Q 109: Find the vector equation of the line passing through the point $\begin{pmatrix} 1 \\ 2 \\ -4 \end{pmatrix}$ and perpendicular to the two lines

$\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ and

$\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{5}$

Sol:

Equation of a $\vec{l}$ passing through $\vec{a}$ and parallel to $\vec{n}$ is given by:

$\vec{l} = \vec{a} + L * \vec{n}$, where L is some constant

Since the line passes through $\begin{pmatrix} 1 \\ 2 \\ -4 \end{pmatrix}$

$\vec{a} = (i + 2j - 4k)$

Let $\vec{n}$ be the normal vector to both lines. If $\vec{m}_1$ and $\vec{m}_2$ are the direction vectors of the lines,then

$\vec{m}_1^T \vec{n} = 0$

$\vec{m}_2^T \vec{n} = 0$

Let $\vec{n} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ $\vec{m}_1 = \begin{pmatrix} 3 \\ -16 \\ 7 \end{pmatrix}$ $\vec{m}_1 = \begin{pmatrix} 3 \\ 8 \\ -5 \end{pmatrix}$

Since $\vec{n}$ is perpendicular to $\vec{m}_1$ and $\vec{m}_2$

$3x - 16y + 7z = 0$

$3x + 8y - 5z = 0$

Solving the equations $\frac{x}{2} = \frac{y}{3} = \frac{z}{6} = K$

$x = 2K, y = 3K, z = 6K$

$\vec{n} = K * (2i + 3j + 6k)$

so the equation of $\vec{l}$ is

$\vec{l} = (i + 2j - 4k) + L * K(2i + 3j + 6k)$,where L is any constant

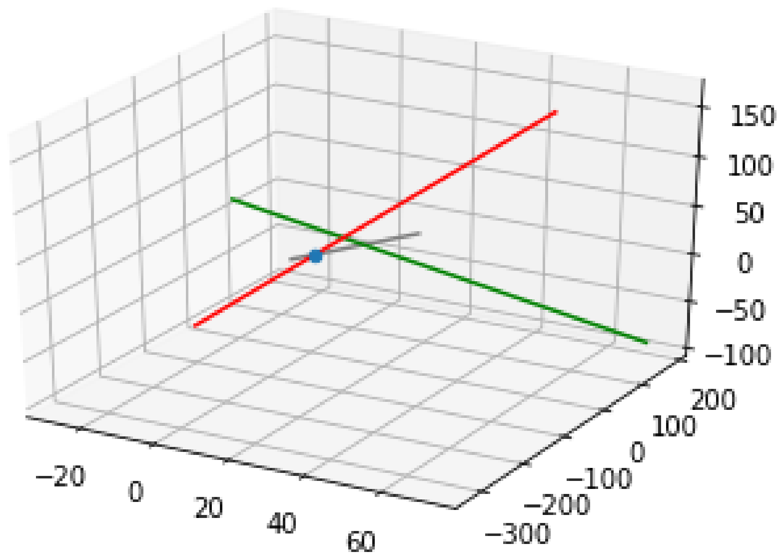

Figure 1: perpendicular