

Assignment 3

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Abstract—This document uses the properties of a parallelogram to prove a statement

Download latex-tikz codes from

<https://github.com/AddagallaSatyanarayana/AI5006/tree/master/Assignment3/Assignment3.tex>

1 PROBLEM

Prove that the sum of the squares of diagonals of parallelogram is equal to the sum of the squares of its sides.

2 EXPLANATION

Given a parallelogram ABCD we have to prove that

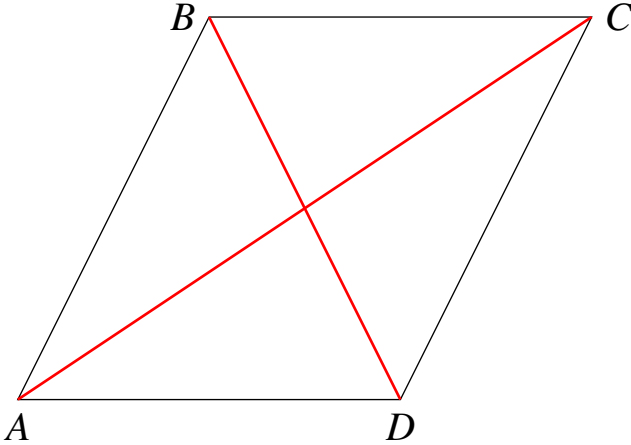


Fig. 0: parallelogram ABCD

$$\begin{aligned} \|A - C\|^2 + \|B - D\|^2 &= \|A - B\|^2 + \|B - C\|^2 + \\ &\quad \|D - C\|^2 + \|A - D\|^2 \end{aligned} \quad (2.0.1)$$

3 SOLUTION

The diagonals of parallelogram are

$$A - C = (A - D) + (D - C) \quad (3.0.1)$$

$$B - D = (A - D) - (A - B) \quad (3.0.2)$$

The sum of the squares of diagonals is

$$\begin{aligned} \|A - C\|^2 + \|B - D\|^2 &= \|(A - D) + (D - C)\|^2 \\ &\quad + \|(A - D) - (A - B)\|^2 \end{aligned} \quad (3.0.3)$$

$$\begin{aligned} &= \|A - D\|^2 + \|D - C\|^2 + 2(A - D)^T(D - C) + \\ &\quad \|A - D\|^2 + \|A - B\|^2 - 2(A - D)^T(A - B) \end{aligned} \quad (3.0.4)$$

$$\begin{aligned} &= \|A - D\|^2 + \|D - C\|^2 + 2\|A - D\|\|D - C\| \\ &\quad \cos(180^\circ - \angle ADC) + \|A - D\|^2 + \|A - B\|^2 \\ &\quad - 2\|A - D\|\|A - B\| \cos \angle DAB \end{aligned} \quad (3.0.5)$$

In the parallelogram ABCD

$$\|A - D\| = \|B - C\| \quad (3.0.6)$$

$$\|A - B\| = \|D - C\| \quad (3.0.7)$$

$$\angle ADC + \angle DAB = 180^\circ \quad (3.0.8)$$

$$\angle DAB = 180^\circ - \angle ADC \quad (3.0.9)$$

From equation (3.0.5), (3.0.6), (3.0.7) and (3.0.9)

$$\begin{aligned} &= \|B - C\|^2 + \|D - C\|^2 + 2\|A - D\|\|A - B\| \cos \angle DAB + \\ &\quad \|A - D\|^2 + \|A - B\|^2 - 2\|A - D\|\|A - B\| \cos \angle DAB \end{aligned} \quad (3.0.10)$$

Simplifying equation (3.0.10)

$$\begin{aligned} \|A - C\|^2 + \|B - D\|^2 &= \|A - B\|^2 + \|B - C\|^2 + \\ &\quad \|D - C\|^2 + \|A - D\|^2 \end{aligned} \quad (3.0.11)$$

The equations (2.0.1) and (3.0.11) are equal, hence proved