AI5006 - Assignment 1

Addagalla Satyanarayana - AIRESCH14003

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Question :

Find the vector equation of the line passing through the point $\begin{pmatrix} 1 \\ 2 \\ -4 \end{pmatrix}$ and perpendicular to the two lines
$\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ and

$\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{5}$

Solution :

Equation of a $\vec{\mathbf{l}}$ passing through $\vec{\mathbf{a}}$ and parallel to $\vec{\mathbf{n}}$ is given by:

$\vec{\mathbf{l}} = \vec{\mathbf{a}} + L * \vec{\mathbf{n}}$, where L is some constant
Since the line passes through $\begin{pmatrix} 1 \\ 2 \\ -4 \end{pmatrix}$
$\vec{\mathbf{a}} = (i + 2j - 4k)$
Let $\vec{\mathbf{n}}$ be the normal vector to both lines. If $\vec{\mathbf{m_1}}$ and $\vec{\mathbf{m_2}}$ are the direction vectors of the lines,then
$\vec{\mathbf{m_1}}^T \vec{\mathbf{n}} = 0$
$\vec{\mathbf{m_2}}^T \vec{\mathbf{n}} = 0$
Let $\vec{\mathbf{n}} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ $\vec{\mathbf{m_1}} = \begin{pmatrix} 3 \\ -16 \\ 7 \end{pmatrix}$ $\vec{\mathbf{m_2}} = \begin{pmatrix} 3 \\ 8 \\ -5 \end{pmatrix}$

Since $\vec{\mathbf{n}}$ is perpendicular to $\vec{\mathbf{m_1}}$ and $\vec{\mathbf{m_2}}$
$3x - 16y + 7z = 0$
$3x + 8y - 5z = 0$

Solving the equations $\frac{x}{2} = \frac{y}{3} = \frac{z}{6} = K$
$x = 2K, y = 3K, z = 6K$
$\vec{\mathbf{n}} = K * (2i + 3j + 6k)$
so the equation of $\vec{l}$ is
$\vec{\mathbf{l}} = (i + 2j - 4k) + L * K(2i + 3j + 6k)$,where L*K is any constant

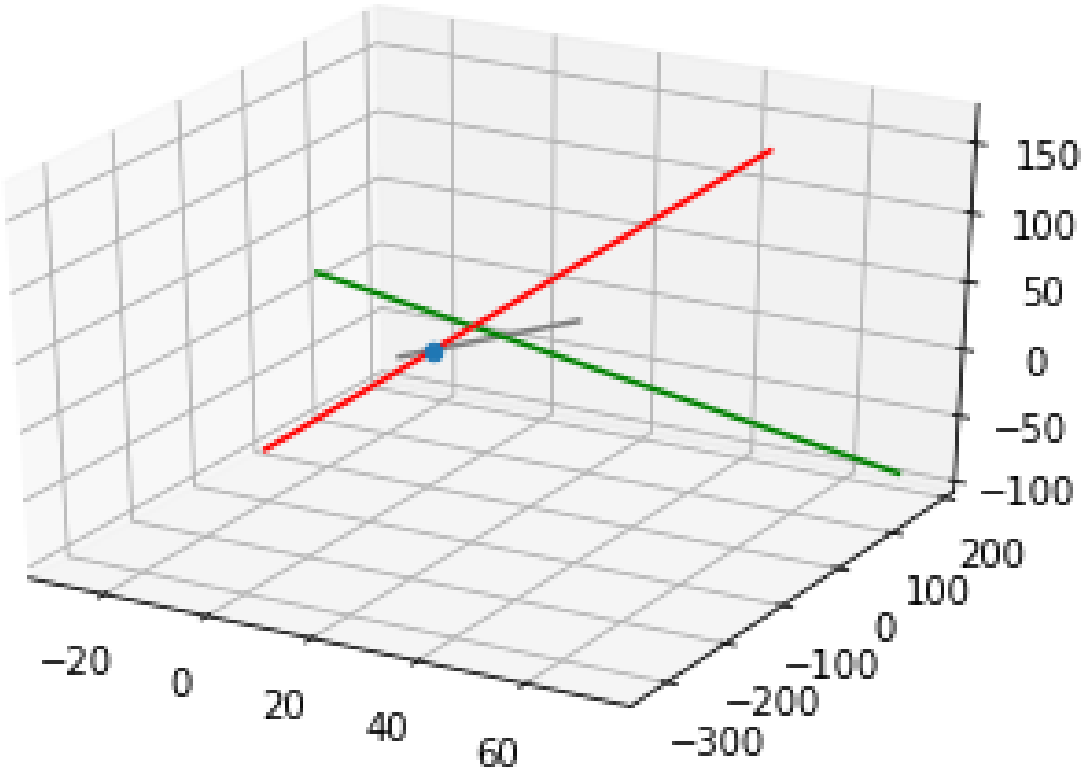

Figure 1: perpendicular