Assignment 9

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***Abstract*—This document shows the concept of markov chain state and tranistion matrices**

https://github.com/AddagallaSatyanarayana/AI5106
/tree/master/Assignment9/Assignment9.tex

1 Problem

Consider a Markov chain with state space {0,1,2,3,4} and transition matrix

$$\mathbf{P} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \tag{1.0.1}$$

Draw the Markov chain and obtain the stationary probabilities

2 Explanation

The Markov chain can be constructed from the transition matrix $\mathbf{P}$ where the states represent the nodes of directed graph and the weights of the edges represent the transition probabilities. For a Markov chain with transition matrix P,a vector $\mathbf{v}$ is called a stationary distribution for $\mathbf{P}$ if and only if

$$\mathbf{vP} = \mathbf{v} \tag{2.0.1}$$

3 Solution

The Markov chain state can be drawn as shown below. The markov chain has

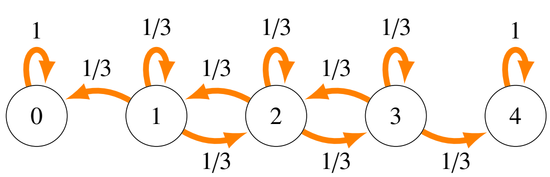

Fig. 0: Markov state diagram

multiple communicating classes namely {0} , {4} which are recurrent(ie remain in the same state) and {1,2,3} which is transient. It can observed that the equation (2.0.1) is similar to the equation $\mathbf{vP} = \lambda\mathbf{v}$ for eigenvalues and eigenvectors, with $\lambda = 1$. Taking the transpose of (2.0.1) we get

$$(\mathbf{vP})^T = \mathbf{v}^T \tag{3.0.1}$$

$$\mathbf{P}^T\mathbf{v}^T = \mathbf{v}^T \tag{3.0.2}$$

The eigenvectors corresponding to the eigenvalue $\lambda = 1$ of the transposed transition matrix $\mathbf{P}^T$ are the stationary distributions of the chain. When there are multiple eigenvectors associated to an eigenvalue of $\lambda = 1$, each such eigenvector is the stationary distribution.

$$\mathbf{P} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \tag{3.0.3}$$

$$\mathbf{P}^T = \begin{pmatrix} 1 & \frac{1}{3} & 0 & 0 & 0 \\ 0 & \frac{1}{3} & \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} & 0 \\ 0 & 0 & 0 & \frac{1}{3} & 1 \end{pmatrix} \tag{3.0.4}$$

The eigen values can be calculated using

$$\left|\mathbf{P}^T - \lambda\mathbf{I}\right| = 0 \tag{3.0.5}$$

$$\begin{vmatrix} 1-\lambda & \frac{1}{3} & 0 & 0 & 0 \\ 0 & \frac{1}{3}-\lambda & \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & \frac{1}{3}-\lambda & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} & \frac{1}{3}-\lambda & 0 \\ 0 & 0 & 0 & \frac{1}{3} & 1-\lambda \end{vmatrix} = 0 \tag{3.0.6}$$

The eigen vectors corresponding to the eigen value $\lambda = 1$ are

$$\mathbf{v}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{3.0.7}$$

$$\mathbf{v}_2 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \tag{3.0.8}$$

The (3.0.7) and (3.0.8) represent the stationary probabilities.