Assignment 1

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***Abstract*—This document explains how to find a line perpendicular to 2 lines and passing through a point.**

Download the python code from

https://github.com/AddagallaSatyanarayana/AI5006 /tree/master/Assignment1

and latex-tikz codes from

https://github.com/AddagallaSatyanarayana/AI5006 /tree/master/Assignment1/Assignment1.tex

1 Problem

Find the vector equation of the line passing through the point $\begin{pmatrix} 1 \\ 2 \\ -4 \end{pmatrix}$ and perpendicular to the two lines $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$

2 Explanation

Equation of a line $\mathbf{l}$ passing through the point $\mathbf{a}$ and parallel to the line $\mathbf{n}$ is given by:

$$\mathbf{l} = \mathbf{a} + \lambda\mathbf{n} \tag{2.0.1}$$

where λ is some constant. Since the line passes through $\begin{pmatrix} 1 \\ 2 \\ -4 \end{pmatrix}$, $\mathbf{a}$ = (1 2 -4)

$$\mathbf{l_1} = \begin{pmatrix} 8 \\ -19 \\ 10 \end{pmatrix} + \lambda_1 \begin{pmatrix} 3 \\ -16 \\ 7 \end{pmatrix}; \tag{2.0.2}$$

$$\mathbf{l_2} = \begin{pmatrix} 15 \\ 29 \\ 5 \end{pmatrix} + \lambda_2 \begin{pmatrix} 3 \\ 8 \\ -5 \end{pmatrix} \tag{2.0.3}$$

3 Solution

Let $\mathbf{n}$ be the normal vector to both lines. If $\mathbf{m_1}$ and $\mathbf{m_2}$ are the direction vectors of the lines,then

$$\mathbf{m_1}^T\mathbf{n} = 0 \tag{3.0.1}$$

$$\mathbf{m_2}^T\mathbf{n} = 0 \tag{3.0.2}$$

Let A be a point on line l_1 and B be point on the line l_2.

The vector passing through the points A and B will be

$$\mathbf{A} - \mathbf{B} = \mathbf{x_1} - \mathbf{x_2} + \begin{pmatrix} \mathbf{m_1} & -\mathbf{m_2} \end{pmatrix} \begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix} \tag{3.0.3}$$

The vectors $\mathbf{m_1}$,$\mathbf{m_2}$ are both perpendicular to the line AB.So the dot product of $\mathbf{m_1}$,$\mathbf{m_2}$ with the line AB is zero.

The dot product of $\mathbf{m_1}$ with the line AB is

$$\mathbf{m_1}^T (\mathbf{A} - \mathbf{B}) = 0 \tag{3.0.4}$$

$$\mathbf{m_1^T}(\mathbf{x_1} - \mathbf{x_2}) + \mathbf{m_1^T}(\mathbf{m_1} - \mathbf{m_2}) \begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix} = 0 \tag{3.0.5}$$

The dot product of $\mathbf{m_2}$ with the line AB is

$$\mathbf{m_2}^T (\mathbf{A} - \mathbf{B}) = 0 \tag{3.0.6}$$

$$\mathbf{m_2^T}(\mathbf{x_1} - \mathbf{x_2}) + \mathbf{m_2^T}(\mathbf{m_1} - \mathbf{m_2}) \begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix} = 0 \tag{3.0.7}$$

Let the matrix $\mathbf{M}$ be

$$\mathbf{M} = \begin{pmatrix} \mathbf{m_1}^T \\ \mathbf{m_2}^T \end{pmatrix} \tag{3.0.8}$$

Combining the equations in matrix form,using equation we get

$$\mathbf{MM}^T \begin{pmatrix} \lambda_1 \\ -\lambda_2 \end{pmatrix} + \mathbf{M}(\mathbf{x_1} - \mathbf{x_2}) = 0 \tag{3.0.9}$$

$$\mathbf{MM}^T \begin{pmatrix} \lambda_1 \\ -\lambda_2 \end{pmatrix} = \mathbf{M}(\mathbf{x_2} - \mathbf{x_1}) \tag{3.0.10}$$

$$\begin{pmatrix} \mathbf{m_1}^T\mathbf{m_1} & \mathbf{m_1}^T\mathbf{m_2} \\ \mathbf{m_2}^T\mathbf{m_1} & \mathbf{m_2}^T\mathbf{m_2} \end{pmatrix} \begin{pmatrix} \lambda_1 \\ -\lambda_2 \end{pmatrix} = \begin{pmatrix} \mathbf{m_1}^T(\mathbf{x_2} - \mathbf{x_1}) \\ \mathbf{m_2}^T(\mathbf{x_2} - \mathbf{x_1}) \end{pmatrix} \tag{3.0.11}$$

$$\begin{pmatrix} \mathbf{m_1}^T\mathbf{m_1} & \mathbf{m_1}^T\mathbf{m_2} & \mathbf{m_1}^T(\mathbf{x_2} - \mathbf{x_1}) \\ \mathbf{m_2}^T\mathbf{m_1} & \mathbf{m_2}^T\mathbf{m_2} & \mathbf{m_2}^T(\mathbf{x_2} - \mathbf{x_1}) \end{pmatrix} \tag{3.0.12}$$

Given

$$\mathbf{x_1} = \begin{pmatrix} 8 \\ -19 \\ 10 \end{pmatrix}, \mathbf{x_2} = \begin{pmatrix} 15 \\ 29 \\ 5 \end{pmatrix}, \mathbf{m_1} = \begin{pmatrix} 3 \\ -16 \\ 7 \end{pmatrix}, \mathbf{m_2} = \begin{pmatrix} 3 \\ 8 \\ -5 \end{pmatrix} \tag{3.0.13}$$

The augmented matrix is

$$\begin{pmatrix} 314 & -154 & -782 \\ -154 & 98 & 430 \end{pmatrix} \tag{3.0.14}$$

$$\begin{pmatrix} 314 & -154 & -782 \\ -154 & 98 & 430 \end{pmatrix} \xleftrightarrow{R_1=\frac{R_1}{2},R_2=\frac{R_2}{2}} \begin{pmatrix} 157 & -77 & -391 \\ -77 & 49 & 215 \end{pmatrix} \tag{3.0.15}$$

$$\begin{pmatrix} 157 & -77 & -391 \\ -77 & 49 & 215 \end{pmatrix} \xleftrightarrow{R_1=R_2+\frac{R_1}{0.49}} \begin{pmatrix} 157 & -77 & -391 \\ 0 & 11.24 & 23.24 \end{pmatrix} \tag{3.0.16}$$

$$\begin{pmatrix} 157 & -77 & -391 \\ 0 & 11.24 & 23.24 \end{pmatrix} \xleftrightarrow{R_1=\frac{R_1}{6.85}+R_2} \begin{pmatrix} 22.92 & 0 & -33.84 \\ 0 & 11.24 & 23.24 \end{pmatrix} \tag{3.0.17}$$

$$\begin{pmatrix} 22.92 & 0 & -33.84 \\ 0 & 11.24 & 23.24 \end{pmatrix} \xleftrightarrow{R_1=\frac{R_1}{22.92},R_2=\frac{R_1}{11.24}} \begin{pmatrix} 1 & 0 & -1.48 \\ 0 & 1 & 2.07 \end{pmatrix} \tag{3.0.18}$$

$$\lambda_1 = -1.48, \lambda_2 = 2.07 \tag{3.0.19}$$

Using the equation (2.0.2) and (2.0.3), we get the points as $A = \begin{pmatrix} 3.56 \\ 4.68 \\ -0.36 \end{pmatrix}$ and $B = \begin{pmatrix} 8.79 \\ 12.44 \\ 15.35 \end{pmatrix}$ on the line l_1,l_2 respectively.

Direction of line AB $= \begin{pmatrix} 5.22449 \\ 7.83673 \\ 15.6735 \end{pmatrix}$

Unit vector along AB

$$\mathbf{n} = \frac{\mathbf{A} - \mathbf{B}}{\|\mathbf{A} - \mathbf{B}\|} \tag{3.0.20}$$

Unit vector of AB $= \begin{pmatrix} 0.285714 \\ 0.428571 \\ 0.857143 \end{pmatrix}$

The equation of required line from equation (2.0.1)

$$\mathbf{l_1} = \begin{pmatrix} 1 \\ 2 \\ -4 \end{pmatrix} + \lambda \begin{pmatrix} 0.285714 \\ 0.428571 \\ 0.857143 \end{pmatrix} \tag{3.0.21}$$

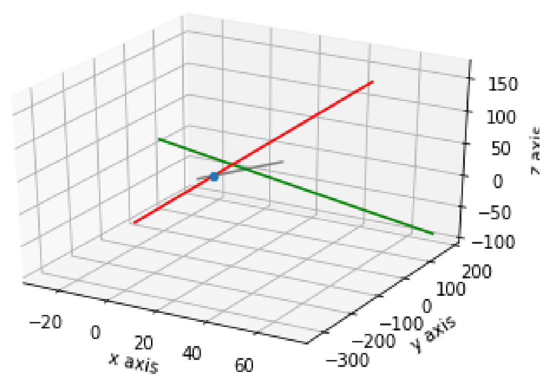

Fig. 0: Perpendicular Line