Assignment 8

Addagalla Satyanarayana

***Abstract*—This document shows the diagonalization of matrices**

https://github.com/AddagallaSatyanarayana/AI5106/tree/master/Assignment8/Assignment8.tex

1 Problem

Which of the following matrices are not diagonalizable over R

$$\mathbf{M_1} = \begin{pmatrix} 2 & 0 & 1 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{pmatrix} \tag{1.0.1}$$

$$\mathbf{M_2} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \tag{1.0.2}$$

$$\mathbf{M_3} = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix} \tag{1.0.3}$$

$$\mathbf{M_4} = \begin{pmatrix} 1 & -1 \\ 2 & 4 \end{pmatrix} \tag{1.0.4}$$

2 Explanation

A matrix $\mathbf{M}$ can be diagonalized if there exists an invertible matrix $\mathbf{P}$ such that

$$\mathbf{M} = \mathbf{P}\mathbf{D}\mathbf{P}^{-1} \tag{2.0.1}$$

The matrix $\mathbf{P}$ is equal to the eigen vectors of matrix $\mathbf{M}$,where eigen values can be calculated using

$$\left|\mathbf{M} - \lambda\mathbf{I}\right| = 0 \tag{2.0.2}$$

2.1 Solving matrix1

$$\begin{pmatrix} 2 & 0 & 1 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{pmatrix} \tag{2.1.1}$$

The eigen values for the matrix $\mathbf{M}$ can be found using (2.0.2)

$$\begin{vmatrix} 2-\lambda & 0 & 1 \\ 0 & 3-\lambda & 0 \\ 0 & 0 & 2-\lambda \end{vmatrix} = 0 \tag{2.1.2}$$

Solving this we get the eigen values of $\mathbf{M}$ as,

$$\lambda_1 = 2 \tag{2.1.3}$$

$$\lambda_2 = 3 \tag{2.1.4}$$

The corresponding eigen vectors are:

$$\mathbf{v}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}; \mathbf{v}_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \tag{2.1.5}$$

For diagonalization the number of eigen vectors should be equal to the matrix dimensions. Since there are only 2 eigen vectors and the dimension of $\mathbf{M}$ is 3 , $\mathbf{M}$ is not diagonalizable.

2.2 Solving matrix2

$$\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \tag{2.2.1}$$

The eigen values for the matrix $\mathbf{M}$ can be found using (2.0.2)

$$\begin{vmatrix} 1-\lambda & 1 \\ 1 & 1-\lambda \end{vmatrix} = 0 \tag{2.2.2}$$

Solving this we get the eigen values of $\mathbf{M}$ as,

$$\lambda_1 = 0 \tag{2.2.3}$$

$$\lambda_2 = 2 \tag{2.2.4}$$

The corresponding eigen vectors are:

$$\mathbf{v}_1 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}; \mathbf{v}_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{2.2.5}$$

The matrix $\mathbf{P}$ is equal to

$$\mathbf{P} = \begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix} \tag{2.2.6}$$

The diagonal matrix $\mathbf{D}$ can be expressed using the eigen values in the form:

$$\mathbf{D} = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \tag{2.2.7}$$

$$\mathbf{D} = \begin{pmatrix} 0 & 0 \\ 0 & 2 \end{pmatrix} \tag{2.2.8}$$

The matrix $\mathbf{P}^{-1}$ can be calculated using (2.2.8)

$$\mathbf{P}^{-1} = \begin{pmatrix} -0.5 & 0.5 \\ 0.5 & 0.5 \end{pmatrix} \tag{2.2.9}$$

Using (2.2.6),(2.2.8) , (2.2.9) and (2.0.1)

$$\mathbf{P}\mathbf{D}\mathbf{P}^{-1} = \begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 0 & 0 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} -0.5 & 0.5 \\ 0.5 & 0.5 \end{pmatrix} \tag{2.2.10}$$

$$\mathbf{M} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \tag{2.2.11}$$

From equation (2.2.11) and equation (2.2.1) we get,$\mathbf{M} = \mathbf{PDP}^{-1}$.Hence the matrix is diagonalizable.

2.3 Solving matrix3

$$\begin{pmatrix} 2 & 1 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix} \tag{2.3.1}$$

The eigen values for the matrix $\mathbf{M}$ can be found (2.0.2)

$$\begin{vmatrix} 2-\lambda & 1 & 0 \\ 0 & 3-\lambda & 0 \\ 0 & 0 & 3-\lambda \end{vmatrix} = 0 \tag{2.3.2}$$

Solving this we get the eigen values of $\mathbf{M}$ as,

$$\lambda_1 = 2 \tag{2.3.3}$$
$$\lambda_2 = 3 \tag{2.3.4}$$

The corresponding eigen vectors are:

$$\mathbf{v}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}; \mathbf{v}_2 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}; \mathbf{v}_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \tag{2.3.5}$$

The matrix $\mathbf{P}$ is equal to

$$\mathbf{P} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{2.3.6}$$

The diagonal matrix $\mathbf{D}$ can be expressed using the eigen values in the form:

$$\mathbf{D} = \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix} \tag{2.3.7}$$
$$\mathbf{D} = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{pmatrix} \tag{2.3.8}$$

The inverse matrix $\mathbf{P}^{-1}$ can be calculated using (2.3.8)

$$\mathbf{P}^{-1} = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{2.3.9}$$

Using (2.3.6),(2.3.8) , (2.3.9) and (2.0.1)

$$\mathbf{PDP}^{-1} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{pmatrix}\begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{2.3.10}$$
$$\mathbf{M} = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix} \tag{2.3.11}$$

From equation (2.3.11) and equation (2.3.1),$\mathbf{M} = \mathbf{PDP}^{-1}$.Hence the matrix is diagonalizable.

2.4 Solving matrix4

$$\begin{pmatrix} 1 & -1 \\ 2 & 4 \end{pmatrix} \tag{2.4.1}$$

The eigen values for the matrix $\mathbf{M}$ can be found using (2.0.2)

$$\begin{vmatrix} 1-\lambda & -1 \\ 2 & 4-\lambda \end{vmatrix} = 0 \tag{2.4.2}$$

Solving this we get the eigen values of $\mathbf{M}$ as,

$$\lambda_1 = 2 \tag{2.4.3}$$
$$\lambda_2 = 3 \tag{2.4.4}$$

The corresponding eigen vectors are:

$$\mathbf{v}_1 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}; \mathbf{v}_2 = \begin{pmatrix} -0.5 \\ 1 \end{pmatrix} \tag{2.4.5}$$

The matrix $\mathbf{P}$ is equal to

$$\mathbf{P} = \begin{pmatrix} -1 & -0.5 \\ 1 & 1 \end{pmatrix} \tag{2.4.6}$$

The diagonal matrix $\mathbf{D}$ can be expressed using the eigen values in the form:

$$\mathbf{D} = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \tag{2.4.7}$$
$$\mathbf{D} = \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix} \tag{2.4.8}$$

The inverse matrix $\mathbf{P}^{-1}$ can be calculated using (2.4.8)

$$\mathbf{P}^{-1} = \begin{pmatrix} -2 & -1 \\ 2 & 2 \end{pmatrix} \tag{2.4.9}$$

Using (2.4.6),(2.4.8) , (2.4.9) and (2.0.1)

$$\mathbf{PDP}^{-1} = \begin{pmatrix} -1 & -0.5 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 0 & 0 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}\begin{pmatrix} -2 & -1 \\ 2 & 2 \end{pmatrix} \tag{2.4.10}$$
$$\mathbf{M} = \begin{pmatrix} 1 & -1 \\ 2 & 4 \end{pmatrix} \tag{2.4.11}$$

From equation (2.4.11) and equation (2.4.1),$\mathbf{M} = \mathbf{PDP}^{-1}$.Hence the matrix is diagonalizable.

2.5 Conclusion:

$$\mathbf{M_1} = \begin{pmatrix} 2 & 0 & 1 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

is the matrix which is not diagonalizable.