

Assignment 6

Addagalla Satyanarayana

Abstract—This document uses the properties of a tangent to a circle

Download latex-tikz codes from

<https://github.com/AddagallaSatyanarayana/AI5106/tree/master/Assignment6/Assignment6.tex>

1 PROBLEM

Find the length of the tangent from the point $\left(\frac{7}{4}\right)$ to the circle

$$\mathbf{x}^T \mathbf{x} - (4 \ 6) \mathbf{x} + 12 = 0 \quad (1.0.1)$$

2 EXPLANATION

The general equation of a second degree can be expressed as :

$$\mathbf{x}^T \mathbf{V} \mathbf{x} + 2\mathbf{u}^T \mathbf{x} + f = 0 \quad (2.0.1)$$

Let the equation of the tangent be

$$(-m \ 1) \mathbf{x} = c \quad (2.0.2)$$

3 SOLUTION

We know that, for a circle,

$$\mathbf{V} = \mathbf{I} \quad (3.0.1)$$

$$\mathbf{c} = -\mathbf{u} \quad (3.0.2)$$

Comparing the equation (1.0.1) and (2.0.1) we get

$$\mathbf{u} = \begin{pmatrix} -2 \\ -3 \end{pmatrix}, f = 12 \quad (3.0.3)$$

$$\mathbf{c} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \quad (3.0.4)$$

The normal vector to the line is obtained as

$$\mathbf{n} = \mathbf{q} + \mathbf{u} \quad (3.0.5)$$

$$\mathbf{q} = \mathbf{n} - \mathbf{u} \quad (3.0.6)$$

Comparing the equation (2.0.2)

$$\mathbf{n} = (-m \ 1)^T \quad (3.0.7)$$

Given

$$\mathbf{u} = \begin{pmatrix} -2 \\ -3 \end{pmatrix} \quad (3.0.8)$$

$$\Rightarrow \mathbf{q} = \begin{pmatrix} -m+2 \\ 4 \end{pmatrix} \quad (3.0.9)$$

The point q also satisfies the equation of the circle at (1.0.1)

$$\mathbf{q}^T \mathbf{q} + 2\mathbf{u}^T \mathbf{q} + f = 0 \quad (3.0.10)$$

$$(\mathbf{n} - \mathbf{u})^T (\mathbf{n} - \mathbf{u}) + 2\mathbf{u}^T (\mathbf{n} - \mathbf{u}) + f = 0 \quad (3.0.11)$$

$$\|\mathbf{n}\|^2 - \mathbf{n}^T \mathbf{u} - \mathbf{u}^T \mathbf{n} + \|\mathbf{u}\|^2 + 2\mathbf{u}^T \mathbf{n} - 2\|\mathbf{u}\|^2 + f = 0 \quad (3.0.12)$$

$$\|\mathbf{n}\|^2 - \|\mathbf{u}\|^2 + f = 0 \quad (3.0.13)$$

$$m^2 + 1 - 13 + 12 = 0 \quad (3.0.14)$$

$$m^2 = 0 \quad (3.0.15)$$

$$m = 0 \quad (3.0.16)$$

Simplifying (3.0.16) and (3.0.9) we get

$$\mathbf{q} = \begin{pmatrix} 2 \\ 4 \end{pmatrix} \quad (3.0.17)$$

Let $\mathbf{p} = \begin{pmatrix} 7 \\ 4 \end{pmatrix}$ The length of tangent is

$$\|p - q\| = \sqrt{(7-2)^2 + (4-4)^2} \quad (3.0.18)$$

$$= \sqrt{25} \quad (3.0.19)$$

$$= 5 \quad (3.0.20)$$

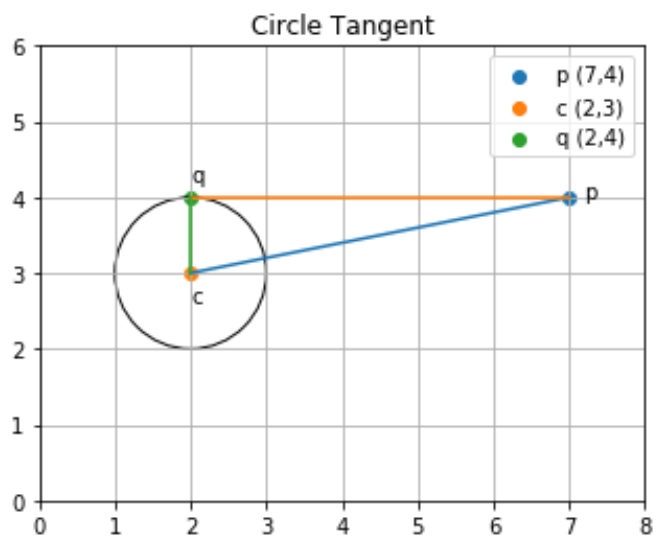


Fig. 0: Perpendicular Line