Assignment 6

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Abstract—**This document uses the properties of a tangent to a circle**

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1 PROBLEM

Find the length of the tangent from the point $\left(\begin{smallmatrix}7\\4\end{smallmatrix}\right)$ to the circle

$$\mathbf{x}^T\mathbf{x} - (\,4\ 6\,)\mathbf{x} + 12 = 0 \qquad (1.0.1)$$

2 EXPLANATION

The general equation of a second degree can be expressed as :

$$\mathbf{x}^T\mathbf{V}\mathbf{x} + 2\mathbf{u}^T\mathbf{x} + f = 0 \qquad (2.0.1)$$

Let the equation of the tangent be

$$(\,-m\ 1\,)\mathbf{x} = c \qquad (2.0.2)$$

3 SOLUTION

We know that, for a circle,

$$\mathbf{V} = \mathbf{I} \qquad (3.0.1)$$
$$\mathbf{c} = -\mathbf{u} \qquad (3.0.2)$$

Comparing the equation (1.0.1) and (2.0.1) we get

$$\mathbf{u} = \left(\begin{smallmatrix}-2\\-3\end{smallmatrix}\right), f = 12 \qquad (3.0.3)$$
$$\mathbf{c} = \left(\begin{smallmatrix}2\\3\end{smallmatrix}\right) \qquad (3.0.4)$$

The normal vector to the line is obtained as

$$\mathbf{n} = \mathbf{q} + \mathbf{u} \qquad (3.0.5)$$
$$\mathbf{q} = \mathbf{n} - \mathbf{u} \qquad (3.0.6)$$

Comparing the equation (2.0.2)

$$\mathbf{n} = (\,-m\ 1\,)^T \qquad (3.0.7)$$

Given

$$\mathbf{u} = \left(\begin{smallmatrix}-2\\-3\end{smallmatrix}\right) \qquad (3.0.8)$$
$$\implies \mathbf{q} = \left(\begin{smallmatrix}-m+2\\4\end{smallmatrix}\right) \qquad (3.0.9)$$

The point q also satisfies the equation of the circle at (1.0.1)

$$\mathbf{q}^T\mathbf{q} + (\,-4\ -6\,)\mathbf{q} + 12 = 0 \qquad (3.0.10)$$
$$m^2 + 4 - 4m + 16 + 4m - 32 + 12 = 0 \qquad (3.0.11)$$
$$m^2 = 0 \qquad (3.0.12)$$
$$m = 0 \qquad (3.0.13)$$

Simplying (3.0.13) and (3.0.9) we get

$$\mathbf{q} = \left(\begin{smallmatrix}2\\4\end{smallmatrix}\right) \qquad (3.0.14)$$

Let $\mathbf{p} = \left(\begin{smallmatrix}7\\4\end{smallmatrix}\right)$ The length of tangent is

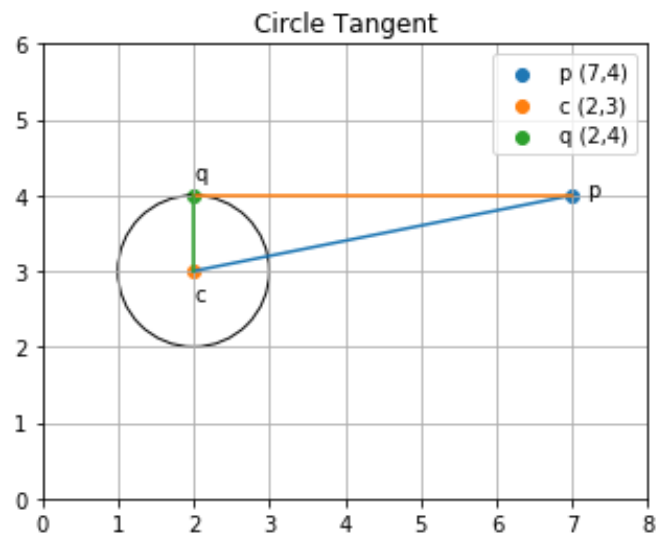$$\|p - q\| = \sqrt{(7-2)^2 + (4-4)^2} \qquad (3.0.15)$$
$$= \sqrt{25} \qquad (3.0.16)$$
$$= 5 \qquad (3.0.17)$$

Fig. 0: Perpendicular Line