Assignment 3

Addagalla Satyanarayana

Abstract—**This document uses the properties of a parallelogram to prove a statement**

Download latex-tikz codes from
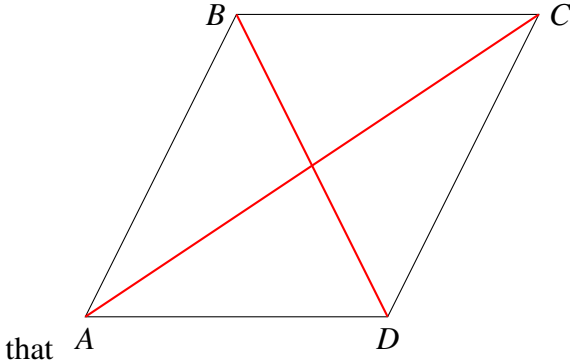
1 PROBLEM

Prove that the sum of the squares of diagonals of parallelogram is equal to the sum of the squares of its sides.

2 EXPLANATION

Given a parallelogram ABCD we have to prove

that

$$\left\|\mathbf{A} - \mathbf{C}\right\|^2 + \|\mathbf{B} - \mathbf{D}\|^2 =$$
$$\|\mathbf{A} - \mathbf{B}\|^2 + \left\|\mathbf{B} - \mathbf{C}\right\|^2 + \left\|\mathbf{D} - \mathbf{C}\right\|^2 + \|\mathbf{A} - \mathbf{D}\|^2 \quad (2.0.1)$$

3 SOLUTION

The diagonals of parallelogram are

$$\mathbf{A} - \mathbf{C} = (\mathbf{A} - \mathbf{D}) + (\mathbf{D} - \mathbf{C}) \quad (3.0.1)$$
$$\mathbf{B} - \mathbf{D} = (\mathbf{A} - \mathbf{D}) - (\mathbf{A} - \mathbf{B}) \quad (3.0.2)$$

The sum of the squares of diagonals is

$$\left\|\mathbf{A} - \mathbf{C}\right\|^2 + \|\mathbf{B} - \mathbf{D}\|^2 =$$
$$\left\|(\mathbf{A} - \mathbf{D}) + (\mathbf{D} - \mathbf{C})\right\|^2 + \left\|(\mathbf{A} - \mathbf{D}) - (\mathbf{A} - \mathbf{B})\right\|^2 \quad (3.0.3)$$

$$\left\|\mathbf{A} - \mathbf{C}\right\|^2 + \|\mathbf{B} - \mathbf{D}\|^2 =$$
$$\|\mathbf{A} - \mathbf{D}\|^2 + \left\|\mathbf{D} - \mathbf{C}\right\|^2 + 2(\mathbf{A} - \mathbf{D})^T(\mathbf{D} - \mathbf{C}) +$$
$$\|\mathbf{A} - \mathbf{D}\|^2 + \|\mathbf{A} - \mathbf{B}\|^2 - 2(\mathbf{A} - \mathbf{D})^T(\mathbf{A} - \mathbf{B}) \quad (3.0.4)$$

$$\left\|\mathbf{A} - \mathbf{C}\right\|^2 + \|\mathbf{B} - \mathbf{D}\|^2 =$$
$$\|\mathbf{A} - \mathbf{D}\|^2 + \left\|\mathbf{D} - \mathbf{C}\right\|^2 + 2\|\mathbf{A} - \mathbf{D}\|\left\|\mathbf{D} - \mathbf{C}\right\|\cos(180° - \angle ADC) +$$
$$\|\mathbf{A} - \mathbf{D}\|^2 + \|\mathbf{A} - \mathbf{B}\|^2 - 2\|\mathbf{A} - \mathbf{D}\|\|\mathbf{A} - \mathbf{B}\|\cos\angle DAB \quad (3.0.5)$$

In the parallelogram *ABCD*

$$\|\mathbf{A} - \mathbf{D}\| = \left\|\mathbf{B} - \mathbf{C}\right\| \quad (3.0.6)$$
$$\|\mathbf{A} - \mathbf{B}\| = \left\|\mathbf{D} - \mathbf{C}\right\| \quad (3.0.7)$$
$$\angle ADC + \angle DAB = 180° \quad (3.0.8)$$
$$\angle DAB = 180° - \angle ADC \quad (3.0.9)$$

From equation (3.0.5),(3.0.6),(3.0.7) and (3.0.9)

$$\left\|\mathbf{A} - \mathbf{C}\right\|^2 + \|\mathbf{B} - \mathbf{D}\|^2 =$$
$$\left\|\mathbf{B} - \mathbf{C}\right\|^2 + \left\|\mathbf{D} - \mathbf{C}\right\|^2 + 2\|\mathbf{A} - \mathbf{D}\|\|\mathbf{A} - \mathbf{B}\|\cos\angle DAB +$$
$$\|\mathbf{A} - \mathbf{D}\|^2 + \|\mathbf{A} - \mathbf{B}\|^2 - 2\|\mathbf{A} - \mathbf{D}\|\|\mathbf{A} - \mathbf{B}\|\cos\angle DAB \quad (3.0.10)$$

Simplifying equation (3.0.10)

$$\left\|\mathbf{A} - \mathbf{C}\right\|^2 + \|\mathbf{B} - \mathbf{D}\|^2 =$$
$$\|\mathbf{A} - \mathbf{B}\|^2 + \left\|\mathbf{B} - \mathbf{C}\right\|^2 + \left\|\mathbf{D} - \mathbf{C}\right\|^2 + \|\mathbf{A} - \mathbf{D}\|^2 \quad (3.0.11)$$

The equations (2.0.1) and (3.0.11) are same,hence proved