Assignment 1

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Abstract—**This document explains how to find a line perpendicular to 2 lines and passing through a point.**

1 PROBLEM

Find the vector equation of the line passing through the point $\begin{pmatrix} 1 \\ 2 \\ -4 \end{pmatrix}$ and perpendicular to the two lines $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$

2 EXPLANATION

Equation of a line **l** passing through the point **a** and parallel to the line **n** is given by:

$$\mathbf{l} = \mathbf{a} + \lambda \mathbf{n} \qquad (2.0.1)$$

where λ is some constant. Since the line passes through $\begin{pmatrix} 1 \\ 2 \\ -4 \end{pmatrix}$, **a** = (1 2 -4)

$$\mathbf{x} = \begin{pmatrix} 8 \\ -19 \\ 10 \end{pmatrix} + \lambda_1 \begin{pmatrix} 3 \\ -16 \\ 7 \end{pmatrix} \qquad (2.0.2)$$

$$\mathbf{x} = \begin{pmatrix} 15 \\ 29 \\ 5 \end{pmatrix} + \lambda_2 \begin{pmatrix} 3 \\ 8 \\ -5 \end{pmatrix} \qquad (2.0.3)$$

3 SOLUTION

Let **n** be the normal vector to both lines. If $\mathbf{m_1}$ and $\mathbf{m_2}$ are the direction vectors of the lines,then

$$\mathbf{m_1}^T \mathbf{n} = 0 \qquad (3.0.1)$$

$$\mathbf{m_2}^T \mathbf{n} = 0 \qquad (3.0.2)$$

Let the matrix **M** be

$$\mathbf{M} = \begin{pmatrix} \mathbf{m_1}^T \\ \mathbf{m_2}^T \end{pmatrix} \qquad (3.0.3)$$

$$\mathbf{m_1} = \begin{pmatrix} 3 \\ -16 \\ 7 \end{pmatrix}, \mathbf{m_2} = \begin{pmatrix} 3 \\ 8 \\ -5 \end{pmatrix}, \mathbf{n} = \begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix}$$

$$\mathbf{Mn} = 0 \qquad (3.0.4)$$

The augmented matrix is

$$\begin{pmatrix} 3 & -16 & 7 & 0 \\ 3 & 8 & -5 & 0 \end{pmatrix} \qquad (3.0.5)$$

$$\begin{pmatrix} 3 & -16 & 7 & 0 \\ 3 & 8 & -5 & 0 \end{pmatrix} \xleftrightarrow{R_2=R_1-R_2} \begin{pmatrix} 3 & -16 & 7 & 0 \\ 0 & -24 & 12 & 0 \end{pmatrix} \qquad (3.0.6)$$

$$\begin{pmatrix} 3 & -16 & 7 & 0 \\ 0 & -24 & 12 & 0 \end{pmatrix} \xleftrightarrow{R_2=\frac{R_2}{2}} \begin{pmatrix} 3 & -16 & 7 & 0 \\ 0 & -2 & 1 & 0 \end{pmatrix} \qquad (3.0.7)$$

$$\begin{pmatrix} 3 & -16 & 7 & 0 \\ 0 & -2 & 1 & 0 \end{pmatrix} \xleftrightarrow{R_1=R_1-8R_2} \begin{pmatrix} 3 & 0 & -1 & 0 \\ 0 & -2 & 1 & 0 \end{pmatrix} \qquad (3.0.8)$$

We have 2 equations and 3 unknowns , we will have parametric solution
Let $n_3 =$ k ,then $n_1 = \frac{k}{3}$ and $n_2 = \frac{k}{3}$

$$\mathbf{n} = \begin{pmatrix} \frac{k}{3} \\ \frac{k}{3} \\ k \end{pmatrix} = \frac{k}{6} \begin{pmatrix} 2 \\ 3 \\ 6 \end{pmatrix} \qquad (3.0.9)$$

The equation of required line from equation (2.0.1)

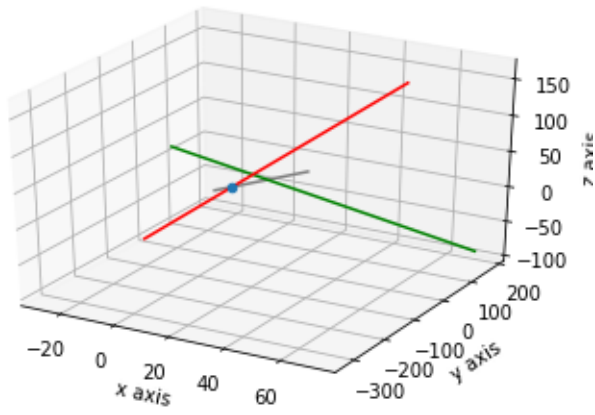$$\begin{pmatrix} 1 \\ 2 \\ -4 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 3 \\ 6 \end{pmatrix} \qquad (3.0.10)$$

Fig. 0: Perpendicular Line