Assignment 8

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***Abstract*—This document shows the diagonalization of matrices**

https://github.com/AddagallaSatyanarayana/AI5106/tree/master/Assignment8/Assignment8.tex

1 Problem

Which of the following matrices are not diagonalizable over R

$$\mathbf{M_1} = \begin{pmatrix} 2 & 0 & 1 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{pmatrix} \tag{1.0.1}$$

$$\mathbf{M_2} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \tag{1.0.2}$$

$$\mathbf{M_3} = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix} \tag{1.0.3}$$

$$\mathbf{M_4} = \begin{pmatrix} 1 & -1 \\ 2 & 4 \end{pmatrix} \tag{1.0.4}$$

2 Explanation

A matrix $\mathbf{M}$ can be diagonalized ,if for every eigenvalue of $\mathbf{M}$, the geometric multiplicity equals the algebraic multiplicity, then A is said to be diagonalizable. Eigen values can be calculated using

$$\left|\mathbf{M} - \lambda\mathbf{I}\right| = 0 \tag{2.0.1}$$

2.1 Solving matrix1

$$\begin{pmatrix} 2 & 0 & 1 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{pmatrix} \tag{2.1.1}$$

The eigen values for the matrix $\mathbf{M}$ can be found using (2.0.1)

$$\begin{vmatrix} 2-\lambda & 0 & 1 \\ 0 & 3-\lambda & 0 \\ 0 & 0 & 2-\lambda \end{vmatrix} = 0 \tag{2.1.2}$$

Solving this we get the eigen values of $\mathbf{M}$ as,

$$\lambda_1 = 2 \tag{2.1.3}$$

$$\lambda_2 = 3 \tag{2.1.4}$$

The algebraic multiplicity of eigen value $\lambda_1 = 2$ is equal to 2.

The eigen vector corresponding to eigen value $\lambda_1 = 2$

$$\mathbf{v} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \tag{2.1.5}$$

$$(\mathbf{M} - \lambda_1\mathbf{I})\mathbf{v} = 0 \tag{2.1.6}$$

$$\begin{pmatrix} 2-\lambda_1 & 0 & 1 \\ 0 & 3-\lambda_2 & 0 \\ 0 & 0 & 2-\lambda_1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0 \tag{2.1.7}$$

$$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0 \tag{2.1.8}$$

solve it by Gaussian Elimination

$$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \xleftrightarrow{R_2<->R_1} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \tag{2.1.9}$$

$$x_2 = 0 \tag{2.1.10}$$

$$x_3 = 0 \tag{2.1.11}$$

$$\mathbf{v} = \begin{pmatrix} x_1 \\ 0 \\ 0 \end{pmatrix} \tag{2.1.12}$$

$$\mathbf{v} = x_1\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \tag{2.1.13}$$

where the scalar x_1 can be arbitrarily chosen. Therefore, the eigenspace of λ_1 is generated by a single vector

$$\mathbf{v} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \tag{2.1.14}$$

Thus the geometric multiplicity of $\lambda_1 = 2$ is 1,but algebraic multiplicity of eigen value $\lambda_1 = 2$ is equal to 2. Since the geometric multiplicity and algebraic multiplicity are not same matrix $\mathbf{M}$ cannot be diagonalized.