BABY RUDIN CHAPTER 1

Problem 1
If $r \in \mathbb{I} = \mathbb{R} \setminus \mathbb{Q}$ and $x \in \mathbb{Q}$ prove that $r + x, rx \in \mathbb{I}$.

By hypothesis there are no integers a, b such that $r = a/b$, but there are integers such that $x = a/b$. In particular let's say $x = a/b$ where $b \neq 0$ and $gcd(a, b) = 1$. For contradiction suppose $r + x = p/q$ for integers p, q. This would imply

$$r = p/q - x = \frac{pb - aq}{bq} \in \mathbb{Q}$$

Also for contradiction assume $rx \in \mathbb{Q}$ and $rx = p/q$ to lazily re-use our old symbols. Then so long as $a \neq 0$ we have

$$r = \frac{p}{q}\frac{1}{x} = \frac{pb}{qa} \in \mathbb{Q}$$

and if $a = 0$ then of course $rx = ra/b = 0 \in \mathbb{Q}$.

Problem 2