unknowns , we will have parametric solution

$$n_1 = \frac{k}{3} \tag{3.0.10}$$

$$n_2 = \frac{k}{2} \tag{3.0.11}$$

$$n_3 = k \tag{3.0.12}$$

$$\mathbf{n} = \frac{k}{6} \begin{pmatrix} 2 \\ 3 \\ 6 \end{pmatrix} \tag{3.0.13}$$

The equation of required line from equation (2.0.1)

$$\begin{pmatrix} 1 \\ 2 \\ -4 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 3 \\ 6 \end{pmatrix} \tag{3.0.14}$$

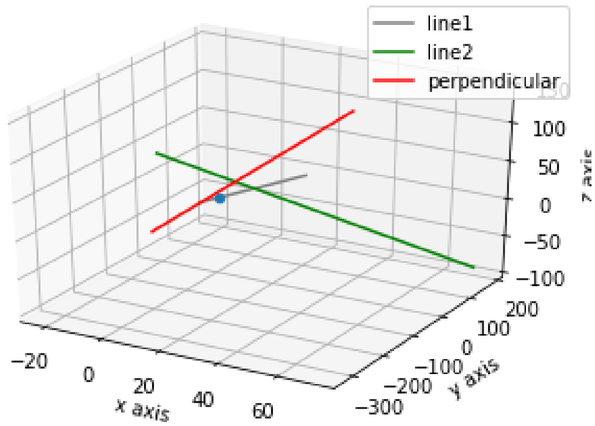

Fig. 0: Perpendicular Line