Assignment 6

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***Abstract*—This document uses the properties of a tangent to a circle**

Download latex-tikz codes from

https://github.com/AddagallaSatyanarayana/AI5106/tree/master/Assignment6/Assignment6.tex

1 Problem

Find the length of the tangent from the point $\begin{pmatrix}7\\4\end{pmatrix}$ to the circle

$$\mathbf{x}^T\mathbf{x} - \begin{pmatrix}4 & 6\end{pmatrix}\mathbf{x} + 12 = 0 \tag{1.0.1}$$

2 Explanation

The general equation of a second degree can be expressed as :

$$\mathbf{x}^T\mathbf{V}\mathbf{x} + 2\mathbf{u}^T\mathbf{x} + f = 0 \tag{2.0.1}$$

Let the equation of the tangent be

$$\begin{pmatrix}-m & 1\end{pmatrix}\mathbf{x} = c \tag{2.0.2}$$

3 Solution

We know that, for a circle,

$$\mathbf{V} = \mathbf{I} \tag{3.0.1}$$

$$\mathbf{c} = -\mathbf{u} \tag{3.0.2}$$

Comparing the equation (1.0.1) and (2.0.1) we get

$$\mathbf{u} = \begin{pmatrix}-2\\-3\end{pmatrix}, f = 12 \tag{3.0.3}$$

$$\mathbf{c} = \begin{pmatrix}2\\3\end{pmatrix} \tag{3.0.4}$$

The normal vector to the line is obtained as

$$\mathbf{n} = \mathbf{q} + \mathbf{u} \tag{3.0.5}$$

$$\mathbf{q} = \mathbf{n} - \mathbf{u} \tag{3.0.6}$$

Comparing the equation (2.0.2)

$$\mathbf{n} = \begin{pmatrix}-m & 1\end{pmatrix}^T \tag{3.0.7}$$

Given

$$\mathbf{u} = \begin{pmatrix}-2\\-3\end{pmatrix} \tag{3.0.8}$$

$$\implies \mathbf{q} = \begin{pmatrix}-m+2\\4\end{pmatrix} \tag{3.0.9}$$

The point q also satisfies the equation of the circle at (2.0.1)

$$\mathbf{q}^T\mathbf{q} + 2\mathbf{u}^T\mathbf{q} + f = 0 \tag{3.0.10}$$

$$(\mathbf{n} - \mathbf{u})^T(\mathbf{n} - \mathbf{u}) + 2\mathbf{u}^T(\mathbf{n} - \mathbf{u}) + f = 0 \tag{3.0.11}$$

$$\|\mathbf{n}\|^2 - \mathbf{n}^T\mathbf{u} - \mathbf{u}^T\mathbf{n} + \|\mathbf{u}\|^2 + 2\mathbf{u}^T\mathbf{n} - 2\|\mathbf{u}\|^2 + f = 0 \tag{3.0.12}$$

$$\|\mathbf{n}\|^2 - \|\mathbf{u}\|^2 + f = 0 \tag{3.0.13}$$

$$m^2 + 1 - 13 + 12 = 0 \tag{3.0.14}$$

$$m^2 = 0 \tag{3.0.15}$$

$$m = 0 \tag{3.0.16}$$

Simplying (3.0.16) and (3.0.9) we get

$$\mathbf{q} = \begin{pmatrix}2\\4\end{pmatrix} \tag{3.0.17}$$

Let $\mathbf{p} = \begin{pmatrix}7\\4\end{pmatrix}$ The length of tangent is

$$\left\|p - q\right\| = \sqrt{(7-2)^2 + (4-4)^2} \tag{3.0.18}$$

$$= \sqrt{25} \tag{3.0.19}$$

$$= 5 \tag{3.0.20}$$

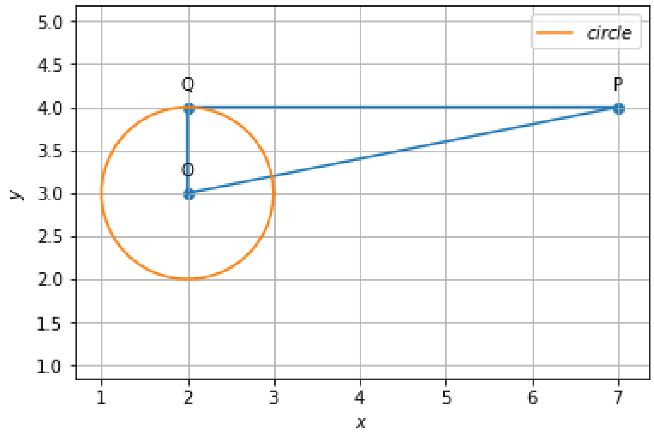

Fig. 0: Perpendicular Line