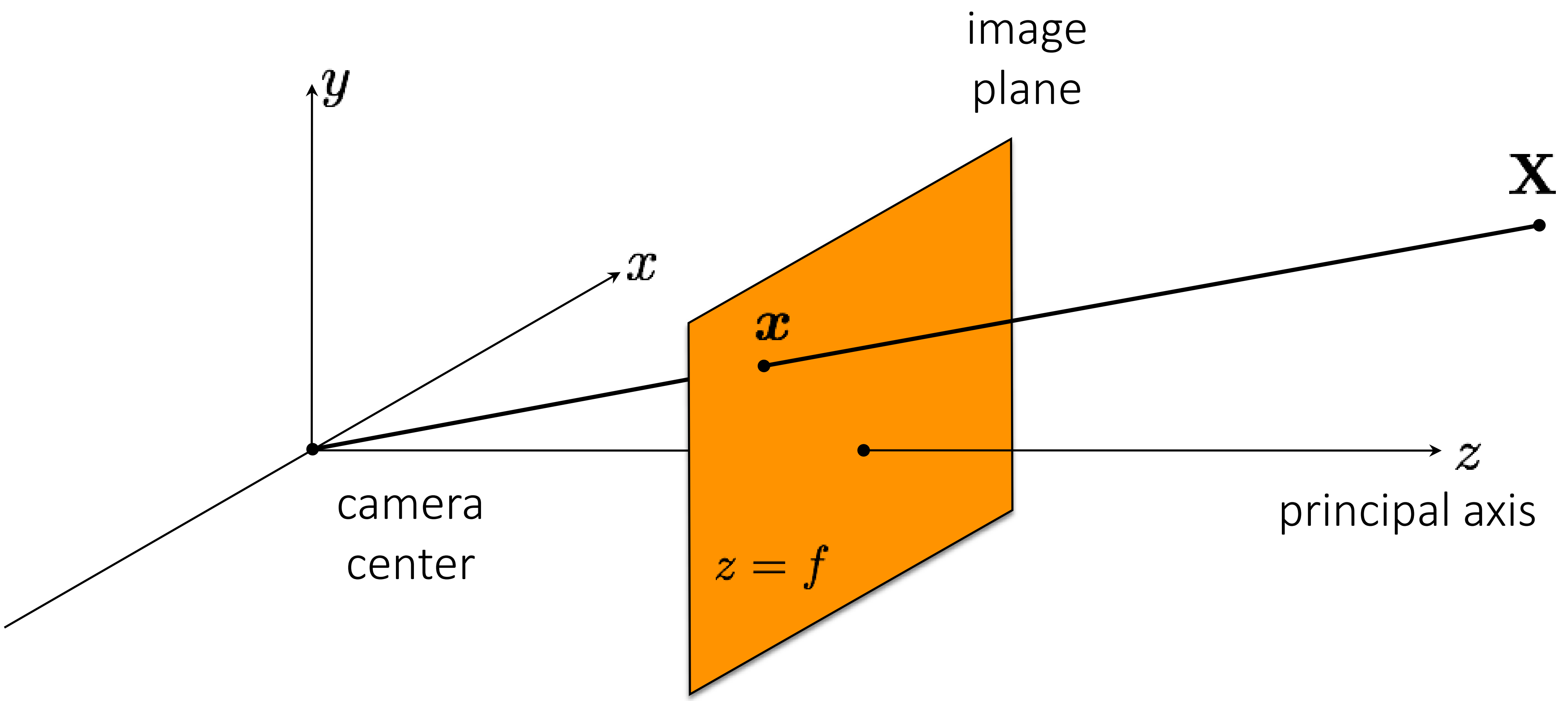
Tutorial 10 – Geometry ee046746

Dahlia Urbach

The (rearranged) pinhole camera



What is the camera matrix **P** for a pinhole camera?

$$\boldsymbol{x} = \mathbf{P}\mathbf{X}$$

General camera matrices

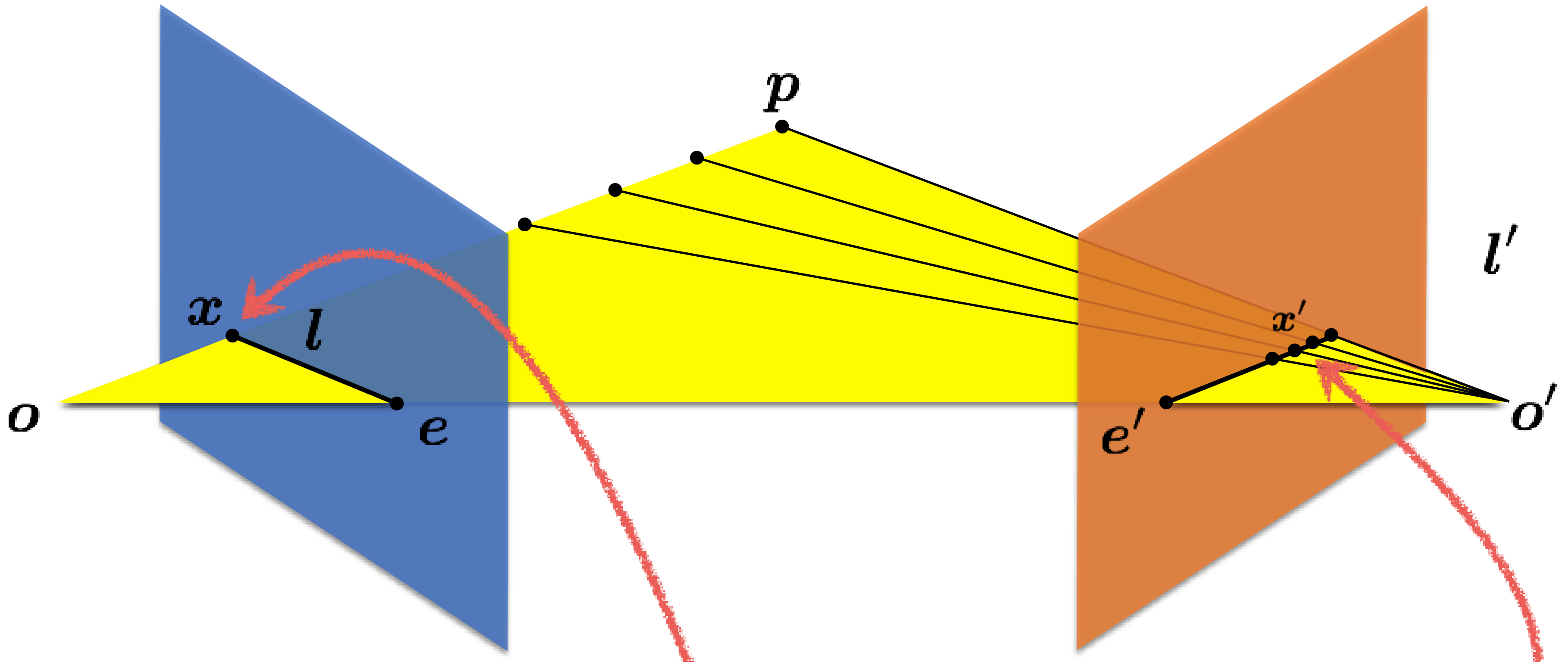
Projective camera

$$\mathbf{P} = \begin{bmatrix} \alpha_x & s & p_x \\ 0 & \alpha_y & p_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{R} & -\mathbf{RC} \end{bmatrix}$$

How many degrees of freedom?

11 DOF

Recall:Epipolar constraint



Potential matches for x lie on the epipolar line l'

properties of the E matrix

Longuet-Higgins equation

$$\boldsymbol{x}'^{\top}\mathbf{E}\boldsymbol{x} = 0$$

Epipolar lines

$$\boldsymbol{x}^{\top}\boldsymbol{l} = 0 \qquad \boldsymbol{x}'^{\top}\boldsymbol{l}' = 0$$

$$\boldsymbol{l}' = \mathbf{E}\boldsymbol{x} \qquad \boldsymbol{l} = \mathbf{E}^{T}\boldsymbol{x}'$$

Epipoles

$$\boldsymbol{e}'^{\top}\mathbf{E} = \mathbf{0} \qquad \mathbf{E}\boldsymbol{e} = \mathbf{0}$$

properties of the ~~E~~ F matrix

Longuet-Higgins equation	$\boldsymbol{x}'^{\top}\mathbf{E}\boldsymbol{x} = 0$
Epipolar lines	$\boldsymbol{x}^{\top}\boldsymbol{l} = 0$ $\boldsymbol{x}'^{\top}\boldsymbol{l}' = 0$ $\boldsymbol{l}' = \mathbf{E}\boldsymbol{x}$ $\boldsymbol{l} = \mathbf{E}^{T}\boldsymbol{x}'$
Epipoles	$\boldsymbol{e}'^{\top}\mathbf{E} = \mathbf{0}$ $\mathbf{E}\boldsymbol{e} = \mathbf{0}$

(points in **image** coordinates)

Example Questions

Question 1

1. Prove that there exists a homography $\mathbf{H}$ that satisfies

$$\mathbf{x}_1 \equiv \mathbf{H} \cdot \mathbf{x}_2, \tag{1}$$

between the 2D points (in homogeneous coordinates) $\mathbf{x}_1$ and $\mathbf{x}_2$ in the images of a *plane* Π captured by two 3×4 camera projection matrices $\mathbf{P_1}$ and $\mathbf{P_2}$, respectively. The $\equiv$ symbol stands for equality *up to scale.* (*Note: A degenerate case happens when the plane* Π *contains both cameras' centers, in which case there are infinite choices of* $\mathbf{H}$ *satisfying Equation* (1). *You can ignore this special case in your answer.*)

(1)

2. Prove that there exists a homography $\mathbf{H}$ that satisfies Equation (1), given two cameras separated by a pure rotation. That is, for camera 1, $\mathbf{x_1} = \mathbf{K_1}\begin{bmatrix}\mathbf{I} & \mathbf{0}\end{bmatrix}\mathbf{X}$, and for camera 2, $\mathbf{x_2} = \mathbf{K_2}\begin{bmatrix}\mathbf{R} & \mathbf{0}\end{bmatrix}\mathbf{X}$. Note that $\mathbf{K_1}$ and $\mathbf{K_2}$ are the 3×3 intrinsic matrices of the two cameras and are different. $\mathbf{I}$ is 3×3 identity matrix, $\mathbf{0}$ is a 3×1 zero vector and $\mathbf{X}$ is a point in 3D space. $\mathbf{R}$ is the 3×3 rotation matrix of the camera.

(2)

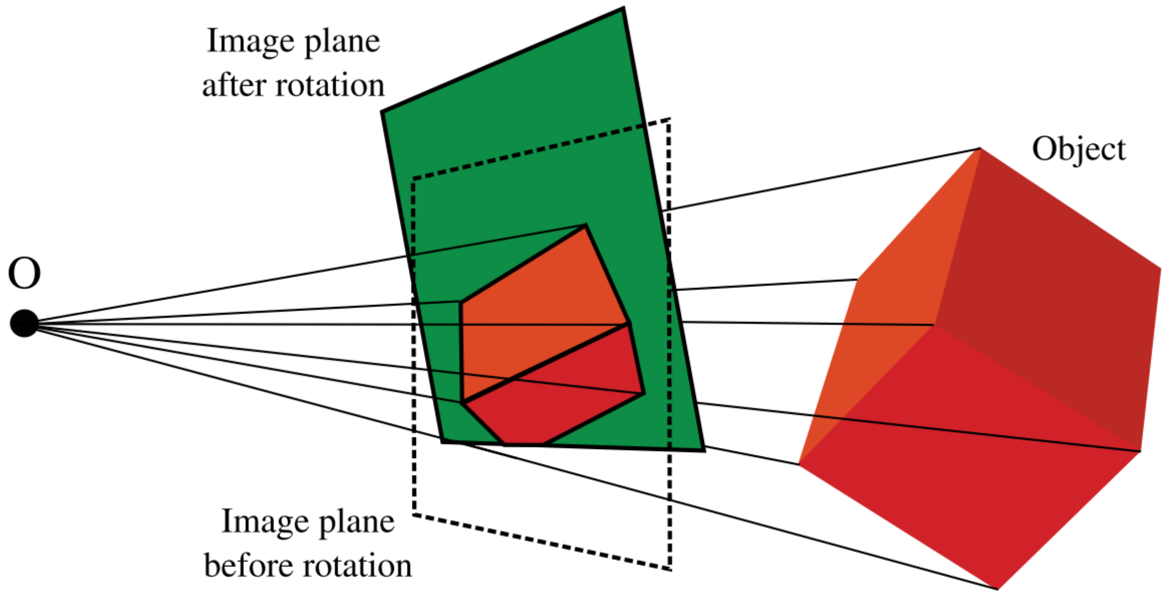


Figure 15.14 Images under pure camera rotation. When the camera rotates but does not translate, the bundle of rays remains the same, but is cut by a different plane. It follows that the two images are related by a homography.

Image source - Prince

3. Suppose that a camera is rotating about its center $\mathbf{C}$, keeping the intrinsic parameters $\mathbf{K}$ constant. Let $\mathbf{H}$ be the homography that maps the view from one camera orientation to the view at a second orientation. Let θ be the angle of rotation between the two. Show that $\mathbf{H^2}$ is the homography corresponding to a rotation of 2θ.

(3)

Question 2

In class we saw that a camera matrix satisfies the equation $\mathbf{x}_i = \mathbf{P}\mathbf{X}_i$, and that six 3D-2D matches $\mathbf{x} \leftrightarrow \mathbf{X}$ are sufficient to recover $\mathbf{P}$ using a linear (non-iterative) algorithm.

1. Find linear algorithms for computing the camera matrix $\mathbf{P}$ in the special cases when: i) the camera location (but not orientation) is known, and ii) the camera location and complete orientation are known.
2. Ignoring degenerate configurations, how many 2D-3D matches are required for there to be a unique solution in each case? Justify your answers.

1. Find linear algorithms for computing the camera matrix **P** in the special cases when: i) the camera location (but not orientation) is known, and ii) the camera location and complete orientation are known.

(i)

(ii)

(2)

Question 3

Consider three images I_1, I_2 and I_3 that have been captured by a system of three cameras, and suppose the fundamental matrices $\mathbf{F}_{13}$ and $\mathbf{F}_{23}$ are known. (Notation: the matrix $\mathbf{F}_{ij}$ satisfies the equation $\mathbf{x}_j^\top \mathbf{F}_{ij}\mathbf{x}_i = 0$ for any correspondence $\mathbf{x}_i \leftrightarrow \mathbf{x}_j$ between images I_i and I_j.) In general, given a point $\mathbf{x}_1$ in I_1 and a corresponding point $\mathbf{x}_2$ in I_2, the corresponding point in $\mathbf{x}_3$ in I_3 is uniquely determined by the fundamental matrices $\mathbf{F}_{13}$ and $\mathbf{F}_{23}$.

1. Write an expression for $\mathbf{x}_3$ in terms of $\mathbf{x}_1$, $\mathbf{x}_2$, $\mathbf{F}_{13}$ and $\mathbf{F}_{23}$.
2. Describe a degenerate configuration of three cameras for which the point $\mathbf{x}_3$ cannot be uniquely determined by this expression.

1. Write an expression for $\mathbf{x}_3$ in terms of $\mathbf{x}_1$, $\mathbf{x}_2$, $\mathbf{F}_{13}$ and $\mathbf{F}_{23}$.

2. Describe a degenerate configuration of three cameras for which the point $\mathbf{x}_3$ cannot be uniquely determined by this expression.

Question 1 – part 4

4. Let I_0 be an image captured by a camera, and I_1 be an image of I_0 captured by another camera (an image of an image). Let the composite image be denoted I'. Show that the apparent camera center of I' is the same as that of I_0. Speculate on how this explains why a portrait's eyes "follow you around the room." (*Hint: The null space of an* $n \times n$ *invertible matrix* $\mathbf{A}$ *is empty, i.e.,* $\mathbf{Ax} = 0$ *if and only if* $\mathbf{x} = 0$.)

Image source - Wikipedia

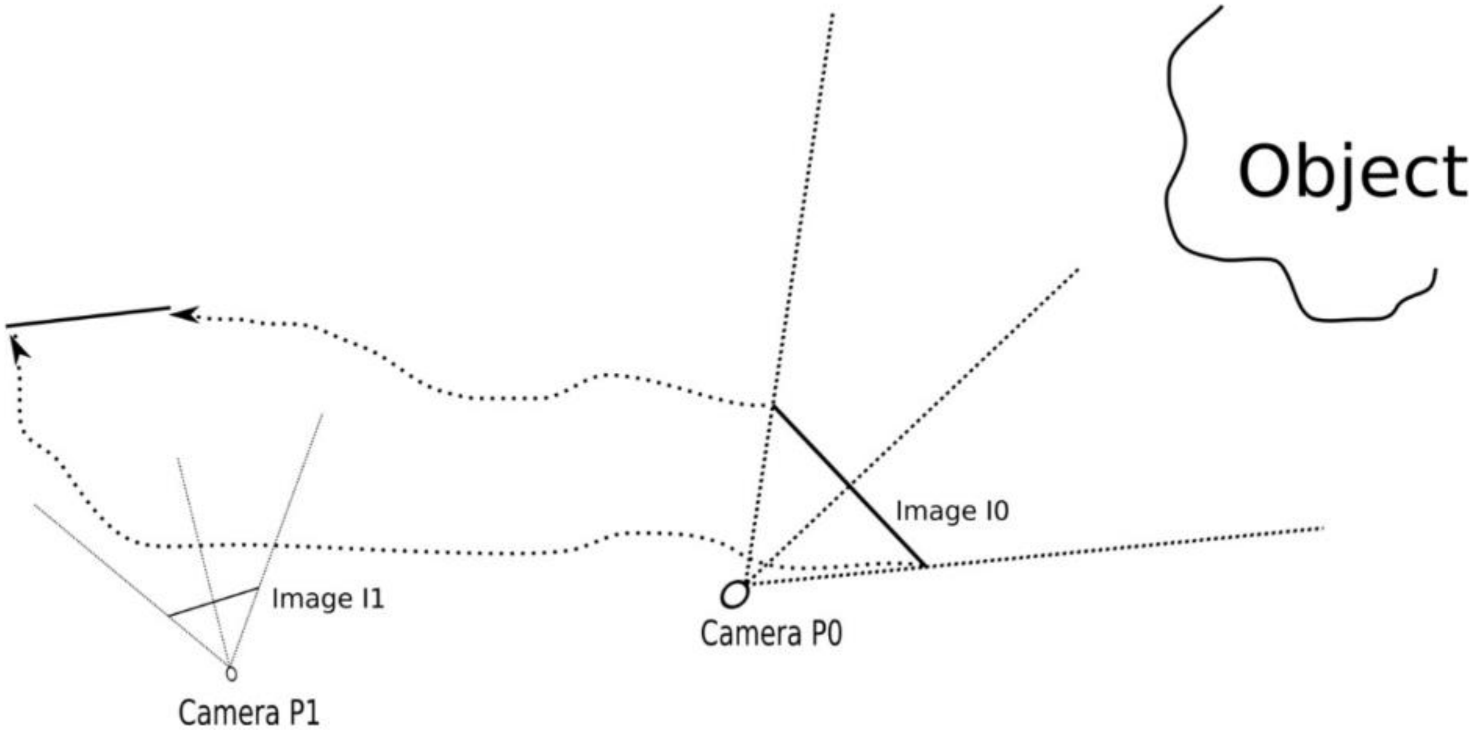


Image source - Wikipedia

Recommended Videos

Warning!

- These videos do not replace the lectures and tutorials.
- Please use these to get a better understanding of the material, and not as an alternative to the written material.

Video By Subject

- Epipolar and Essential matrix https://www.youtube.com/watch?v=Opy8xMGCDrE
- Fundamental matrix https://www.youtube.com/watch?v=wb9245ZAoaE

- The Fundamental Matrix Song https://www.youtube.com/watch?v=DgGV3l82NTk

Read More

Basic reading:

- Szeliski textbook, Section 2.1.5, 6.2. , 7.1, 7.2, 11.1.
- Hartley and Zisserman, Chapters 9, 11, 12, 18.
- Prince- Computer vision: models, learning and inference, Chapter 15

Additional reading:

- Hartley and Zisserman, “Multiple View Geometry in Computer Vision,” Cambridge University Press 2004.

chapter 6 of this book has a very thorough treatment of camera models.