Assignment 4

Addagalla Satyanarayana

***Abstract*—This document uses the properties of a tangent to a circle**

Download latex-tikz codes from

https://github.com/AddagallaSatyanarayana/AI5006/tree/master/Assignment4/Assignment4.tex

1 Problem

Find the length of the tangent from the point $\begin{pmatrix}7\\4\end{pmatrix}$ to the circle

$$\mathbf{x}^T\mathbf{x} - \begin{pmatrix}4 & 6\end{pmatrix}\mathbf{x} + 12 = 0 \tag{1.0.1}$$

2 Explanation

Let the point of intersection of the tangent and the circle be denoted by $\mathbf{p}$ as shown in figure below.

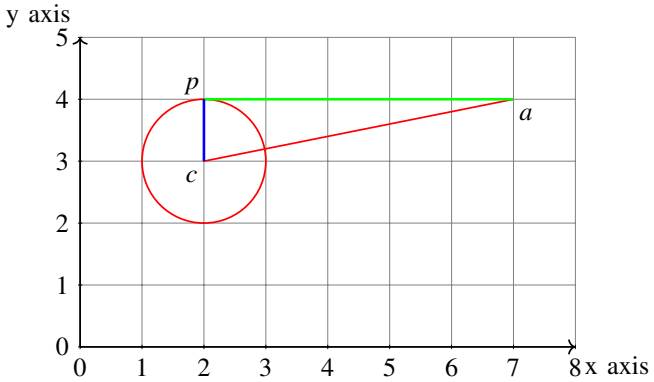

Fig. 0: Tangent to Circle

Then $\mathbf{p}-\mathbf{a}$ and $\mathbf{c}-\mathbf{p}$ are $\perp$, $\triangle cpa$ is a right angled triangle .

3 Solution

The equation of circle with center $\mathbf{c}$ can be expressed as

$$\mathbf{x}^T\mathbf{x} - 2\mathbf{c}^T\mathbf{x} + f = 0 \tag{3.0.1}$$

Comparing the equation (1.0.1) and (3.0.1) we get

$$\mathbf{c} = \begin{pmatrix}2\\3\end{pmatrix} \tag{3.0.2}$$

$$f = 12 \tag{3.0.3}$$

$$r = \sqrt{\|\mathbf{c}\|^2 - f} = \sqrt{13-12} = 1 \tag{3.0.4}$$

let $\mathbf{a} = \begin{pmatrix}7\\4\end{pmatrix}$, then

$$\|\mathbf{a}-\mathbf{c}\| = \sqrt{26} \tag{3.0.5}$$

from equation (3.0.4) and (3.0.5) By pythagoras theorem , length of tangent $\left\|\mathbf{a}-\mathbf{p}\right\|$ is

$$\left\|\mathbf{a}-\mathbf{p}\right\| = \sqrt{\|\mathbf{a}-\mathbf{c}\|^2 - \|\mathbf{r}\|^2} \tag{3.0.6}$$

$$\left\|\mathbf{a}-\mathbf{p}\right\| = \sqrt{26-1} \tag{3.0.7}$$

$$\left\|\mathbf{a}-\mathbf{p}\right\| = \sqrt{25} \tag{3.0.8}$$

$$\left\|\mathbf{a}-\mathbf{p}\right\| = 5 \tag{3.0.9}$$