Assignment 3

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***Abstract*—This document uses the properties of a parallelogram to prove a statement**

Download latex-tikz codes from

https://github.com/AddagallaSatyanarayana/AI5006/tree/master/Assignment3/Assignment3.tex

1 Problem

Prove that the sum of the squares of diagonals of parallelogram is equal to the sum of the squares of its sides.

2 Explanation

Given a parallelogram ABCD we have to prove that

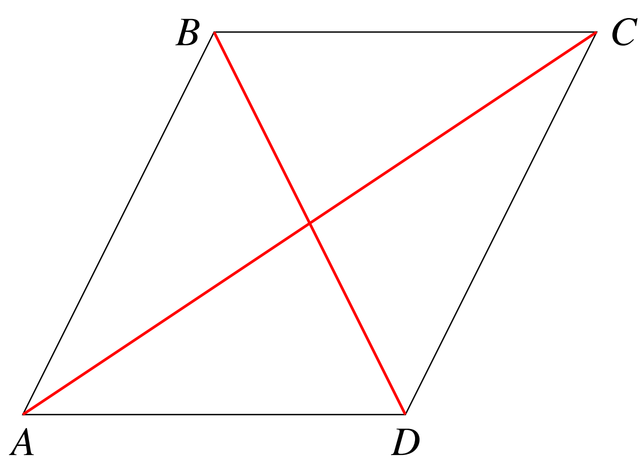

Fig. 0: parallelogram $ABCD$

$$\left\|\mathbf{A}-\mathbf{C}\right\|^2+\|\mathbf{B}-\mathbf{D}\|^2=\|\mathbf{A}-\mathbf{B}\|^2+\left\|\mathbf{B}-\mathbf{C}\right\|^2+\left\|\mathbf{D}-\mathbf{C}\right\|^2+\|\mathbf{A}-\mathbf{D}\|^2 \tag{2.0.1}$$

3 Solution

The diagonals of parallelogram are

$$\mathbf{A}-\mathbf{C}=(\mathbf{A}-\mathbf{D})+(\mathbf{D}-\mathbf{C}) \tag{3.0.1}$$

$$\mathbf{B}-\mathbf{D}=(\mathbf{A}-\mathbf{D})-(\mathbf{A}-\mathbf{B}) \tag{3.0.2}$$

The sum of the squares of diagonals is

$$\left\|\mathbf{A}-\mathbf{C}\right\|^2+\|\mathbf{B}-\mathbf{D}\|^2=\left\|(\mathbf{A}-\mathbf{D})+(\mathbf{D}-\mathbf{C})\right\|^2+\left\|(\mathbf{A}-\mathbf{D})-(\mathbf{A}-\mathbf{B})\right\|^2 \tag{3.0.3}$$

$$=\|\mathbf{A}-\mathbf{D}\|^2+\left\|\mathbf{D}-\mathbf{C}\right\|^2+2(\mathbf{A}-\mathbf{D})^T(\mathbf{D}-\mathbf{C})+\|\mathbf{A}-\mathbf{D}\|^2+\|\mathbf{A}-\mathbf{B}\|^2-2(\mathbf{A}-\mathbf{D})^T(\mathbf{A}-\mathbf{B}) \tag{3.0.4}$$

$$=\|\mathbf{A}-\mathbf{D}\|^2+\left\|\mathbf{D}-\mathbf{C}\right\|^2+2\|\mathbf{A}-\mathbf{D}\|\left\|\mathbf{D}-\mathbf{C}\right\|\cos(180^\circ-\angle ADC)+\|\mathbf{A}-\mathbf{D}\|^2+\|\mathbf{A}-\mathbf{B}\|^2-2\|\mathbf{A}-\mathbf{D}\|\|\mathbf{A}-\mathbf{B}\|\cos\angle DAB \tag{3.0.5}$$

In the parallelogram $ABCD$

$$\|\mathbf{A}-\mathbf{D}\|=\left\|\mathbf{B}-\mathbf{C}\right\| \tag{3.0.6}$$

$$\|\mathbf{A}-\mathbf{B}\|=\left\|\mathbf{D}-\mathbf{C}\right\| \tag{3.0.7}$$

$$\angle ADC+\angle DAB=180^\circ \tag{3.0.8}$$

$$\angle DAB=180^\circ-\angle ADC \tag{3.0.9}$$

From equation (3.0.5),(3.0.6),(3.0.7) and (3.0.9)

$$=\left\|\mathbf{B}-\mathbf{C}\right\|^2+\left\|\mathbf{D}-\mathbf{C}\right\|^2+2\|\mathbf{A}-\mathbf{D}\|\|\mathbf{A}-\mathbf{B}\|\cos\angle DAB+\|\mathbf{A}-\mathbf{D}\|^2+\|\mathbf{A}-\mathbf{B}\|^2-2\|\mathbf{A}-\mathbf{D}\|\|\mathbf{A}-\mathbf{B}\|\cos\angle DAB \tag{3.0.10}$$

Simplifying equation (3.0.10)

$$\left\|\mathbf{A}-\mathbf{C}\right\|^2+\|\mathbf{B}-\mathbf{D}\|^2=\|\mathbf{A}-\mathbf{B}\|^2+\left\|\mathbf{B}-\mathbf{C}\right\|^2+\left\|\mathbf{D}-\mathbf{C}\right\|^2+\|\mathbf{A}-\mathbf{D}\|^2 \tag{3.0.11}$$

The equations (2.0.1) and (3.0.11) are equal,hence proved