

Assignment 4

Addagalla Satyanarayana

Abstract—This document uses the properties of a tangent to a circle

Download latex-tikz codes from

<https://github.com/AddagallaSatyanarayana/AI5006/tree/master/Assignment4/Assignment4.tex>

1 PROBLEM

Find the length of the tangent from the point $\begin{pmatrix} 7 \\ 4 \end{pmatrix}$ to the circle

$$\mathbf{x}^T \mathbf{x} - (4 \ 6) \mathbf{x} + 12 = 0 \quad (1.0.1)$$

2 EXPLANATION

The general equation of a second degree can be expressed as :

$$\mathbf{x}^T \mathbf{V} \mathbf{x} + 2\mathbf{u}^T \mathbf{x} + f = 0 \quad (2.0.1)$$

Let the point of intersection of the tangent and the circle be denoted by \mathbf{p} as shown in figure below.

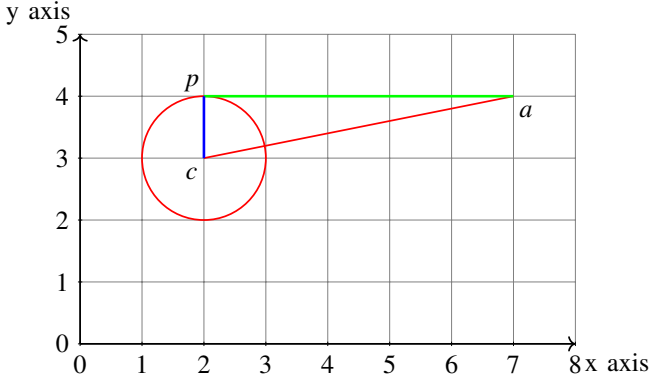


Fig. 0: Tangent to Circle

3 SOLUTION

We know that, for a circle,

$$\mathbf{V} = \mathbf{I} \quad (3.0.1)$$

$$\mathbf{c} = -\mathbf{u} \quad (3.0.2)$$

Comparing the equation (1.0.1) and (2.0.1) we get

$$\mathbf{u} = \begin{pmatrix} -2 \\ -3 \end{pmatrix}, f = 12 \quad (3.0.3)$$

$$\mathbf{c} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \quad (3.0.4)$$

$$radius = \sqrt{\mathbf{u}^T \mathbf{u} - f} \quad (3.0.5)$$

$$radius = \sqrt{\|\mathbf{u}\|^2 - f} = 1 \quad (3.0.6)$$

let $\mathbf{a} = \begin{pmatrix} 7 \\ 4 \end{pmatrix}$, then

$$\|\mathbf{a} - \mathbf{c}\| = \sqrt{26} \quad (3.0.7)$$

from equation (3.0.6) and (3.0.7) By pythagoras theorem , length of tangent $\|\mathbf{a} - \mathbf{p}\|$ is

$$\|\mathbf{a} - \mathbf{p}\| = \sqrt{\|\mathbf{a} - \mathbf{c}\|^2 - \|\mathbf{r}\|^2} \quad (3.0.8)$$

$$\|\mathbf{a} - \mathbf{p}\| = \sqrt{26 - 1} \quad (3.0.9)$$

$$\|\mathbf{a} - \mathbf{p}\| = \sqrt{25} \quad (3.0.10)$$

$$\|\mathbf{a} - \mathbf{p}\| = 5 \quad (3.0.11)$$