Assignment 3

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***Abstract*—This document uses the properties of a parallelogram to prove a statement**

Download latex-tikz codes from

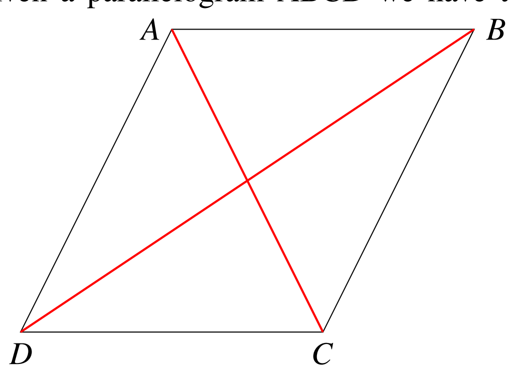
https://github.com/AddagallaSatyanarayana/AI5006/tree/master/Assignment3/assignment3.tex

1 Problem

Prove that the sum of the squares of the diagonals of parallelogram is equal to the sum of the squares of its sides.

2 Explanation

Given a parallelogram ABCD we have to prove that

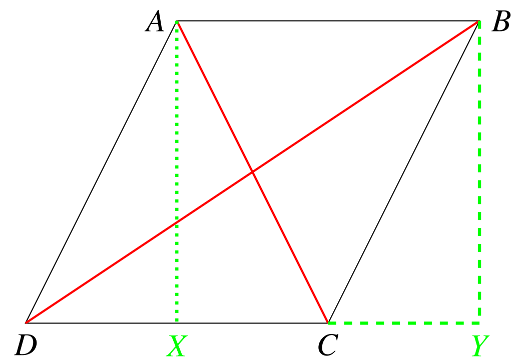


$$\left\|\mathbf{A}-\mathbf{C}\right\|^2+\|\mathbf{B}-\mathbf{D}\|^2 = \|\mathbf{A}-\mathbf{B}\|^2+\left\|\mathbf{B}-\mathbf{C}\right\|^2+\left\|\mathbf{D}-\mathbf{C}\right\|^2+\|\mathbf{A}-\mathbf{D}\|^2 \tag{2.0.1}$$

In the parallelogram $ABCD$

$$\|\mathbf{A}-\mathbf{B}\| = \left\|\mathbf{D}-\mathbf{C}\right\| \tag{2.0.2}$$
$$\|\mathbf{A}-\mathbf{D}\| = \left\|\mathbf{B}-\mathbf{C}\right\| \tag{2.0.3}$$

Draw perpendiculars from A to $\left\|\mathbf{D}-\mathbf{C}\right\|$ and B to $\left\|\mathbf{D}-\mathbf{C}\right\|$ extended as shown



3 Solution

From $\triangle AXD$ and $\triangle BYD$

$$\left\|\mathbf{A}-\mathbf{C}\right\|^2 = \|\mathbf{A}-\mathbf{X}\|^2+\left\|\mathbf{C}-\mathbf{X}\right\|^2 \tag{3.0.1}$$
$$\|\mathbf{B}-\mathbf{D}\|^2 = \|\mathbf{B}-\mathbf{Y}\|^2+\|\mathbf{D}-\mathbf{Y}\|^2 \tag{3.0.2}$$

From equation (3.0.1)

$$\left\|\mathbf{A}-\mathbf{C}\right\|^2 = \|\mathbf{A}-\mathbf{X}\|^2 + \left(\left\|\mathbf{D}-\mathbf{C}\right\| - \|\mathbf{D}-\mathbf{X}\|\right)^2 \tag{3.0.3}$$

$$\left\|\mathbf{A}-\mathbf{C}\right\|^2 = \|\mathbf{A}-\mathbf{X}\|^2+\left\|\mathbf{D}-\mathbf{C}\right\|^2+\|\mathbf{D}-\mathbf{X}\|^2 - 2\left\|\mathbf{D}-\mathbf{C}\right\|.\|\mathbf{D}-\mathbf{X}\| \tag{3.0.4}$$

$$\left\|\mathbf{A}-\mathbf{C}\right\|^2 = (\|\mathbf{A}-\mathbf{X}\|^2+\|\mathbf{D}-\mathbf{X}\|^2)+\left\|\mathbf{D}-\mathbf{C}\right\|^2 - 2\left\|\mathbf{D}-\mathbf{C}\right\|.\|\mathbf{D}-\mathbf{X}\| \tag{3.0.5}$$

$$\left\|\mathbf{A}-\mathbf{C}\right\|^2 = \|\mathbf{A}-\mathbf{D}\|^2+\left\|\mathbf{C}-\mathbf{D}\right\|^2 - 2\left\|\mathbf{D}-\mathbf{C}\right\|.\|\mathbf{D}-\mathbf{X}\| \tag{3.0.6}$$

From equation (3.0.2)

$$\|\mathbf{B}-\mathbf{D}\|^2 = \|\mathbf{B}-\mathbf{Y}\|^2 + \left(\left\|\mathbf{D}-\mathbf{C}\right\| + \left\|\mathbf{C}-\mathbf{Y}\right\|\right)^2 \tag{3.0.7}$$

$$\|\mathbf{B}-\mathbf{D}\|^2 = \|\mathbf{B}-\mathbf{Y}\|^2+\left\|\mathbf{D}-\mathbf{C}\right\|^2+\left\|\mathbf{C}-\mathbf{Y}\right\|^2 + 2\left\|\mathbf{D}-\mathbf{C}\right\|.\left\|\mathbf{C}-\mathbf{Y}\right\| \tag{3.0.8}$$

$$\|\mathbf{B}-\mathbf{D}\|^2 = (\|\mathbf{B}-\mathbf{Y}\|^2 + \left\|\mathbf{C}-\mathbf{Y}\right\|^2) + \left\|\mathbf{D}-\mathbf{C}\right\|^2 + \\ + 2\left\|\mathbf{D}-\mathbf{C}\right\|.\left\|\mathbf{C}-\mathbf{Y}\right\| \quad (3.0.9)$$

$$\|\mathbf{B}-\mathbf{D}\|^2 = \left\|\mathbf{B}-\mathbf{C}\right\|^2 + \left\|\mathbf{D}-\mathbf{C}\right\|^2 + 2\left\|\mathbf{D}-\mathbf{C}\right\|.\left\|\mathbf{C}-\mathbf{Y}\right\| \quad (3.0.10)$$

In $\triangle AXD$ and $\triangle BYC$

$$\|\mathbf{A}-\mathbf{X}\| = \|\mathbf{B}-\mathbf{Y}\| \quad (3.0.11)$$

$$\angle AXD = \angle BYC \quad (3.0.12)$$

$$\|\mathbf{A}-\mathbf{D}\| = \left\|\mathbf{B}-\mathbf{C}\right\| \quad (3.0.13)$$

Therefore by RHS Congruency

$$\|\mathbf{D}-\mathbf{X}\| = \left\|\mathbf{C}-\mathbf{Y}\right\| \quad (3.0.14)$$

Substituting the value of DX from (3.0.14) to equation (3.0.6)

$$\left\|\mathbf{A}-\mathbf{C}\right\|^2 = \|\mathbf{A}-\mathbf{D}\|^2 + \left\|\mathbf{D}-\mathbf{C}\right\|^2 - 2\left\|\mathbf{D}-\mathbf{C}\right\|.\left\|\mathbf{C}-\mathbf{Y}\right\| \quad (3.0.15)$$

Combining equation (3.0.15) and (3.0.10) and simplifying

$$\left\|\mathbf{A}-\mathbf{C}\right\|^2 + \|\mathbf{B}-\mathbf{D}\|^2 = \|\mathbf{A}-\mathbf{D}\|^2 + \left\|\mathbf{D}-\mathbf{C}\right\|^2 + \\ \left\|\mathbf{B}-\mathbf{C}\right\|^2 + \left\|\mathbf{D}-\mathbf{C}\right\|^2 \quad (3.0.16)$$

From equation (2.0.2) $\|\mathbf{A}-\mathbf{B}\| = \left\|\mathbf{D}-\mathbf{C}\right\|$

$$\left\|\mathbf{A}-\mathbf{C}\right\|^2 + \|\mathbf{B}-\mathbf{D}\|^2 = \\ \|\mathbf{A}-\mathbf{B}\|^2 + \left\|\mathbf{B}-\mathbf{C}\right\|^2 + \left\|\mathbf{D}-\mathbf{C}\right\|^2 + \|\mathbf{A}-\mathbf{D}\|^2 \quad (3.0.17)$$